DEMAND SYSTEMS WITH NONSTATIONARY PRICES

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Abstract—Relative prices are nonstationary and standard root-T inference is invalid for demand systems. But demand systems are nonlinear functions of relative prices, and standard methods for dealing with nonstationarity in linear models cannot be used. Demand system residuals are also frequently found to be highly persistent, further complicating estimation and inference. We propose a variant of the translog demand system, the NTLOG, and an associated estimator that can be applied in the presence of nonstationary prices with possibly nonstationary errors. The errors in the NTLOG can be interpreted as random utility parameters. The estimates have classical root-T limiting distributions. We also propose an explanation for the observed nonstationarity of aggregate demand errors, based on aggregation of consumers with heterogeneous preferences in a slowly changing population. Estimates using U.S. data are provided.

I. Introduction

In most industrialized economies real per capita income trends upward and inflation rates are positive. More precisely, the tendency for prices and standards of living to rise makes the time series of prices and real income nonstationary. Less obviously, relative prices are also nonstationary (see, for example, Ng, 1995, and Lewbel, 1996b). This is recognized informally in the observation that the prices of some goods, such as higher education and medical care, have been rising significantly faster than the average rate of inflation for many years. More broadly, debates over which price measure to use to index social security or to assess monetary policy are based on the fact that different measures diverge over time, which can only occur if there are differences in growth rates of the prices of different goods.

Almost every empirical demand system study suffers from a severe econometric flaw, namely, failure to cope with this nonstationarity of prices.[1] The usual techniques for handling nonstationary regressors, such as cointegration or linear error correction models, cannot be applied to demand system estimation, because any nontrivial demand system that is consistent with utility maximization must be nonlinear in relative prices (see section IV below). But very few estimators exist for nonlinear structural models of any form containing nonstationary data. The problem is further exacerbated by the facts that demands are multiple-equation systems with nonlinear cross-equation restrictions mandated by utility maximization, and that demand systems with dimensions large enough to be empirically interesting involve a large number of parameters relative to the number of available time periods, T. These problems affect demand systems estimated using individual-, household-, panel-, cohort-, or aggregate-level data, because all depend upon utility-maximizing agents facing nonstationary relative prices.[2]

Because of these many difficulties, existing demand system studies either ignore the problem entirely, or deal with nonstationarity using linear model cointegration methods.[3] Even if one could overcome the problems of nonlinearity and high dimension, cointegration methods might still not be appropriate because the errors in demand systems (particularly those estimated with aggregate data) tend to be highly autocorrelated.[4] As is well known, standard asymptotic theory provides a poor guide to finite-sample inference when the errors are highly persistent. In cases when a unit root in the residuals cannot be rejected, the regressions are spurious and the parameter estimates are inconsistent.

In this paper, we provide a solution to the problem of estimating utility-derived demand systems with nonstationary prices. The methodology also takes care of possible nonstationarity of the errors. The key is a new functional form that, by interacting budget shares with prices, produces a model that is both consistent with utility maximization and, when differenced, enables nonlinear estimation of the demand parameters by instrumental variables. Classical root-T consistency and asymptotic normality of the estimates then follows from Hansen's (1992) theory for the generalized method of moments (GMM). The model, which we call NTLOG (nonstationary translog), is a variant of Jorgenson, Lau, and Stoker's (1982) translog demand system. Unlike the translog system in which the errors are appended to budget shares, the error terms in the aggregate NTLOG model equal the average values of utility-function parameters that vary across consumers. Thus, persistence in the error of the aggregate model can be attributed to preferences in a slowly changing heterogeneous population.

Our NTLOG model and the associated estimator provide a solution to the generic empirical problem of demand system estimation with nonstationary relative prices and possibly nonstationary errors. We apply the model to aggregate data, and focus on $T \to \infty$ asymptotics. However, we also also show how the NTLOG could be applied using data at the level of individual households, which also suffer from

Received for publication June 18, 2003. Revision accepted for publication November 2, 2004.

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This research was supported in part by the National Science Foundation through grant SES-9905010. We would like to thank Jean-Marc Robin and Cliff Attfield for helpful comments and discussions. All errors are our own.

[1] See, for example, Stock (1994) and Watson (1994) for a review of econometric issues relating to nonstationary variables.

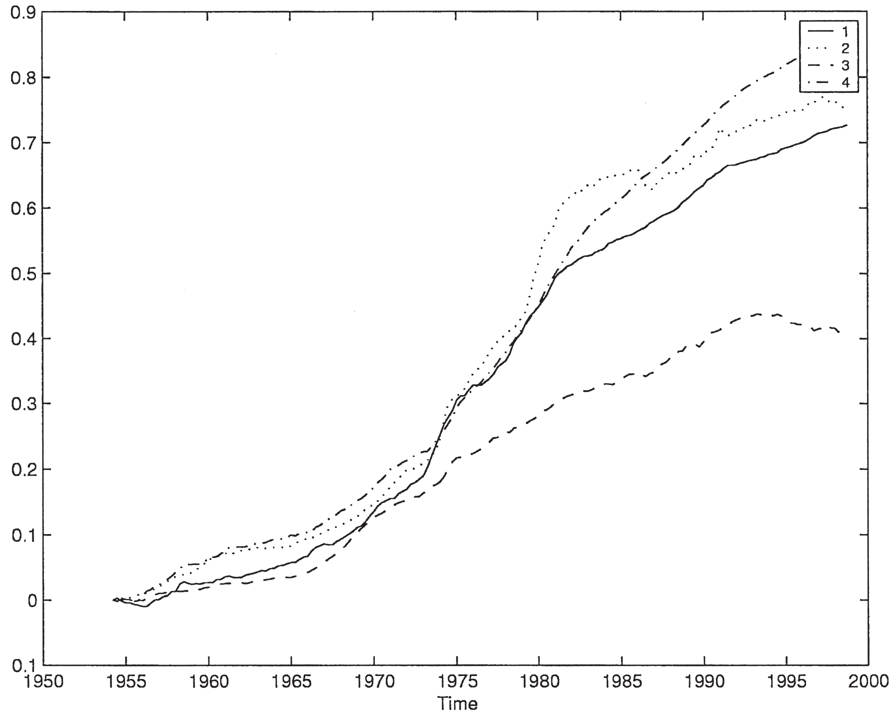
[2] Other issues, including the lack of variation in prices, arise with estimation using cross-section data.

[3] For example, Attfield (1997, 2004) applies linear cointegration techniques to Deaton and Muellbauer's (1980) *almost ideal* model, replacing the true nonlinear (quadratic) price deflator terms with an approximate linear index. Ogaki (1992) employs a two-good demand systems along with a functional form that restricts cross price effects to obtain a linear model for cointegration. Adda and Robin (1996) provide conditions for unbiased multiple-cross-section demand system estimates with nonstationary prices, but they also assume a linear model.

[4] See, e.g., Berndt and Savin (1995), Stoker (1986), Lewbel (1991, 1996a), and Pollak and Wales (1992).

The Review of Economics and Statistics, August 2005, 87(3): 479–494

FIGURE 1.—LOG PRICES



the same nonstationarity of relative prices. In addition, the NTLOG model allows for individual-specific fixed effects that are consistent with utility maximization and, if sufficient cross-sectional price variation is present, can be used to obtain consistent demand system estimates with fixed-T panels and fixed effects.

The plan of this paper is as follows. In the next section, we use U.S. demand data to provide additional evidence of nonstationarity of demand system regressors. We also show that nonstationarity in the residuals is not due simply to missing variables. In section III we propose a possible explanation for these nonstationary errors, by showing that error persistence could arise as the result of aggregation across utility-maximizing individuals with heterogeneous preferences in a slowly changing population. We later provide empirical evidence that this explanation is at least plausible, using household-level data on food demand from the Michigan PSID surveys. Sections IV and V of the paper give the derivation of the NTLOG functional form. Section VI provides our estimator of this NTLOG model and the associated empirical results, and section VII concludes.

II. Nonstationary Demands, Prices, and Incomes

We begin in this section with an exploratory empirical analysis of quarterly, seasonally adjusted data for the United States, documenting nonstationarity of regressors and a high degree of persistence in demand model residuals. We also provide evidence that residual nonstationarity is not due to omitted variables. We use aggregate data because that is where nonstationarity problems are most obvious and severe, and because price effects can be accurately estimated using long time series with a great deal of relative price variation.

Let p_{it} be the price of good or service (or group of goods and services) i at time t, $i = 1, \ldots N$, $t = 1, \ldots T$. Let M_t be per capita expenditures on nondurable goods and services at time t, W_{it} be the fraction of M_t spent on group i at time t, and $r_{it} = \ln(p_{it}/M_t)$. By homogeneity, demands are functions of r_t, the vector of elements r_{it}. Figures 1 and 2 present graphs of log prices and r_{it} for four groups of nondurable goods and services: food (good 1), energy (good 2), clothing (good 3), and all other nondurable goods and services (good 4). Even after taking logarithms, the graphs clearly show the drifts and trends of nonstationary behavior. Similar results are obtained when deflating by an overall price index like the CPI instead of M_t. Figure 3 shows the corresponding graph for aggregate budget shares W_{it}, indicating that at least some of these shares may also appear nonstationary. Budget shares must by construction lie between 0 and 1, and so cannot remain nonstationary forever, but as long as the magnitudes of changes from year to year are small (relative to the range 0 to 1), shares can closely approximate a nonstationary process for decades, as may be the case for some shares in these U.S. data.

