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TESTING FOR HOMOGENEITY IN DEMAND SYSTEMS WHEN THE REGRESSORS ARE NONSTATIONARY

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SUMMARY

An implication of optimizing theory is that demand functions are homogeneous of degree zero in prices and nominal income. Evidence based on estimations of demand systems has repeatedly found this restriction to be rejected by the data. However, the hypothesis is often formulated in terms of regressors that are non-stationary. This paper re-examines the evidence for homogeneity in light of recent developments in time-series econometrics with special emphasis on the treatment of trends. We find the demand system to be stochastically but not deterministically cointegrated. Using techniques developed for estimating cointegrating vectors in the presence of deterministic trends, we re-estimate the demand system and find that homogeneity holds in many cases.

1. INTRODUCTION

A function is said to be homogeneous of degree zero in nominal variables when proportional increases in the nominal variables do not change the behaviour of the real variable described by the function. When estimates from demand systems are used to test whether real expenditures are invariant to proportional increases in prices and income, the homogeneity hypothesis is often found to be rejected by the data of several industrialized countries. Evidence based on US data presented in this study is typical of such results.[1]

This analysis suggests that time-series issues are partly responsible for these rejections. Problems pertaining to dynamic misspecification have been noted by some authors in their analyses. However, most studies that estimate demand systems precede the recent development on the treatment of trends and integrated variables in the time-series literature. This paper capitalizes on this development and re-examines the evidence with special focus on two issues. First, if the hypothesis is formulated in terms of variables that are nonstationary, classical inference could be invalid in the sense that tests based on standard asymptotic results will have the wrong size. We analyse the time-series properties of US data, tabulate the finite sample distribution of the statistic for testing homogeneity, and find a significant departure of the statistic from normality.

Second, the demand system that is typically estimated is found not to be deterministically cointegrated (in a sense to be made precise). While all variables of the demand system are $I(1)$ with drifts, the first differences of some variables evolve around higher-order deterministic

[1] See, for example, Barten (1969) for an application of Dutch data and Deaton and Muellbauer (1980) for the case of the UK.

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terms than others. The typical formulation of the demand system does not allow for these higher-order trends, which are absorbed in the regression residuals, and are responsible for our inability to reject the null hypothesis of no 'deterministic cointegration'. However, we can reject the null hypothesis of no 'stochastic cointegration'. This allows us to apply recently developed techniques for estimating cointegrating vectors to re-estimate the demand system and to test for homogeneity. The evidence for homogeneity and the model is more favourable. We conclude that time-series issues can account for some of the rejections found in previous studies.

This study uses the Almost Ideal Demand Model (AIDM) developed by Deaton and Muellbauer (1980) as the framework for analysis. A brief description of the model and the estimates based on US data are presented in the next subsection. The rest of the paper discusses and analyses the econometric issues involved.

1.1 The Model and Typical Results

One of the basic principles of economics is that agents make their decisions on a rational basis. According to the theory of consumer choice, the demand of a utility-maximizing consumer should satisfy (1) the adding up of budget shares, (2) the negativity of compensated own-price elasticities, (3) the symmetry of the Slutsky matrix, and (4) the homogeneity of degree zero with respect to prices and income.

Of these conditions, the adding-up constraint is often imposed in empirical demand analyses rather than tested. Symmetry implies restrictions across equations, while the negative semi-definiteness of the Slutsky matrix can be used to test for negativity. Homogeneity can be tested on an equation-by-equation basis, and given the simplicity of its implementation, it is the most commonly tested of the four restrictions. There is another reason why homogeneity is an interesting proposition in its own right. Homogeneity is an important assumption underlying classical macroeconomic theory. For example, money will be neutral if the real and monetary sectors are dichotomized, a condition which requires the real side of the economy to be homogeneous of degree zero in the nominal variables. Evidence which contradicts the homogeneity assumption therefore has important implications from both a micro and a macro perspective. This paper deals specifically with issues involved in testing for homogeneity.

Numerous models have been developed to test whether or not homogeneity holds in the data. A convenient framework is the Almost Ideal Demand Model (AIDM) developed by Deaton and Muellbauer (1980). The empirical equation for aggregate data, valid under the assumption of intertemporal separability, is

$$w_{it} = \alpha_i^* + \sum_{j=1}^{n} \gamma_{ij} \log p_{jt} + \beta_i \log(X_t/P_t\Upsilon_t) + e_{it} \tag{1}$$

where n is the number of goods, w_{it} is the aggregate expenditure share of good i in period t, p_j is the price of good j, $\log P = \sum_k w_k \log p_k$ is the general price index as defined in Stone (1953), X is the level of expenditure averaged over households, and α_i^*, γ_{ij}, and β_i are parameters.

The variable Υ_t is an index of inequality across households arising from variations in tastes, earning capacities, and demographic characteristics. Since this index is unobserved, a solution is to subsume it in the constant, giving $\alpha_i = \alpha_i^* - \beta_i \log \Upsilon$. Deaton and Muellbauer argued that this reparameterization is valid if Υ is constant, or if idiosyncratic movements in Υ are independent of prices and total expenditure so that no bias is induced by omitting such variations. Implicit in this omitted variables argument is that movements in Υ are stationary. This assumption seems harmless when working with cross-sectional data, but is questionable in a time-series context, an issue to which we will return.

Estimation of equation (1) in its restricted and unrestricted form provides the basis of an F-test for the restriction $\sum_{j=1}^{n} \gamma_{ij} = 0$. An equivalent but somewhat more convenient approach is to choose a numeraire and write the model as

$$w_{it} = \alpha_i + \sum_{j=1}^{n-1} \gamma_{ij} \log(p_{jt}/p_{nt}) + \left(\sum_{j=1}^{n} \gamma_{ij}\right) \log p_{nt} + \beta_i \log(X_t/P_t) + e_{it} \tag{2}$$

An immediate test for homogeneity is provided by the t-statistic on $\log p_n$, the numeraire. Of course, the t-statistic in equation (2) is simply the square root of the F-statistic in equation (1).

In their original work on AIDM, Deaton and Muellbauer (1980) took annual averages of quarterly data from 1954 to 1974 for Britain and tested the model for eight groups of nondurable commodities. They rejected the homogeneity restriction on the basis of the F-statistic. As Laitinen (1978) noted, the F-distribution is not a good approximation to the finite sample distribution of the statistic for testing homogeneity when the number of commodity groups is large relative to the sample size and will over-reject homogeneity in small samples. While it could be argued that the small sample used in Deaton and Muellbauer's analysis might bias their results towards rejecting homogeneity, the hypothesis is still frequently rejected when estimated using larger samples. For example, in a recent study, Attfield (1991) rejected homogeneity when a subset of quarterly British data was tested over the period 1963 to 1988. Evidence based on alternative formulations of the demand system also leads to the same conclusion. See, for example, Barten (1969), Deaton (1974), Deaton and Muellbauer (1980), and Deaton (1986) for a survey of related work.

