Lecture 10: Logistical Regression II—Multinomial Data

Prof. Sharyn O'Halloran

Sustainable Development U9611

Econometrics II

Logit vs. Probit Review

- Use with a dichotomous dependent variable
- Need a link function F(Y) going from the original Y to continuous Y′
  - Probit: $F(Y) = \Phi^{-1}(Y)$
  - Logit: $F(Y) = \log[Y/(1-Y)]$
- Do the regression and transform the findings back from Y′ to Y, interpreted as a probability
  - Unlike linear regression, the impact of an independent variable X depends on its value
  - And the values of all other independent variables

Classical vs. Logistic Regression

- Data Structure: continuous vs. discrete
  - Logistic/Probit regression is used when the dependent variable is binary or dichotomous.
- Different assumptions between traditional regression and logistic regression
  - The population means of the dependent variables at each level of the independent variable are not on a straight line, i.e., no linearity.
  - The variance of the errors are not constant, i.e., no homogeneity of variance.
  - The errors are not normally distributed, i.e., no normality.

Logistic Regression Assumptions

1. The model is correctly specified, i.e.,
   - The true conditional probabilities are a logistic function of the independent variables;
   - No important variables are omitted;
   - No extraneous variables are included; and
   - The independent variables are measured without error.
2. The cases are independent.
3. The independent variables are not linear combinations of each other.
   - Perfect multicollinearity makes estimation impossible,
   - While strong multicollinearity makes estimates imprecise.

About Logistic Regression

- It uses a maximum likelihood estimation rather than the least squares estimation used in traditional multiple regression.
- The general form of the distribution is assumed.
- Starting values of the estimated parameters are used and the likelihood that the sample came from a population with those parameters is computed.
- The values of the estimated parameters are adjusted iteratively until the maximum likelihood value for the estimated parameters is obtained.
  - That is, maximum likelihood approaches try to find estimates of parameters that make the data actually observed "most likely."

Interpreting Logistic Coefficients

- Logistic slope coefficients can be interpreted as the effect of a unit of change in the X variable on the predicted logits with the other variables in the model held constant.
  - That is, how a one unit change in X effects the log of the odds when the other variables in the model held constant.

Interpreting Odds Ratios

- Odds ratios in logistic regression can be interpreted as the effect of a one unit of change in X in the predicted odds ratio with the other variables in the model held constant.

$$\frac{\text{odds(if the corresponding variable is incremented by 1)}}{\text{odds(if variable not incremented)}}$$

$$\frac{\mathrm{P}(\text{event} \mid x+1) \,/\, \left(1-\mathrm{P}(\text{event} \mid x+1)\right)}{\mathrm{P}(\text{event} \mid x) \,/\, \left(1-\mathrm{P}(\text{event} \mid x)\right)}$$

Interpreting Odds Ratios

- An important property of odds ratios is that they are constant.
  - It does not matter what values the other independent variables take on.
- For instance, say you estimate the following logistic regression model:
  - $-13.70837 + .1685\ x_1 + .0039\ x_2$
  - The effect of the odds of a 1-unit increase in x_1 is exp(.1685) = 1.18
  - Meaning the odds increase by 18%
- Incrementing x_1 increases the odds by 18% regardless of the value of x_2 (0, 1000, etc.)

Example: Admissions Data

- 20 observations of admission into a graduate program
- Data collected includes whether admitted, gender (1 if male) and the student’s aptitude on a 10 point scale.

aptitude	*gender*	*admit*
8	1	1
7	1	0
5	1	1
3	1	0
3	1	0
5	1	1
7	1	1
8	1	1
5	1	1
5	1	1
4	0	0
7	0	1
3	0	1
2	0	0
4	0	0
2	0	0
3	0	0
4	0	1
3	0	0
2	0	0

Admissions Example – Calculating the Odds Ratio

- Example: admissions to a graduate program
  - Assume 70% of the males and 30% of the females are admitted in a given year
  - Let P equal the probability a male is admitted.
  - Let Q equal the probability a female is admitted.
    - Odds males are admitted: odds(M) = P/(1-P) = .7/.3 = 2.33
    - Odds females are admitted: odds(F) = Q/(1-Q) = .3/.7 = 0.43
  - The odds ratio for male vs. female admits is then
    - odds(M)/odds(F) = 2.33/0.43 = 5.44
- The odds of being admitted to the program are about 5.44 times greater for males than females.

Ex. 1: Categorical Independent Var.

```
. logit admit gender
Logit estimates                                   Number of obs   =         20
                                                  LR chi2(1)      =       3.29
                                                  Prob > chi2     =     0.0696
Log likelihood = -12.217286                       Pseudo R2       =     0.1187
------------------------------------------------------------------------------
   admit |      Coef.   Std. Err.       z     P>|z|       [95% Conf. Interval]
---------+--------------------------------------------------------------------
  gender |   1.694596    .9759001     1.736   0.082       -.2181333    3.607325
   _cons |  -.8472979    .6900656    -1.228   0.220       -2.199801    .5052058
------------------------------------------------------------------------------
```

Formula to back out Y from logit estimates: $Y = \frac{\exp(X\beta)}{1+\exp(X\beta)}$

```
.dis exp(_b[gender]+_b[_cons])/(1+exp(_b[gender]+_b[_cons]))
.7
. dis exp(_b[_cons])/(1+exp(_b[_cons]))
.3
```

Ex. 1: Categorical Independent Variable

To get the results in terms of odds ratios:

```
logit admit gender, or

Logit estimates                                   Number of obs   =         20
                                                  LR chi2(1)      =       3.29
                                                  Prob > chi2     =     0.0696
Log likelihood = -12.217286                       Pseudo R2       =     0.1187

------------------------------------------------------------------------------
   admit | Odds Ratio   Std. Err.       z     P>|z|       [95% Conf. Interval]
---------+--------------------------------------------------------------------
  gender |   5.444444   5.313234      1.736   0.082       .8040183    36.86729
------------------------------------------------------------------------------
```

Translates original logit coefficients to odds ratio on gender
Same as the odds ratio we calculated by hand above

Ex. 1: Categorical Independent Variable

To get the results in terms of odds ratios:

```
logit admit gender, or
Logit estimates                                   Number of obs   =         20
                                                  LR chi2(1)      =       3.29
                                                  Prob > chi2     =     0.0696
Log likelihood = -12.217286                       Pseudo R2       =     0.1187
------------------------------------------------------------------------------
   admit | Odds Ratio   Std. Err.       z     P>|z|       [95% Conf. Interval]
---------+--------------------------------------------------------------------
  gender |   5.444444   5.313234      1.736   0.082       .8040183    36.86729
------------------------------------------------------------------------------
```

So 5.4444 is the "exponentiated coefficient"
Don't confuse this with the logit coefficient (1.6945)

Ex. 1: Categorical Independent Variable

To get the results in terms of odds ratios:

```
logit admit gender, or

Logit estimates                                   Number of obs   =         20
                                                  LR chi2(1)      =       3.29
                                                  Prob > chi2     =     0.0696
Log likelihood = -12.217286                       Pseudo R2       =     0.1187

------------------------------------------------------------------------------
   admit | Odds Ratio   Std. Err.      z     P>|z|       [95% Conf. Interval]
---------+--------------------------------------------------------------------
  gender |   5.444444   5.313234     1.736   0.082       .8040183    36.86729
------------------------------------------------------------------------------
```

That is, exp(1.694596) = 5.444444

Ex. 2: Continuous Independent Var.

Look at the probability of being admitted to graduate school given the candidate's aptitude

```
logit admit apt

Iteration 0:   log likelihood = -13.862944
Iteration 1:   log likelihood = -9.6278718
Iteration 2:   log likelihood = -9.3197603
Iteration 3:   log likelihood = -9.3029734
Iteration 4:   log likelihood = -9.3028914
Logit estimates                                   Number of obs   =         20
                                                  LR chi2(1)      =       9.12
                                                  Prob > chi2     =     0.0025
Log likelihood = -9.3028914                       Pseudo R2       =     0.3289
------------------------------------------------------------------------------
   admit |      Coef.   Std. Err.       z     P>|z|       [95% Conf. Interval]
---------+--------------------------------------------------------------------
     apt |   .9455112    .422872      2.236   0.025       .1166974    1.774325
   _cons |  -4.095248    1.83403     -2.233   0.026      -7.689881   -.5006154
------------------------------------------------------------------------------
```

Ex. 2: Continuous Independent Var.

```
logit admit apt

Iteration 0:   log likelihood = -13.862944
Iteration 1:   log likelihood = -9.6278718
Iteration 2:   log likelihood = -9.3197603
Iteration 3:   log likelihood = -9.3029734
Iteration 4:   log likelihood = -9.3028914
Logit estimates                                   Number of obs   =         20
                                                  LR chi2(1)      =       9.12
                                                  Prob > chi2     =     0.0025
Log likelihood = -9.3028914                       Pseudo R2       =     0.3289
------------------------------------------------------------------------------
   admit |      Coef.   Std. Err.       z     P>|z|       [95% Conf. Interval]
---------+--------------------------------------------------------------------
     apt |   .9455112    .422872     2.236   0.025       .1166974    1.774325
   _cons |  -4.095248    1.83403    -2.233   0.026      -7.689881   -.5006154
------------------------------------------------------------------------------
```

Look at the probability of being admitted to graduate school given the candidate's aptitude

Aptitude is positive and significantly related to being admitted into the graduate program

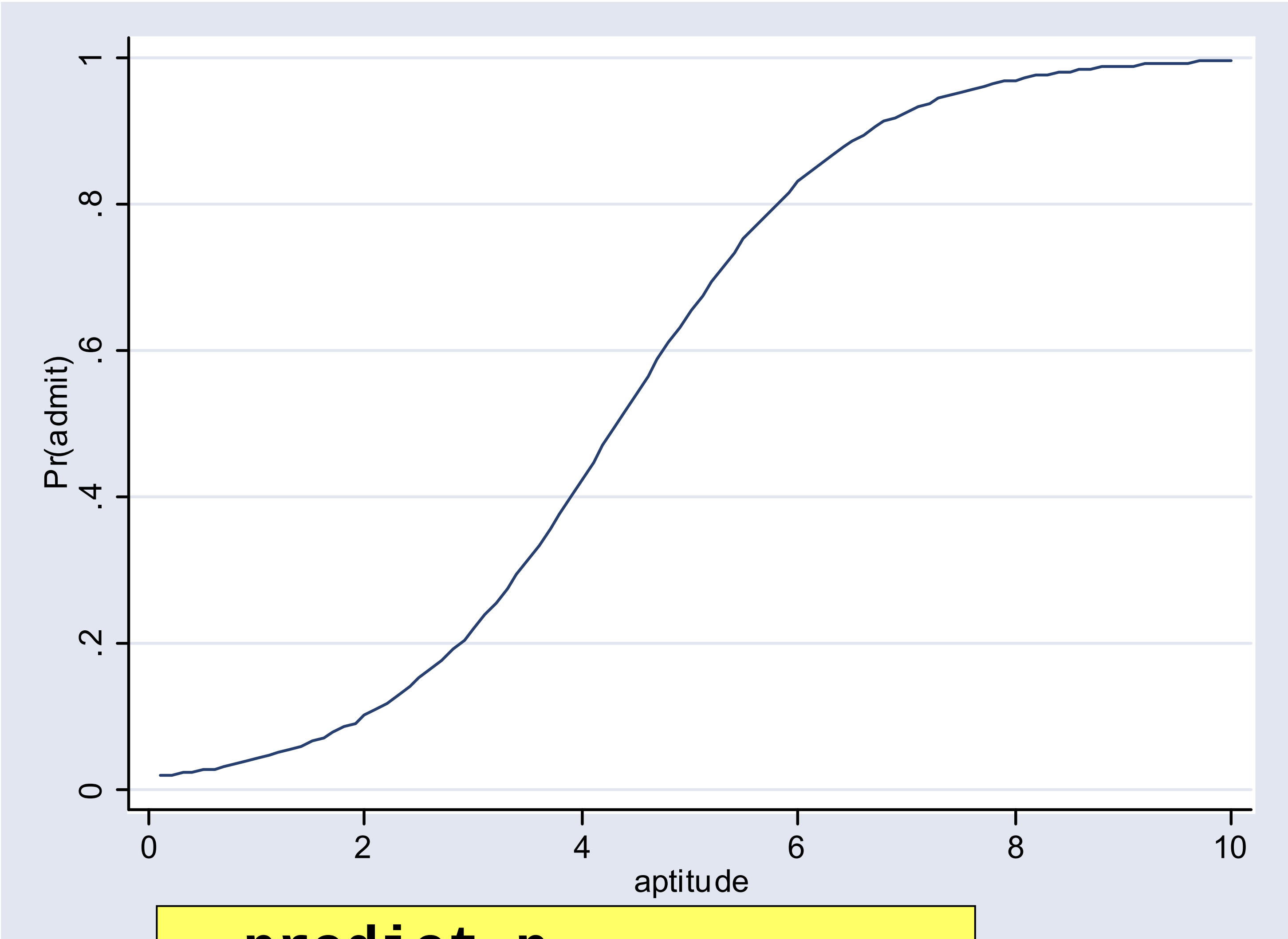
Ex. 2: Continuous Independent Var.

```
logit admit apt, or

Logit estimates                                   Number of obs   =         20
                                                  LR chi2(1)      =       9.12
                                                  Prob > chi2     =     0.0025
Log likelihood = -9.3028914                       Pseudo R2       =     0.3289

------------------------------------------------------------------------------
   admit | Odds Ratio   Std. Err.      z     P>|z|       [95% Conf. Interval]
---------+--------------------------------------------------------------------
     apt |   2.574129   1.088527     2.236   0.025       1.123779      5.8963
------------------------------------------------------------------------------
```

