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Testing for unit roots in flow data sampled at different frequencies

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Abstract

Perron (1989) finds that increasing the span of the point sampled data always increases the power of unit root tests. We extend the analysis to analyze flow data which have a moving average error structure. We examine five unit root tests which correct for serial correlation and find the power of some tests not to be monotonic in the span of the data.

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1. Motivation

Consider a time series of data y_{th} sampled at a discrete interval h over a span of S time units with the data generating process given by

$$y_{th} = \alpha_h y_{(t-1)h} + u_{th} . \tag{1}$$

This discrete time representation of y_{th} is the exact solution to an Ornstein–Uhlenbeck diffusion process

$$\mathrm{d}y_t = \gamma y_t \,\mathrm{d}t + \sigma \,\mathrm{d}w_t \tag{2}$$

where $-\infty < \gamma < \infty$, $0 < t < S$, w_t is a unit Weiner process, and γ, σ, and $y(0)$ are constants. The discrete time model (1) is related to the continuous time model (2) by $\alpha_h = \exp(h\gamma)$, $u_{th} = \sigma \int_{(t-1)h}^{th} \exp(\gamma(th - s))\,\mathrm{d}w_s$, with $u_{th} \sim N(0, \sigma^2(\exp^{2\gamma h} - 1)/2\gamma)$. Given the correspondence between the two models, testing $\alpha_h = 1$ against the stationary alternative that $\alpha_h < 1$ in (1) is equivalent to testing $\gamma = 0$ against the alternative of $\gamma < 0$ in (2).

Simulations of Shiller and Perron (1985) and Perron (1989) showed that when the data are sampled at discrete points the power of unit root tests is more influenced by the span of the

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data and less so by the number of observations. The span of the data, S, plays an important role in unit root tests because the sampling interval is tied to the number of observations, T, by the relationship $h = S/T$. The sampling interval evidently falls as the number of observations increases if the span of the data is held fixed. In the limit, $h \to 0$ as $T \to \infty$ for fixed S. Since $\alpha_h = \exp(\gamma h)$, this means that $\alpha_h \to 1$ as $h \to 0$ regardless of the true value of γ. In other words the value of α_h under the alternative tends to that under the null as $h \to 0$. The implication is that unit root tests based on (1) have lower power when applied to higher frequency data and are outright inconsistent in the limit when $h \to 0$. Increasing the span of the data yields more power for a given number of observations because it moves h away from zero. A formal treatment of these issues can be found in Perron (1990).

The assumption that data are sampled at discrete points in time is appropriate for variables such as interest rates and the money supply. However, many economic time series such as consumption are observed only as integrals over a fixed sampling interval. That is, these data are *flows*. Suppose that y_t is not observed at every point but over an interval of length h. Integrating (2) from $t - h$ to t and rearranging terms gives

$$Y_{th} = \int_{t-1}^{t} y_r \, \mathrm{d}r = \exp(\gamma h) Y_{(t-1)h} + v_{th}$$

$$v_{th} = \sigma h \int_{t-1}^{t} \int_{0}^{h} \exp(s\gamma) w(\tau h - s) \, \mathrm{d}s \, \mathrm{d}\tau \, .$$

The double integral in the error term v_{th} is the primary cause of serial correlation. Phillips (1974) formally showed that v_{th} is an MA(1) and that the exact discrete representation can be approximated by

$$\begin{aligned} Y_{th} &= \alpha_h Y_{(t-1)h} + v_{th} \\ v_{th} &= e_{th} + 0.268 e_{(t-1)h} \end{aligned} \tag{3}$$

where $\alpha_h = \exp(\gamma h)$. In other words, if y_t is a diffusion process like (2), then observed Y_{th} will be an ARMA(1,1). When $\alpha_h = 1$, the first order autocorrelation coefficient for ΔY_{th} equals 0.25. This is the special case considered by Working (1960). In general, the first difference of Y_{th} is an ARMA(1,2).

The objective of this paper is to see how the span and the number of observations affect the power of unit root tests on flow data. Hypothesis testing is complicated by the presence of serial correlation, and statistics analyzed in Perron (1989) are inappropriate in these cases. Choi (1992) analyzed the implications of data aggregation over k periods on the power of some unit root tests with the sampling frequency held fixed. The DGP considered in this paper is a special case of data aggregation with $k \to \infty$, but we consider the power function at different sampling frequencies.