Results from estimation based on US data are typical of such analyses. Table I(a) presents the unconstrained OLS (ordinary least squares) estimates for five groups of non-durables using 148 quarterly, seasonally adjusted, observations for the period 1954Q1 to 1990Q4. The data are obtained from CITIBASE. The dependent variables are the share of food (w_f), clothing (w_c), other services (w_s), other goods (w_g), and housing (w_h) in total expenditure on non-durables (X). The price variables are constructed as the nominal expenditure on the commodity divided by the corresponding real component. Total real non-durable expenditure is expressed in per capita terms and is deflated by Stone's aggregate price index. The estimates are based on equation (2) using the price of housing as the numeraire.

According to the sign of the coefficient on $\log X/P$, food and clothing are necessary goods while the remaining three commodities are luxuries. The coefficient of interest here is on the (log) price of housing which should not be statistically different from zero if homogeneity holds. Applying the two-tailed 5% critical values of $\pm 1{\cdot}96$, the t-statistic overwhelmingly rejects homogeneity in every equation. According to the estimates, a proportional increase in total expenditure and prices will increase the expenditure share of food, clothing, and other services, but reduce the expenditure share of other goods and housing. Table I(b) reports the estimates for the constrained model. It is clear from the Durbin–Watson statistic that there is severe serial correlation in the residuals whether or not homogeneity is imposed, but there is indication that imposing homogeneity changes the dynamics of the residuals.[2] Furthermore, the coefficient on total expenditure switches sign in the case of housing and is no longer significant in the goods equation. On the basis of these results, one would be led to conclude that either the homogeneity

[2] Earlier work dealt with the problem of serial correlation by first differencing the model and allowing for a non-zero intercept. As we now know, this omits important information on the levels of the variables if the variables are cointegrated.

Table I(a). OLS estimates of the unconstrained model

$$w_i = \alpha + \sum_{j=1}^{n-1} \gamma_{ij} \ln(p_j/p_n) + \left(\sum_{j=1}^{n} \gamma_{ij}\right) \ln p_n + \beta \ln(X/P) + e_{ui}$$

	α	$\ln(p_f/p_h)$	$\ln(p_c/p_h)$	$\ln(p_s/p_h)$	$\ln(p_g/p_h)$	$\ln(p_h)$	$\ln(X/P)$	DW	SE E+02
w_f	0·553	0·118	0·066	−0·134	−0·019	0·052	−0·161	0·202	0·381
	(19·93)	(8·46)	(3·82)	(−4·83)	(−2·21)	(5·28)	(−11·96)		
w_c	0·079	0·024	0·052	−0·125	−0·009	0·021	−0·004	0·325	0·186
	(5·81)	(3·61)	(6·12)	(−9·19)	(−2·14)	(4·33)	(−0·67)		
w_s	0·209	−0·107	0·019	0·143	−0·053	0·049	0·090	0·337	0·288
	(9·96)	(−10·21)	(1·48)	(6·79)	(−8·18)	(6·48)	(8·83)		
w_g	0·032	0·002	−0·057	−0·002	0·089	−0·072	0·052	0·567	0·137
	(3·15)	(0·45)	(−9·21)	(−0·24)	(28·90)	(−20·11)	(10·62)		
w_h	0·127	−0·037	−0·080	0·119	−0·008	−0·050	0·024	0·155	0·261
	(6·66)	(−3·90)	(−6·72)	(6·24)	(−1·45)	(−7·35)	(2·59)		

Note: The subscripts f, c, s, g, h stand for food, clothing, nonhousing services, other goods, and housing respectively. w_i and p_i are the expenditure share and the price of commodity i. X/P is total expenditure on nondurables deflated by Stone's (1953) price index. The numeraire, p_n, is the price of housing. The estimation period is 1954:1–1990:4. The t-statistics are in parenthesis. DW denotes the Durbin Watson statistic and SE is the standard error of the regression.

Table I(b). OLS estimates of the constrained model

$$w_i = \alpha + \sum_{j=1}^{n-1} \gamma_{ij} \ln(p_j/p_n) + \beta \ln(X/P) + e_{ri}$$

	α	$\ln(p_f/p_h)$	$\ln(p_c/p_h)$	$\ln(p_s/p_h)$	$\ln(p_g/p_h)$	$\ln(p_h)$	$\ln(X/P)$	DW	SE e+02
w_f	0·491	0·157	−0·021	−0·097	−0·028	0·0	−0·130	0·166	0·415
	(17·90)	(12·45)	(−7·00)	(−3·33)	(−3·15)		(−9·84)		
w_c	0·054	0·040	0·016	−0·110	−0·012	0·0	0·007	0·288	0·197
	(4·14)	(6·74)	(6·54)	(−7·89)	(−2·97)		(1·21)		
w_s	0·151	−0·069	−0·061	0·176	−0·061	0·0	0·117	0·506	0·327
	(7·01)	(−7·00)	(−14·70)	(7·65)	(−8·64)		(11·26)		
w_g	0·116	−0·052	0·627	−0·052	0·102	0·0	0·101	0·219	0·269
	(6·54)	(6·44)	(18·13)	(−2·76)	(17·37)		(1·18)		
w_h	0·185	−0·075	0·003	0·083	0·0008	0·0	−0·0048	0·625	0·306
	(9·18)	(−8·08)	(0·98)	(3·88)	(−1·45)		(−0·49)		

restriction is invalid, and/or the model suffers from dynamic misspecification. The subsequent sections consider some time-series issues which might affect the interpretation of these results.

2. INFERENCE ISSUES

A main finding in the econometrics literature during the past decade is that the least squares estimator associated with non-stationary, or $I(1)$, regressors is super (order T) consistent but that the distribution of the estimator will not, in general, be asymptotically normal.[3] Standard

[3]See, for example, Park and Phillips (1988), Park and Phillips (1989), and Sims *et al.* (1990).

inference applies only if the innovations driving the $I(1)$ regressors are serially uncorrelated and independent of the regression innovations, or if there are sufficient deterministic trends in the DGP to nullify the stochastic trends. In general, the asymptotic distribution of the least squares estimator will contain nuisance parameters that depend on the covariance between the innovations underlying the integrated regressors and the regression error. The empirical implication is that the t- and F-statistics will, in general, have the wrong size when the regressors have unit roots (or stochastic trends).