This means:

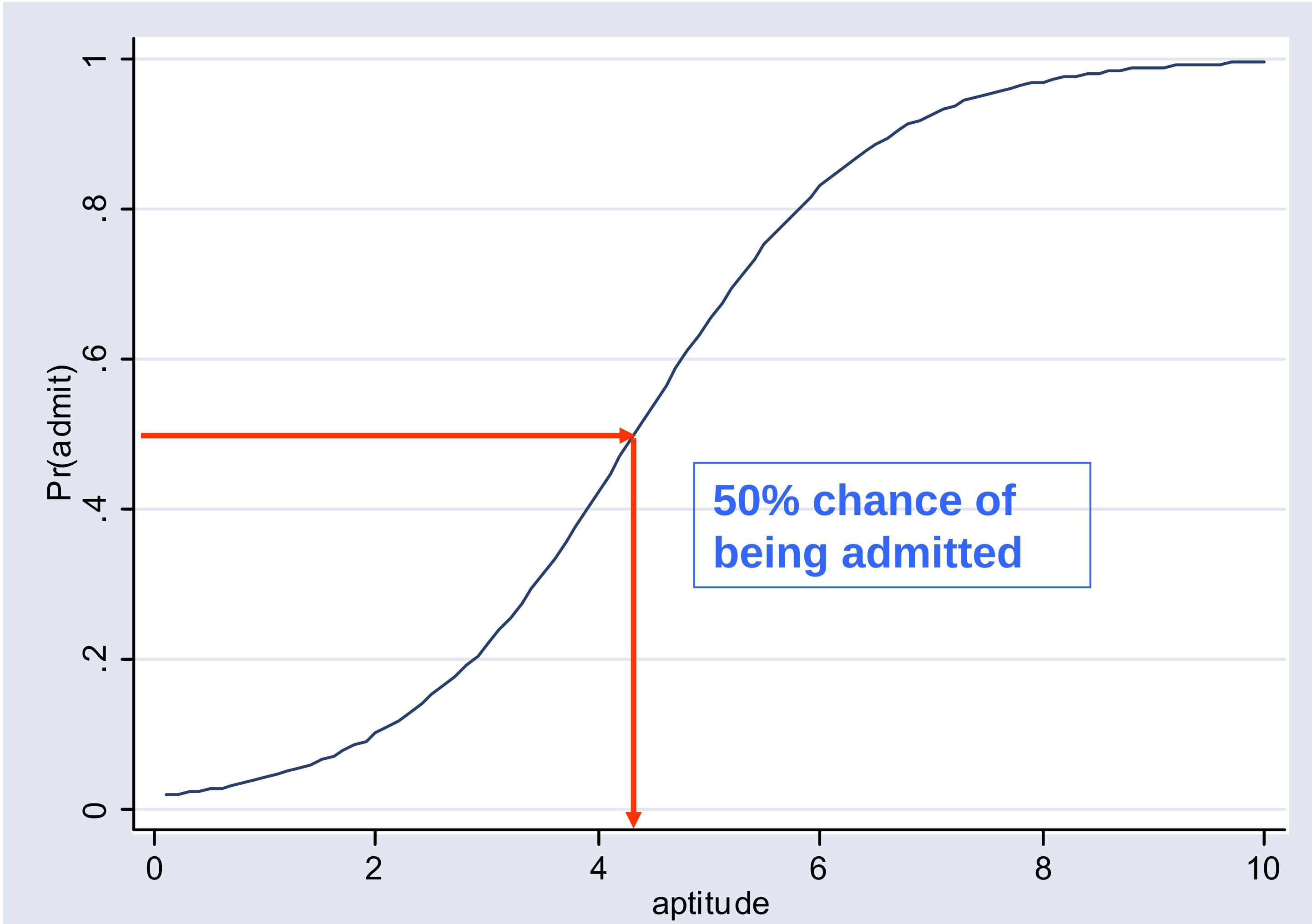
$$\frac{\Pr(\text{admit} \mid \text{apt}+1)/1-\Pr(\text{admit} \mid \text{apt}+1)}{\Pr(\text{admit} \mid \text{apt})/1-\Pr(\text{admit} \mid \text{apt})} = 2.57$$

Ex. 2: Continuous Independent Var.




```
. predict p
. line p aptitude, sort
```

Ex. 2: Continuous Independent Var.




```
. predict p
. line p aptitude, sort
```

Example 3: Categorical & Continuous Independent Variables

```
logit admit gender apt

Logit estimates                                   Number of obs   =         20
                                                  LR chi2(2)      =       9.16
                                                  Prob > chi2     =     0.0102
Log likelihood = -9.2820991                       Pseudo R2       =     0.3304
------------------------------------------------------------------------------
   admit |      Coef.   Std. Err.       z     P>|z|       [95% Conf. Interval]
---------+--------------------------------------------------------------------
  gender |   .2671938   1.300899     0.205   0.837      -2.282521    2.816909
     apt |   .8982803   .4713791     1.906   0.057      -.0256057    1.822166
   _cons |  -4.028765   1.838354    -2.192   0.028      -7.631871   -.4256579
------------------------------------------------------------------------------
```

Gender is now insignificant!
Once aptitude is taken into account gender plays no role

Likelihood Ratio Test

- Log-likelihoods can be used to test hypotheses about nested models.
- Say we want to test the null hypothesis H_0 about one or more coefficients
  - For example, H_0: $x_1 = 0$, or H_0: $x_1 = x_2 = 0$
- Then the likelihood ratio is the ratio of the likelihood of imposing H_0 over the likelihood of the unrestricted model:
  - $\mathcal{L}$(model restricted by H_0)/ $\mathcal{L}$(unrestricted model)
- If H_0 is true, then this ratio should be near 1

Likelihood Ratio Test

- Under general assumptions,

  $-2 * (\text{log of the likelihood ratio}) \sim \chi^2(k)$

  - Where the k degrees of freedom are the number of restrictions specified in H_0
- This is called a likelihood ratio test
- Call the restricted likelihood $\mathcal{L}_0$, and the unrestricted likelihood $\mathcal{L}$.
- Then we can rewrite the equation above as:

  $-2*\log(\mathcal{L}_0 / \mathcal{L}) = -2*\log(\mathcal{L}_0) - 2*\log(\mathcal{L}) \sim \chi^2(k)$
- The difference of the log-likelihoods will be

Likelihood Ratio Test

- In our admissions example, take
  - Pr(admit) = $\beta_0 + \beta_1$*gender + β_2*aptitude
  - The log-likelihood of this model was -9.282

Likelihood Ratio Test

- In our admissions example, take
  - Pr(admit) = $\beta_0 + \beta_1$*gender + β_2*aptitude
  - The log-likelihood of this model was -9.282

```
logit admit gender apt
Logit estimates                                   Number of obs   =         20
                                                  LR chi2(2)      =       9.16
                                                  Prob > chi2     =     0.0102
Log likelihood = -9.2820991                       Pseudo R2       =     0.3304
------------------------------------------------------------------------------
   admit |      Coef.   Std. Err.       z     P>|z|       [95% Conf. Interval]
---------+--------------------------------------------------------------------
  gender |   .2671938   1.300899      0.205   0.837      -2.282521    2.816909
     apt |   .8982803   .4713791      1.906   0.057      -.0256057    1.822166
   _cons |  -4.028765   1.838354     -2.192   0.028      -7.631871   -.4256579
------------------------------------------------------------------------------
```

Likelihood Ratio Test

- In our admissions example, take
  - Pr(admit) = $\beta_0 + \beta_1$*gender + β_2*aptitude
  - The log-likelihood of this model was -9.282

```
logit admit gender apt
Logit estimates                                   Number of obs   =         20
                                                  LR chi2(2)      =       9.16
                                                  Prob > chi2     =     0.0102
Log likelihood = -9.2820991                       Pseudo R2       =     0.3304
------------------------------------------------------------------------------
   admit |      Coef.   Std. Err.       z     P>|z|       [95% Conf. Interval]
---------+--------------------------------------------------------------------
  gender |   .2671938   1.300899      0.205   0.837      -2.282521    2.816909
     apt |   .8982803   .4713791      1.906   0.057      -.0256057    1.822166
   _cons |  -4.028765   1.838354     -2.192   0.028      -7.631871   -.4256579
------------------------------------------------------------------------------
```

Log-likelihood with no restrictions

Likelihood Ratio Test

- In our admissions example, take
  - Pr(admit) = $\beta_0 + \beta_1$*gender + β_2*aptitude
  - The log-likelihood of this model was -9.282
- First look at H_0: $\beta_2 = 0$

```
logit admit gender, or
Logit estimates                                   Number of obs   =         20
                                                  LR chi2(1)      =       3.29
                                                  Prob > chi2     =     0.0696
Log likelihood = -12.217286                       Pseudo R2       =     0.1187
------------------------------------------------------------------------------
   admit | Odds Ratio   Std. Err.       z     P>|z|       [95% Conf. Interval]
---------+--------------------------------------------------------------------
  gender |   5.444444   5.313234      1.736   0.082       .8040183    36.86729
------------------------------------------------------------------------------
```

Likelihood Ratio Test

- In our admissions example, take
  - Pr(admit) = $\beta_0 + \beta_1$*gender + β_2*aptitude
  - The log-likelihood of this model was -9.282
- First look at H_0: $\beta_2 = 0$

```
logit admit gender, or

Logit estimates                                   Number of obs   =         20
                                                  LR chi2(1)      =       3.29
                                                  Prob > chi2     =     0.0696
Log likelihood = -12.217286                       Pseudo R2       =     0.1187
------------------------------------------------------------------------------
   admit | Odds Ratio   Std. Err.       z     P>|z|       [95% Conf. Interval]
---------+--------------------------------------------------------------------
  gender |   5.444444   5.313234      1.736   0.082       .8040183    36.86729
------------------------------------------------------------------------------
```

Log-likelihood with aptitude=0

Likelihood Ratio Test

- In our admissions example, take
  - Pr(admit) = $\beta_0 + \beta_1$*gender + β_2*aptitude
  - The log-likelihood of this model was -9.282
- First look at H_0: $\beta_2 = 0$
  - The likelihood of the regression with gender but not aptitude was -12.217
- Likelihood ratio test:
  - [-2 * (-12.217)] – [-2 * (-9.282)] = 5.87
  - From Stata
    - `dis 1- chi2(1, 5.87)`
    - `.01540105`

Significant at 5% level. Therefore we can reject the null hypothesis that $\beta_2 = 0$.

