Higher-Dimension Markov Models

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Iron Laws of the Political Science Profession

Tolstoy's Law of Journal Reviews

All good reviews are good in the same way; all bad reviews are bad in different ways.

Corollary

Never revise a paper for a new journal submission in response to comments by a bad reviewer at the previous journal.

The James Bond Law of Previous Literature

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Basics of Markov Models

- Given: a system that can exist in any of a finite number of states in each period.
- Markov models estimate the probabilities π_{ab} of transitions from state a at time $t-1$ to state b at time t.

$$\begin{pmatrix} \pi_{11} & \pi_{12} & \cdots & \pi_{1N} \\ \pi_{21} & \pi_{22} & \cdots & \pi_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ \pi_{N1} & \pi_{N2} & \cdots & \pi_{NN} \end{pmatrix}$$

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Typical Uses

1. International Relations: Friendly ↔ War
2. Election Challenger: Unchallenged ↔ Challenger
3. Transitions: Autocracy ↔ Democracy

- This is the simplest 2x2 case
- But one can imagine higher dimension categories...

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- Previous implementations in political science include:
  1. Dean and Moran (1977)
  2. Jones, Kim and Starz (2005)
  3. Walker (2005)
  4. Others??

Models of Transition Dynamics

Given C categories for the dependent variable, let π_{ab} be the transition probabilities from state a to state b be, where $0 \leq a, b \leq C-1$ and $\sum_b \pi_{ab} = 1$.

The simplest Markov process consists of a two-state system:

$$\begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{pmatrix} \pi_{00} & \pi_{01} \\ \pi_{10} & \pi_{11} \end{pmatrix} \end{array}.$$

Using a logit link, this two-state case could be estimated by a single regression:

$$\Pr(Y_t = 1) = \text{logit}(X_{t-1}\beta).$$

Problems With the Single-Equation Approach

- This formulation implicitly assumes that the factors moving the state from 0 to 1 are equal and opposite from those that move it from 1 to 0.
- In many substantive applications, we would not wish to assume this *a priori*:
  - Religious factionalization starts ethnic wars; international intervention stops them
  - A bad economy makes people want to elect Democrats; war makes them want to elect Republicans
  - Good economic conditions foster transitions out of autocracy; group-based politics and violence trigger reversals to autocracy

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The Markov Approach to Transition Dynamics

The Markov approach is to estimate the 2x2 system by a pair of logit regressions, each depending explicitly on the prior state of the system:

$$\begin{aligned} \Pr(Y_t = 1 | Y_{t-1} = 0) &= \text{logit}(X_{t-1}\beta) \\ \Pr(Y_t = 1 | Y_{t-1} = 1) &= \text{logit}(X_{t-1}\alpha) \end{aligned}$$

which can be written more compactly as

$$\Pr(Y_t = 1) = \text{logit}(X_{t-1}\beta + Y_{t-1}X_{t-1}\gamma)$$

where

$$\gamma = \alpha - \beta.$$

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Higher-Dimension Processes

For the 3-state case we want to estimate the probabilities π_{ab} in the transition matrix

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We could run nine regular logits for each entry in the matrix; this is known as the "fully saturated" model.

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Cumulative Probabilities

First, it is easier to work with *cumulative* transition probabilities: $Y_a^* = 1$ if $Y \leq a$.

Given the cumulative probabilities, we can recover the cell probabilities since $\Pr(Y \leq a) = \Pr(Y \leq a-1) + \Pr(Y = a)$.

In the 3-state case the translation from Y to Y^* is:

Y:	0	1	2
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Note that $Y_2^* = 1$.

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Example

As a simple example, the log-odds model of cumulative probabilities is:

$$\text{logit}\,\Pr(Y \leq a) = \log \frac{\Pr(Y \leq a)}{\Pr(Y > a)} = \theta_a + X\beta.$$

If $X = 0$, then $\Pr(Y \leq a) = e^{\theta_a}/(1 + e^{\theta_a})$, which is non-decreasing in a, so $\theta_0 \leq \theta_1 \leq \ldots \leq \theta_{C-2}$.

If $\theta_a = \theta_{a+1}$, then $\Pr(Y \leq a) = \Pr(Y \leq a + 1)$, and categories a and $a + 1$ can therefore be collapsed.