If one were given a set of data to estimate the demand system, one would expect relative prices and total real expenditure to be constant in a no-growth economy. Expenditure shares should also be constant absent taste shifts. Things are more ambiguous when the economy grows. Total real expenditure could be trend stationary or difference stationary depending on the growth process, and expenditure shares will vary over time unless preferences are homothetic. Otherwise, one would observe the share of luxury goods to rise and that of necessary goods to fall as the economy becomes richer. Relative prices may also be nonconstant as the pattern of demand changes.

Previous studies of demand systems have been based on the data of industrialized countries which, evidently, have experienced economic growth, structural and demographic changes. The explanatory variables of AIDM are the logarithms of total real expenditure and prices. Casual inspection of the data for the USA suggests that these variables are non-stationary. As well, the share of food has been trending down while that of services has been rising over the sample. Issues pertaining to nonstationary regressors could therefore be relevant.

To verify the time series properties of these regressors, the $Z_{\tau\alpha}$-statistic due to Phillips and Perron (1988) and the $t_{\tau\alpha}$-statistic of Said and Dickey (1984) are used to test the null hypothesis of a unit root against the alternative of trend stationarity.[4] The inclusion of a trend in the regressions ensures that the unit root hypothesis is not falsely accepted when the series is really trend stationary. These results are reported in Table II. The unit root tests indicate that all the variables are better characterized as difference rather than trend stationary at the 5% level. For completeness, we use the more powerful $Z_{\mu\alpha}$- and $t_{\mu\alpha}$-statistics to confirm the presence of a unit root (with drift) in all the variables. In all cases, the null hypothesis of a unit root cannot be rejected and the drift terms are significant. That prices and total expenditure contain unit roots is not surprising. If preferences are not homothetic, then by implication of Engel's Law, the share of necessary goods in total expenditure should fall and that of luxury goods should rise as the economy grows. The finding that expenditure shares also contain unit roots not only suggests preferences are not homothetic, but also that shifts in the composition of demand have stochastic trends.

We then proceed to test the null hypothesis of two unit roots against the alternative of one unit root. When a trend term is excluded from the regression, the null of a unit root in the first differenced data is rejected in all cases except the first differences of the log price of housing and other services. However, when a time trend is included in the regression, the $Z_{\tau\alpha}$-statistic indicates that both $\Delta \log p_s$ and $\Delta \log p_h$ are stationary around deterministic trends. This implies that the (log) level of these two price variables have trend functions consisting of t and t^2.[5]

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[4] Definitions of the statistics are given in Table II.

[5] Although $t_{\alpha\tau}$ cannot reject the hypothesis of two unit roots in these two prices from an autoregression with four lags of Δp, the hypothesis is rejected when fewer lags are included in the regression. The $t_{\alpha\tau}$-statistic is $-3{\cdot}6$ and $-4{\cdot}5$ for services and housing respectively for zero lags.

Table II(a). Tests for one unit root

	$Z_{\mu\alpha}$ [a]	$Z_{\tau\alpha}$ [b]	$t_{\mu\alpha}$ [c]	$t_{\tau\alpha}$ [d]
$\log p_f$	0·82	−4·07	0·78	−2·09
$\log p_c$	0·60	−5·38	0·78	−2·17
$\log p_s$	0·65	−2·56	0·28	−2·20
$\log p_g$	0·75	−3·76	0·97	−1·69
$\log p_h$	1·06	−1·68	−0·17	−2·13
w_f	−0·93	−10·22	−1·02	−2·66
w_c	−1·39	−13·98	−1·11	−2·95
w_s	0·20	−6·22	0·73	−2·09
w_g	0·28	−5·10	−0·09	−1·94
w_h	−4·37	−7·27	−2·39	−2·52
X/P	−0·52	−3·66	−1·05	−1·59

Table II(b). Tests for two unit roots

	$Z_{\mu\alpha}$ [a]	$Z_{\tau\alpha}$ [b]	$t_{\mu\alpha}$ [c]	$t_{\tau\alpha}$ [d]
$\Delta \log p_f$	−60·12	−72·23	−3·50	−3·69
$\Delta \log p_c$	−114·50	−119·80	−3·83	−3·96
$\Delta \log p_s$	−14·83	−20·86	−2·00	−2·10
$\Delta \log p_g$	−61·66	−65·93	−3·62	−3·86
$\Delta \log p_h$	−15·46	−33·29	−1·76	−1·76
Δw_f	−147·81	−148·03	−5·39	−5·41
Δw_c	−148·72	−148·54	−5·20	−5·20
Δw_s	−176·87	−176·47	−5·41	−5·15
Δw_g	−105·44	−105·52	−4·65	−4·79
Δw_h	−112·83	−114·22	−4·76	−4·92
$\Delta X/P$	−127·33	−128·99	−4·71	−4·82
CV (5%)	−14·10	−21·80	−2·86	−3·41
CV (10%)	−11·30	18·30	−2·57	−3·12

[a] $Z_{\tau\alpha} = T(\hat{\alpha}-1) - (s^2 - s_u^2)T^6/24D_x$, where $D_x = \det(X'X)$, X is the matrix of regressors in the least squares regression $y_t = \hat{\mu} + \hat{\tau}t + \hat{\alpha}y_{t-1} + \hat{u}_t$, $s_u^2 = T^{-1}\sum_{i=1}^{T}\hat{u}_t^2$, and $s^2 = s_u^2 + 2T^{-1}\sum_{j=1}^{k}(1 - j/(k+1))\sum_{t=j+1}^{T}\hat{u}_t\hat{u}_{t-j}$. The truncation lag, k, is set to four in the tests.

[b] $t_{\tau\alpha}$ is the t-statistic on y_{t-1} in the regression $\Delta y_t = \mu + \tau t + \alpha y_{t-1} + \sum_{j=1}^{k}\delta_j\Delta y_{t-j} + e_t$.

[c] $Z_{\mu\alpha} = T(\hat{\alpha}-1) - 1/2(s^2 - s_u^2)[T^{-2}\sum_{t=1}^{T}(y_{t-1}-\bar{y})^2]^{-1}$, where $\hat{\alpha}$ is the least squares estimate in the regression $y_t = \hat{\mu} + \hat{\alpha}y_{t-1} + \hat{u}_t$, s_u^2 and s^2 are defined as above.