Likelihood Ratio Test

- In our admissions example, take
  - Pr(admit) = $\beta_0 + \beta_1$*gender + β_2*aptitude
  - The log-likelihood of this model was -9.282
- Now look at H_0: $\beta_1 = 0$

Likelihood Ratio Test

- In our admissions example, take
  - Pr(admit) = $\beta_0 + \beta_1$*gender + β_2*aptitude
  - The log-likelihood of this model was -9.282
- Now look at H_0: $\beta_1 = 0$

```
logit admit apt, or
Logit estimates                                   Number of obs   =         20
                                                  LR chi2(1)      =       9.12
                                                  Prob > chi2     =     0.0025
Log likelihood = -9.3028914                       Pseudo R2       =     0.3289
------------------------------------------------------------------------------
   admit | Odds Ratio   Std. Err.       z     P>|z|       [95% Conf. Interval]
---------+--------------------------------------------------------------------
     apt |   2.574129   1.088527     2.236   0.025       1.123779      5.8963
------------------------------------------------------------------------------
```

Likelihood Ratio Test

- In our admissions example, take
  - $Pr(admit) = \beta_0 + \beta_1 * gender + \beta_2 * aptitude$
  - The log-likelihood of this model was -9.282
- Now look at H_0: $\beta_1 = 0$

Log-likelihood with gender=0

```
logit admit apt, or

Logit estimates                                   Number of obs   =         20
                                                  LR chi2(1)      =       9.12
                                                  Prob > chi2     =     0.0025
Log likelihood = -9.3028914                       Pseudo R2       =     0.3289
------------------------------------------------------------------------------
   admit | Odds Ratio   Std. Err.       z     P>|z|       [95% Conf. Interval]
---------+--------------------------------------------------------------------
     apt |   2.574129   1.088527      2.236   0.025       1.123779      5.8963
------------------------------------------------------------------------------
```

Likelihood Ratio Test

- In our admissions example, take
  - Pr(admit) = $\beta_0 + \beta_1$*gender + β_2*aptitude
  - The log-likelihood of this model was -9.282
- Now look at H_0: $\beta_1 = 0$
  - The likelihood of the regression with gender but not aptitude was -9.303
- Likelihood ratio test:
  - [-2 * (-9.303)] – [-2 * (-9.282)] = 0.042
  - From Stata
    - `dis 1- chi2(1, .042)`
    - `.83761977`

Not significant at 5% level. Therefore we fail to reject the null hypothesis that $\beta_1 = 0$.

Example 4: Honors Composition using High School and Beyond Dataset

use http://www.gseis.ucla.edu/courses/data/hsb2

Variable	Obs	Mean	Std. Dev.	Min	Max
id	200	100.5	57.87918	1	200
female	200	.545	.4992205	0	1
race	200	3.43	1.039472	1	4
ses	200	2.055	.7242914	1	3
schtyp	200	1.16	.367526 1	2	
prog	200	2.025	.6904772	1	3
read	200	52.23	10.25294	28	76
write	200	52.775	9.478586	31	67
math	200	52.645	9.368448	33	75
science	200	51.85	9.900891	26	74
socst	200	52.405	10.73579	26	71
honors	200	.265	.4424407	0	1
ses1	200	.235	.4250628	0	1
ses2	200	.475	.5006277	0	1
ses3	200	.29	.4549007	0	1

Example 4: Categorical and continuous independent variables

```
generate honors = (write>=60)

/* create dummy coding for ses */
tabulate ses, generate(ses)
        ses |      Freq.     Percent        Cum.
------------+-----------------------------------
        low |         47       23.50       23.50
     middle |         95       47.50       71.00
       high |         58       29.00      100.00
------------+-----------------------------------
      Total |        200      100.00

tabulate honors
     honors |      Freq.     Percent        Cum.
------------+-----------------------------------
          0 |        147       73.50       73.50
          1 |         53       26.50      100.00
------------+-----------------------------------
      Total |        200      100.00
```

Example 4: Categorical and continuous independent variables

```
generate honors = (write>=60)

/* create dummy coding for ses */
tabulate ses, generate(ses)
        ses |      Freq.     Percent        Cum.
------------+-----------------------------------
        low |         47       23.50       23.50
     middle |         95       47.50       71.00
       high |         58       29.00      100.00
------------+-----------------------------------
      Total |        200      100.00

tabulate honors
     honors |      Freq.     Percent        Cum.
------------+-----------------------------------
          0 |        147       73.50       73.50
          1 |         53       26.50      100.00
------------+-----------------------------------
      Total |        200      100.00
```

Creates new variables ses1, ses2, and ses3

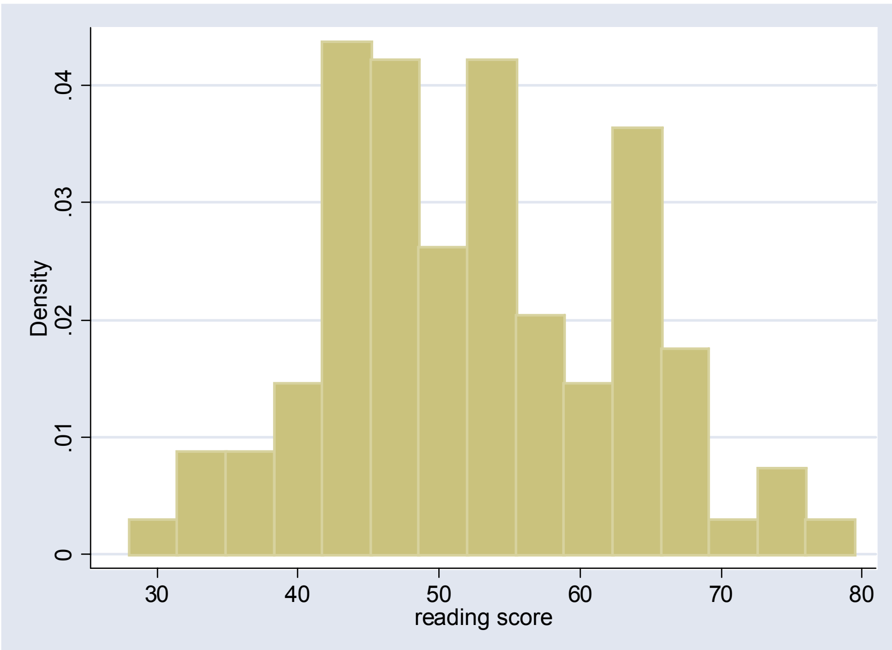
Example 4: Categorical and continuous independent var.

```
describe honors female ses1 ses2 read math

              storage  display     value
variable name   type   format      label      variable label
-------------------------------------------------------------------------------
honors          float  %9.0g
female          float  %9.0g       fl
ses1            byte   %8.0g                  ses==low
ses2            byte   %8.0g                  ses==middle
read            float  %9.0g                  reading score
math            float  %9.0g                  math score
```

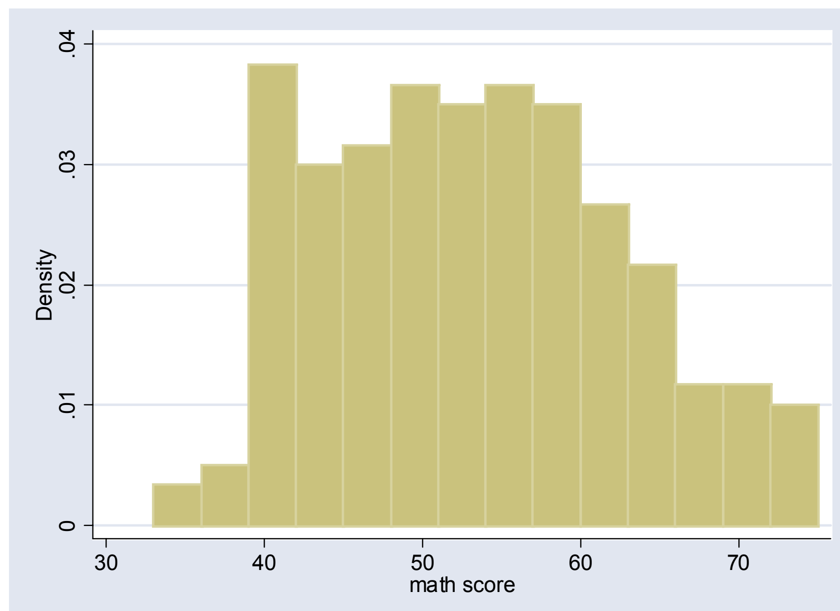
```
tab1 honors female ses1 ses2 read math

-> tabulation of honors

     honors |      Freq.     Percent        Cum.
------------+-----------------------------------
          0 |        147       73.50       73.50
          1 |         53       26.50      100.00
------------+-----------------------------------
      Total |        200      100.00

-> tabulation of female

     female |      Freq.     Percent        Cum.
------------+-----------------------------------
       male |         91       45.50       45.50
     female |        109       54.50      100.00
------------+-----------------------------------
      Total |        200      100.00
```

```
-> tabulation of ses1

    ses==low |      Freq.     Percent        Cum.
-------------+-----------------------------------
           0 |        153       76.50       76.50
           1 |         47       23.50      100.00
-------------+-----------------------------------
       Total |        200      100.00

-> tabulation of ses2

 ses==middle |      Freq.     Percent        Cum.
-------------+-----------------------------------
           0 |        105       52.50       52.50
           1 |         95       47.50      100.00
-------------+-----------------------------------
       Total |        200      100.00
```

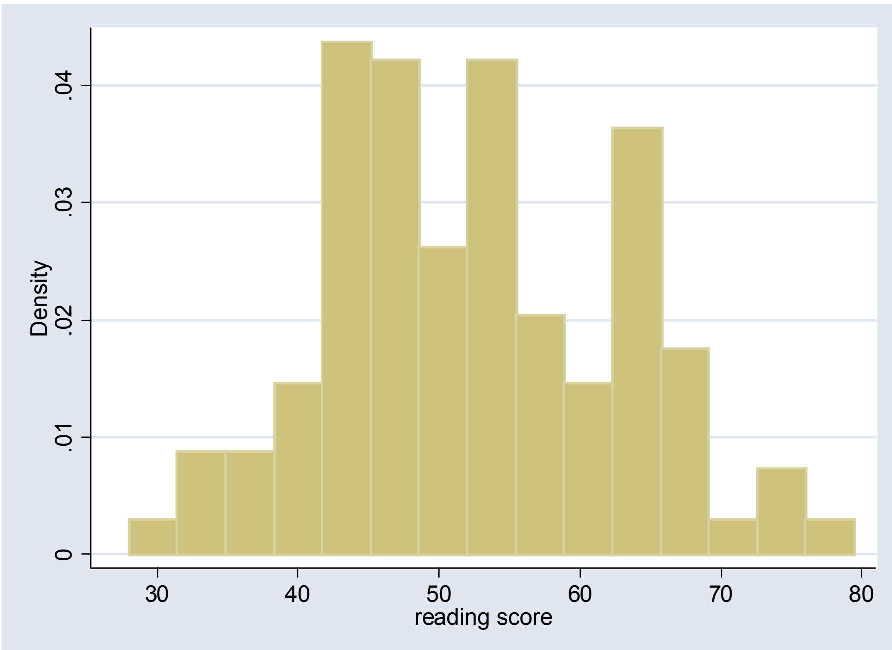
Example 4: Categorical and continuous independent var.



We would normally worry about this but….

