

The 45% Solution: Racial Gerrymandering and Representative Democracy*

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Abstract

We present a model of optimal redistricting schemes to promote minority interests, incorporating both electoral and legislative stages. In the model, after districting, candidates from majority and minority groups run for office in each district. The election winners then form a legislature, which in turn passes a redistributive policy. The results show that minorities with relatively little political power prefer to concentrate their voters in a few districts, while more powerful minorities do best by distributing their voters more evenly across districts. Furthermore, declining majority racism has two effects on minorities: it helps them by making it easier to elected minorities to office, but it may also hurt them by making majority voters more pivotal and therefore increasing their relative power at minorities' expense. In addition, the impact of adding more minority voters to a given district is non-monotonic, and in some cases can have the perverse effect of electing a candidate less favored by the minority community.

1 Introduction

For political minorities to exercise influence over policy in a majoritarian system, they must, perforce, form coalitions. These coalitions can be constructed at the electoral stage, by working through political parties or peak organizations that amalgamate a variety of interests, or at the legislative stage, through vote trading and logrolling with legislators representing other groups. At times, one of these strategies may be more effective than the other.

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Polarized voting and sharp policy differences among subgroups of voters may make electoral coalitions harder to negotiate and maintain, for example, or ideological divisions in the legislature may make cross-party policy coalitions rare and ineffective.

Furthermore, electoral institutions can play an important role in favoring one type of coalition building over the other. The classic debate over consociational democracy (Lijphardt 1969, Horowitz 1985) is in this vein, asking whether in divided societies it is best to have broad parties that coalesce interests, or parliamentary systems that allow each group to elect representatives from their caste/tribe/subgroup and move the locus of debate off the street and into the legislature. The same clearly describes the debate over two-party vs. multi-party systems; the former creates broad parties with less legislative vote trading, while the latter has more, narrow parties that form policy coalitions in the parliament. And districting schemes, the topic of this paper, can spread minority voters across many districts, so as to maximize the number of elected officials on which they can exert direct influence via the ballot box, or concentrate minority voters in fewer districts so as to shift the weight of the coalition building to the legislature.

If one were to design political institutions to protect minority interests, then, what institutions would be most appropriate under a given set of circumstances? This theoretical question has been brought into sharp relief by the recent Supreme Court ruling in *Georgia v. Ashcroft*. The question before the Court was whether a proposed redistricting scheme violated Section 5 of the 1965 Voting Rights Act (VRA) just because it “unpacked” minority voters, spreading them out more evenly across districts. The Court ruled that state legislators were attempting to increase blacks’ overall influence on policy, and so the scheme was overall not “retrogressive” even though it might result in fewer minority representatives elected to office.

Although the question of protecting minority rights through political institutions is an important one in much of the comparative, civil liberties, and voting rights literatures, it has received scant theoretical attention. This paper therefore presents a formal model of policy making with a political minority, incorporating both electoral and legislative stages. In the analysis, a districting scheme is implemented which divides minority voters across districts. Candidates from majority and minority groups then present themselves for election in each district, offering platforms designed to maximize the probability of being elected. The winners in each district then form a legislature, which in turn passes a redistributive policy.

Voters’ payoffs are a function of both their redistributive gains and their ideological attachment to the legislator elected from their district. In other contexts, this ideological attachment might stem from a candidate’s expected voting record on issues aligned along a left-right spectrum; here, it can also

encompass the degree of racism or in-group racial preferences across voters. We can thus analyze the impact of changing ideological attachments for voters of one group in relation to candidates from another group; i.e., increasing or declining racism.

The results show that minorities with relatively little political power prefer to concentrate their voters in a few districts and shift the weight of the bargaining problem to the legislature. Conversely, as minorities gain power, they do best by distributing their voters more evenly across districts. Furthermore, declining majority racism has two effects on minorities: it helps them by making it easier to elected minorities to office, but it may also hurt them by making majority voters more pivotal and therefore increasing their relative power at minorities' expense. In addition, the impact of adding more minority voters to a given district is non-monotonic, and in some cases can have the perverse effect of electing a candidate less favored by the minority community.