[d] $t_{\mu\alpha}$ is the t-statistic on y_{t-1} in the least squares regression $\Delta y_t = \mu + \alpha y_{t-1} + \sum_{j=1}^{k}\delta_j\Delta y_{t-j} + e_t$.

As we have seen, all the variables of the demand system are non-stationary. If one were interested in consistent estimates of price and income elasticities, then the presence of these non-stationary regressors is actually advantageous since the least squares estimates are super-consistent. However, testing for homogeneity will be a problem because the test statistics will likely have non-standard properties. In fact, this will be the case because the dependent variables in this model are expenditure shares; they are, by construction, correlated with prices and total expenditure on the right-hand side of the regressions. Furthermore, we have more stochastic trends than deterministic trends in the data. The two conditions for classical inference to be valid when the regressors are integrated therefore do not apply. Other formulations of the demand system are likely to have a similar inference problem.[6]

It is of some interest to ask whether the homogeneity restriction can be formulated in terms of a coefficient on a stationary regressor whose asymptotic distribution would have been normal. Theorem 2 of Sims *et al.* (1990) shows that if a restriction involves estimated coefficients that exhibit different rates of convergence, then the estimated coefficient with the slowest rate of convergence will dominate the test statistic. Since all the variables in our system are $I(1)$, the F- and t-tests will have non-standard properties.[7]

2.1 The Finite Sample Properties of the Test Statistic

While the results of the previous subsection suggest the t-statistic will have non-standard properties, we do not know whether the distributions of the t-statistic are being shifted to the left or to the right, and hence the extent to which the tests are under- or oversized. The discrepancy between the finite sample and the Student t distribution is an empirical issue which depends on the strength of the current and past correlations among the innovations. This section provides an assessment of size distortions by tabulating the empirical distribution of the t-statistic for each equation.

A maintained assumption throughout this section is that equation (2) is correctly specified. The objective is to simulate samples of the regressors and the dependent variables which can replicate the covariance structure of the data, and then compute the t-statistic for each set of simulated data. The data-generating processes for the regressors are based on results from the univariate unit root tests. More specifically,

$$\begin{aligned}\Delta \log p_{ft} &= \mu_f + e_{ft}\\ \Delta \log p_{ct} &= \mu_c + e_{ct}\\ \Delta \log p_{gt} &= \mu_g + e_{gt}\\ \Delta \log p_{st} &= \mu_s + \tau_s t + e_{st}\\ \Delta \log p_{ht} &= \mu_h + \tau_h t + e_{ht}\\ \Delta \log(X/P)_t &= \mu_x + e_{xt}\end{aligned} \tag{3}$$

where μ and τ are obtained by least squares estimation. Let $e_2 = \{e_{ft}, e_{ct}, e_{gt}, e_{st}, e_{ht}, e_{xt}\}$ with elements assumed to evolve according to the univariate autoregression:

$$e_{2jt} = \phi e_{2jt-1} + v_{2jt}$$

To obtain random samples of the dependent variable, it is necessary to be specific about what the null hypothesis is. Under the maintained assumptions of intertemporal separability and the conditions require for exact aggregation as discussed in Deaton and Muellbauer (1980), the AIDM is a local first-order approximation to any demand system satisfying utility maximization. But there are many aspects of the demand system that can be tested. If we test a composite

[6] In a recent paper, Attfield (1991) focused on the issue of simultaneity bias in estimation of demand systems, but the variables are assumed stationary in his analysis.

[7] Critical values for the multivariate trace and maximum λ tests of Johansen (1988) are not available when the variables have quadratic time trends, but using the critical values tabulated in Perron and Campbell (1993) for a linear trend (which would be too liberal), we cannot reject the hypothesis that there is less than one cointegrating vector in a system consisting of the five prices. The prices are therefore individually integrated but not cointegrated. This is confirmed by our unit root tests for the $n(n-1)/2$ relative prices, all of which are found to contain a unit root regardless of the choice of the numeraire.

hypothesis, say, symmetry and homogeneity, we would not be able to tell whether homogeneity fails just because symmetry is violated or if it is an outright invalid restriction. We therefore formulate the homogeneity restriction as a simple hypothesis, i.e.:

$$\mathrm{H}_0: w_{it} = \alpha_i + \sum_{j=1}^{n-1} \gamma_{ij} \log\,(p_{jt}/p_{nt}) + \beta_i \log(X_t/P_t) + e_{irt}$$

$$\mathrm{H}_1: w_{it} = \alpha_i + \sum_{j=1}^{n-1} \gamma_{ij} \log\,(p_{jt}/p_{nt}) + \left(\sum_{j=1}^{n} \gamma_{ij}\right)\log p_{nt} + \beta_i \log(X_t/P_t) + e_{iut} \tag{4}$$

$$e_{ij,t} = \rho_j e_{ij,t-1} + v_{ijt} \qquad j = r, u$$

The dependent variable is then constructed using the specification under the null with random draws of v_{ir}.

The next step is to ensure that the simulated data replicate the sample covariance. Let $v_i = (v_f, v_c, v_g, v_s, v_h, v_x, v_{ir})$, $\Psi_i = P_i'P_i$ be the contemporaneous covariance of the innovations relevant to the ith equation, and P_i be the Cholesky decomposition of Ψ_i. For each i, $\tilde{v}_i$ is generated as $\varepsilon_i P_i$, where ε_i is a 148×7 matrix of $N(0, 1)$ variates generated by the *rndn*() function in Gauss. In each run, T is set to 148, the sample size that produced the results in Table I. By construction,

$$\lim_{T\to\infty} \frac{1}{T} \sum_{1}^{T} \tilde{v}_i'\tilde{v}_i \to \Psi_i$$

The regression is based on the unrestricted model, H_1, and the t-statistic on log p_n is calculated in each of the 1000 replications. The calculations are performed using GAUSS V 3.0.

Empirical quantiles of the t-statistic corresponding to $\sum_j \gamma_{ij}$ for each ith equation are presented in Table III. A general observation to be made is that the Student t distribution provides poor approximations to the empirical distributions of the t-statistic on log p_n. Large size distortions are recorded in all cases. Also, the finite sample distributions of the t-statistic are noncentral, and, in many cases, nonsymmetric. The tails of the distributions are more spread out than the normal distribution so that at conventional significant levels, the standard t-test will reject homogeneity too often.