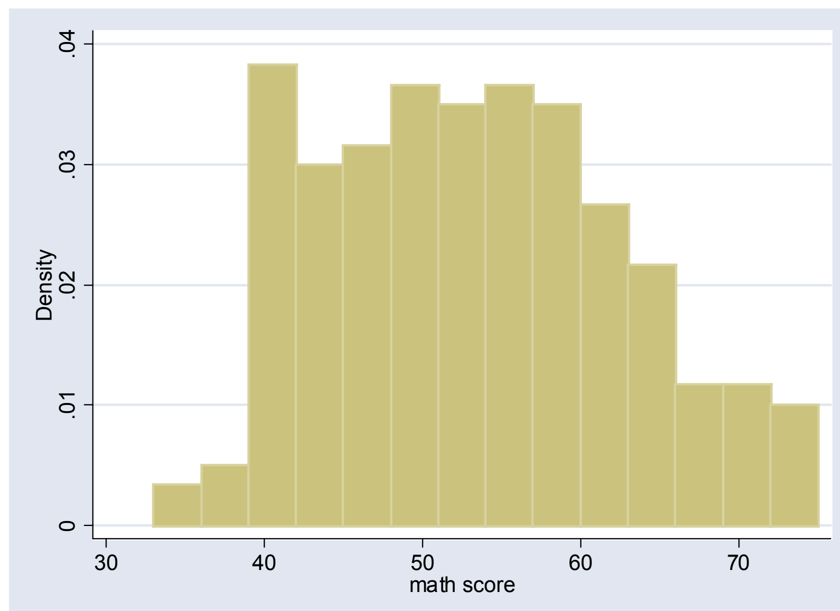


Example 4: Categorical and continuous independent var.



We would normally worry about this but….

The logit link function takes logs of the series.



Example 4: Categorical and continuous independent variables

```
logit honors female ses1 ses2 read math

Logit estimates                                   Number of obs   =        200
                                                  LR chi2(5)      =      87.30
                                                  Prob > chi2     =     0.0000
Log likelihood = -71.994756                       Pseudo R2       =     0.3774

------------------------------------------------------------------------------
  honors |      Coef.   Std. Err.       z     P>|z|       [95% Conf. Interval]
---------+--------------------------------------------------------------------
  female |   1.145726   .4513589      2.538   0.011       .2610792    2.030374
    ses1 |  -.0541296   .5945439     -0.091   0.927      -1.219414    1.111155
    ses2 |  -1.094532   .4833959     -2.264   0.024       -2.04197   -.1470932
    read |   .0687277   .0287044      2.394   0.017       .0124681    .1249873
    math |   .1358904   .0336874      4.034   0.000       .0698642    .2019166
   _cons |  -12.49919   1.926421     -6.488   0.000      -16.27491   -8.723475
------------------------------------------------------------------------------
```

```
test ses1 ses2
 ( 1)  ses1 = 0.0
 ( 2)  ses2 = 0.0
           chi2(  2) =    6.13
         Prob > chi2 =    0.0466
```

So the socioeconomic variables are significant as a group.

Example 4: Categorical and continuous independent variables

```
logistic honors female ses1 ses2 read math

Logit estimates                                   Number of obs   =        200
                                                  LR chi2(5)      =      87.30
                                                  Prob > chi2     =     0.0000
Log likelihood = -71.994756                       Pseudo R2       =     0.3774

------------------------------------------------------------------------------
  honors | Odds Ratio   Std. Err.       z     P>|z|       [95% Conf. Interval]
---------+--------------------------------------------------------------------
  female |   3.144725     1.4194     2.538   0.011        1.29833    7.616932
    ses1 |   .9473093    .563217    -0.091   0.927       .2954031    3.037865
    ses2 |   .3346963   .1617908    -2.264   0.024       .1297728    .8632135
    read |   1.071145   .0307466     2.394   0.017       1.012546    1.133134
    math |   1.145556   .0385909     4.034   0.000       1.072363    1.223746
-------------------------------------------------------------------------------
```

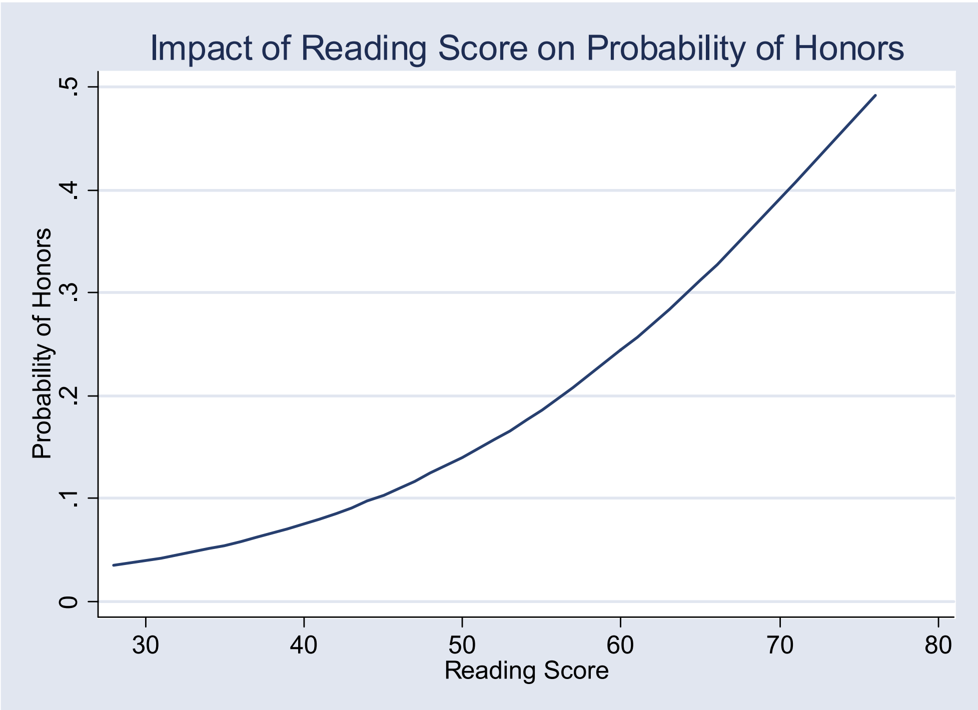
Same as logit, or

```
test ses1 ses2
 ( 1)  ses1 = 0.0
 ( 2)  ses2 = 0.0
           chi2(  2) =    6.13
         Prob > chi2 =    0.0466
```

So the socioeconomic variables are significant as a group.

Graphing the Results

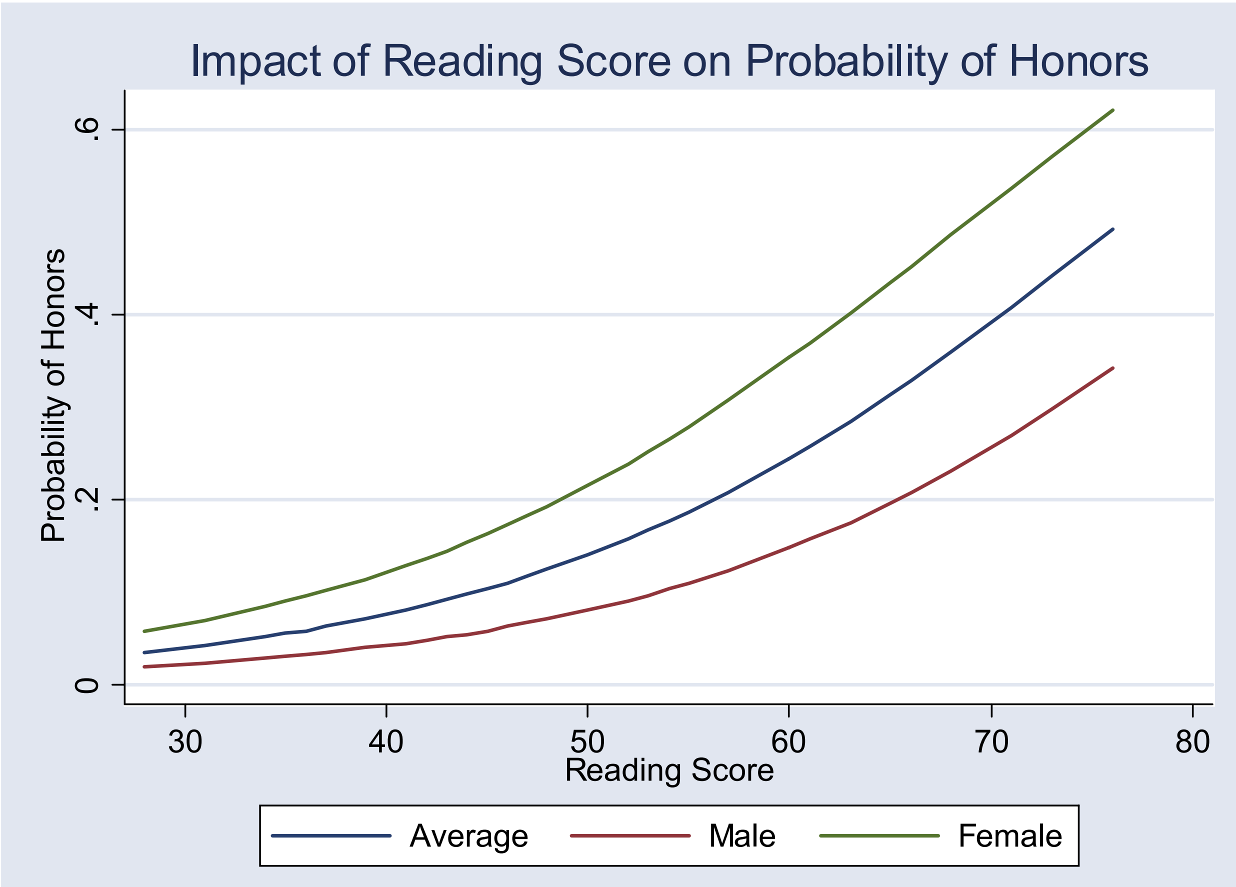
- Let's say we want to see how the probability of honors changes with the reading score
- Stata's postgr3 command will create a new variable giving the probability after a logit




```
. postgr3 read, gen(avg)
. line avg read, sort
```