The next section defines the districting problem in a spatial context and provides some preliminary results. We then develop the formal model in more detail, provide strategies for the players and an objective function for social planners drawing districts, and solve the game. The next section explores the policy implications of our results, analyzing optimal districting schemes under varying circumstances. The final section concludes and compares our results to others in the field.

2 Triangles: Districting Made Simple(x)

We first provide a graphical context in which to analyze districting alternatives. For simplicity, we take the electorate to consist of three groups: White Democrats (WD), Black Democrats (BD) and Republicans (R). These three groups exist in certain proportions in the state overall, and we look at possible districting schemes that divide these voters up into some number of equally-sized districts.

2.1 Districting Alternatives

A valid districting scheme is a matrix, like the one illustrated in Table 1. Each row is a district, giving the number of WD, BD, and R voters (where each of these quantities is, naturally, non-negative). The sum of the entries in each row must equal the total population N divided by the number of districts K . And the sums of the entries in each column are the group populations N_i .

To visualize the alternatives, we use an equilateral triangle, as in Figure 1, which represents the two-dimensional simplex of possible percentages of each group in a given district. The corners thus indicate districts that contain

$$\begin{array}{ccc}
 \left(\begin{array}{ccc}
 WD_1 & BD_1 & R_1 \\
 WD_2 & BD_2 & R_2 \\
 \vdots & \vdots & \vdots \\
 WD_K & BD_K & R_K
 \end{array} \right) & \begin{array}{c} \frac{N}{K} \\ \frac{N}{K} \\ \vdots \\ \frac{N}{K} \end{array} & \\
 N_{WD} & N_{BD} & N_R
 \end{array}$$

Table 1: Sample districting matrix

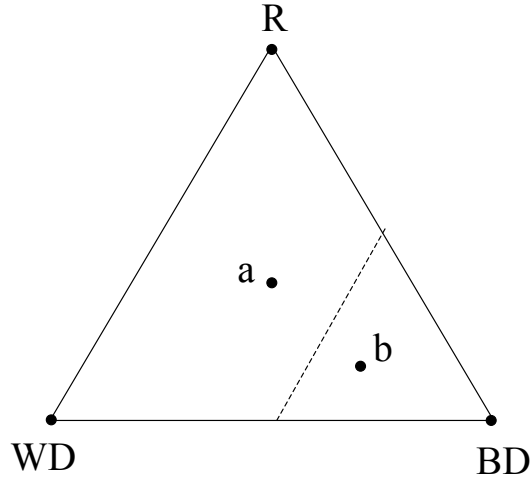


Figure 1: Graphical representation of possible voting districts: a = equal proportions of each type; b = majority-minority district

only one type of voter: WD in the bottom left, BD in the bottom right, and R on top. Point a in the center stands for a district with an equal division of all three types, so that each comprise $1/3$ of the district population. The diagonal line is drawn where the BD voters are $1/2$ of the district, so all points down and to the right of it are majority-minority; one such point is labelled b .

The state as a whole has a given percent of each type of voter, so it can also be represented as a point on the triangle. Then a valid districting scheme is a set of points that average to the state-wide population proportions. Figure 2 illustrates a state with five districts. The statewide distribution of voters is marked with a large dot labelled “S,” while the other five points represent the districts, one of which is majority-minority.