Results of formal tests of nonstationarity using the *DFGLS* test of Elliot, Rothenberg, and Stock (1996) and the

FIGURE 2.—LOG NORMALIZED PRICES (*R*)

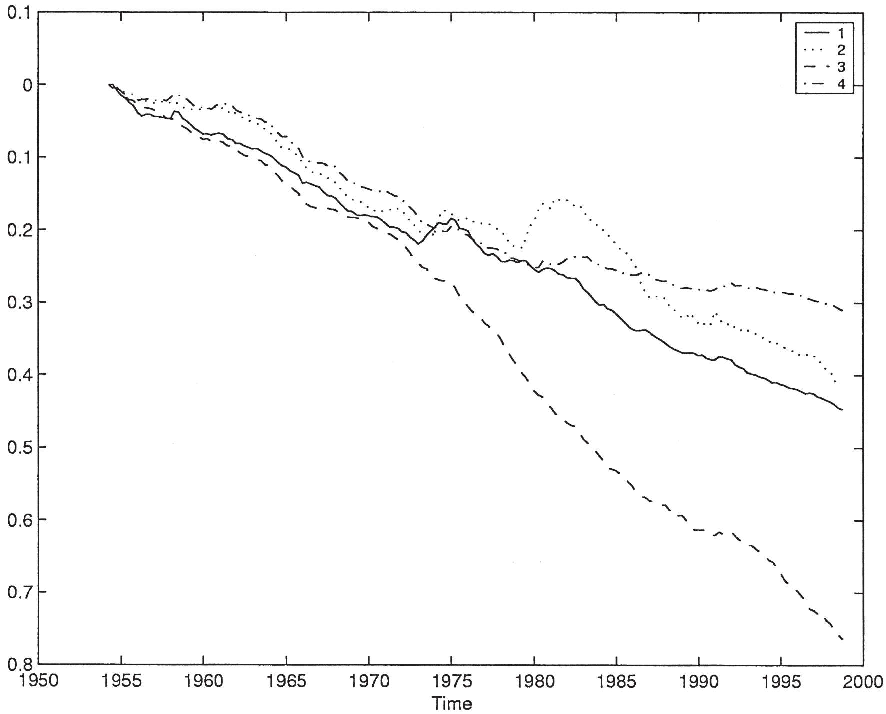
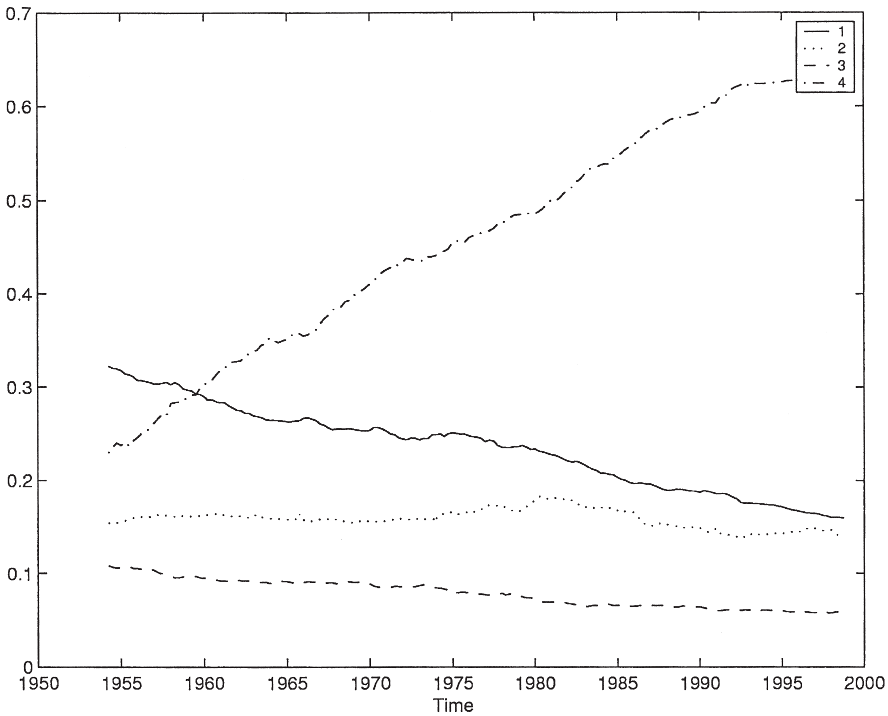


FIGURE 3.—SHARES



MZGLS test of Ng and Perron (2001) are summarized in table 1.[5] For a system of four goods, we consider log prices, log normalized prices, total expenditure, budget shares, and some cross terms for a total of 35 variables over the period

[5] The *DFGLS* and the *MZGLS* tests estimate the trend parameters more efficiently and are more powerful than the Dickey-Fuller (DF) test and the modified Phillips-Perron *MZ* tests of Perron and Ng (1996). The *MZ* tests have better size properties than the Phillips-Perron *Z* tests. Results are reported for MZ_α and $MZGLS_\alpha$, which are improved versions of Z_α.

TABLE 1.—TESTS FOR NONSTATIONARITY OF PRICES AND TOTAL EXPENDITURE

Series	Levels			First Differences		
	$DFGLS_\tau$	$MZGLS_{\alpha\tau}$	Lags	$DFGLS_\mu$	$MZGLS_{\alpha\mu}$	Lags
$\log p_1$	−1.541	−5.506	3	−3.153	−17.191	2
$\log p_2$	−1.268	−4.901	3	−3.813	−25.618	2
$\log p_3$	−1.107	−5.574	4	−2.613	−11.675	4
$\log p_4$	−1.442	−4.650	3	−2.044	−8.031	2
$\log\tau_1$	−2.384	−11.371	1	−2.509	−11.856	3
$\log\tau_2$	−1.842	−9.818	4	−4.525	−32.102	2
$\log\tau_3$	−0.975	−2.408	2	−1.568	−5.173	4
$\log\tau_4$	−0.924	−2.141	2	−4.580	−33.948	2
$\log(p_1/p_2)$	−1.624	−7.065	4	−5.813	−56.045	2
$\log(p_1/p_3)$	−1.370	−4.183	2	−4.620	−36.045	3
$\log(p_1/p_4)$	−1.200	−3.713	3	−3.944	−25.392	3
$\log(p_2/p_3)$	−1.182	−2.903	1	−3.757	−24.921	4
$\log(p_2/p_4)$	−0.732	−2.091	1	−6.030	−62.858	4
$\log(p_3/p_4)$	−0.773	−2.028	4	−3.594	−17.646	4
$\log M$	−1.284	−3.937	3	−1.958	−7.532	4
W_1	−1.707	−5.887	1	−4.321	−30.873	4
W_2	−0.953	−2.237	0	−6.644	−68.654	2
W_3	−1.701	−6.441	0	−5.944	−58.423	3
W_4	0.058	0.2149	0	−2.551	−8.446	4
$w_1\log p_1$	−0.756	−5.204	3	−2.060	−8.524	2
$w_1\log p_2$	−0.664	−3.189	3	−3.647	−26.940	4
$w_1\log p_3$	0.179	−.085	2	−2.339	−10.237	4
$w_1\log p_4$	−0.878	−5.478	3	−3.726	−21.141	2
$w_2\log p_1$	−0.858	−1.662	1	−4.522	−33.241	3
$w_2\log p_2$	−1.168	−4.446	3	−2.804	−12.830	3
$w_2\log p_3$	−0.994	−4.832	4	−3.158	−12.997	4
$w_2\log p_4$	−0.739	−1.057	1	−2.258	−7.651	4
$w_3\log p_1$	−0.458	−1.236	1	−4.564	−28.905	3
$w_3\log p_2$	−0.7751	−2.652	2	−4.172	−29.847	2
$w_3\log p_3$	−0.169	−.813	2	−4.752	−35.906	3
$w_3\log p_4$	−0.880	−2.968	2	−4.012	−19.959	3
$w_4\log p_1$	−0.946	−2.344	3	−2.441	−9.365	3
$w_4\log p_2$	−1.042	−2.278	3	−2.927	−15.130	4
$w_4\log p_3$	−1.046	−3.110	3	−2.743	−10.189	4
$w_4\log p_4$	−1.175	−3.110	4	−0.947	−1.518	4

The 5% critical values for $DFGLS_T$ and $MZGLS_{\alpha\tau}$ (which include a constant and a linear time trend) are −2.9 and −19.1, respectively. The critical values for $DFGLS_\mu$ and $MZGLS_{\alpha\mu}$ (which include a constant) are −1.9 and −8.1, respectively.

1954–1998. Neither test can reject the null hypothesis of a unit root around a linear trend.[6] However, when the first difference of each series is tested for a unit root, the tests reject nonstationarity in 33 of the 35 series being tested. Prices r_t are faced by all consumers, so estimates of demand equations at any level of aggregation or disaggregation will need to deal with nonstationarity of prices.

Though our later estimates will permit budget shares to be stationary, we assume for now that budget shares and logged scale prices are I (1), as indicated by the tests in table 1. In that case, demand equations linear in N,

$$W_{it} = a_{0i} + a_{li}t + b_i'r_t + e_{it}, \tag{1}$$

could be consistently estimated by standard least squares methods if the errors e_{it} in equation (1) were stationary. Stationary errors for these equations would require that W_{it} and r_t be cointegrated for each commodity group i. To test for cointegration, we include a deterministic time trend in equation (1), for linear trends are found to be significant in, for example, Banks, Blundell, and Lewbel (1997). Tests for the null hypothesis of no cointegration in table 2 indicate that these errors are not stationary (and remain nonstationary even when a quadratic trend is included as a regressor). We use two variants of the residuals-based cointegration test developed in Phillips and Ouliaris (1991). The 5% critical value with four regressors and a linear trend is −4.49 for the Dickey-Fuller (DF) test, and −37.7 for the modified Phillips-Perron test (MZ_α). For three of the four consumption groups, the evidence of no cointegration is overwhelming. In the case of clothing, the DF test is −4.885 and rejects a unit root in e_{it}, but the MZ_α test is −31.804 and does not reject the null hypothesis of no cointegration.