Graphing the Results

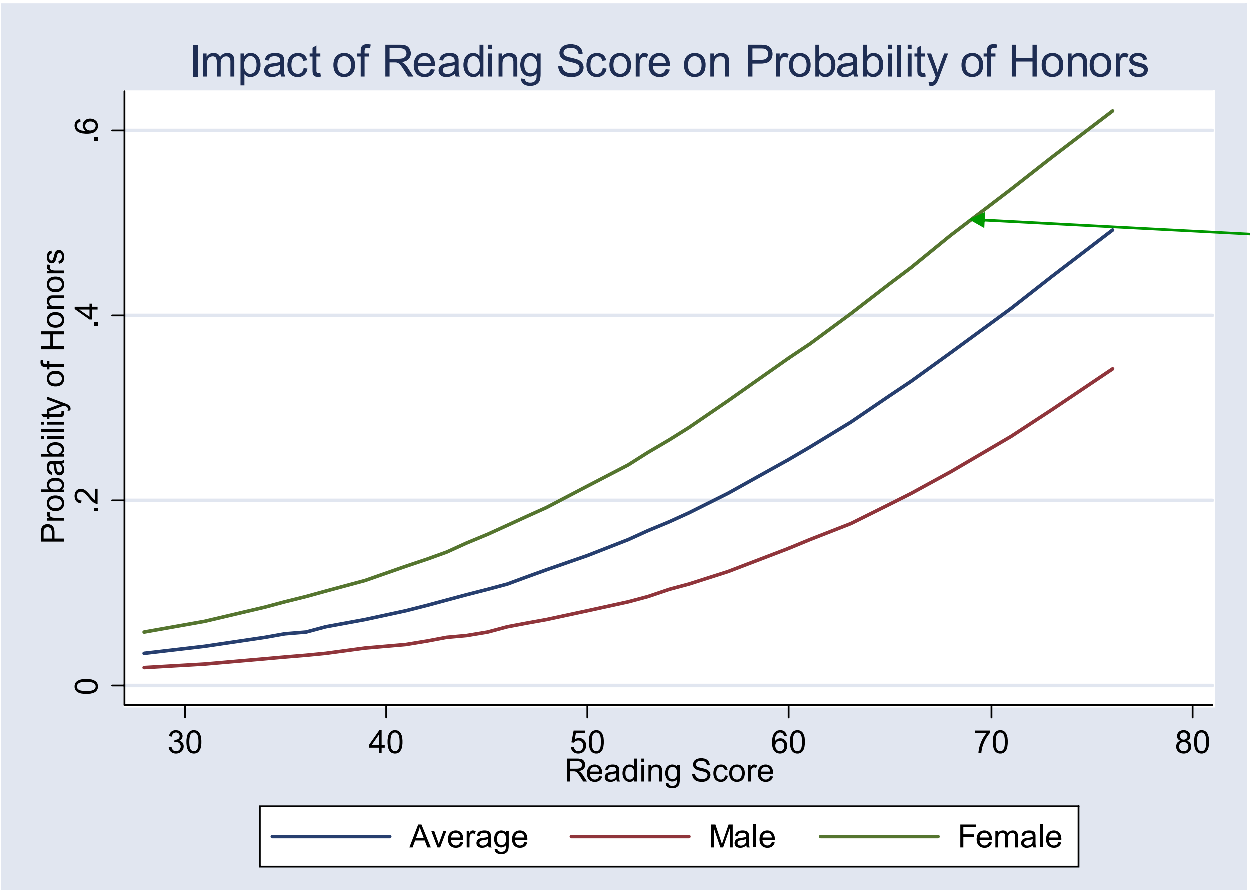
- Can do this separately for males & females




```
. postgr3 read, gen(male) x(female=0) nodraw
. postgr3 read, gen(fem) x(female=1) nodraw
. graph twoway (line avg read, sort) (line male read, sort) (line fem read, sort)
```

Graphing the Results

- Can do this separately for males & females



Marginal impact is higher for females than for males

```
. postgr3 read, gen(male) x(female=0) nodraw
. postgr3 read, gen(fem) x(female=1) nodraw
. graph twoway (line avg read, sort) (line male read, sort) (line fem read, sort)
```

Assessing Model Fit

- How good a job does the model do of predicting outcomes?
- General answer is “hits and misses”
  - What percent of the observations the model correctly predicts
- How to calculate:
  - Use model to generate the probability p that each observation will have Y=1
    - If $p \geq 0.5$, predict Y=1
    - If $p < 0.5$, predict Y=0
  - Check predictions against the actual outcomes in the data

Assessing Model Fit

- Can do this by checking predictions
  - Events that happened that were predicted to happen
    - E.g., model correctly predicts honors
  - Events that didn't happen that were predicted not to happen
    - E.g., model correctly predict no honors
- Or can go the other way around
  - The probability of a positive prediction given honors
    - This is the model's <u>sensitivity</u>
  - The probability of a negative prediction given

Example 4: Categorical and continuous independent variables

```
lstat

Logistic model for honors

              -------- True --------
Classified |         D            ~D  |      Total
-----------+--------------------------+-----------
     +     |        31            12  |         43
     -     |        22           135  |        157
-----------+--------------------------+-----------
   Total   |        53           147  |        200

Classified + if predicted Pr(D) >= .5
True D defined as honors ~= 0
--------------------------------------------------
Sensitivity                     Pr( +| D)   58.49%
Specificity                     Pr( -|~D)   91.84%
Positive predictive value       Pr( D| +)   72.09%
Negative predictive value       Pr(~D| -)   85.99%
--------------------------------------------------
False + rate for true ~D        Pr( +|~D)    8.16%
False - rate for true D         Pr( -| D)   41.51%
False + rate for classified +   Pr(~D| +)   27.91%
False - rate for classified -   Pr( D| -)   14.01%
--------------------------------------------------
Correctly classified                        83.00%
--------------------------------------------------
```

Definition of D as student getting honors

Example 4: Categorical and continuous independent variables

```
lstat

Logistic model for honors

              -------- True --------
Classified |         D            ~D  |      Total
-----------+--------------------------+-----------
     +     |        31            12  |         43
     -     |        22           135  |        157
-----------+--------------------------+-----------
   Total   |        53           147  |        200

Classified + if predicted Pr(D) >= .5
True D defined as honors ~= 0
--------------------------------------------------
Sensitivity                     Pr( +| D)   58.49%
Specificity                     Pr( -|~D)   91.84%
Positive predictive value       Pr( D| +)   72.09%
Negative predictive value       Pr(~D| -)   85.99%
--------------------------------------------------
False + rate for true ~D        Pr( +|~D)    8.16%
False - rate for true D         Pr( -| D)   41.51%
False + rate for classified +   Pr(~D| +)   27.91%
False - rate for classified -   Pr( D| -)   14.01%
--------------------------------------------------
Correctly classified                        83.00%
--------------------------------------------------
```

Summary of correct predictions

Example 4: Categorical and continuous independent variables

```
lstat

Logistic model for honors

              -------- True --------
Classified |         D            ~D  |      Total
-----------+--------------------------+-----------
     +     |        31            12  |         43
     -     |        22           135  |        157
-----------+--------------------------+-----------
   Total   |        53           147  |        200

Classified + if predicted Pr(D) >= .5
True D defined as honors ~= 0
--------------------------------------------------
Sensitivity                     Pr( +| D)   58.49%
Specificity                     Pr( -|~D)   91.84%
Positive predictive value       Pr( D| +)   72.09%
Negative predictive value       Pr(~D| -)   85.99%
--------------------------------------------------
False + rate for true ~D        Pr( +|~D)    8.16%
False - rate for true D         Pr( -| D)   41.51%
False + rate for classified +   Pr(~D| +)   27.91%
False - rate for classified -   Pr( D| -)   14.01%
--------------------------------------------------
Correctly classified                        83.00%
--------------------------------------------------
```

Summary of correct predictions

Summary of incorrect predictions

Example 4: Categorical and continuous independent variables

```
lstat

Logistic model for honors

              -------- True --------
Classified |         D            ~D  |      Total
-----------+--------------------------+-----------
     +     |        31            12  |         43
     -     |        22           135  |        157
-----------+--------------------------+-----------
   Total   |        53           147  |        200

Classified + if predicted Pr(D) >= .5
True D defined as honors ~= 0
--------------------------------------------------
Sensitivity                     Pr( +| D)   58.49%
Specificity                     Pr( -|~D)   91.84%
Positive predictive value       Pr( D| +)   72.09%
Negative predictive value       Pr(~D| -)   85.99%
--------------------------------------------------
False + rate for true ~D        Pr( +|~D)    8.16%
False - rate for true D         Pr( -| D)   41.51%
False + rate for classified +   Pr(~D| +)   27.91%
False - rate for classified -   Pr( D| -)   14.01%
--------------------------------------------------
Correctly classified                        83.00%
--------------------------------------------------
```

Overall success rate:

(31 + 135) / 200

Example 4: Categorical and continuous independent variables

```
lstat

Logistic model for honors

              -------- True --------
Classified |         D            ~D  |      Total
-----------+--------------------------+-----------
     +     |        31            12  |         43
     -     |        22           135  |        157
-----------+--------------------------+-----------
   Total   |        53           147  |        200

Classified + if predicted Pr(D) >= .5
True D defined as honors ~= 0
--------------------------------------------------
Sensitivity                     Pr( +| D)   58.49%
Specificity                     Pr( -|~D)   91.84%
Positive predictive value       Pr( D| +)   72.09%
Negative predictive value       Pr(~D| -)   85.99%
--------------------------------------------------
False + rate for true ~D        Pr( +|~D)    8.16%
False - rate for true D         Pr( -| D)   41.51%
False + rate for classified +   Pr(~D| +)   27.91%
False - rate for classified -   Pr( D| -)   14.01%
--------------------------------------------------
Correctly classified                        83.00%
--------------------------------------------------
```

Overall success rate:

(31 + 135) / 200 = 83%

Assessing Model Fit

- This is all calculated using 50% as a cutoff point for positive predictions
- But this isn't set in stone; depending on your application, you might want to change it
- You might want to avoid false positives
  - For example, don't convict innocent people
  - Then you would set the cutoff higher than 50%
- Or you might want to avoid false negatives
  - For example, don't report that someone who has a disease is actually healthy
  - Then you would set the cutoff lower than 50%

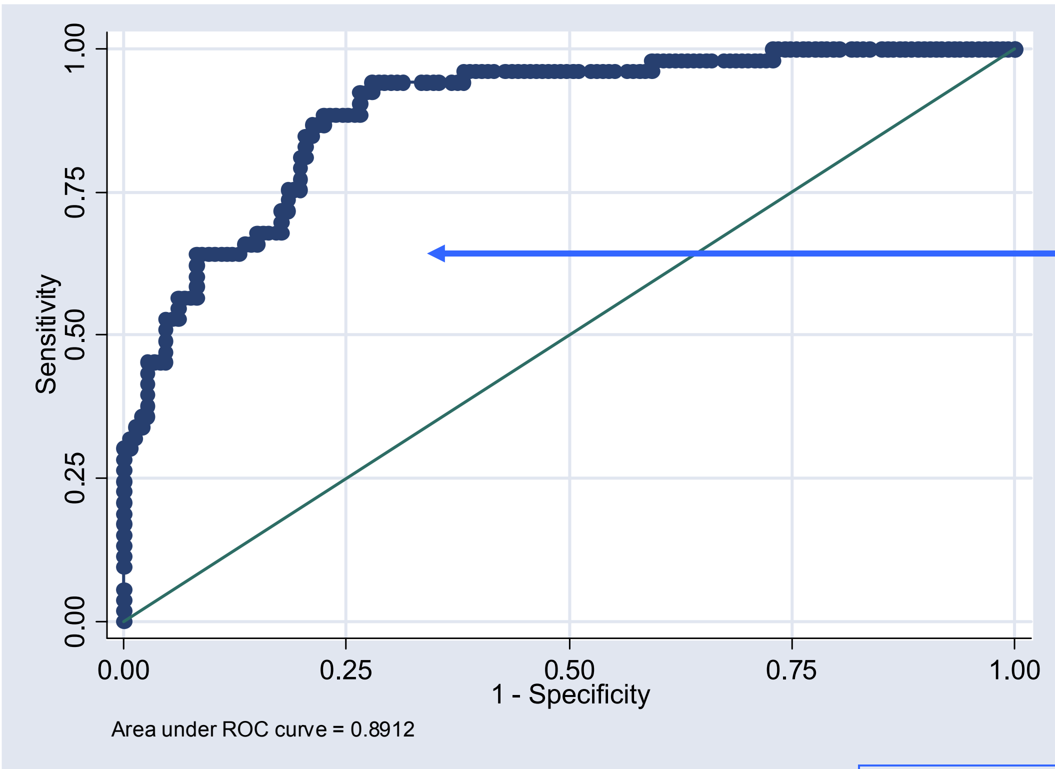
Assessing Model Fit

- We can imagine changing the cutoff point π continuously from 0 to 1
- Recall that
  - Sensitivity = Prob(+ | D)
  - Specificity = Prob (- | ~D)
- At π=0, everything is predicted to be positive
  - That means you will misclassify all the negatives
  - So the sensitivity=1, specificity=0
- At π=1, everything is predicted to be negative
  - That means you will misclassify all the positives
  - So the sensitivity=0, specificity=1

Assessing Model Fit

- In between, you can vary the number of false positives and false negatives
  - If your model does a good job of predicting outcomes, these should be low for all π
- The ROC curve plots the sensitivity and 1-specificity as π goes from 0 to 1
  - The better the model does at predicting, the greater will be the area under the ROC curve
- Produce these with Stata command **"`lroc`"**