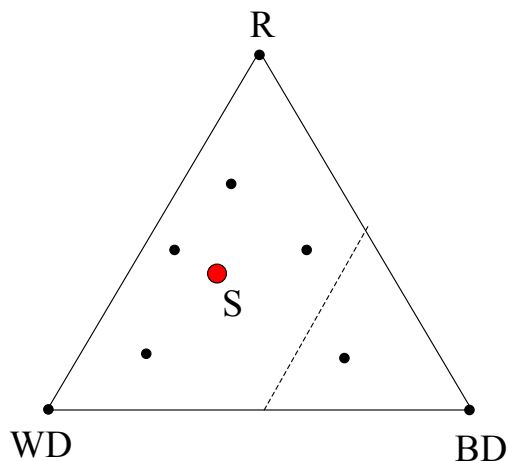


Figure 2: Sample state with five districts, one of which is majority-minority

The figure also gives some hints as to districting strategy. Say for example that at S a white democrat is expected to win. Then by making all districts in a state have demographics equal to S — making them microcosms of the state as a whole — white democrats will be the favorites in all races. But if the probability of success for a white democrat in such a district is close to 50%, then risk-averse legislators might prefer to make some districts heavily republican (toward the top of the triangle), allowing them to move the other districts to safer democratic regions (towards the bottom of the triangle).

Similarly, the requirement that some majority-minority districts be created means that these districts must be located near the bottom right, pushing the other districts up and to the left, possibly increasing the likelihood that Republicans will win elsewhere.

2.2 Voting and Elections

Let us now turn to the question of which type of candidate will win, given district characteristics. We analyze a two-stage electoral cycle, consisting of a primary pitting a White Democratic candidate against a Black Democrat, with the winner facing a Republican in the general election. For the time being, we make the following simplifying assumptions.

1. In the primary election, all BD voters cast their ballots for the BD candidate;
2. In the general election, all BD voters cast their ballots against the R candidate, so they vote for whichever type of Democrat won the primary;

3. In the general election, all R voters cast their ballots for the R candidate;
4. In the primary, a fraction a of WD voters cross over to cast their ballots for the BD candidate, with $0 \leq a \leq \frac{1}{2}$;
5. In a BD vs. R general election, a fraction b of WD voters cast their ballots for the BD candidate, with $a \leq b \leq 1$; and
6. In a BD vs. WD general election, a fraction c of WD voters cast their ballots for the WD candidate, with $b \leq c \leq 1$.

These conditions are fairly natural: the BD and R voters are extreme and so will vote only for candidates of their own party. Furthermore, the BD voters are homogeneous enough that they cross over and vote for WD candidates at a lower rate than WD voters will vote for a black candidate.¹ WD voters in a BD vs. R general election will vote for a BD candidate at a higher rate than they crossed over in the primary, since they in general prefer a WD candidate to a Republican. And WD voters will vote for a WD candidate versus a Republican opponent at a higher rate than they voted for a BD candidate.

Given these assumptions, the BD candidate will win the primary in a district with n_{BD} black democrats and n_{WD} white democrats if:

$$\begin{aligned} n_{BD} + an_{WD} &\geq (1 - a)n_{WD} \\ \frac{n_{BD}}{n_{WD}} &\geq 1 - 2a. \end{aligned} \tag{1}$$

The greater the value of a , the fewer the number of BD voters relative to WD voters are necessary for a BD to win the election. And the critical ratio reaches 0 when $a = 1/2$; if a were any greater, then a BD candidate could win even with no black voters in the district (which would be nice, of course, but this does not yet describe reality).

The set of points in the triangle satisfying Equation 1 is illustrated in Figure 3. In general, this region will be to the right of a line starting at the top apex and going down to the bottom side. At $a = 0$ it will bisect the triangle, and then grow smoothly until it contains the entire triangle at $a = 1/2$.

For the minority candidate to then win the general election against a Republican opponent, normalizing the district size to 1 and using the fact that $n_R = 1 - n_{WD} - n_{BD}$, we need:

¹The results below do not change qualitatively if we allow for black crossover as well; see the analysis in Section 4.3 below.