We might not expect cointegration in equation (1), because utility maximization, and in particular Slutsky sym-

[6] The lag lengths of the augmented autoregressions are selected by the MAIC developed in Ng and Perron (2001). The BIC (not reported) leads to the same conclusion that the levels of the series are not stationary.

TABLE 2.—TESTS FOR THE NULL HYPOTHESIS OF NO COINTEGRATION

Good	DF	MZ_α	Lags
Equation (1): $W_{it} = a_{0i} + a_{1i}t + \sum_{j=1}^{N} b_{ij}r_{jt} + e_{it}$			
1	−2.835	−11.512	0
2	−2.446	−13.545	1
3	−4.885	−31.804	0
4	−2.665	−10.673	0
CV	−4.49	−37.7	
Equation (2): $W_{it} = a_{0i} + a_{1i}t + b_i'r_t + c_i\log(M_t/P_t^*) + e_{it}$			
1	−3.139	−18.098	0
2	−3.226	−20.042	1
3	−5.210	−30.300	0
4	−3.732	−14.835	0
CV	−4.74	−42.5	
Equation (4): $W_{it} = a_{0i} + a_{1i}t + b_i'r_t + [c_{1i}\log(M_t/P_t^*) + c_{2i}\log(M_t/P_t^*)]^2 + e_{it}$			
1	−3.568	−22.125	0
2	−2.608	−13.831	1
3	−5.426	−43.275	0
4	−2.242	−9.785	0
CV	−5.04	−47.5	
Equation (14): $W_{it} = a_{0i}t + a_{1i}t + \sum_{j=1}^{N} b_{ij}r_{jt} + \sum_{j=1}^{N} c_j z_{ijt} + e_{it}$			
1	−4.356	−28.923	0
2	−4.430	−35.425	1
3	−6.029	−50.875	0
4	−4.344	−32.421	0
CV	−6.5	−67.5	

metry, would impose implausibly strong cross-equation restrictions on the coefficients (see section IV below). A much more reasonable class of demand equations is

$$W_{it} = a_{0i} + a_{1i}t + b_i'r_t + c_i g(x_t) + e_{it}, \tag{2}$$

where $g(x_t)$ is some function that is common to all of the demand equations, and x_t is a vector of observed or unobserved variables (which could include t and r_t) that affect demand. In particular, one of the most frequently employed demand systems in empirical work, Deaton and Muellbauer's (1980) *almost ideal demand system* (AIDS), is a special case of equation (2) in which g is a constrained quadratic in t and r_t. We tested for cointegration in the approximate AIDS model, which uses Stone's price index (P^*) to deflate total expenditure, as is common practice in this literature (see, for example, Deaton & Muellbauer, 1980). This amounts to using $g(x_t) = -\sum_{j=1}^{N} w_{jt}r_{jt}$ in equation (2). The results are given in the second panel of table 2. The critical values for the two tests with five regressors are −4.74 and −42.5 respectively. Once again, there is strong evidence for noncointegration in three of the four cases, with clothing being the possible exception.

Having equation (2) hold for every group i implies that for $i \neq 1$,

$$W_{it} = \tilde{a}_i + \tilde{a}_{1i}t + \tilde{b}_i'r_t + \tilde{c}_i W_{1t} + \tilde{e}_{it}, \tag{3}$$

where $\tilde{e}_{it} = e_{it} - c_i e_{1t}/c_1$, and the other tilde parameters are similarly defined. Therefore, if the errors e_{it} in equation (2) were stationary, then the errors $\tilde{e}_{it}$ in equation (3) would also be stationary. This means that a necessary condition for the AIDS model, or for any other demand equation in the form of equation (2), to have well-behaved (that is, stationary) errors is that W_{it}, r_t, and W_{jt} must be cointegrated for each group $i \neq j$. However, the test statistics in table 3 indicate that W_{it}, r_t, and W_{jt} (for any $j \neq i$) are not cointegrated, and hence any model in the form of equation (2), including the exact AIDS model, will yield inconsistent parameter estimates.

More generally, the test results based on equation (3) show that failure of cointegration is not due to *any* single missing variable or regressor. This is because utility maximization would require that any omitted variable appear in the demand equations for all goods i. Cointegration of equation (2) with any variable or function $g(x_t)$ would imply cointegration of equation (3), which is rejected.

An even more general class of demand systems is

$$W_{it} = a_{0i} + a_{1i}t + b_i'r_t + c_{1i}g(x_t) + c_{2i}g_2(x_t) + e_{it} \tag{4}$$

for arbitrary functions $g(x_t)$ and $g_2(x_t)$. Examples are the approximate quadratic AIDS (QUAIDS) model of Blundell, Pashardes, and Weber (1993) and the exact, integrable QUAIDS model of Banks, Blundell, and Lewbel (1997). The third panel of table 2 reports cointegration tests for equation (4), taking g to be log deflated income, and $g_2 = g^2$, corresponding to the approximate QUAIDS model.[7] In all cases, MZ_a is less than the approximate critical value of −47.5. The approximate critical value for the DF is −5.05. Again, clothing is the only good for which there is some support for cointegration.

Similar to equation (3), having equation (4) hold for every group i implies that for $i \neq 1$ and 2,

$$W_{it} = \tilde{a}_i + \tilde{a}_{1i}t + \tilde{b}_i'r_t + \tilde{c}_{1i}W_{1t} + \tilde{c}_{2i}W_{2t} + \tilde{e}_{it}, \tag{5}$$

where $\tilde{e}_{it}$ is linear in e_{it}, e_{1t}, and e_{2t}. Therefore, if the errors e_{it} in equation (4) were stationary, then the errors $\tilde{e}_{it}$ in equation (5) would also be stationary, so a necessary condition for any demand equation in the form of equation (5) to have stationary errors is that W_{it}, r_t, W_{jt}, and W_{kt} are

[7] Exact critical values have not been tabulated for systems of such high dimensions. Ng (1993) finds that an approximate guide is to raise the critical value of the DF by 0.35, and of MZ_α by 5, for each added regressor.

TABLE 3.—TESTS FOR THE NULL HYPOTHESIS OF NO COINTEGRATION

Equation (3): $W_{it} = \tilde{a}_{0i} + \tilde{a}_{1i} + \tilde{b}_i' r_t + \tilde{c}_i W_{jt} + \tilde{e}_{it}, j \neq i$				
Good i	Good j	DF	$M Z_\alpha$	Lags
2	1	−2.450	−13.525	1
3	1	−4.883	−32.419	0
4	1	−2.317	−9.802	0
1	2	−2.881	−11.895	0
3	2	−5.456	−37.449	0
4	2	−4.180	−20.909	0
1	3	−2.825	−12.175	0
2	3	−3.012	−17.663	1
4	3	−3.524	−23.658	0
1	4	−2.487	−10.667	0
2	4	−3.724	−21.577	1
3	4	−5.479	−42.895	0
CV		−4.74	−42.5	
Equation (5): $W_{it} = \tilde{a}_{0i} + \tilde{a}_{1i} + \tilde{b}_i' r_t + \tilde{c}_{ij} W_{jt} + \tilde{c}_{ik} W_{kt} + \tilde{e}_{it}, j, k \neq i$				
Good i	Goods j, k	*DFGLS*	MZGLS	Lags
1	2,3	−2.851	−12.200	0
	2,4	−2.411	−10.487	0
	3,4	−2.127	−7.509	1
2	1,3	−2.997	−17.252	1
	1,4	−3.722	−22.032	1
	3,4	−3.558	−22.159	1
3	1,4	−5.404	−41.440	0
	1,2	−5.438	−37.798	0
	2,4	−5.050	−38.951	0
4	1,2	−3.825	−19.744	0
	1,3	−2.889	−18.44	1
	2,3	−3.660	−21.983	0
CV		−5.04	−47.5	

cointegrated for each group $i \neq j, k$. As with all the other models tested, the test statistics in the second panel of table 3 indicate that W_{it}, r_t, W_{jt} and W_{kt} (for any ordering of the goods) are not cointegrated.

Analogously to the discussion regarding equation (3), failure of cointegration of equation (5) implies that nonstationarity of the demand system errors could not be due to any two missing regressors. Other evidence of nonstationarity is provided by Ng (1995), Lewbel (1996a), and Attfield (1997). We will later give one more example of demands that are linear in variables, based on the translog system, and show that it too appears to have nonstationary errors.

The test statistics used in this section are based on asymptotic theory assuming that T is extremely large, and also that T is large relative to the number of regressors, which is not the case here. The small-sample distortions in some of these tests could therefore be substantial. Nevertheless, the evidence of trends or drifts in relative prices, aggregate total expenditures, and aggregate demand system errors seems strong, even if exact p-values for many of these tests might be in doubt.

III. Aggregation and Nonstationary Errors

In this section, we propose one possible explanation for the empirically observed high autocorrelation and possible nonstationarity of aggregate demand system errors. We show that this persistence could be caused by aggregation across a slowly changing population of consumers with heterogeneous preferences. Our NTLOG model does not depend on the validity of this explanation, and in fact can be applied to deal with nonstationary prices even if the demand system errors are stationary, but it is useful to understand why demand errors could be persistent.

Blundell, Pashardes, and Weber (1993) suggest that aggregation over consumers with time-varying individual-specific effects can lead to omitted variations in the aggregate demand system. Here, we show that even if the individuals have specific effects that are time-invariant, aggregating over an evolving population with heterogeneous preferences will induce omitted variations (that is, aggregate errors). Moreover, because the population evolves slowly over time, these omitted effects are likely to be highly persistent.