We saw in Table I(a) that homogeneity was strongly rejected in the 'other goods' equation. Since $\sum_{j=1}^{n} \hat{\gamma}_{ij}$ for that equation is negative, the relevant critical values are those from the left tail. Although the finite sample critical values for the lower percentage points are much larger (in absolute value) than those of the Student t distribution, they are well below the t-statistic reported in Table I(a). Thus, for this equation, classical inference turns out to have been correct. However, for the remaining four equations, we cannot reject homogeneity at the two-tailed 10%

Table III. Empirical quantiles of the t-statistic for $(\sum_i \gamma_{ii})$

	0·01	0·025	0·05	0·10	0·90	0·95	0·975	0·99
w_f	−8·9937	−7·4995	−6·2839	−4·9259	4·4294	5·6506	6·9922	8·2742
w_c	−7·6742	−6·3565	−4·9389	−3·8251	3·8496	5·0065	6·0373	6·9114
w_s	−6·6903	−4·7070	−3·1234	−1·9009	7·5700	9·1029	10·3664	11·9827
w_g	−11·6497	−10·2262	−8·6700	−7·2563	1·8191	2·8416	3·9103	5·0244
w_h	−10·4783	−8·2552	−6·8992	−5·5962	4·5365	5·7899	6·8795	7·8807
t	−2·32	−1·96	−1·64	−1·28	1·28	1·64	1·96	2·32

level using the finite sample critical values. Size distortions are therefore responsible for many of the rejections reported in Table I(a).

The results of this section can be summarized as follows. All regressors used in the estimation of AIDM are nonstationary; the validity of classical inference is therefore questionable. The finite sample critical values obtained from a simulation exercise confirm that the Student t distribution provides an inadequate guide to the empirical distributions of the t-statistic. Using the finite sample critical values, we reject homogeneity in one equation only. Other rejections of homogeneity based on time-series data could also have been overstated.

3. SPECIFICATION ISSUES

The analysis of the previous section is based on the assumption that equation (2) is properly specified. Some might doubt its validity because there are symptoms of dynamic misspecification. There are several reasons why dynamic misspecification might arise. These include (1) lacking an account of the stock aspect of commodities such as housing, (2) omission of inflationary expectations, (3) the variable Υ is not constant, and (4) violation of weak intertemporal separability. In spite of these problems, there is a sense in which estimation of equation (2) remains meaningful in its own right. To motivate this viewpoint, it is necessary to be clear about the horizon over which the demand system is expected to hold.

In view of market and information imperfections, it is possible for demand theory not to hold on a period-by-period basis.[8] The AIDM is therefore best viewed as a description of long-run (equilibrium) behaviour. In econometric terms, the variables in equation (2) should form a 'cointegrated' system if the theory is consistent with the data. Imposing homogeneity is analogous to a zero restriction on an element of the cointegrating vector which can be consistently estimated from the static equation (2). However, estimation of equation (2) serves the additional purpose of allowing us to test for cointegration. If the variables in the demand system are not cointegrated, tests for homogeneity would not be very meaningful because of the inherent conflict between the model and the data even in the absence of constraints. Almost all empirical studies of the demand system have proceeded to test for homogeneity without testing whether the demand system is cointegrated. We propose to study the properties of the demand system from a different perspective.

A precise definition of cointegration is in order because there are two definitions of cointegration in the literature. Under deterministic cointegration, defined in Engle and Granger (1987), the same vector which removes stochastic trends also has to remove the deterministic trends in the system. Under stochastic cointegration, introduced by Ogaki and Park (1990), the deterministic trends do not have to be cotrending. In other words, non-zero time trends are allowed to remain after unit roots are eliminated by linear combinations of the variables. The distinction between these two concepts is important in hypothesis testing. The issue is discussed in a recent paper by Perron and Campbell (1993).

3.1 Testing for Deterministic Cointegration

The absence of trends in equation (2) implies that the demand system is to be cointegrated in a deterministic sense if demand theory holds. Testing for deterministic cointegration amounts to testing for unit roots in the residuals of equation (2). The results of cointegration tests based on

[8] Anderson and Blundell (1982) discussed the difficulties that might arise in imposing restrictions on dynamic demand systems.

Z_α and t_α are given in the first two columns of Table IV. Critical values for the tests depend on the number of regressors which enter the regression and are given in Phillips and Ouliaris (1990) for up to five variables. As we can see, the null of no cointegration cannot be rejected in any of the equations if homogeneity is imposed. Even when the homogeneity restriction is not imposed, the null hypothesis of no cointegration—in a deterministic sense—cannot be rejected at either the 5% or the 10% significance level in four of the five equations. The only case in which deterministic cointegration appears to hold, and only when the homogeneity restriction is relaxed, is the goods equation. This last result is particularly troublesome because it implies that even in the absence of restrictions, demand behaviour does not seem to conform to theory. Not surprisingly, imposing the homogeneity restriction will only make the empirical model more unacceptable to the data.[9]

Given the joint time series properties of the log p_i's and $\log(X/P)$, the failure of deterministic cointegration should not come as a surprise. Recall that by the definition of

Table IV(a). Cointegration tests on the unrestricted model

$$w_i = \sum_{j=0}^{d} \alpha_j t^j + \sum_{j=1}^{n-1} \gamma_{ij} \ln(p_j/p_n) + \left(\sum_{j=1}^{n} \gamma_{ij}\right) \ln p_n + \beta \ln(X/P) + e_{ui}$$

	Z_α $(d=0)$	t_α $(d=0)$	Z_α $(d=1)$	t_α $(d=2)$	$Z_\alpha (d=2)$	t_α $(d=2)$
w_f	−15·80	−2·72	−44·82	−5·46	−55·85	−5·77
w_c	−25·17	−3·36	−47·41	−4·29	−48·94	−4·31
w_s	−21·22	−2·15	−75·49	−4·90	−86·79	−6·00
w_g	−43·52	−5·09	−45·46	−5·25	−47·81	−5·26
w_h	−16·19	−3·51	−17·64	−3·54	−37·58	−4·02

Table IV(b). Cointegration tests on the restricted model

$$w_i = \sum_{j=0}^{d} \alpha_j t^j + \sum_{j=1}^{n-1} \gamma_{ij} \ln(p_j/p_n) + \beta \ln(X/P) + e_{ri}$$

	Z_α $(d=0)$	t_α $(d=0)$	Z_α $(d=1)$	t_α $(d=2)$	Z_α $(d=2)$	t_α $(d=2)$
w_f	−14·48	−2·58	−28·68	−3·79	−54·26	−5·79
w_c	−22·92	−3·23	−39·05	−3·72	−48·87	−4·31
w_s	−18·71	−2·41	−36·92	−4·03	−86·82	−5·97
w_g	−21·04	−3·57	−21·44	−3·59	−45·21	−5·10
w_h	−12·92	−3·00	−12·39	−2·51	−20·44	−3·36
CV (5%)	−42.57	−4·76	−47·51	−5·02	−52·13	−5·29
CV (10%)	−37·55	−4·47	−42·14	−4·73	−47·11	−4·99

Note: The cointegration tests are unit root tests on the residuals from the restricted and the unrestricted models. The Z_α and t_α tests are the same as $Z_{\mu\alpha}$ and $t_{\mu\alpha}$ defined in Table II(a) but without the constant in the regression. The critical values are tabulated using the procedure as described in Phillips and Ouliaris (1990) and are available on request.