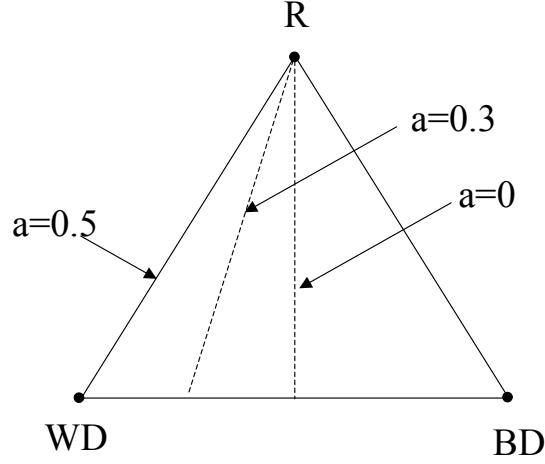


Figure 3: Regions in which minority candidate wins primary, by degree of white crossover (a)

$$\begin{aligned}
 n_{BD} + bn_{WD} &\geq (1 - b)n_{WD} + n_R \\
 n_{BD} &\geq (1 - 2b)n_{WD} + (1 - n_{WD} - n_{BD}) \\
 2n_{BD} &\geq 1 - 2bn_{WD} \\
 n_{BD} + bn_{WD} &\geq 1/2.
 \end{aligned} \tag{2}$$

This denotes a region demarcated by a line starting at the midpoint of the right side of the triangle, where $n_{BD} = n_R = 1/2$. At $b = 1$, the line is horizontal, meaning that black and white democratic voters are perfect substitutes, so republicans can only win if they comprise more than half the district. At $b = 0$, the line goes down to the midpoint of the bottom edge of the triangle, meaning that blacks can only win if they are over half the population. These possibilities, along with $b = 1/2$, are illustrated in Figure 4.

Calculations similar to equation 2 show that a white democrat will win the general election if $n_{BD} + cn_{WD} \geq 1/2$. Combining the primary and general election effects, Figure 5 shows a typical scenario for who wins the overall election, drawn for $a = 0.3$, $b = 0.8$, and $c = 1$. Note the asymmetry on the left-hand side of the figure, due to the fact that it is easier for a white democrat to defeat a republican opponent than it is for a black democrat to win, given greater white support for WD candidates as opposed to BD candidates.

One last issue concerns strategic voting, or lack thereof, in the primary. The assumptions above imply that no one votes strategically; the net crossover is a regardless of what happens in the general election. For most points in

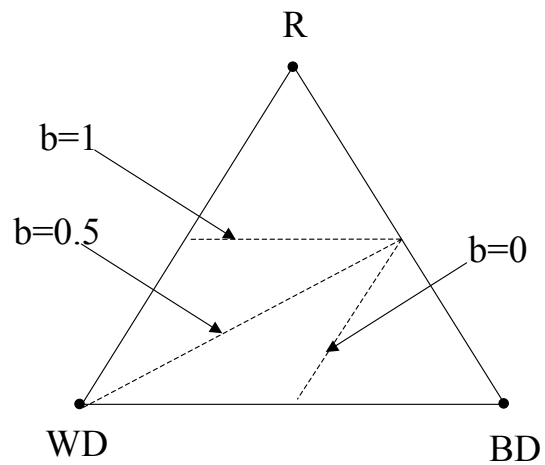


Figure 4: Regions in which minority candidate wins general election, by degree of white crossover (b)