To see how aggregation across consumers could cause persistence in aggregate demand system errors, let a_{hi} be a fixed effect of consumer h for good i. This fixed effect can be interpreted as a taste parameter, that is, a parameter in consumer h's utility function. Let $\mathcal{H}_t$ be the set of all consumers in the economy in time t, and $H_t = \sum_{h \in \mathcal{H}_t} 1$ be the enumeration of $\mathcal{H}_t$. Note that $\mathcal{H}_t = \mathcal{H}_{t-1} + \mathcal{H}_t^+ - \mathcal{H}_{t-1}^-$, where $\mathcal{H}_t^+$ is the set of consumers who enter the economy in period t, and $\mathcal{H}_{t-1}^-$ is the set of consumers that leave the economy in period $t - 1$. Then $a_{it} = (1/H_t) \sum_{h \in \mathcal{H}_t} a_{hi}$ is the simple average of a_{hi} across the consumers. We can write

$$a_{it} = \frac{H_{t-1}}{H_t} a_{i\,t-1} + \frac{\sum_{h \in \mathcal{H}_t^+} a_{hi} - \sum_{h \in \mathcal{H}_{t-1}^-} a_{hi}}{H_t}$$
$$= \nu_t a_{i\,t-1} + \eta_{it},$$

where ν_t is the relative size of the population between the two periods, and η_{it} is the average difference between the preferences of the consumers that dropped out and those that were added in time t. The dynamic properties of a_{it} thus depend on ν_t and η_{it}. Consider first the latter. Taste parameters a_{hi} depend in part on age, family size, and other demographic characteristics. All these variables change slowly over time. Also, to the extent that taste parameters vary across households and cohorts, the average taste parameter of those who drop out will generally differ from the average taste parameter of those who enter the sample in any given period. Both considerations suggest that η_{it} should exhibit random variations.

Now ν_t depends on the number of consumers in two consecutive periods and does not depend on i. Because the set of consumers in an economy changes slowly over time, the large majority of consumers in $\mathcal{H}_t$ are also in $\mathcal{H}_{t-1}$. If η_{it}

is uncorrelated with $a_{i,t-1}$, then this implies that a_{it} is a highly persistent, near-unit-root process. More generally, η_{it} can be correlated with $a_{i,t-1}$, which could increase or decrease the persistence in a_{it}. In postwar quarterly data, ν_t ranged from a low of 0.9921 to a high of 0.9982 with a standard deviation of 0.0009, so empirically ν_t is very close (but not exactly equal) to 1.

Whether a_{it} is a near-unit-root process or not depends on both the evolution of the population $\mathcal{H}_{t-1}$ and the distribution of the demand system errors a_{hi} in each time period. These are not directly observed, but we will later provide empirical evidence that substantial persistence in a_{it} is at least plausible, based on an analysis of food demand at the household level using PSID data.

The above argument for persistence in the average fixed effect assumes that each household receives the same weight of $1/H_t$, but the argument also holds when a_{it} is defined as an unequally weighted average. Let ω_{ht} be the weight applied to household h at time t. Then for $a_{it} = \sum_{h\in\mathcal{H}_t}\omega_{iht}a_{iht}$, it can be shown that

$$a_{it} = \nu_t a_{i\,t-1} + \eta_{it} + \left[\sum_{h\in\mathcal{H}_t}\left(\omega_{ht} - \frac{1}{H_t}\right)a_{hi}\right] - \nu_t\left[\sum_{h\in\mathcal{H}_{t-1}}\left(\omega_{h\,t-1} - \frac{1}{H_{t-1}}\right)a_{hi}\right].$$

In addition to heterogeneous preferences, time variations in weights (the last two terms) will also introduce randomness into a_{it}. If the weights ω_{ht} are budget shares, then the changes in the income distribution between periods will be the additional source of randomness. In consequence, one would still expect a_{it} to be an autoregressive process with a root very close to unity.

More generally, a fixed effect can be the sum of an aggregate component which is unaffected by aggregation over households (for example, common trends in tastes) and a household-specific component. Then the aggregate fixed effect, α_{it}, is

$$\alpha_{it} = a_{i0} + a_{i1}t + a_{it}. \tag{6}$$

The implications of a slowly increasing but heterogeneous population for the aggregate fixed effect are threefold. First, a model which approximates α_{it} by a deterministic trend function $a_{i0} + a_{i1}t$ will have omitted the random variations a_{it}. Second, given the size of ν_t in the data, the aggregate fixed effect is likely to be well approximated by a random walk with drift. The magnitude of ν_t also implies that even if we were to observe α_{it}, unit root tests would have very low power in rejecting the null hypothesis of nonstationarity. Third, when demand system errors have autoregressive roots so close to the unit circle, the distribution of the estimated parameters will not be well approximated by the normal distribution even asymptotically, and hence standard inference will be inaccurate (this is in addition to the problems stemming from nonstationary prices). Persistence arising from time aggregation of fixed effects is consistent with the empirical evidence of nonstationarity and non-cointegration documented in the previous section, and with the high degree of serial correlation found in the errors of estimated demand systems cited in the introduction. We will later present evidence from the PSID to show that persistence can indeed arise from aggregation.

IV. A Linear Form for Translog Demands

In the time series literature, nonstationarity is readily handled in the context of linear models. The difficulty for demand systems is that in linear models the Slutsky symmetry implied by utility maximization results in extremely restrictive and implausible constraints on cross-price elasticities. Linear models are also resoundingly rejected empirically.

To illustrate the problem, suppose the demands of an individual household were given by the general linear model $\omega_{it} = a_i + \sum_{j=1}^{N} b_{ij}\ln p_{jt} + c_i \ln m_t$ for goods $i = 1, \ldots, N$, where m_t is the consumer's total expenditures on goods and services in time t, and w_{it} is the fraction of m_t spent on good i in time t. To be consistent with utility maximization, this demand model must satisfy homogeneity and Slutsky symmetry. Homogeneity requires $c_i = -\sum_{j=1}^{N} b_{ij}$ for $i = 1, \ldots, N$, which is not overly restrictive. However, it can be directly verified that Slutsky symmetry requires either that $c_i = 0$ for all goods i, implying homothetic demands (budget shares independent of the total expenditure level), or that $a_i = 0$ and $b_{ij} = \beta_i\beta_j$ for some scalars $\beta_1, \ldots, \beta_N$, so that all cross price elasticities are forced to be proportional to own price elasticities. Virtually all empirical demand studies reject these restrictions. Of course, results of empirical tests of symmetry and homogeneity will depend in part on the precision with which the associated parameters are estimated. Estimates of Slutsky matrix terms are often very imprecise.

Similar restrictions arise in linear models expressed in terms of quantities rather than budget shares, as observed by Deaton (1975), who raised these objections in the context of the Stone-Geary linear expenditure system. Phlips (1974) describes similar restrictions regarding the Rotterdam model (see Barten, 1967, and Theil, 1971), which is a linear demand system based on time differencing of quantities and prices. Deaton and Muellbauer (1980) provide further discussion of these points (they attribute the Rotterdam model objection to unpublished results by McFadden).

To see how we construct a model that is linear in variables while overcoming these constraints, consider the translog indirect utility function of Christensen, Jorgenson, and Lau (1975),

$$U(p_t, m_t) = \sum_{i=1}^{N}\left(\alpha_i + \frac{1}{2}\sum_{j=1}^{N} b_{ij}\ln\frac{p_{jt}}{m_t}\right)\ln\frac{p_{it}}{m_t}. \tag{7}$$

The function U here is the indirect utility function for the household. Without loss of generality, assume $\sum_{i=1}^{N} \alpha_i = 1$ and $b_{ij} = b_{ji}$. Define $c_i = \sum_{j=1}^{N} b_{ij}$ with $\sum_{i=1}^{N} c_i = 0$ to make the translog exactly aggregable; see Muellbauer (1975), Jorgenson, Lau, and Stoker (1982), and Lewbel (1987). By Roy's identity, the resulting translog budget shares are

$$w_{it} = \frac{\alpha_i + \sum_{j=1}^{N} b_{ij} \ln p_{jt} - c_i \ln m_t}{1 + \sum_{j=1}^{N} c_j \ln p_{jt}} \tag{8}$$

Unlike the severe restrictions on elasticities implied by linear models, equation (8) satisfies Slutsky symmetry and homogeneity without constraints on own price, cross price, or total expenditure elasticities at a point. This feature of unrestricted elasticities at a point is known as Diewert (1974) flexibility, and was one of the motivations for the derivation of both the popular translog and the almost ideal demand model. Diewert and Wales (1987) show that imposing negative definiteness on the translog does limit its flexibility at some points (see also Moschini, 1999), but the resulting constraints on elasticities are minimal compared to the above-described constraints required of linear models.

Now observe that equation (8) can be rewritten as

$$w_{it} = \alpha_i + \sum_{j=1}^{N} b_{ij} \ln (p_{jt}/m_t) - \sum_{j=1}^{N} c_j w_{it} \ln p_{jt}. \tag{9}$$

Equation (9) is a model for a single household, but can be readily extended to a panel of households by adding appropriate household subscripts h. The relevant point for estimation is that equation (9) is linear in the variables $\ln(p_{jt}/m_t)$ and $w_{it} \ln p_{jt}$ for $j = 1, \dots, N$. Hence, if some or all of these variables (in particular, log prices) are nonstationary, the model is at least in principle amenable to estimation using time series methods, which we will make precise in section VI. Furthermore, α_i could be random, implying that if we were to estimate equation (9) in the cross section, the errors could be interpreted as random utility function parameters. The next section provides details for our particular estimation method in the context of an aggregate version of this model.