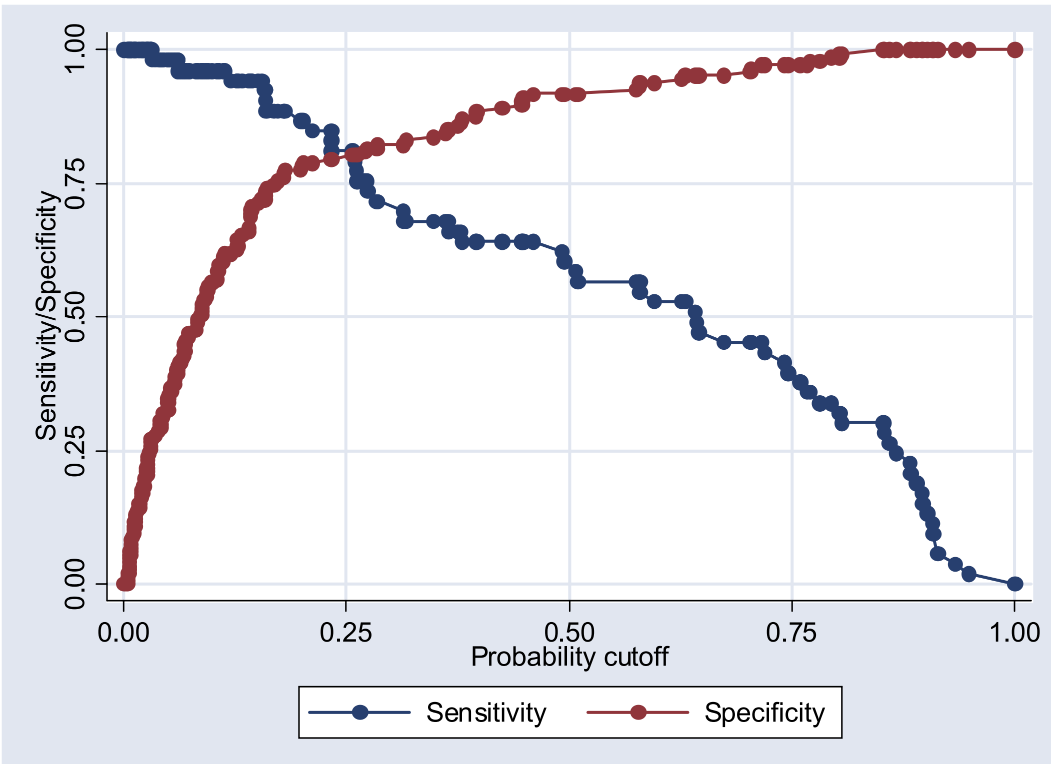
Example 4: Categorical and continuous independent variables



Area under the ROC curve is .8912

```
lroc
Logistic model for honors
number of observations =       200
area under ROC curve   =    0.8912
```

Example 4: Categorical and continuous independent variables



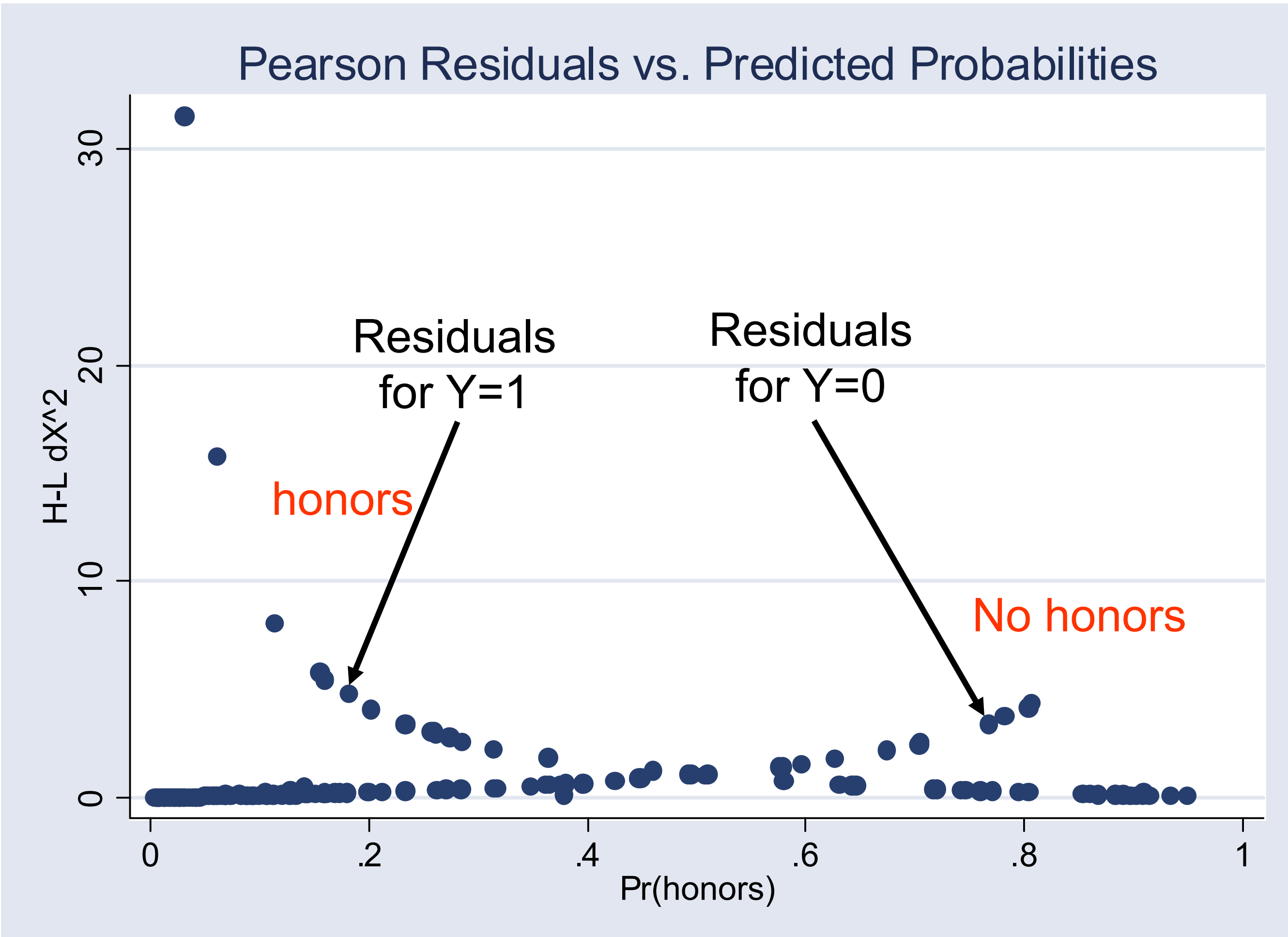

```
. lsens
```

Or, you can use the **"lsens"** function to directly plot the sensitivity and specificity as your cutoff changes from 0 to 1.

Diagnostic Plots

- Can obtain predicted values in the usual way, with command "**`predict p`**"
- Two methods to calculate residuals
  - Pearson residuals: "**`predict x, dx2`**"
  - Deviance residuals: "**`predict z, ddeviance`**"
- Leverage: "**`predict b, dbeta`**"
- Draw the graphs:
  - Pearson residuals vs. predicted probabilities
  - Deviance residuals vs. predicted probabilities
  - Leverage residuals vs. predicted probabilities

Diagnostic Plots



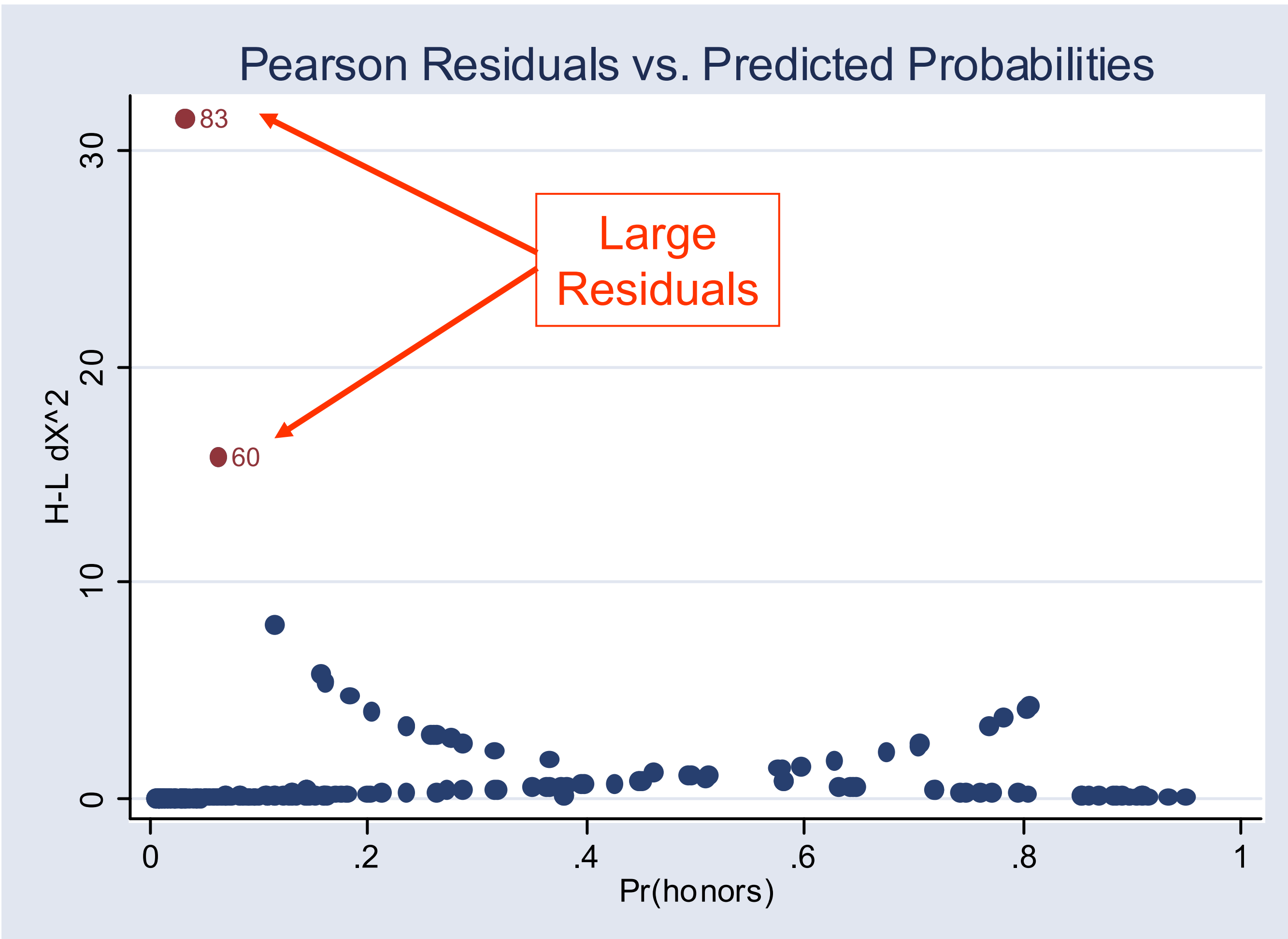
Two distinct patterns of residuals

One for Y=1, the other for Y=0

As with all logits and probits, the residuals are definitely heteroskedastic

```
scatter x p, ti(Pearson Residuals vs. Predicted Probabilities)
```