2. Results

Our Monte Carlo analysis is based on the regression model

$$y_{th} = \mu + \alpha y_{(t-1)h} + u_{th} \tag{4}$$

using (3) as DGP. We examine h in the range 40 and 1/40 by allowing S and T to vary between 25 and 1000. All simulations are based on 1000 replications with y_0 normalized to zero and σ^2 chosen such that $e_t \sim N(0,1)$.

We consider five unit root tests that deal with serial correlation. They are the Phillips–Perron Z_α and Z_t tests, the Said–Dickey t_ρ test, Hall's IV test, and the test for randomness based on high order autocorrelation coefficient on the first difference data.

The t_ρ statistic due to Said and Dickey (1984) is based on the t statistic on ρ_h in the regression

$$\Delta y_{th} = \mu + \rho_h y_{(t-1)h} + \sum_{j=2}^{l} \theta_j \, \Delta y_{(t+j-1)h} + e_{th} \, .$$

The truncation lag is determined by $l = \text{int}(4(T/100)^{1/3})$ as described in Schwert (1989).

The Z_α and Z_t tests, due to Phillips and Perron (1988), are defined as

$$Z_\alpha = T(\hat{\alpha} - 1) - 0.5(s_{tl}^2 - s_u^2)\Big(T^{-2} \sum_{t=1}^{T} (y_{(t-1)h} - y_{1h})^2\Big)^{-1}$$

$$Z_t = (s_u / s_{tl}) t_{\hat{\alpha}} - (0.5 s_{tl}(s_{tl}^2 - s_u^2))\Big(s_{tl}^2 T^{-2} \sum_{t=1}^{T} (y_{(t-1)h} - y_{1h})^2\Big)^{-1/2}$$

where $y_{1h} = T^{-1} \Sigma_{t=1}^{T} y_{(t-1)h}$, $\hat{u}_{th} = y_{th} - \hat{\mu} - \hat{\alpha} y_{(t-1)h}$ are the OLS residuals, $s_u^2 = T^{-1} \Sigma_{t=1}^{T} \hat{u}_{th}^2$, $s_{tl}^2 = T^{-1} \Sigma_{t=1}^{T} \hat{u}_{th} + 2T^{-1} \Sigma_{\tau=1}^{l} \omega(\tau, l) \Sigma_{t=+1}^{T} \hat{u}_{th} \hat{u}_{(t-\tau)h}$, $\omega(\tau, l)$ is a lag window with truncation lag l. The quadratic spectral window with automatic bandwidth as described in Andrews (1991) is used to estimate s_{tl}^2.[1]

The IV test, due to Hall (1989), is based on the idea that since $y_{(t-1)h} = y_{(t-m)h} + y_{(t-1)h} - y_{(t-m)h}$, the sample variation in $y_{(t-1)h}$ is dominated by variations in $y_{(t-m)h}$. Therefore $y_{(t-m)h}$ can be used as an instrument to remove the bias caused by correlation between $y_{(t-1)h}$ and u_{th}. When the serial correlation is an MA(q) Hall suggests setting $m = q + 1$. Thus, we have

$$\hat{\alpha}_h(IV) = \Big(\sum_{t=m+1}^{T} y_{(t-m)h} y_{(t-1)h}\Big)^{-1} \sum_{t=m+1}^{T} y_{(t-m)h} y_{th}$$

and the statistic of interest is $T(\hat{\alpha}_h(IV) - 1)$. Since the error term of the DGP in question is an MA(1), the second lag of y_{th} is used as an instrument.

The Z_α statistic has the same asymptotic distribution as Hall's test, while the Z_t test and t_ρ are asymptotically identical. Critical values for these tests are given in Fuller (1976).

The last statistic looks for randomness in the first differenced data. Under the null of a unit root, the noise component is an MA(1); the first differenced data therefore have second and higher order autocorrelation coefficients of zero. Under the alternative, the first differenced data is an ARMA(1,2). The test statistic is $\sqrt{T} R_h(k) \sim N(0, \text{avar} R(k))$, where

[1] The results based on the Bartlett window and the autoregressive spectral window are quite similar and are available on request.

$$R_h(j) = \left(\sum_{t=j+1}^{T} \Delta y_{th} \, \Delta y_{(t-j)h} \right) \left(\sum_{t=2}^{T} \Delta y_{th}^2 \right)^{-1}$$

and avar $R(j)$ is the asymptotic variance based on Bartlett's formula as defined in Priestley

Table 1
Power of unit root tests for flow data DGP: $y_{th} = \exp(\gamma h) y_{(t-1)h} + e_{th} + 0.268 e_{(t-1)h}$
$\gamma = -0.20$, $h = S/T$