V. The Nonstationary Translog Demand System

A convenient implication of the linearity of equation (9) is that it facilitates aggregation across households (for estimation with household-level data, the aggregation step below can be ignored). Let m_{ht} be consumer (or household) h's total expenditures on goods and services in time t, w_{hit} be the fraction of m_{ht} spent on goods i in time t, and $r_{hit} = \ln (p_{it}/m_{ht})$. Also, for each good i let α_{hi} denote the value of the parameter α_i for household h, so the vector of utility function parameters $(\alpha_{h1}, \dots, \alpha_{hN})$ embody preference heterogeneity. The household level translog budget shares from equation (8) are

$$w_{hit} = \frac{\alpha_{hi} + \sum_{j=1}^{N} b_{ij} \ln p_{jt} - c_i \ln m_{ht}}{1 + \sum_{j=1}^{N} c_j \ln p_{jt}} \tag{10}$$

Let $M_t = 1/H_t \sum_{h \in \mathcal{H}_t} m_{ht}$, $W_{it} = \sum_{h \in \mathcal{H}_t} w_{hit} m_{ht} / \sum_{h \in \mathcal{H}_t} m_{ht}$, and $\delta_t = \sum_{h \in \mathcal{H}_t} m_{ht} \ln m_{ht} \sum_{h \in \mathcal{H}_t} - \ln M_t$. Then

$$\tilde{a}_{it} = \frac{\sum_{h \in \mathcal{H}_t} \alpha_{hi} m_{ht}}{\sum_{h \in \mathcal{H}_t} m_{ht}} - c_i \delta_t, \equiv \alpha_{it} - c_i \delta_t. \tag{11}$$

Notice that α_{it} is the average fixed effect for good i using expenditure shares as weights. It then follows that the aggregate budget shares are given by

$$W_{it} = \frac{\tilde{a}_{it} + \sum_{j=1}^{N} b_{ij} \ln p_{jt} - c_i \ln M_t}{1 + \sum_{j=1}^{N} c_j \ln p_{jt}},$$

$$\equiv \frac{\tilde{a}_{it} + \sum_{j=1}^{N} b_{ij} r_{jt}}{1 + \sum_{j=1}^{N} c_j \ln p_{jt}}, \tag{12}$$

because $r_{it} = \ln (p_{it}/M_t)$. Models like this aggregate translog would usually be estimated as in Jorgenson, Lau, and Stoker (1982), that is, by replacing $\tilde{a}_{it}$ with a linear combination of trend or demographic variables, and appending an additive error to equation (12).

We propose to estimate the aggregate analog of equation (9) instead. Define

$$z_{ijt} = W_{it} \ln p_{jt}, \tag{13}$$

and let $e_{it} = a_{it} - c_i \delta_t$. Substituting equations (6) and (13) into (12) then gives

$$W_{it} = a_{i0} + a_{i1} t + \sum_{j=1}^{N} b_{ij} r_{jt} - \sum_{j=1}^{N} c_j z_{ijt} + e_{it}. \tag{14}$$

Separate from any considerations of nonstationarity or aggregation, one advantage of defining the model this way is that the errors e_{it} are by definition equal to $a_{it} - c_i \delta_t$ and so can be directly interpreted as preference heterogeneity (taste) parameters, as in McElroy (1987) and Brown and Walker (1989). More importantly, equation (14) is linear in r_t and z_{it}. Nonstationarity in the variables and the errors can now be dealt with, as described in the next section.

We call the system of equations (14) for all goods i the *nonstationary translog demand system* (NTLOG), for it is based on demands derived from translog utility functions, and some or all of its component variables may be nonstationary. If the system (14) were cointegrated for every good i, then the demand equations could be estimated using an error correction model. This would require that e_{it} be stationary. Lewbel (1991) found, using both U.K. and U.S. data, that δ_t varies very little over time, with little or no trend or drift. Thus e_{it} is stationary if a_{it} is stationary. However, the analysis in section III suggests that e_{it} is likely to be nonstationary (or nearly so), because there are likely random variations in preferences and in the income distribution over time. Therefore, tests of equation (14) should find no cointegration. In addition, even in our small system with few goods, 10 regressors in each of the equations now have to be tested for cointegration, and the power of cointegration tests is known to decrease with increasing number of regressors. Based on approximate critical values of -6.5 and -67.5, the fourth panel of table 2 shows that the variables in the system (14) do not appear to be cointegrated.

Thus, both theory and empirical tests are consistent with e_{it} being an integrated or nearly integrated process. In the time series literature, it is recognized that imposing a unit root on nearly integrated processes can be desirable when the limiting distributions of estimators and test statistics are not well approximated by the normal distribution. In the present context, the unit root restriction can be justified given both the test results and the magnitude of v_t. Consistent with unit roots, the first differences of W_{it}, r_{jt}, and z_{ijt} for all goods i and j all appear to be I(0), and thus standard tools for inference can be applied.

The above NTLOG model is designed for estimation with aggregate data, but it or some similar variant of equation (9) could be applied to cohort- or household-level data. At disaggregate levels, errors and income may well be stationary. But relative prices, which are faced by all households, will still be nonstationary, so the NTLOG will be useful with disaggregate data also.[8]

A key feature of equation (13) is that it is linear in variables. These variables include z_{ijt}, which is the product of the nonstationary log prices, and budget shares. Although the budget shares appear nonstationary in the data, they are bounded between 0 and 1. The nonstationarity in the cross-product term can thus be expected to be weaker. Evidently, the variables z_{ijt} all appear stationary when differenced.[9] In contrast, Deaton and Muellbauer's (1980) almost ideal demand model was designed to be nearly linear, but misses that ideal because of the presence of a quadratic price deflator, which includes terms like r_{jt}^2. First differences of r_{jt}^2 terms are not close to stationary. Thus, even in first-differenced form, the correct limiting distribution for the AIDS model may still be nonstandard. Recognizing problems of high autocorrelation in levels, Deaton and Muellbauer reported estimates from differencing the AIDS model, but assumed a standard limiting distribution for the result.

It is also of some interest to compare the nonstationary translog with the Rotterdam model (see, for example, Barten, 1967, and Theil, 1971). The Rotterdam model consisted of regressing differenced quantities on differenced prices and income. The Rotterdam model has the virtue of making the regressors stationary. Its shortcoming is that it is not consistent with utility maximization without imposing extreme restrictions on its coefficients, as described in the previous section. Unlike the Rotterdam model, the NTLOG is derived from a utility function that has flexible demands. Furthermore, the error terms of the Rotterdam model, like the errors in the ordinary aggregate Translog and AIDS models, are appended to demands with no economic interpretation. In contrast, the error terms of the NTLOG are directly derived from heterogeneity in taste parameters and variations in the income distribution.

VI. Estimation and Results

Equation (14) manages full linearity, but at the cost of having some of the regressors (the z_{ijt}) depend on W_{it}, and hence those regressors could be correlated with the errors e_{it}. This issue must be dealt with upon estimation. Assume we have a vector of stationary instrumental variables s_t that are uncorrelated with the stationary difference $\Delta e_{it} = e_{it} - e_{it-1}$. Then

$$E[s_t(\Delta W_{it} - a_{i1} - \sum_{j=1}^{N} b_{ij}\,\Delta r_{jt} + \sum_{j=1}^{N} c_j\,\Delta z_{ijt})] = 0. \qquad (15)$$

[8] A limitation on using NTLOG for disaggregate data is that the translog is a rank-two demand system, with budget shares linear in log income, whereas empirical evidence on household-level data suggests demands are quadratic and of rank three. See Howe, Pollak, and Wales (1979), Gorman (1981), Lewbel (1991), Blundell, Pashardes, and Weber (1993), and Banks, Blundell, and Lewbel (1997). Although demands for individual households appear to be of rank three, there is evidence that aggregate demands may be adequately modeled as rank two. Lewbel (1991) shows that rank-three curvature arises primarily from households at the extremes of the income distribution, and that excluding a small percentage of households in these tails results in demands that are empirically of rank two. If the contribution of these few extreme households to the aggregate is small, then the aggregate will appear to be of rank two. Also, the range of observed aggregate (per capita) income is small relative to the range of incomes that exists across households. The effect of these rank-three households in the aggregate is therefore small. In our empirical application later we find that the rank-two NTLOG is satisfactory for aggregate data. Nonetheless, rank-three extensions of the NTLOG could be constructed, and might be desirable for future applications using disaggregate data.

[9] Ogaki and Reinhart (1998) encountered a similar problem and also argued that first-differencing is likely to make nonstationarity in a ratio term empirically unimportant.

The set of equations (15) for all goods i can be stacked to yield a collection of moment conditions for the parameters, which can be estimated using the standard GMM. The instruments and the differenced variables in the equations (15) are all stationary, so the coefficients in this GMM will have the standard root-T normal limiting distribution. Because these are demand equations, and the errors arise from preference heterogeneity, suitable instruments will be variables that affect the supply side of the economy.

To check sensitivity to the choice of instruments, we consider two sets of instruments. The first simply uses differences in the lags of the variables in the system: $\Delta W_{i\,t-2}$, $i = 1, \ldots, N-1$; $\Delta \ln p_{i\,t-2}$, $i = 1, \ldots, N$; $\Delta \ln M_{t-2}$, $\Delta z_{i\,j\,t-2}$, $j = 1, 2$; the lag of the differenced log population; a constant; and a time trend. These instruments deal with the dependence of z on endogenous budget shares, but fail to control for classical simultaneity of demand with supply

The second set of instruments, which should be suitable for both these problems, consist of supply variables, like those used, for example, by Jorgenson, Lau, and Stoker (1982). These instruments are the deflator for civilian compensation of government employees, government purchases and its deflator, imports of goods and services, wages and salaries, unit labor costs and participation rate, government transfers to individuals, unemployment, and population. These are also differenced to stationarity. Also included are a constant and a time trend. The first set of instruments has 13 variables and the second has 14, yielding a total of 39 and 42 moment conditions, respectively. It is not feasible to use both sets of instruments simultaneously, because doing so will result in too many moment conditions relative to the sample size.