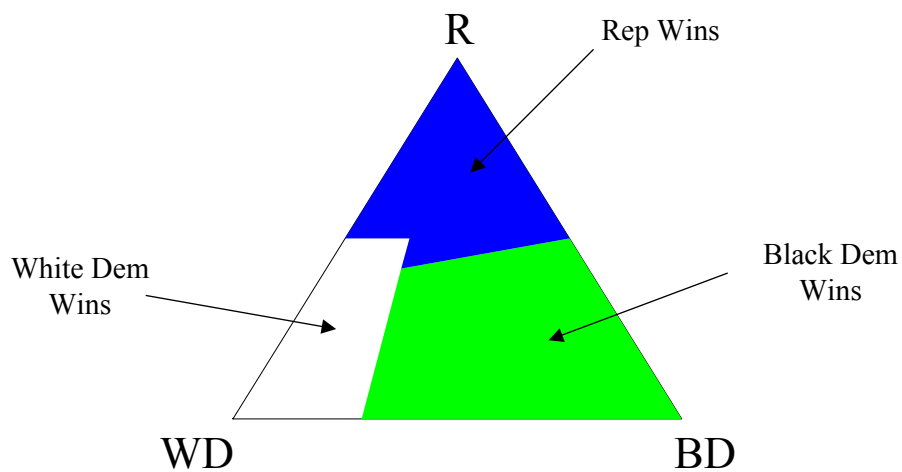


Figure 5: Overall election winners

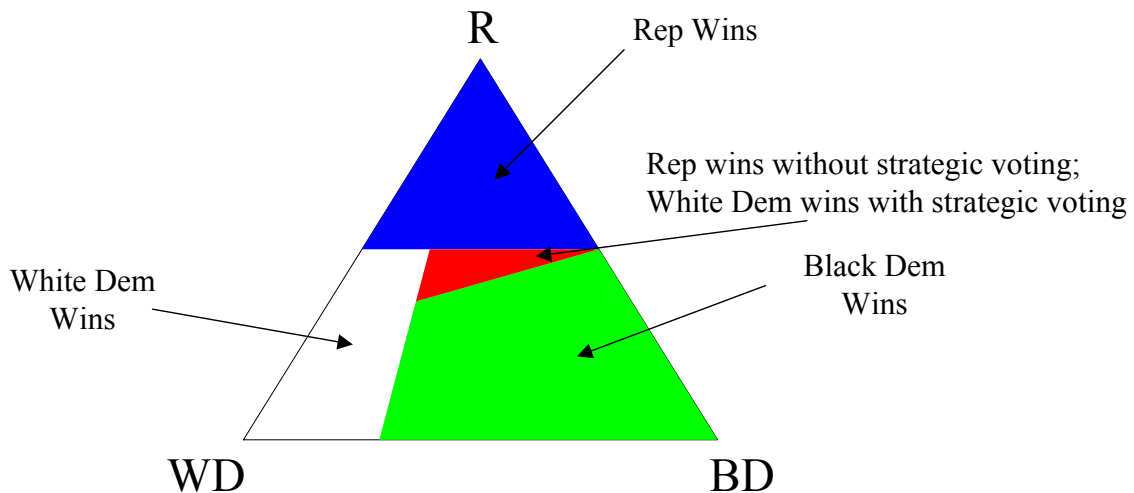


Figure 6: Region in which strategic voting in the primary can affect the election winner

the triangle this makes no difference, but there is a region in which the lack of strategic voting does alter the results, as indicated in Figure 6. For points in the shaded region in the middle of the triangle, naive voting would have a BD winning the primary and then losing to a Republican in the general election. Were the primary voters sophisticated, they would elect a WD instead, who would go on to beat the Republican in the general. This type of sophisticated voting is not much in evidence, so we will assume sincere primary voting hereafter and indicate which of our results depend on this lack of sophistication.

2.3 Some Preliminary Analysis

The triangle diagrams also offer a few interesting and cautionary tales, as further illustrated in Figure 7. One diagonal line, running along a contour of constant n_{BD} , goes from an area where a WD wins, to BD, on to R. So there is more to districting than just specifying the percent of black voters; the composition of the rest of the district matters as well. Even more to the point, adding Republicans to a district can be good for black voters; as the figure shows, this can allow a black democrat to win the primary, and then the general, whereas a white democrat would have won before. So supposedly conservative shifts in districting can aid minority constituents.