$T \downarrow \rightarrow S$	25	50	100	250	500	1000
Z_α						
25	0.041	0.060	0.114	0.144	0.155	0.172
50	0.103	0.180	0.248	0.316	0.368	0.402
100	0.375	0.555	0.681	0.844	0.866	0.896
250	0.824	0.984	1.000	1.000	1.000	1.000
500	0.921	1.000	1.000	1.000	1.000	1.000
1000	0.930	1.000	1.000	1.000	1.000	1.000
Size	0.014	0.021	0.025	0.036	0.054	0.041
Z_t						
25	0.030	0.035	0.055	0.078	0.090	0.091
50	0.082	0.092	0.125	0.187	0.247	0.279
100	0.285	0.367	0.496	0.681	0.701	0.773
250	0.776	0.946	0.996	1.000	1.000	1.000
500	0.894	1.000	1.000	1.000	1.000	1.000
1000	0.900	1.000	1.000	1.000	1.000	1.000
Size	0.035	0.028	0.028	0.040	0.058	0.054
$T(\alpha_h(IV) - 1)$						
25	0.223	0.169	0.149	0.150	0.139	0.146
50	0.436	0.370	0.330	0.341	0.348	0.358
100	0.633	0.742	0.798	0.855	0.868	0.883
250	0.770	0.963	1.000	1.000	1.000	1.000
500	0.704	0.934	1.000	1.000	1.000	1.000
1000	0.690	0.850	0.992	1.000	1.000	1.000
Size	0.081	0.052	0.030	0.035	0.047	0.037
t_ρ						
25	0.072	0.074	0.074	0.108	0.113	0.116
50	0.110	0.160	0.165	0.245	0.270	0.312
100	0.163	0.274	0.399	0.623	0.719	0.773
250	0.254	0.515	0.799	0.981	1.000	1.000
500	0.270	0.615	0.947	1.000	1.000	1.000
1000	0.289	0.648	0.973	1.000	1.000	1.000
Size	0.050	0.047	0.034	0.045	0.058	0.049
$R_h(3)$						
25	0.077	0.080	0.079	0.055	0.038	0.032
50	0.119	0.120	0.110	0.065	0.049	0.039
100	0.093	0.186	0.260	0.150	0.095	0.052
250	0.074	0.157	0.356	0.398	0.280	0.142
500	0.052	0.110	0.264	0.636	0.668	0.468
1000	0.055	0.097	0.141	0.580	0.880	0.911
Size	0.022	0.040	0.042	0.041	0.025	0.016

(1981). Although in theory, $R_h(2)$ ought to be an appropriate test for randomness since Δy_{th} is an MA(1), we find the size of the test to be distorted. We therefore report the results based on $R_h(3)$.

Table 1 gives the power of the statistics of interest. In general, the power of all statistics is lower than that reported in Perron (1990) for point sampled observations. The two Z tests are undersized while the *IV* test is oversized at small samples. Note also that the two normalized least squares tests are more powerful than the two t tests.

The Z_α, Z_t and t_ρ tests share the properties that (i) increasing T while keeping S fixed increases power but at a diminishing rate; (ii) power decreases when T increases as S falls. Thus, none of the tests considered here seem to be small-S consistent in the sense of Perron (1990).[2] Increasing the number of observations while reducing the span always leads to a power loss; (iii) power increases with S for fixed T. These findings are similar to those of Perron (1990) for point sampled data.

The *IV* test appears not to be monotonic in the S when T is small. While the test is the most powerful for small values of S and T, power actually falls as T increases holding S fixed at small values. The power of $R(1)$ was found to peak at $T \approx 50$ in the case of point sampled data. Here in the case of flow data, the power function is still non-monotonic in T but it peaks at a higher value of 250 for every S. As well, increasing the span of flow data when $T \leq 100$ could decrease the power of the statistic.

3. Conclusion

This paper uses simulations to show that while increasing the span of the data unambiguously increases the power of unit root tests in point sampled observations, some unit root tests which correct for serial correlation in time averaged data do not share the same property. In particular, the *IV* based normalized least squares statistic and the test for randomness show non-monotonicity of power in the span of the data. Other unit root tests not examined here may have similar properties. As far as Z_α, Z_t and t_ρ are concerned, increasing the span of the data always yields power gain.

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[2] Small S consistency is defined as $lim_{S \to 0} lim_{h \to 0} P_{Sh} = 1$, where P is the power function.

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