A nonstandard feature of our application of GMM is the following. The adding-up constraint means that the condition $\sum_{i=1}^{N} e_{it} = 0$ must be satisfied. This imposes strong cross-equation restrictions on the dynamic structure of the errors if the Δe_{it} terms are serially correlated. See, for example, Berndt and Savin (1975) and Moschini and Moro (1994). We first estimate the parameters with the White-Huber correction for heteroskedasticity, and then test for serial correlation in the residuals. First differencing appears to be sufficient to render $\hat{e}_{it}$ approximately white noise, and the Box-Ljung statistic with six lags cannot reject the null hypothesis of no serial correlation at the 5% level for equations (1) and (2), or at the 10% level for equation (3). We also tried quasi-differencing the first-differenced data to estimate a common AR(1) parameter for the differenced residuals, corresponding to Berndt and Savin's (1975) error specification after differencing. The autocorrelation parameter estimate is numerically small and insignificant, so those results are not reported.[10]

[10] If we had seen stronger evidence of serial correlation, then a more flexible treatment of autocorrelation could have been used, as in Moschini and Moro (1994).

TABLE 4.—RESTRICTED AND UNRESTRICTED ESTIMATES OF THE PARAMETERS BY GMM

	INST1		INST2	
	Unrestricted	Restricted	Unrestricted	Restricted
b_{11}	0.1186	0.0838	0.1665	0.0988
S.e.	0.1077	0.0886	0.1086	0.0883
b_{12}	0.0020	—	−0.0062	—
S.e.	0.0563	—	0.0482	—
b_{13}	0.0484	—	−0.0602	—
S.e.	0.0403	—	0.0483	—
b_{22}	0.1661	0.1206	0.1408	0.1048
S.e.	0.0487	0.0410	0.0509	0.0390
b_{23}	−0.0373	—	−0.0438	—
S.e.	0.0287	—	0.0310	—
b_{24}	−0.1672	−0.1835	−0.1474	−0.1768
S.e.	0.0623	0.0532	0.0621	0.0466
b_{33}	−0.0617	−0.0715	−0.1121	−0.1387
S.e.	0.0361	0.0392	0.0407	0.0415
b_{34}	−0.0191	—	−0.0181	—
S.e.	0.0427	—	0.0533	—
b_{44}	0.4851	0.3153	0.5064	0.4075
S.e.	0.1214	0.1230	0.1475	0.1350
a_{11}	−0.0009	−0.0008	−0.0008	−0.0007
S.e.	0.0003	0.0002	0.0003	0.0002
a_{12}	0.0001	—	−0.0001	—
S.e.	0.0002	—	0.0002	—
a_{13}	−0.0007	−0.0005	−0.0010	−0.0008
S.e.	0.0002	0.0002	0.0002	0.0002
χ^2	25.095	32.360	25.924	30.323
D.f.	27	32	30	35

For a system of N consumption groups, only $N - 1$ equations need to be estimated given the adding-up constraint. After imposing the symmetry condition $b_{ij} = b_{ji}$, the homogeneity condition $ci = \sum_{j=1}^{N} b_{ij}$, and the exact aggregation condition $\sum_{i=1}^{N} c_i = 0$, we still have 12 parameters in a model with four goods. We first obtain unrestricted estimates of all parameters, and then restrict those b_{ij}, $i \neq j$, that are statistically insignificant to 0 to improve precision of the estimates. These results are reported in table 4. Overall, the χ^2 test for overidentifying restrictions cannot reject the orthogonality conditions.

A. Testing the Model

We consider two additional tests of the empirical adequacy of the NTLOG model. The first is a test for stability of the coefficients (that are not statistically different from 0 in the full sample). For both sets of instruments, the sup LM test of Andrews (1993) is maximized at $\pi = 0.2$, where πT is the breakpoint for a sample of size T. The test statistic is 16.38 and 11.43 for the two sets of instruments, respectively, and the 5% critical value for seven parameters is 21.07. Thus, we cannot reject the null hypothesis of parameter constancy.

The second is a general test for any omitted factors, analogous to our earlier use of equation (3) to test for the existence of any function g in equation (2). Suppose the nonstationary translog omits some variable, or some function of variables, g_t, which could be price-related because of flexible regularity, income related due to rank considerations, or some other source of misspecification such as omitted dynamic or demographic effects. Then

$$W_{it} \equiv d_i g_t + a_{i0} + a_{i1}t + \sum_{j=1}^{N} b_{ij} r_{jt} - \sum_{j=1}^{N} c_j z_{ijt} + e_{it}. \quad (16)$$

For example, $d_i g_t$ could be a component of α_{it}, or equation (16) could arise from the aggregation of demands of a potentially rank-three utility function.

Analogously to how equation (2) implies equation (3), we have that if equation (16) holds for any g_t, then

$$W_{it} \equiv d_i W_{kt} + \tilde{a}_{i0} + \tilde{a}_{i1}t + \sum_{j=1}^{N} \tilde{b}_{ij} r_{jt} - \sum_{j=1}^{N} c_j (z_{ijt} - d_i z_{kjt}) + \tilde{e}_{it}. \quad (17)$$

Each equation (17) for $i = 2, \ldots, N - 1$ is linear in the observables, and so can be estimated by differencing and GMM, again using our instruments s_t. We may thereby indirectly test for the existence of any omitted factor g_t by testing whether the coefficients d_i are statistically significant. We may similarly test for two omitted variables by including two different budget shares as regressors in place of just W_{kt} in equation (17), analogously to using equation (5) to test for the structure of equation (4). A total of 27 variations of the model exist, depending on which budget shares are modeled and which are used as regressors. To conserve space, table 5 only reports results for 12 configurations. When good 4 (others) is added to the food equation, the t-statistic is sometimes significant at the two-tailed 5% level, suggesting some (though not overwhelming) evidence of omitted variables. But for both sets of instruments, the t-statistics on other d_i are generally insignificant. The J-test for overidentifying restrictions is reported in the last column of table 5. Compared with the J test in table 4 (25.095 and 25.924), the difference never exceeds 7.814, the critical value from the χ^2 distribution with three degrees of freedom. Thus, we cannot reject that the d_i are jointly 0.

TABLE 5.—SPECIFICATION TESTS

(i,j,k)	t_i	t_j	t_k	χ^2_{24} / χ^2_{27}
(A) With INST1				χ^2_{24}
(2,1,1)	−0.6336	−0.3359	−0.3005	26.0097
(3,1,1)	−0.7389	−0.1753	−0.2321	25.8998
(4,1,1)	−2.2732	−0.4706	0.3154	23.2479
(2,3,4)	−0.3955	−1.9064	−0.3559	26.5953
(3,1,4)	−0.6823	−0.1775	−0.5276	25.8385
(4,1,4)	−2.1087	−0.4602	−0.0459	23.2554
(3,4,1)	−0.6355	−0.7701	−0.0733	26.9111
(3,4,2)	−0.6164	−0.7175	−0.1656	26.8183
(3,4,4)	−0.5854	−0.6251	−0.3532	26.7020
(4,3,1)	−2.2817	−1.7406	0.3635	24.2229
(4,3,2)	−2.2068	−1.7463	0.2535	24.2755
(4,3,4)	−2.1489	−1.8058	0.0476	24.2617
(B) With INST2				χ^2_{27}
(2,1,1)	−0.9181	0.5734	0.8393	21.2570
(3,1,1)	−1.4878	0.6800	0.9548	21.2513
(4,1,1)	−1.9244	0.1801	0.9574	20.2703
(2,3,4)	−1.2099	0.2463	0.3557	22.6511
(3,1,4)	−1.4857	0.6412	0.6393	21.8971
(4,1,4)	−1.8274	0.1964	0.6944	20.2437
(3,4,1)	−1.0192	−2.2839	0.6290	24.8406
(3,4,2)	−1.0342	−2.1522	0.4570	25.0925
(3,4,4)	−0.9126	−2.2949	0.3867	25.3738
(4,3,1)	−2.0192	−0.1576	0.7604	20.9346
(4,3,2)	−2.0149	−0.1076	0.7502	20.9106
(4,3,4)	−1.9207	−0.1403	0.5289	21.1045

The first column shows the variables being added to the equation for goods 1, 2, and 3, respectively. The next three columns are the t-statistic on the variable being added.

TABLE 6.—ESTIMATES OF PRICE AND INCOME ELASTICITIES FOR NTLOG

Good	Price 1	Price 2	Price 3	Price 4	Income
(A) Using INST1					
1 (food)	−0.6808	0.0638	0.0724	−0.2682	0.8128
S.e.	0.3847	0.0734	0.0722	0.2995	0.6090
2 (energy)	−0.0438	−0.1661	0.0724	−1.2644	1.4019
S.e.	0.1424	0.9802	0.0722	1.2932	0.4626
3 (clothing)	−0.0438	0.0638	−1.8580	−0.0924	1.9305
S.e.	0.1424	0.0734	0.8553	0.0901	0.9275
4 (others)	−0.1326	−0.3379	0.0724	−0.4024	0.8004
S.e.	0.1361	0.3798	0.0722	0.3885	0.1946
(B) Using INST2					
1 (food)	−0.6068	0.0739	0.1423	−0.4368	0.8274
S.e.	0.3803	0.0646	0.0987	0.2507	0.4359
2 (energy)	−0.0404	−0.2482	0.1423	−1.3193	1.4655
S.e.	0.1019	0.6554	0.0987	0.9694	0.4069
3 (clothing)	−0.0404	0.0739	−2.6858	−0.1759	2.8282
S.e.	0.1019	0.0646	1.1687	0.1211	1.2674
4 (others)	−0.1722	−0.3180	0.1423	−0.2724	0.6202
S.e.	0.1486	0.2576	0.0987	0.3949	0.2615