The other diagonal line moves straight towards the BD corner, so it represents adding more black voters to a district while keeping the ratio of WD to R voters constant. Again, the impact is non-monotonic. We first move from a situation where a republican wins, to a white democrat, back to a

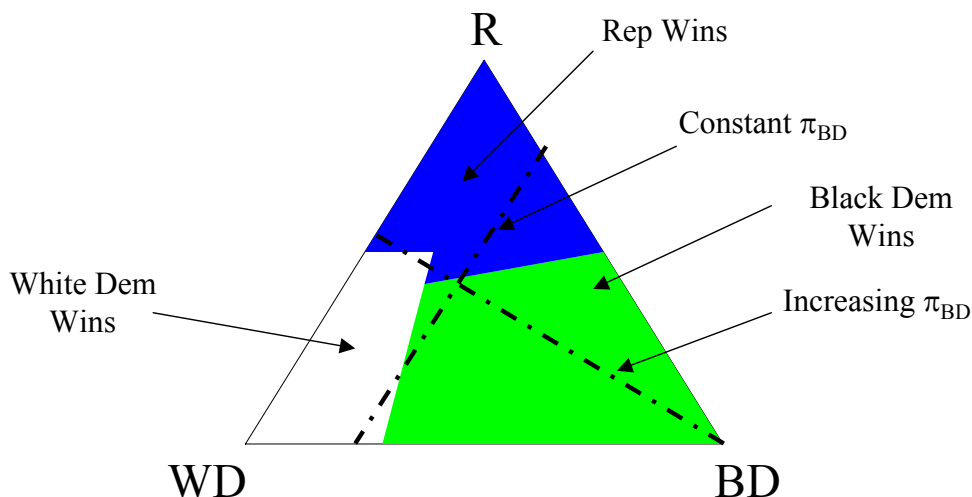


Figure 7: Analysis of election winners

republican, and finally to a black democrat. So a pro-minority shift can have adverse consequences if the entire district composition is not taken into account.²

3 General Model

The triangle plots help us understand the districting problem and some of its subtleties. We now move to a more systematic analysis of redistricting, specifically from the perspective of the policy gains that can accrue to minority voters. The question is, given a state with a certain percentage of BD , WD , and R voters, which districting plans maximize substantive minority representation? Is it better for minorities to have a lot of influence in a few districts, or more modest influence over a wider area? Is the election of black representatives a necessary part of substantive representation, or can black constituents have their policy interests championed by non-minority representatives?

To address these questions, we generalize the triangle analysis above by adapting the Dixit-Londregan (1996) model of electoral competition. In this model, voters have certain ideological attachments to different candidates, and these candidates then compete for office by promising group-specific policy benefits. This model seems well-suited to our purposes: it captures both the fact that voters of one race may prefer representatives of the same race, as well as competition over policy outcomes. As we will see, it allows us

²This non-monotonicity requires some degree of sincere voting; if all voters were sophisticated, then the line could not re-enter the Republican region.

to address the impact of increasing or decreasing racism, greater registration and turnout by minority constituents, and changing partisan attachments in the electorate.

3.1 Districts

Assume a population of voters, represented by a continuum of types X . Within the total population there are a given number of identifiable groups G , possibly divided along ethnic, economic, political, or any other set of dimensions. Thus there is a partition from the set of voters $\{i\}$ to groups, $\nu : i \rightarrow G$.

For simplicity, we assume here a state population divided along racial and partisan lines with voter types $\Theta = \{BD, WD, R\}$, for Black Democrat, White Democrat, and Republican, respectively. Their statewide populations are N_{BD} , N_{WD} , and N_R , with $\sum_i N_i = N$, the total state population. The set of possible population proportions in any given district is the two-dimensional simplex, S^2 .