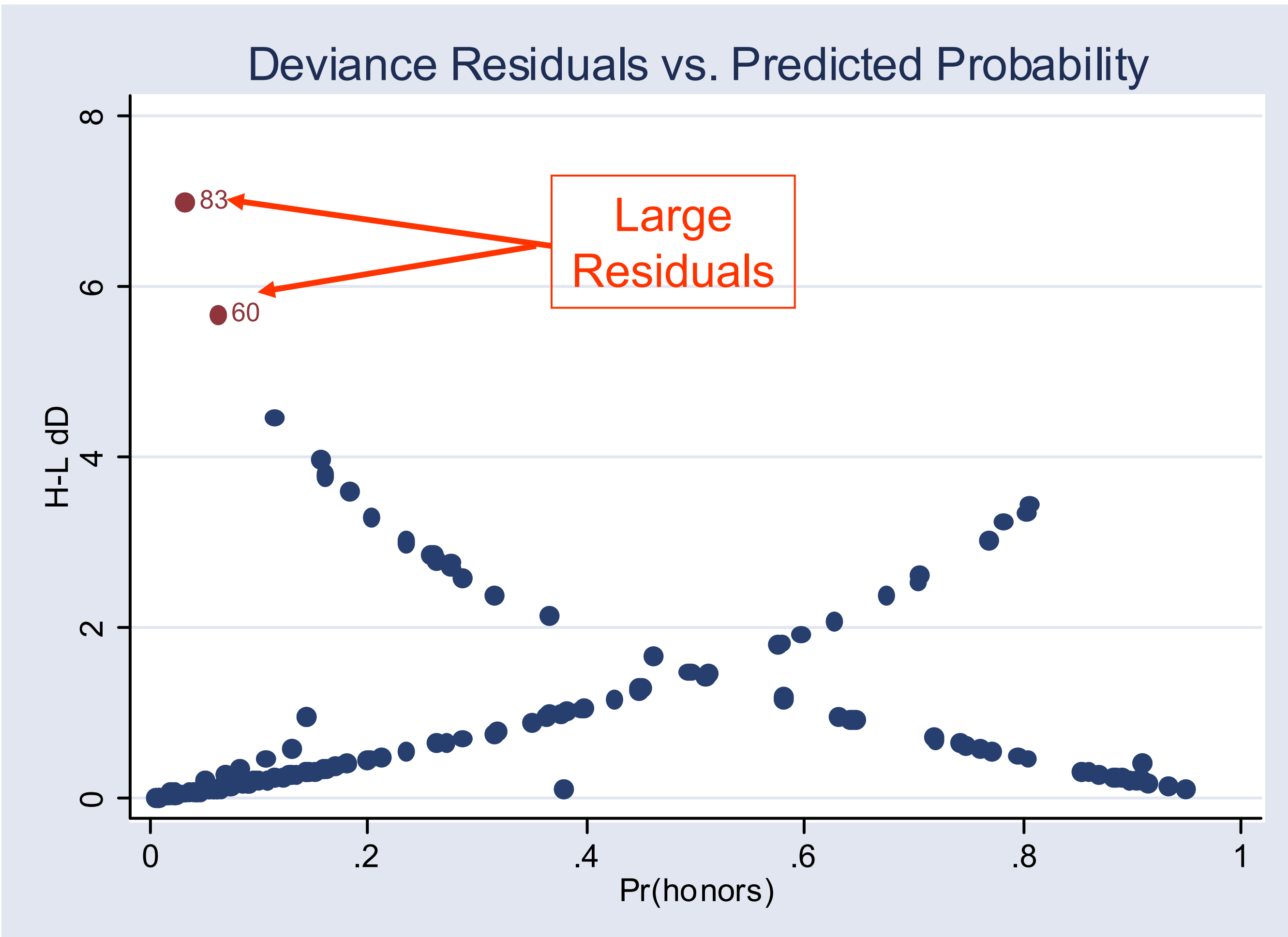
Diagnostic Plots



High residual points were predicted to be Y=0, but got honors anyway

```
scatter x p, ti(Pearson Residuals vs. Predicted Probabilities)
```

Diagnostic Plots

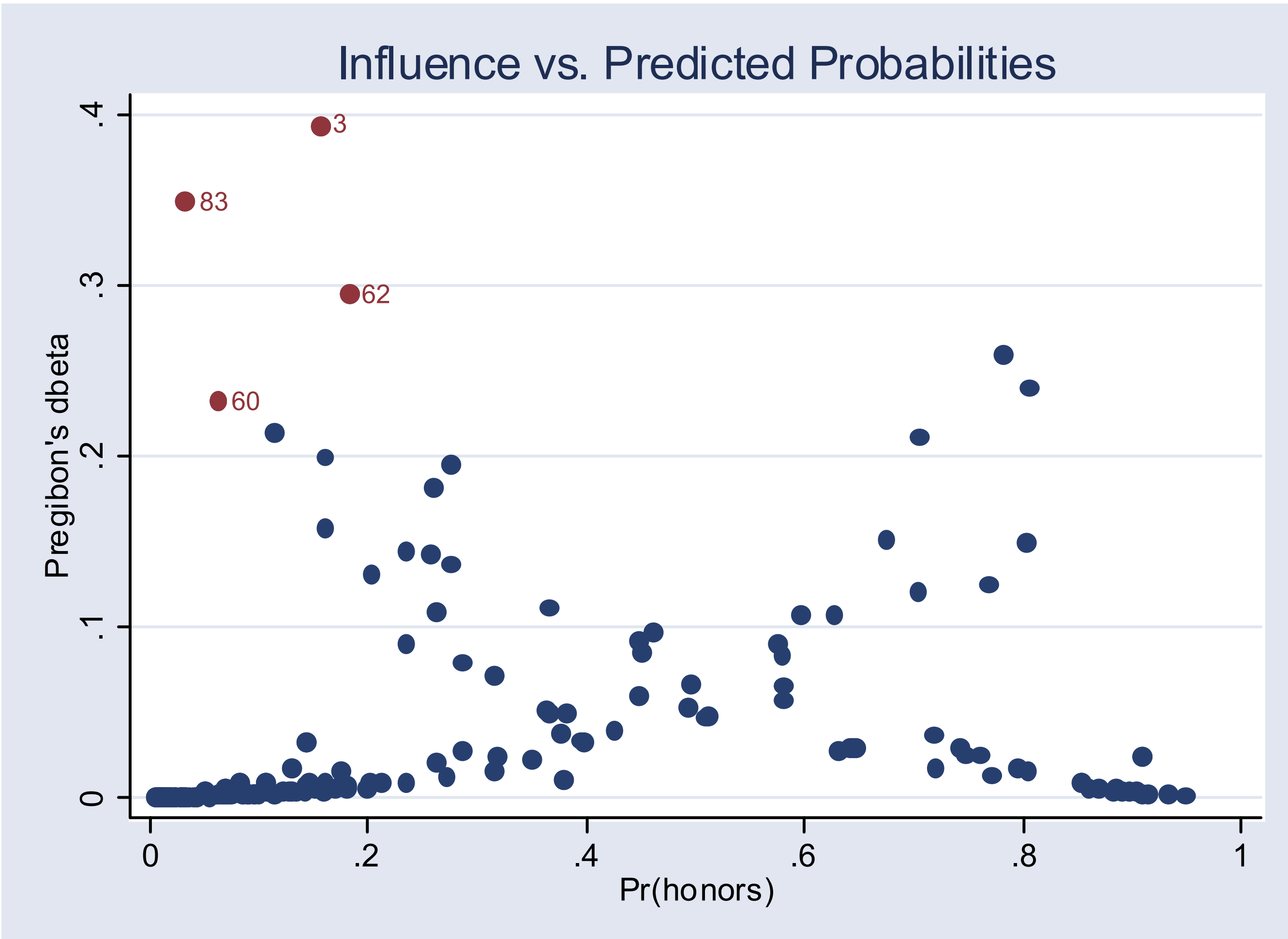


Same pattern as before.

Same two points as outliers

```
scatter x p, ti(Deviance Residuals vs. Predicted Probabilities)
```

Diagnostic Plots

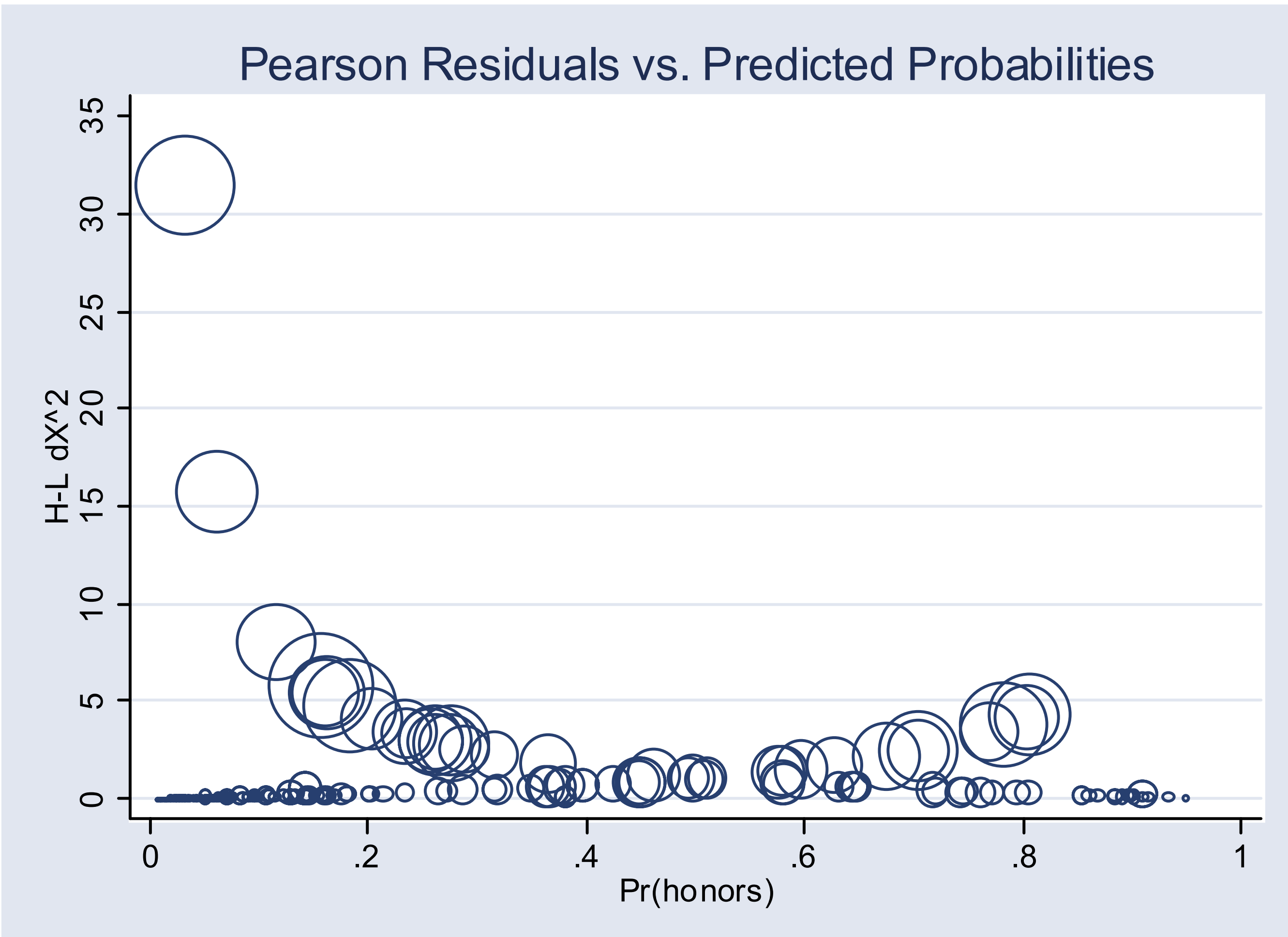


Different points have large influence.