B. *Elasticities*

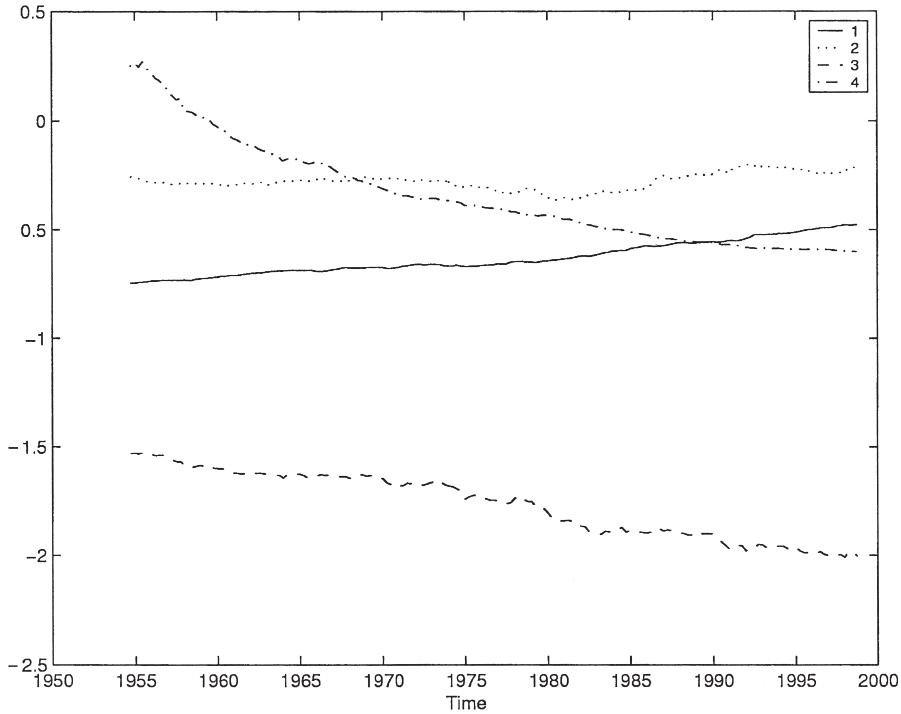
Aggregate quantities are given by $Q_{it} = M_t W_{it}/p_{it}$. One can verify from equation (12) that the corresponding aggregate price and income elasticities are given by

$$\frac{\partial \ln Q_{it}}{\partial \ln p_{jt}} = \frac{(b_{ij}/W_{it}) - c_j}{1 + \sum_{k=1}^{N} c_k \ln p_{kt}} - 1_{ij}, \quad (18)$$

$$\frac{\partial \ln Q_{it}}{\partial \ln M_t} = \frac{- c_i/W_{it}}{1 + \sum_{k=1}^{N} c_k \ln p_{kt}} + 1, \quad (19)$$

where 1_{ij} is the Kronecker delta, which equals 1 if $i = j$, and 0 otherwise. The constants a_{0i} are not identified when differencing as in equation (15). The elasticity formulas given in equations (18) and (19) do not make use of a_{0i}, and

FIGURE 4.—OWN PRICE ELASTICITIES



so are identified. We present estimates of price and income elasticities (evaluated at the mean) in table 6. The standard errors are calculated using the delta method.

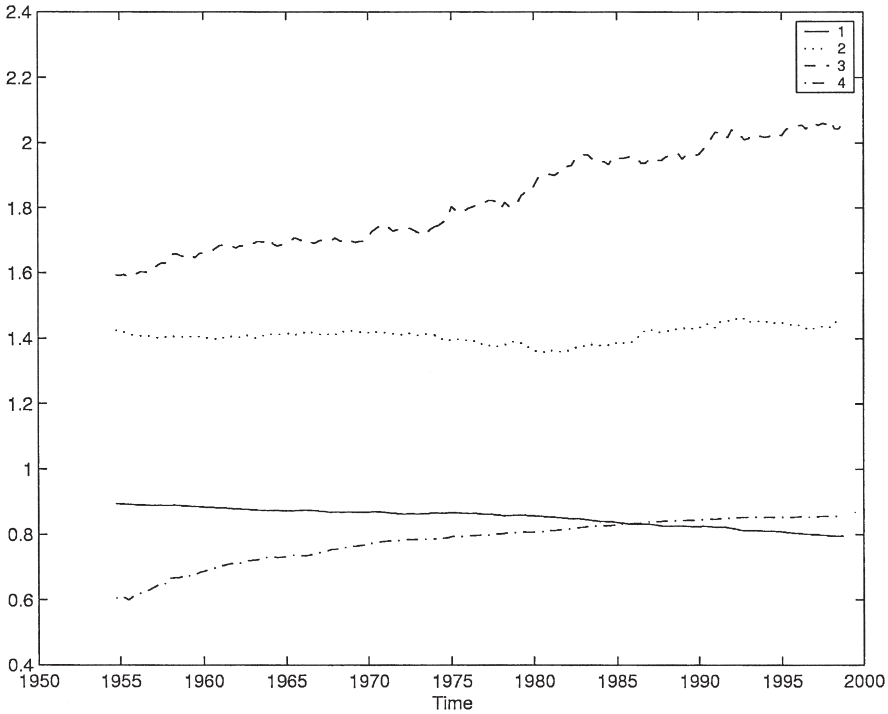
We find that spending on energy and other goods is not price-sensitive. The income elasticities for energy and for clothing are above 1, whereas food and other goods are income-inelastic. Also, according to the NTLOG estimates, a 1% increase in the price of food reduces expenditure on food by 0.68%, and a 1% increase in the price of clothing reduces expenditure on clothing by around 2%. These elasticities are statistically significant and are larger than most others based on time series data in the literature, which are generally estimated over a shorter sample. See, for example, Denton, Mountain, and Spencer (1999) for a survey of estimates. Using the standard translog model, Jorgenson, Lau, and Stoker (1982) found a very large price elasticity for a combined food and clothing group. In results not reported, we find that estimation of the standard translog model with our data set over the same time period yields a positive own price elasticity for food, and income elasticities for food and clothing that are approximately double those based on the NTLOG. One cannot make inference about the statistical significance of the standard translog estimates, because the standard translog model is expressed in terms of nonstationary variables. The asymptotic normality of the NTLOG estimates, on the other hand, allows for standard inference.

The elasticities evaluated at the sample means reported in table 6 have reasonable magnitudes and signs. An interesting implication of nonstationarity of prices is that elasticities may drift over time. This is illustrated in figure 4, with estimates taken from INST2. The price elasticity for energy appears to change little over time and has historically been quite small. The price elasticity for food has fallen somewhat during the course of the past forty years, but the variations around the mean elasticity of −0.6 are rather small. The price elasticity for clothing has increased in recent years. The price elasticity for other goods seems to have increased since the mid-sixties, when these goods became a much larger share of total spending (see figure 1). The time series of income elasticities are presented in figure 5. A notable feature is that not only has clothing become more price-sensitive over time, but its income elasticity has also gone up. These time variations in price and income elasticities may reflect substantial changes in the composition of these categories over time.

C. *Aggregate Fixed-Effect Estimates*

Our empirical analyses provide evidence that aggregate demand system errors are nonstationary. We have suggested that nonstationarity of errors could be due to aggregation of consumers' fixed effects across a slowly evolving population of consumers with heterogeneous preferences. We now provide some empirical evidence to suggest that this explanation is at least plausible. The ideal data for this exercise would be consumer-level information over a long span, but detailed information on consumption by households (such as the CEX) is generally available only in the form of short

FIGURE 5.—INCOME ELASTICITIES



panels or rotating panels that do not track the same households, neither of which are suited for the estimation of individual specific effects. The best available data for our purpose appear to be the PSID, which tracks households' food consumption and income since 1968.

We begin by estimating the food demand equation

$$w_{ht} = a_h + \gamma f_{ht} + \delta_t + \beta_t \log m_{ht} + \varepsilon_{ht}, \quad (20)$$

where f_{ht} is age and family size to control for observed sources of heterogeneity, and a_h is the fixed effect for each household h. The regression includes year dummies both additively and interacted with log income to obtain the time-varying coefficients δ_t and β_t. This is equivalent to estimating a separate Engel curve for each time period, so this analysis does not require measuring prices or specifying how prices affect food demands at the household level.

We only consider male-headed households with at least 10 observations over our estimation sample of 1974 to 1992, and whose heads are between ages 25 and 55.[11] By performing fixed effect estimation, we can obtain $\hat{a}_h$ for 1308 households.

Given a sample, we cannot observe households entering or leaving the true population of households $\mathcal{H}_t$, so we proxy changes in the population by changes in subpopulations, defined by age. As a first check, we aggregate the estimated fixed effect corresponding to the 147 households that are in the sample all 17 years. In this first example there is, by construction, no change over time in the composition of this subpopulation of households, so the aggregate is constant over time (see table 7). This first example corresponds to the extreme case of $v_t = 1$ and $\eta_{it} = 0$ in section III. We then construct three estimated aggregate fixed effects, based on different subpopulations. The first averages the fixed effect across those households whose heads are between ages 30 and 50 in each time period, the second between ages 30 and 40, and the third between ages 40 and

TABLE 7.—ESTIMATED AGGREGATE FIXED EFFECT

Year	Balanced Panel	Age 30–50	Age 30–40	Age 40–50
74	.0757	.1915	.1176	.3239
75	.0757	.1755	.1019	.2938
76	.0757	.1510	.0731	.2775
77	.0757	.1286	.0505	.2566
78	.0757	.1083	.0265	.2487
79	.0757	.0806	.0010	.2243
80	.0757	.0555	−.0189	.2029
81	.0757	.0364	−.0301	.1729
82	.0757	.0171	−.0444	.1489
83	.0757	−.0082	−.0690	.1312
84	.0757	−.0301	−.0874	.1003
85	.0757	−.0478	−.1092	.0892
86	.0757	−.0607	−.1270	.0699
87	.0757	−.0699	−.1427	.0473
90	.0757	−.0914	−.1894	−.0192
91	.0757	−.0955	−.1999	−.0335
92	.0757	−.1011	−.2180	−.0526
AR(1)		.9545	.9945	1.015

The first column gives the fixed effect aggregated over a fixed set of households. The remaining columns are based on aggregation over household heads in each year that are between ages 30 and 50, 30 and 40, and 40 and 50, respectively. The estimated individual fixed effects are from estimates of the household-level food demand equation (20) using the fixed-effect estimator as implemented in Stata.