Combining Equations

Second, we can run each column of the matrix as a single estimation equation, as in the 2x2 case.

Assume that for any given a, the model to be estimated is

$$\Pr(Y_t = b | Y_{t-1} = a) = \text{logit}\left(\theta_{ab} + X\beta_a\right)$$

Then we can write:

$$\Pr(Y_t = b) = \text{logit}\left(X_{t-1}\beta + \sum_a Y^*_{at-1} X_{t-1}\gamma_a\right)$$

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Example

Say we have three states and one independent variable. Then we estimate:

$$\Pr(Y_{it} = b) = \beta_0 + \beta_1 y_0^* + \beta_2 y_1^* + \gamma_0 X + \gamma_1 X y_0^* + \gamma_2 X y_1^*$$

Now if $X = 0$, then

$$\begin{aligned} \Pr(Y_t = b|Y_{t-1} = 2) &= \beta_0 \\ \Pr(Y_t = b|Y_{t-1} = 1) &= \beta_0 + \beta_2 \\ \Pr(Y_t = b|Y_{t-1} = 0) &= \beta_0 + \beta_2 + \beta_1 \end{aligned}$$

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Interpretation of Interaction Terms

To summarize, for any given independent variable X:

- The γ_0 coefficient on the un-interacted X term gives the impact of X_{t-1} on Y_t when $Y_{t-1} = Y_{C-1}$, the "last" category of Y.
- The γ_a coefficients on the interaction terms Xy_a^* give the *differential* impact of X_{t-1} on Y_t between $Y_{t-1} = Y_{a+1}$ and $Y_{t-1} = Y_a$.
- The cell probabilities giving the impact of X on $\Pr(Y_t = b|Y_{t-1} = a)$ when $a < C - 1$ can be recovered as the sums of the γ coefficients, in the order $\gamma_0 + \gamma_{C-1} + \gamma_{C-2} + \ldots + \gamma_{a+1}$.

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Insignificant values of coefficients on interactions of X with Y_a^* mean that X has a similar impact on Y for categories a and $a+1$, so we can collapse those categories in the analysis.

We can then test down to a more parsimonious model, eliminating some of the interactive terms.

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Summary

Key points in higher-dimension Markov models:

- Work with cumulative probabilities.
- Combine cases for transitions to state b using interactions with the y^* terms.
- Start with the saturated model with all interactive terms and test down.

To overcome small-N cell problems, you can also combine the analysis for all values of b into a single ordered probit/logit.

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Application: Democratic Transitions

- Modernization theory (Lipset 1959) says that as countries get richer, they get democratic.
- This was always thought of as a causal relationship, although there has always been a dispute about the mechanics.
- Przeworski, et. al. ("PACL" 2000) challenge this, saying that the process could be:
  1. Countries become democratic randomly.
     Once there, higher GDP per capita helps keep them there.
- So a GDP-democracy relationship could develop, even though modernization doesn't *cause* democracy.

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Application: Democratic Transitions

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- This was always thought of as a causal relationship, although there has always been a dispute about the mechanics.
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Results from PACL Table 2.12

Indep. Var.	$D \rightarrow A$	$A \rightarrow D$ (Original)	$A \rightarrow D$ (Corrected)
Constant	-1.144** (0.000)	-2.524** (0.000)	-2.524** (0.000)
GDP	-0.201 (0.162)	0.329 (0.484)	0.329** (0.004)
GDP^2	-0.003 (0.874)	-0.029 (0.191)	-0.029 (0.069)
GDP Growth	-0.042** (0.003)	-0.021** (0.000)	-0.021* (0.015)