[9] The Johansen (1988) likelihood ratio test (using approximate critical values) suggests there are three or more non-stationary components in each equation. However, as discussed in Campbell and Perron (1991), a process with r cointegrating vectors can be well approximated by a process with $m > r$ cointegrating vectors in finite samples. The choice of m or r is one of efficiency, and is not the focus of this analysis. The crucial implication of the tests is that there exist non-stationary elements in the residuals. We therefore choose $r = 1$ for analytic convenience.

deterministic cointegration, the same cointegrating vector is supposed to remove the stochastic and the deterministic trends that are common among the variables in the system. Deterministic cointegration will fail even if there are common stochastic trends as long as there are non-common deterministic trends. The parameterization of equation (4) implies that although all regressors contain a linear trend, the term t^2 appears only in the (log) price of services and housing. This non-common deterministic component cannot be annihilated by any cointegrating vector. Although deterministic trend functions are often assumed to be linear for analytic convenience, our application shows that this may not always be adequate.

Omitting time trends from the regression has been known to lead to inconsistent tests. This result arises because if the data are trend stationary but trends are excluded from the regression, the only way to fit the model is to accept the stochastic trends. The consequence is that the residuals will absorb the omitted deterministic trends, and unit root tests based on those residuals will bias the tests in favour of the unit root hypothesis. In a cointegration context, the tests will tend to accept the null hypothesis of no cointegration. It is therefore important to include at least as many deterministic trends in cointegration tests as there are in the DGP. This result is a generalization of the phenomenon analysed in Perron (1988) for the univariate case.

There is, potentially, another source of misspecification in the model. Recall that the index Υ, subsumed in the constant α_i, is supposed to capture demographic changes, taste shifts, and skewness of the income distribution. Although it is difficult to quantify the evolution of Υ, few would believe that Υ is constant over the sample. For example, Cutler and Katz (1992) found the consumption distribution to have changed in the 1980s alone. Time variations over the entire sample are likely to be more profound. To the extent that Υ is a measure of inequality across households, it should vary over time. If it is assumed constant, as is the case in the regressions previously reported, this misspecification can also induce a time-varying component in the residuals.

3.2 Testing for Stochastic Cointegration

Given that the variables in the demand system all contain unit roots, the issue arises as to whether these stochastic trends move in tandem. As we have seen, two of the prices in the demand system contain quadratic trends. Accordingly, the appropriate null hypothesis is that of no 'stochastic cointegration'. There are two ways to control for deterministic trends when testing for cointegration. One can remove the deterministic trends from all the variables prior to the regression, or one can detrend them in the regression by including a polynomial in time of sufficient order. As discussed in Campbell and Perron (1991), these two approaches have the same effect asymptotically. In the present context, the latter approach has the additional advantage that it can pick up possible time variations in demographics and tastes, and is the approach that this paper will follow.

The results for testing the null hypothesis of no stochastic cointegration are given in columns 3 to 6 of Table IV. Homogeneity is still rejected with the inclusion of the variable t, but the model is much improved with the addition of t^2 as seen from the larger test statistics. A formal test for stochastic cointegration requires some extra work. Critical values for the statistics of cointegration tests depend on the number of regressors and the order of the deterministic trend function. Phillips and Ouliaris (1990) tabulated critical values for one to five regressors with a polynomial trend function of orders 0 and 1, but critical values for the model being tested are not readily available from the literature. Following the same procedure discussed in Phillips and Ouliaris, we tabulated critical values for Z_α and t_α for one to six regressors and extended the polynomial trend to second order. These critical values are available upon request.[10]

Using these critical values we tested the null of no stochastic cointegration and rejected the hypothesis in the equations for food and other services at the 5% level, and for clothing at the 10% level. The goods equation is found not to be cointegrated, but only marginally, at the 10% level. However, the data suggest that housing is clearly not stochastically cointegrated. Note that when cointegration holds, it seems not to matter whether homogeneity is imposed. This result is important because it suggests that when trends in the data are properly taken into account, the power of the model in explaining long-run demand behaviour is, in most cases, unaffected by the imposition of the homogeneity restriction. While the statistical cost of imposing the restriction may be small, the analytical benefit from being able to impose homogeneity cannot be overlooked since it validates a crucial assumption of many optimizing models.

Recapitulating the results of this section, we tested and cannot reject the null of no deterministic cointegration in both the unrestricted model and the model with homogeneity imposed. We argue that equation (2) is overly restrictive and that it is appropriate to incorporate deterministic trends into the regressions. We reject the null of no stochastic cointegration in three of the five equations, marginally reject the hypothesis in one case, and cannot reject in one equation only, namely, housing. The same results obtain with the restricted model. The empirical demand model with trends is therefore more suitable for the data in question.

4. TESTING FOR HOMOGENEITY ONCE AGAIN

The finding that the demand system is stochastically cointegrated opens up new possibilities for estimating and testing the model. The estimation of cointegrating vectors has been an active area of research in recent years. Phillips (1991) discussed the theoretical conditions under which efficient estimators that belong to the LAMN (locally asymptotically mixed normal) class can be obtained. These estimators are efficient compared to OLS (which is consistent) because they take into account the properties of the $I(1)$ variables in the estimation.