A district is a vector $\mathbf{d} = \{n_{BD}, n_{WD}, n_R\}$ of voters, $n_i \geq 0$. Let \mathcal{D} be the set of all possible districts, and assume that the state will be divided into K districts, K odd, with n_{ik} representing the number of voters of type i in district k . Then a *districting scheme* is a function $\mathbf{D} : S^2 \rightarrow \mathcal{D}^K$, yielding a vector $\{\mathbf{d}_1, \mathbf{d}_2, \dots, \mathbf{d}_K\}$ of districts. Furthermore, a *valid districting scheme* is a districting scheme such that in any given district, $\sum_i n_{ik} = N/K$, and across districts $\sum_k n_{ik} = N_i$. Equivalently, as in the triangle analysis above, the average of the percentages of each group in the K districts must equal their statewide population proportion N_i/N .

3.2 Candidates and Elections

In each of the K districts there are three candidates competing for a seat in the legislature, and these candidates are also of types BD , WD , and R . Candidates care only about winning office, and they run on platforms consisting of redistributive promises, offering a proportion T_i of the district's redistributive benefits to voters of type i . Denote the redistributive platform of candidate j 's party towards group i in district k as T_{ijk} , so that campaign platforms satisfy $\sum_i T_{ijk} = 1$ for each j and k . Represent a candidate by a vector $c = \{\theta, T_{BD}, T_{WD}, T_R\}$, where $\theta \in \{BD, WD, R\}$ is the candidate's type, and let \mathcal{C} be the set of all possible candidates. Let \mathbf{c}_k be the vector of three candidates from district k , and $\mathbf{C} = \{\mathbf{c}_1, \mathbf{c}_2, \dots, \mathbf{c}_K\}$ be the entire set of $(3K)$ candidates in all districts. Then an election is a mapping $\mathbf{L} : \mathcal{D}^K \times \mathcal{C}^{3K} \rightarrow \mathcal{C}^K$, producing a representative for each district with a given type and committed to a given platform.

The winners of the K general elections then go to a legislature $\mathbf{L} \in \mathcal{C}^K$. Let L_k be the legislator elected from district k , and let $\theta(L_k)$ be her type. If we take candidates' equilibrium strategies as given, then elections transform a districting scheme into a legislature; that is, $\mathbf{L} = \mathbf{L}(\mathbf{D}(S^2))$.

Candidates reach office according to a two-stage electoral cycle. Each district first holds a primary election, in which the BD candidate faces a WD opponent, and then a general election where the primary winner squares off against the Republican. Only WD and BD voters cast ballots in the primary, whereas all voters take part in the general election.

3.3 The Legislature and Policy Outcomes

The winning candidates form a legislature, which passes a redistributive policy. In particular, the legislature has K dollars to distribute across all districts. They do so via a Baron-Ferejohn (1989) bargaining process: a legislator is selected at random to offer a proposed division of the legislative pie. The entire legislature then votes on the proposal, and if it is adopted then the game ends. If it is rejected by majority vote, then discounting occurs (all payoffs are lowered by a factor of δ , $0 < \delta \leq 1$), and the game starts again with another member chosen at random to make an offer. In this game, members try to maximize the expected benefits going to their district.

The final outcome of this legislative game will be a vector $\{B_1, B_2, \dots, B_K\}$ of district-specific benefits, with $B_k \geq 0$ and $\sum_k B_k \leq K$. So the legislative policy function is $\mathbf{P} : \mathcal{C}^K \rightarrow \mathbb{R}_+^K$. This follows from the results of the elections, which in turn depend on the districting scheme, so $\mathbf{P} = \mathbf{P}(\mathbf{L}(\mathbf{D}(S^2)))$.

Any funds allocated to district k in the legislative process are divided according to the platform adopted by that district's representative. So if the type j representative from district k ran on a platform promising T_{ijk} to members of group i , then voters in this group will receive $T_{ijk} * B_k$ in total benefits, with individual benefits $b_i = (T_{ijk} * B_k) / n_{ik}$.

3.4 Voters

Voters receive utility both from this redistributive policy outcome and from their ideological attachment to the winning candidate in their district.