Could eliminate these and see if results change.

```
scatter b p, ti(Influence vs. Predicted Probabilities)
```

Diagnostic Plots



One way to show both residuals and influence on one graph is to weight each residual marker by the value of its influence.

```
scatter x p [weight=b], msymbol(oh) ylab(0 (5) 35)
```

Multinomial Data

- We now move on to study logits when there are more than 2 possible outcomes
- There are two major categories of analysis: ordered and unordered outcomes
- Examples of unordered outcomes
  - Religion: Protestant, Catholic, or other
  - Mode of transportation: bus, car, subway, walking
- Examples of ordered outcomes
  - Regime type: Autocracy, Partial Dem., Full Dem.
  - Socioeconomic status: High, Medium, Low

Unordered Outcomes

- Pick a base category and calculate the odds of the other possible outcomes relative to it
  - For example, say a student can enter a general, vocational, or academic program
  - Use academic as the base category
- Then we will use <u>multinomial logit</u> to estimate
  - Prob(general)/Prob(academic)
  - Prob(vocational)/Prob(academic)
- That is, the probability of choosing general or vocational relative to an academic program

Unordered Outcomes

- Pick a base category and calculate the odds of the other possible outcomes relative to it
  - For example, say a student can enter a general, vocational, or academic program
  - Use academic as the base category
- Then we will use <u>multinomial logit</u> to estimate
  - Prob(general)/Prob(academic)
  - Prob(vocational)/Prob(academic)

  Two separate regressions
- That is, the probability of choosing general or vocational relative to an academic program

Unordered Outcomes

- Can interpret the results from a multinomial logit as relative risk ratios (RRR)

$$\text{RRR} = \frac{\text{P}(y = 1 \,|\, x + 1) \,/\, \text{P}(y = \text{base category} \,|\, x + 1)}{\text{P}(y = 1 \,|\, x) \,/\, \text{P}(y = \text{base category} \,|\, x)}$$

- Or they can be interpreted as Conditional Odds Ratios

$$\text{COR}_1 = \frac{\text{odds}(y = 1 \,|\, x + 1 \text{ and } (y = 1 \text{ or } y = \text{base category}))}{\text{odds}(y = 1 \,|\, x \text{ and } (y = 1 \text{ or } y = \text{base category}))}$$

$$\text{COR}_2 = \frac{\text{odds}(y = 2 \,|\, x + 1 \text{ and } (y = 1 \text{ or } y = \text{base category}))}{\text{odds}(y = 1 \,|\, x \text{ and } (y = 1 \text{ or } y = \text{base category}))}$$

Multinomial Logit Example

```
. mlogit prog female math socst

Multinomial logistic regression                   Number of obs   =        200
                                                  LR chi2(6)      =      65.51
                                                  Prob > chi2     =     0.0000
Log likelihood = -171.34162                       Pseudo R2       =     0.1605

------------------------------------------------------------------------------
        prog |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
general      |
      female |  -.0840263   .3806826    -0.22   0.825    -.8301505    .6620979
        math |  -.0739045   .0254512    -2.90   0.004    -.1237879   -.0240211
       socst |  -.0370939   .0217034    -1.71   0.087    -.0796319    .0054441
       _cons |   5.130723   1.392646     3.68   0.000     2.401188    7.860258
-------------+----------------------------------------------------------------
vocation     |
      female |  -.0177488   .4085162    -0.04   0.965    -.8184258    .7829282
        math |  -.1127775   .0289322    -3.90   0.000    -.1694836   -.0560714
       socst |   -.079675   .0227946    -3.50   0.000    -.1243516   -.0349984
       _cons |   9.106635   1.545711     5.89   0.000     6.077098    12.13617
------------------------------------------------------------------------------
(Outcome prog==academic is the comparison group)
```

Multinomial Logit Example

```
. mlogit, rrr

Multinomial logistic regression                   Number of obs   =        200
                                                  LR chi2(6)      =      65.51
                                                  Prob > chi2     =     0.0000
Log likelihood = -171.34162                       Pseudo R2       =     0.1605

------------------------------------------------------------------------------
        prog |        RRR   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
general      |
      female |   .9194071   .3500023    -0.22   0.825     .4359837    1.938856
        math |   .9287604    .023638    -2.90   0.004     .8835673    .9762651
       socst |   .9635856   .0209131    -1.71   0.087     .9234562    1.005459
-------------+----------------------------------------------------------------
vocation     |
      female |   .9824078   .4013295    -0.04   0.965     .4411255     2.18787
        math |   .8933494   .0258466    -3.90   0.000     .8441006    .9454716
       socst |   .9234164   .0210489    -3.50   0.000     .8830693    .9656069
------------------------------------------------------------------------------
(Outcome prog==academic is the comparison group)
```

Same results, but with RRR interpretation

Multinomial Logit Example

```
. listcoef

mlogit (N=200): Factor Change in the Odds of prog

Variable: female (sd=.4992205)

    Odds comparing|
Group 1 vs Group 2|      b         z     P>|z|     e^b   e^bStdX
------------------+----------------------------------------------
general -vocation |  -0.06628   -0.155   0.877   0.9359   0.9675
general -academic |  -0.08403   -0.221   0.825   0.9194   0.9589
vocation-general  |   0.06628    0.155   0.877   1.0685   1.0336
vocation-academic |  -0.01775   -0.043   0.965   0.9824   0.9912
academic-general  |   0.08403    0.221   0.825   1.0877   1.0428
academic-vocation |   0.01775    0.043   0.965   1.0179   1.0089
-----------------------------------------------------------------

(similar results for other two independent variables omitted)
```

"listcoef" gives all the relevant comparisons
Also gives p-values and exponentiated coefficients

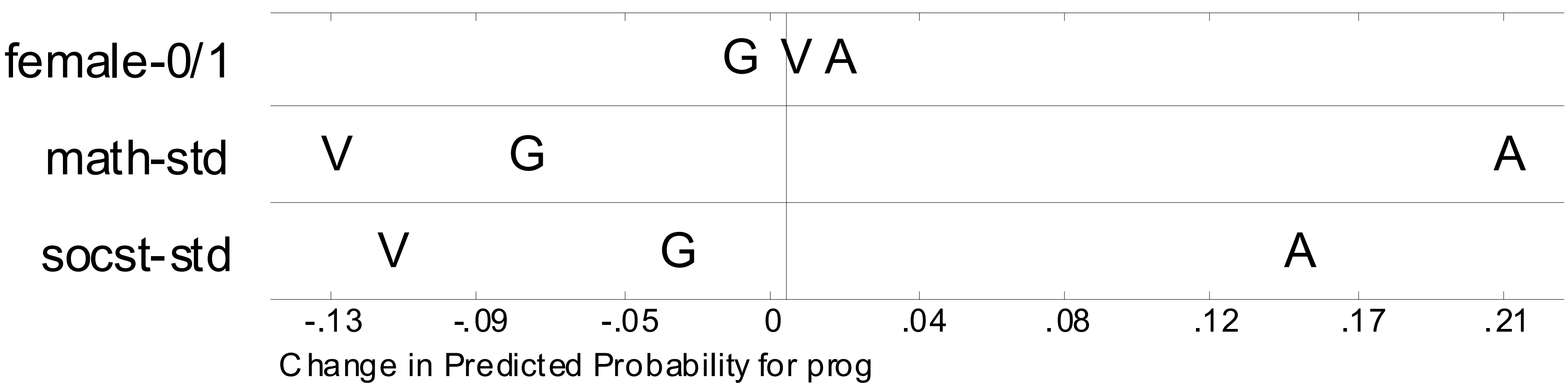
Multinomial Logit Example

```
. prchange

mlogit: Changes in Predicted Probabilities for prog

female
             Avg|Chg|     general    vocation    academic
    0->1     .0101265  -.01518974   .00147069   .01371908

math
             Avg|Chg|     general    vocation    academic
Min->Max    .49023263  -.23754089  -.49780805   .73534894
   -+1/2    .01500345   -.0083954  -.01410978   .02250516
  -+sd/2    .13860906  -.07673311  -.13118048   .20791358
MargEfct    .01500588  -.00839781  -.01411102   .02250882

(socst omitted)

            general   vocation   academic
Pr(y|x)  .25754365  .19741122  .54504514

          female      math     socst
    x=      .545    52.645    52.405
sd(x)=    .49922   9.36845   10.7358
```

“**prchange**” gives the probability changes directly

Multinomial Logit Example

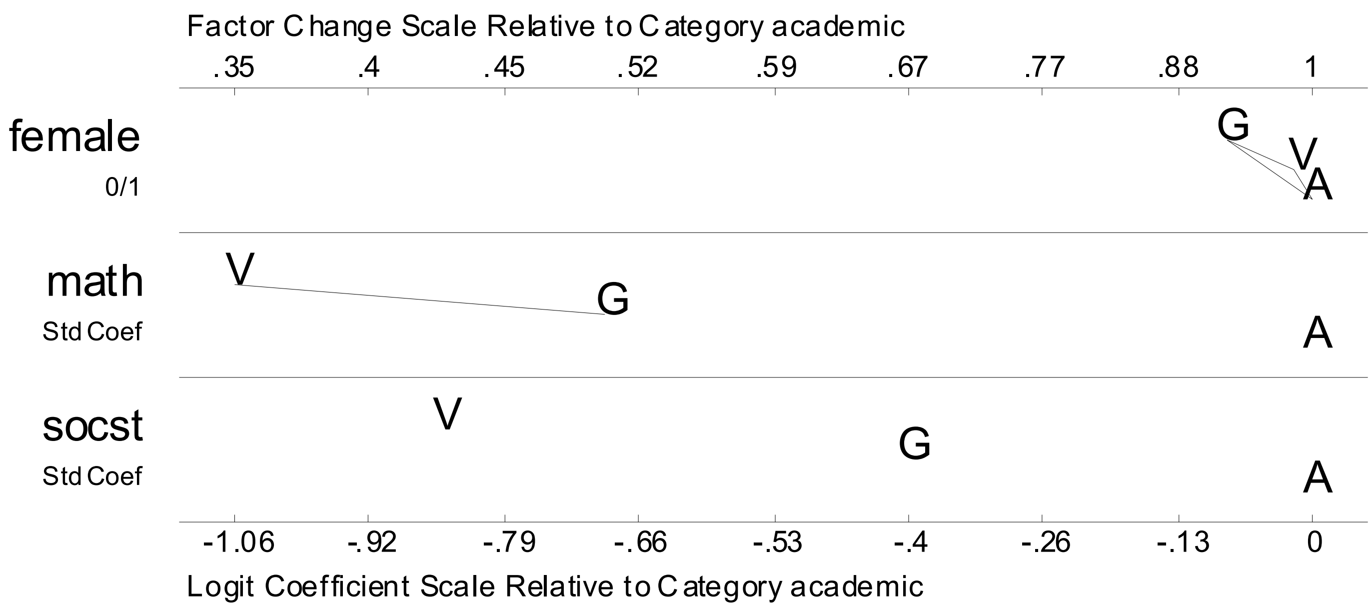
- Stata's **"`mlogplot`"** illustrates the impact of each independent variable on the probabilities of each value of the dependent variable




```
mlogplot  female math socst, std(0ss) p(.1)  dc  ntics(9)
```

Multinomial Logit Example

- Same plot, with odds ratio changes rather than discrete changes




```
mlogplot  female math socst, std(0ss) p(.1) or ntics(9)
```

Multinomial Logit Example

- Use **"prgen"** to show how probabilities change with respect to one variable

```
. mlogit prog math science, nolog

(output omitted)

------------------------------------------------------------------------------
        prog |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
general      |
        math |  -.1352046   .0305449    -4.43   0.000    -.1950716   -.0753376
     science |   .0602744   .0254395     2.37   0.018     .0104139    .1101348
       _cons |   3.166452   1.298818     2.44   0.015     .6208165    5.712088
-------------+----------------------------------------------------------------
vocation     |
        math |  -.1690188   .0331945    -5.09   0.000    -.2340789   -.1039588
     science |   .0170098   .0250403     0.68   0.497    -.0320684    .0660879
       _cons |   7.053851    1.37717     5.12   0.000     4.354647    9.753055
------------------------------------------------------------------------------
(Outcome prog==academic is the comparison group)

. prgen math, gen(m) x(science=50) from(25) to(75) n(100)
```

Multinomial Logit Example

- Use prgen to show how probabilities change with respect to one variable

```
mlogplot  female math socst, std(0ss) p(.1) or ntics(9)
```