One efficient estimator that seems suited for the demand system in the DOLS (Dynamic Ordinary Least Squares) estimator developed independently by Phillips (1991), Phillips and Loretan (1991), and Stock and Watson (1993).[11] The approach begins by expressing a system of variables, (y_{1t}, y_{2t}), in a triangular form such that $\Delta y_{2t} = \sum_{i=0}^{k} d_{2i} t^i + e_{2t}$, and $y_{1t} = \sum_{i=0}^{k} d_{1i} t^i + \beta y_{2t} + e_{1t}$, where β is the cointegrating vector. A DOLS regression augments the least squares regression of y_{1t} on y_{2t} with leads and lags of e_2. These additional regressors essentially orthogonalize e_{1t} and have the effect of removing those terms which pose problems for asymptotic inference. The Wald statistic for testing restrictions on the cointegrating vector so estimated is asymptotically distributed as χ^2 provided the long-run variance of the regression error is used to construct the statistic.[12] Accordingly, the statistic for testing the coefficient on one regressor is asymptotically normal.

[10] It is interesting to note a regularity in the tabulated critical values. Each additional regressor tends to increase the critical value for Z_α and t_α by 5 and 0·5, respectively. Furthermore, increasing the order of the deterministic trend by one also tends to increase the critical values by 5 and 0·5, respectively. For example, if the critical value for Z_α is $-14{\cdot}8$ for one regressor and no deterministic term, a rough and ready estimate of the critical value when there are three regressors with a constant and a trend included in the regression would be -35.

[11] Johansen (1988)'s FIML approach and other vector autoregressive models such as that of Saikkonen (1991) are less suited for our purpose because a complete system specification would have required expressing total expenditure and prices as lags of the shares which may not be appropriate.

[12] Stock and Watson's approach differs from that of Phillips and Loretan (1991) in that the latter adds lags of the equilibrium error to the regression and entails estimation by nonlinear least squares.

Expressing our demand system in a triangular form, the expenditure shares would correspond to y_{1t}, while y_{2t} would consist of the prices and real total expenditure. We therefore apply DOLS to the following model:

$$w_{it} = \alpha_{0i} + \alpha_{1i}t + \alpha_{2i}t^2 + \sum_{j=1}^{n-1} \gamma_{ij} \log(p_{jt}/p_{ht}) + \left(\sum_{j=1}^{n} \gamma_{ij}\right) p_{ht} + \beta_i \log(X_t/P_t) + d(L)e_{2t} + e_{iut} \tag{5}$$

where we recall that e_2, defined in equation (3), are the residuals after the stochastic and deterministic trends are removed from the regressors, and $d(L)$ is a two-sided lag polynomial. The equation can be interpreted as the stochastic relationship between expenditure shares and the regressors after controlling for deterministic trends. In the empirical work, one lead and one lag of e_{2t} are included in the regression and a Bartlett window truncated at four lags is used to construct the long-run variance of the regression error. As we recall, the purpose of adding leads and lags of e_2 in the regression is to orthogonalize e_{iu}. Setting the order of $d(L)$ to one raises the question of whether or not e_{iu} is sufficiently orthogonalized. There is no practical guideline as to how many leads and lags of e_2 to include, but with six regressors in the model, the degrees of freedom fall rapidly even with a modest number of leads and lags.[13] We experimented with four leads and lags and found the estimates of the cointegrating vector to be robust but the standard errors are larger.

There were two interesting results in the analysis of Deaton and Muellbauer (1980). They found that imposing homogeneity aggravated the problem of serial correlation when the model was estimated in level form, and when the model was estimated in first-difference form, the constant term was significant with the problem of serial correlation much improved. This amounts to saying that the model in level form fits the data better when a trend is included. The specification above is also motivated by the fact that introducing time trends can remove much of the conflict between the data and the theory, but we find a t^2 term is necessary for the data in question. The value added of the present analysis is to exploit the results for estimating cointegrating vectors to obtain more efficient estimates and to note the theoretical conditions under which standard statistics can be used to test for homogeneity. It is important to remark, however, that application of DOLS to the model without trends (i.e. results in Table I(a)) would have been invalid since the individual equations are not cointegrated; the DOLS estimator is developed specifically for cointegrated systems.

The coefficients of interest in the estimation of equation (5) are those on prices and expenditure as they form the cointegrating vector of the demand system. The DOLS estimates are presented in Table V. The presence of t and t^2 in the regression causes the coefficient on total real expenditures to flip sign in most cases. To the extent that $\log X/P$ is effectively detrended by t and t^2, it is more appropriate to interpret β_i as the response by expenditure shares to the stochastic innovations of $\log X/P$. An increase in the stochastic component of total expenditure has a positive effect on clothing and other goods, a negative effect on housing, and small effects on food and services. Perhaps more interesting is that a proportional increase in prices and total expenditure (as summarized by $\sum_{j=1}^{n} \gamma_{ij}$) now leads to a statistically significant increase in the expenditure share of housing, rather than a decrease as found in Table I(a) when the model was estimated without trends. Thus, an inadequate treatment of trends can mislead our understanding of the stochastic relationships in the system.

[13] The standard errors are calculated on the basis of the degrees of freedom in the regression, rather than the sample size, to control for any bias that might arise from increasing the number of regressors.

Table V. DOLS estimates for the unconstrained model

$$w_i = \alpha_0 + \alpha_1 t + \alpha_2 t^2 + \sum_{j=1}^{n-1} \gamma_{ij} \ln(p_j/p_n) + \left(\sum_{j=1}^{n} \gamma_{ij}\right) \ln p_n + \beta \ln(X/P) + d(L)e_2 + e_{ui}$$

	α_0	$\alpha_1 e+3$	$\alpha_2 e+6$	$\ln p_f/p_h$	$\ln p_c/p_h$	$\ln p_s/p_h$	$\ln p_g/p_h$	$\ln p_h$	$\ln X/P$	DW / SEe+2
w_f	0·285	−1·590	4·400	0·069	−0·053	0·102	−0·020	−0·035	0·028	0·873
	(10·37)	(−14·02)	(4·87)	(5·95)	(−2·98)	(2·97)	(−2·81)	(−1·87)	(1·70)	0·152
w_c	−0·044	−0·740	1·201	0·011	0·028	−0·061	0·005	0·007	0·089	0·847
	(−2·33)	(−9·37)	(1·91)	(1·43)	(2·27)	(−2·57)	(1·07)	(0·55)	(7·56)	0·109
w_s	0·313	0·769	2·142	−0·093	0·024	0·126	−0·037	0·006	−0·017	1·306
	(14·93)	(8·91)	(3·14)	(−10·51)	(1·78)	(4·83)	(−6·61)	(0·45)	(−1·37)	0·133
w_g	0·086	0·221	−1·937	0·006	−0·033	−0·044	0·079	−0·038	0·025	1·093
	(4·71)	(2·92)	(−3·24)	(0·87)	(−2·84)	(−1·93)	(16·17)	(−3·10)	(2·02)	0·110
w_h	0·358	1·340	−5·763	0·005	0·034	−0·122	−0·026	0·059	−0·126	0·628
	(24·94)	(22·55)	(−12·28)	(0·88)	(3·70)	(−6·80)	(−6·96)	(6·12)	(−14·12)	0·070