PACL's Extended Transition Model

- PACL also run a regression adding a number of covariates, but without GDP^2.
- Actual results here are more favorable to their hypothesis:
  - GDP helps keep democracies from backsliding, but has no effect on autocracy.
  - These results are somewhat fragile to specification.
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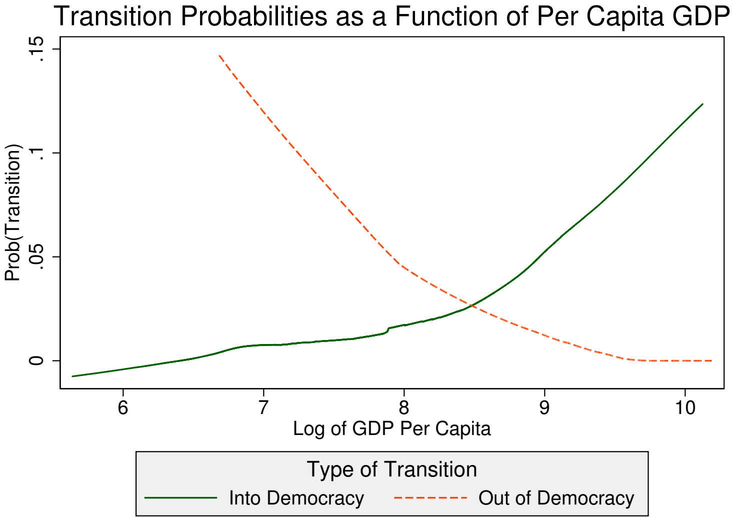
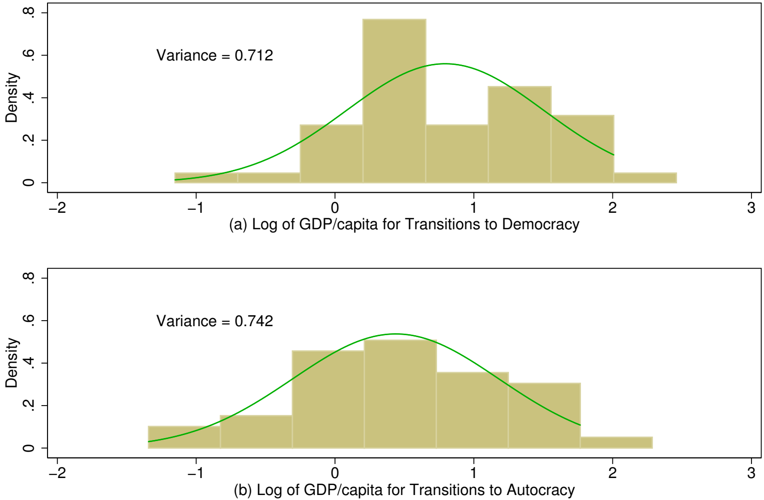
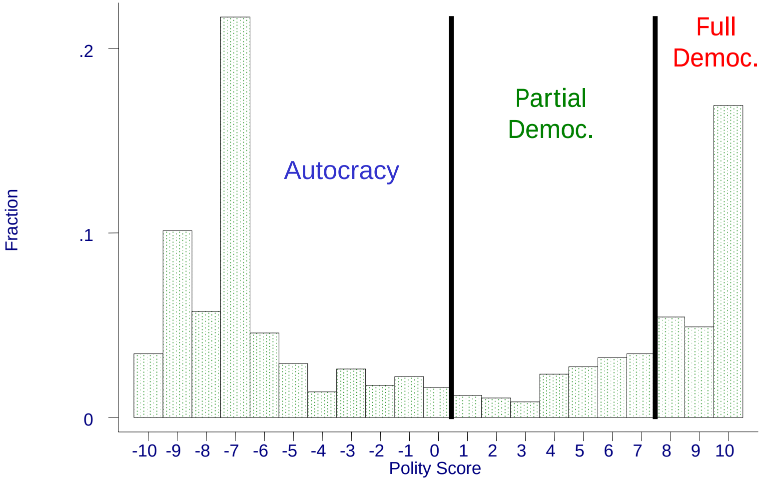
Impact of GDP on Transition Probabilities