[11] Food is the sum of food consumed at home plus food consumed outside of home. Food data were not collected in 1973, 1987, and 1988. The SEO sample was excluded from the analysis. Households who reported zero income and/or consumption are dropped.

50. In all three cases, the sample size changes over time both as the size of the subpopulation changes and as households drop in and out of the interviews. The average numbers of observations used in the aggregations are 803, 505, and 337, respectively, with standard deviations of 147, 243, and 146, respectively.

The estimates reported in table 7 suggest strong trends in the resulting aggregate fixed effects, consistent with our conjecture that these series can be highly persistent. The first-order autoregressive parameter is estimated to be near unity in every case.[12]

This simple exercise is subject to many caveats due to data limitations. For example, there is likely to be more period-to-period change in the survey respondents than in the population at large. Nonetheless, the results suggest that aggregation of demand equation fixed effects over a slowly evolving heterogeneous population could be a plausible cause of apparent nonstationarity of errors in aggregate demand systems.

VII. Household-Level Data

We implemented NTLOG with aggregate rather than household-level data, because that is the context in which the nonstationarity problem is most obvious and severe. In this section we briefly describe how the estimator could be applied at the household level. Assume household h has the time t indirect utility function

$$U_{ht}(p_t, m_t,) = \sum_{i=1}^{N} \left(\alpha_{hi} + d_i' f_{ht} + e_{hit} + \frac{1}{2} \sum_{j=1}^{N} b_{ij} \ln \frac{p_{hjt}}{m_{ht}} \right) \ln \frac{p_{hit}}{m_{ht}},$$

where f_{ht} is a vector of observed characteristics of household h that can affect utility and change over time; α_{hi} is a constant preference parameter for each household h and good i; e_{hit} embodies time-varying unobserved preference heterogeneity; $\sum_{i=1}^{N} d_i = 0$, $\sum_{i=1}^{N} e_{hit} = 0$, and $\sum_{i=1}^{N} \alpha_{hi} = 1$; and we drop the law-of-one-price assumption and allow prices to vary across households. If the utility function is over nondurables and services, then f_{ht} could include stocks of durables, yielding conditional demand functions. U_{ht} will then be a conditional rank-two utility function, which is equivalent to an unconditional rank-three model (see Lewbel, 2002).

Let $z_{hijt} = w_{hit} \ln p_{hjt}$ and $r_{hit} = \ln(p_{hit}/m_{ht})$, and follow the same steps used to derive equation (9), to obtain

$$w_{hit} = \alpha_{hi} + d_i' f_{ht} + \sum_{j=1}^{N} b_{ij} r_{hjt} - \sum_{j=1}^{N} c_j z_{hijt} + e_{hit}.$$

Instead of (or in addition to) appearing in the utility function, the error term e_{hit} can embody measurement error in w_{hit} or optimization error on the part of household h. The preference parameter α_{hi} is a household specific fixed effect for good i. Assuming that each household is observed in at least two time periods, estimation is then GMM based on the moment conditions

$$E\left[s_{ht} \left(\Delta w_{hit} - d_i' \, \Delta f_{ht} - \sum_{j=1}^{N} b_{ij} \, \Delta r_{hjt} + \sum_{j=1}^{N} c_j \, \Delta z_{hijt} \right) \right] = 0.$$

The instruments s_{ht} can be lags of $\Delta \ln p_{hit}$, $\Delta \ln m_{ht}$, and possibly Δf_{ht}, which are assumed to be uncorrelated with Δe_{hit} This specification incorporates observed and unobserved constant sources of heterogeneity in preferences across households into the fixed-effect parameters α_{hi}. These fixed effects are differenced out, so the incidental-parameters problem does not arise even when each household is only observed for a small number of time periods. Consistency requires that either the number of households or the number of time periods go to infinity.

Although this model gives consistent estimates with few time periods when the number of households goes to infinity, it should be noted that in industrialized economies, factors including price competition and antidiscrimination laws result in limited variation in the prices faced by different households for identical goods in the same time period. Therefore, in short panel data sets (where nonstationarity might not be a problem), the available price variation will generally be very limited, and hence price effects will be estimated very imprecisely. For example, when prices only vary by time and region (so p_{hit} is the same for all households h in a region), vastly increasing the number of households in each region provides no increase in observed relative price variation. At least for some goods, long time series, such as are available with aggregate data, are needed to observe substantial relative price variation.

If observed prices vary by region as well as time, then our aggregate model could be applied, by adding a region subscript and using region-specific aggregates W and M. These aggregates could be constructed from panel or from repeated cross-section data. The time trends and aggregate errors can now be due to aggregation of f_{ht} in addition to trends in the (regional) population means of α_{hi}.

VIII. Conclusions

Price and income elasticities are important statistics which characterize consumers' behavior and are fundamental to the evaluation of tax policies and welfare programs. Demand systems provide a conceptually coherent framework for estimating these elasticities. Utility maximization requires any reasonable specification of demand systems to be nonlinear in relative prices, and relative prices themselves are nonstationary.

[12] Formal tests of nonstationarity or unit roots in the data in table 7 are not practical, because the number of time periods is very short, no data are available in some years, and ordinary tests would fail to allow for estimation errors in the generation of these data.

Very few techniques exist for estimation of structural nonlinear models with nonstationary data. The vast majority of existing empirical demand system studies, with either household- or more aggregate-level data, simply ignore this problem, treating the data as if they were stationary. The few empirical studies that do consider price nonstationarity assume linearity by, for example, estimating an almost ideal model while ignoring its nonlinear component, which is a quadratic price index.

To deal with price nonstationarity, we propose a reformulation of the utility-derived translog model that can be written in a linear form (albeit with endogeneity in the regressors caused by interacting prices with budget shares), thereby avoiding the severe constraints of ordinary utility-derived linear demand models, while preserving sufficient linear structure to deal with nonstationarity. Our NTLOG model provides a solution to the empirical problem, which exists at both the household and the aggregate level, of demand system estimation with nonstationary relative prices. At the household data level, the NTLOG also permits consistent estimation in the presence of preference heterogeneity that takes the form of utility-derived household and good-specific fixed effects.

In addition to handling nonstationarity of relative prices, our NTLOG model can also cope with possible nonstationarity of demand system errors, a feature commonly found in models using aggregate data. We show theoretically that nonstationarity of demand system errors could arise from aggregation across heterogeneous consumers in a slowly changing population, and we provide some empirical evidence for this effect based on a panel of household demands for food. Other possible sources of nonstationarity are omitted variables, omitted dynamics, and aggregation across goods as in Lewbel (1996a). We provide some empirical evidence against the omitted variables explanation.

We estimate this NTLOG model using aggregate U.S. data over the sample 1954–1998. The model is subjected to and passes a variety of specification tests. Estimates of the model parameters and elasticities are also reported, and are found to be economically plausible. Unlike other demand system estimates in the literature, given nonstationary data and nonstationary errors, these NTLOG estimates have root-T asymptotically normal distributions and so allow for standard inference.

An open problem in all time series estimation of demand systems is to reconcile the apparent nonstationary behavior of budget shares with the fact that budget shares are bounded between 0 and 1. In this paper, we first-difference the model, because the resulting GMM estimator has standard root-T limiting distribution. However, as discussed in Davidson and Terasvirta (2002), fractional instead of first differencing could be an appealing alternative when the variables displaying strong persistence are strictly bounded. It remains to be seen whether fractional cointegration estimation, as in Davidson (2002), can be applied to the NTLOG and the estimates remain root-T consistent and asymptotically normal. Alternatively, one might construct models where budget share behavior changes from nonstationary to stationary in the neighborhood of boundaries. Persistent movements in budget shares could be a result of changes in demographics, tastes, and the composition of goods. A further decomposition of these effects might provide a better understanding of the sources of apparent nonstationarity.

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APPENDIX

Data Sources

The data are from the U.S. National Income and Product Accounts, obtained via Citibase.

The sample period is 1954Q1–1998Q4. In Citibase mnemonics, $M = GC - GCD$. Nominal expenditures on the four groups are:

1. *GCFO* (food),
2. $GCNF + GCNG + GCST + GCSHO$ (energy),
3. *GCNC* (clothing),
4. Others $= M -$ food $-$ energy $-$ clothing.

Price indices are obtained by dividing nominal by real expenditures in these groups. Following many other authors (such as Campbell & Mankiw, 1990), data from before the mid-1950s are excluded to avoid the effects of both the Korean war and measurement errors in the first few years of data collection.

In Citibase, the second set of instruments are *GGE, GDGE, GGCGE, GIMQ, GW, GMPT, GPOP, LBLCPU, LHUR*, and *LHP16*. We take logs of the first six of these variables before first-differencing them.

For table 7, household-level data from 1974 to 1992 are taken from the Panel Study of Income Dynamics, excluding the SEO sample. We use observations with male household heads who are between age 25 and 55, and have no missing data on age, sex, marital status, number of children, or income. Income is defined as the sum of earned and transfer income of the husband, wife, and other family members. Food is defined as food consumed at home and outside the home. Consumption data are not available for 1973, 1987, and 1988. A total of 17,568 observations over 17 years were used in the fixed-effect estimation.