Empirical quantiles of the t-statistic for $(\sum_i \gamma_{ii})$

	0·01	0·025	0·05	0·10	0·90	0·95	0·975	0·99
w_t	−3·1768	−2·5441	−2·0168	−1·5961	1·6372	2·0698	2·5535	3·0065
w_c	−3·6410	−2·84062	−2·2644	−1·7485	1·8370	2·5875	3·3257	4·0621
w_s	−2·7900	−2·4724	−1·9430	−1·4487	1·4982	1·9204	2·2502	2·8479
w_g	−3·2360	−2·7379	−2·2179	−1·5233	1·6029	1·9979	2·4248	3·0061
w_h	−4·4621	−3·8114	−3·1743	−2·3488	2·1439	2·8011	3·5598	4·6785
t	−2·32	−1·96	−1·64	−1·28	1·28	1·64	1·96	2·32

Note: e_2 is the vector of detrended residuals of the regressors as defined in equation (3), $d(L)$ is a two-sided lag polynomial. The results reported are for one lead and lag.

According to asymptotic theory, the t-statistics associated with the DOLS estimator should have standard properties. We therefore use critical values from the normal distribution to test for homogeneity. We cannot reject homogeneity in the food, clothing, and services equations. This is consistent with the cointegration tests which find decisive evidence against the null hypothesis of no stochastic cointegration in the three equations. Furthermore, consistent with our inability to reject the null of no stochastic cointegration in the housing equation, we also find the evidence against homogeneity to be strongest. The only case when ambiguity arises is with other goods; while homogeneity is decisively rejected, the rejection of no stochastic cointegration is only marginal.

While inference based on asymptotic theory rejects homogeneity in two of the five cases, it remains to consider the accuracy of asymptotic inference. This is a useful exercise in its own right because the optimality of the DOLS estimator is based on large-sample considerations, but the practical issues associated with the estimator are not known. Furthermore, the empirical properties of the estimator have been gauged in the context of simulations of small scale models. See, for example, Stock and Watson (1993) and Inder (1993). The performance of the estimator in larger models remains to be scrutinized. The demand system analysed here provides a practical framework for analysing these properties.

As in the previous section, we tabulate the finite sample critical values of the t-statistic. The DGP is given by the restricted model:

$$w_{it} = \alpha_{0i} + \alpha_{1i}t + \alpha_{2i}t^2 + \sum_{j=1}^{n-1} \gamma_{ij} \log(p_{jt}/p_{nt}) + \beta_i \log(X_t/P_t) + e_{irt} \quad (6)$$

where the coefficients are the efficient estimates obtained by applying DOLS to the restricted variant of equation (5) with $\sum_{j=1}^{n} \gamma_{ij}$ constrained to zero. The Monte Carlo simulations are based on random draws to the innovations of e_{irt} and those of the regressors as given by equation (3) with cross-correlations taken into account by Cholesky decomposition as discussed earlier.

Quantiles of the finite sample distributions of the t-statistic are given in the lower panel of Table V. As we can see, the distributions are significantly less spread out than those reported in Table III and appears symmetric. Introducing leads and lags of e_2 succeed, to a large extent, in restoring symmetry and centrality of the t-statistic relative to the normal distribution. However, the critical values are still quite different from those of the normal distribution. In the case of clothing and housing, the departure from normality is significant.[14] Although our cointegrated system falls within the LAMN framework which suggests that standard inference should be valid, our results reveal that the approximations provided by large-sample theory are not always adequate. While Stock and Watson (1993) also noted size distortions (e.g. $t_{0\cdot05} = -1\cdot98$) in a two-variable system, our equations with seven variables reveals that size distortions can be much larger. In view of the potential usefulness of the estimator in applied work, it would be useful to explore the relationship between size distortions and the dimension of the model.[15]

The finding that there are size distortions in the statistic for testing homogeneity should not be seen as a discredit to the DOLS estimator because it produces consistent and more efficient estimates of price and income elasticities. However, it would be more appropriate in this case to use the finite sample critical values for inference. Using the finite sample critical values, the homogeneity hypothesis cannot be rejected in three cases, is marginally rejected in the other goods equation, and is strongly rejected in the housing equation only. Note that while asymptotic inference strongly rejects homogeneity in the other goods equation, the evidence based on the finite sample critical values is less convincing. As for housing, the evidence is clear. The homogeneity restriction can be rejected regardless what critical values we use.

5. DISCUSSIONS AND CONCLUSIONS

Previous estimations of demand systems, which precede the literature on unit roots, do not take into account that the regressors are nonstationary when testing the homogeneity postulate. The treatment of trends in most specifications also appears inadequate. Using the AIDM, we showed that (1) critical values from the normal distribution over-reject homogeneity in demand systems estimated without trends, (2) none of the equations in the demand systems are cointegrated in a deterministic sense, but the null of no stochastic cointegration can be rejected in all but the housing equation, and (3) based on finite sample critical values, homogeneity is strongly rejected in just the housing equation once deterministic trends are taken into account. The evidence against the demand system is much weaker when time-series issues are taken into

[14] Varying the number of leads and lags does not eliminate the size problem. In fact, size distortions are larger when $d(L)$ is of the fourth order, and occasional singularity of the moment matrix of regressors was encountered in estimations using simulated data.

[15] An earlier version of this paper uses the modified t-statistic of Phillips and Hansen (1990) to test for homogeneity. Symptoms of size distortions are also revealed.

account. Other formulations of the demand system are also susceptible to such inference and specification problems.

This analysis raises three issues worth further investigation. First, to the extent that demographic shifts and the degree of inequality across households might not be constant over time, it would be useful to model the precise mechanism by which demand is affected by these structural factors. Second, the demand system does not seem capable of explaining the expenditure share of housing services. One possibility is that the assumption of weak intertemporal separability is invalid. Relaxing this assumption is also a promising avenue of research. Third, estimation by DOLS yields statistics whose finite sample distributions are found to depart from normality. Monte Carlo simulations which analyse the procedures for estimating cointegrating vectors are typically based on bivariate systems. It would be useful to see if the extent of size distortions is related to the dimension of the regression model.

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