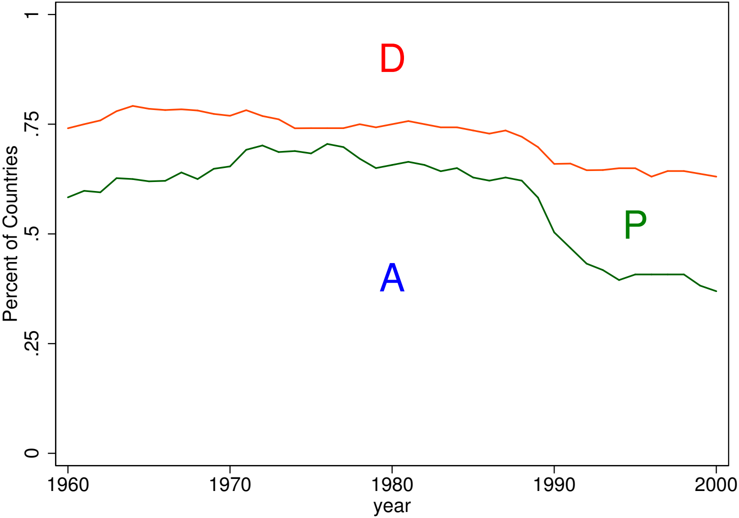
Distribution of GDP for Transition Years



Definition of Partial Democracy



World Democratization Trends, 1960-2000



Transition Marginals

	Current Year		
Previous Year	Autocracy	Partial Democracy	Democracy
Autocracy	97.3%	2.1%	0.7%
	(3,121)	(66)	(22)
Partial Democracy	6.4%	90.4%	3.3%
	(49)	(695)	(25)
Democracy	1.1%	0.8%	98.2%
	(16)	(12)	(1,496)
Total	3,186	773	1,543

Note: Numbers in parentheses are cell counts.

Summary Statistics

Variable	Mean	Std. Dev.	Min.	Max.	N
Polity Score	-0.45	7.58	-10	10	5671
Regime Category	0.70	0.88	0	2	5671
Log of Per Capita GDP	8.14	1.04	5.64	10.21	4417
Percent Change in GDP	0.02	0.06	-0.52	1.01	4475
Percent Urban Pop.	44.94	24.29	2.3	100	5245
Log of Population Density	3.61	1.46	-0.49	8.77	5600
Log of Trade Openness	3.98	0.62	0.43	6.16	4902
Previous Transitions	3.96	6.41	0	31	5671
Resource Curse	0.23	0.42	0	1	5671

Regression Results

Adding Partial Autocracies

	Polity Range
Autocracy	(-10,-7)
Partial Aut.	(-6,0)
Partial Dem.	(1,7)
Democracy	(8,10)

Check to see if we should split the autocracies as well.

Adding Partial Autocracies

			Polity Range
		Autocracy	(-10,-7)
Y_0^*	$\longrightarrow$		
		Partial Aut.	(-6,0)
Y_1^*	$\longrightarrow$		
		Partial Dem.	(1,7)
Y_2^*	$\longrightarrow$		
		Democracy	(8,10)

Use the Y^* variables to test for collapsing adjacent categories.

Adding Partial Autocracies

			Polity Range
		Autocracy	(-10,-7)
		$\updownarrow$	
		Partial Aut.	(-6,0)
Y_1^*	$\longrightarrow$		
		Partial Dem.	(1,7)
Y_2^*	$\longrightarrow$		
		Democracy	(8,10)

Only Y_0^* is insignificant, lending support to our three-way classification vs. four-way classification with partial autocracies.

Comparison with PACL

Autocracy

Partial Dem. a

Partial Dem. b

Democracy

Split our partial democracies into PACL autocracies (a) and PACL democracies (b).

Comparison with PACL

Autocracy
$\updownarrow$ PACL
Partial Dem. a

Partial Dem. b
$\updownarrow$ PACL
Democracy

PACL collapse the partial democracies into the full autocracies and full democracies.

Comparison with PACL

Autocracy

Partial Dem. a

$\updownarrow$ O'Epstein

Partial Dem. b

Democracy

We collapse the partial democracies into a single category.

Comparison with PACL

Autocracy

$Y_0^* \longrightarrow$

Partial Dem. a

$Y_1^* \longrightarrow$

Partial Dem. b

$Y_2^* \longrightarrow$

Democracy

Again use the Y^* variables to discriminate.

Comparison with PACL

Autocracy

$Y_0^* \quad \longrightarrow$

Partial Dem. a

$\updownarrow$

Partial Dem. b

$Y_2^* \quad \longrightarrow$

Democracy

Only Y_1^* is insignificant, lending support to our three-way classification vs. PACL's dichotomous classification.