Each voter is assumed to receive an ideological benefit X^j for a candidate of type j . Thus, for instance, a voter with ideological preference of X^{BD} for black candidates and X^R for republicans gets extra utility $X^{BD} - X^R$ from seeing a black democrat win office instead of a republican.³ The voter will therefore vote for the Black Democrat unless the Republican offers her

³This difference may, of course, be negative.

sufficiently greater consumption value:

$$U_i(b_{iR}) - U_i(b_{iBD}) > X^{BD} - X^R.$$

Define the critical value, or “cutpoint” X_i for group i in an election between candidates of types 1 and 2 by:

$$X_i^e = U_i(b_{i1}) - U_i(b_{i2}),$$

where e is the type of election being contested. Then voters with values of X less than X_i^e will vote for candidate 1, while the others will vote for candidate 2. Let $\Phi_i^e(\cdot)$ be the cumulative distribution of voters of group i in an election of type e , so that, given the campaign platforms, a proportion $\Phi_i^e(X_i)$ will vote for candidate 1. Given n_i voters of type i , then this candidate will receive $n_i\Phi_i^e(X_i)$ votes from group i , with total votes of:

$$V_1^e = \sum_{i \in \Theta} n_i \Phi_i^e(X_i).$$

Similarly, the opposing candidate will get votes:

$$V_2^e = \sum_{i \in \Theta} n_i [1 - \Phi_i^e(X_i)] = \sum_{i \in \Theta} n_i - V_1^e.$$

The distribution functions $\Phi_i^e(X_i)$ play an important role in the analysis to follow. They indicate the ideological preference of a given voter i for one candidate over another. These preferences could arise partly from a spatial policy model, measuring the degree to which voters agree with the policy choices of their representative. But they could also arise to some degree from racial voting preferences: voters might want to support candidates of one race over those of another race. In the legal literature, this is what is meant by polarized voting; the willingness, or lack thereof, of voters to cross over and vote for candidates of another race. We assume for simplicity that if the distribution of type i voters in the entire population is $\Phi_i(\cdot)$, then this is also the distribution of the type i voters in any given district.

We assume that the utility from consumption is given by:

$$U_i(b) = \kappa_i \frac{b^{1-\epsilon}}{1-\epsilon}$$

where $\epsilon > 0$. Then the marginal utility of an extra dollar of consumption is

$$U_i'(b) = \kappa_i(b^{-\epsilon}).$$

As b increases from 0 to ∞ the marginal utility falls from ∞ to 0. A one percent increase in b causes an ϵ percent decrease in marginal utility, so ϵ captures the degree of diminishing returns in private consumption. Furthermore, the parameter κ captures the tradeoff between ideological and consumption benefits; higher values of κ imply that voters are more responsive to distributive as opposed to ideological benefits.

3.5 Evaluating Plans

To summarize, the order of play is as follows:

1. Given statewide population parameters N_{BD} , N_{WD} , and N_R and number of districts K , a valid districting scheme \mathbf{D} is enacted.
2. Candidates of each type in each district adopt platforms offering consumption shares T_i for members of group i .
3. Voters elect candidates in primary and general elections, yielding a legislature \mathbf{L} .
4. The legislature passes a redistributive policy \mathbf{P} .
5. All players receive their utilities, and the game ends.

We will evaluate districting plans according to their impact on minority voters, assuming that a social planner wishes to implement a plan that maximizes minority voters' overall welfare. Given the utility functions above, this means selecting

$$\mathbf{D}^* = \operatorname{argmax}_{\mathbf{D} \in \mathcal{D}^K} \sum_{i=1}^{N_{BD}} X_i^{\theta[L_k(\mathbf{D})]} + E(U_i(b_i) | \mathbf{P}(\mathbf{L}(\mathbf{D}))).$$

4 Analysis

An equilibrium must specify the voting and proposal strategies of legislators, the offers made to each group in the distributive policies, the voting behavior of each voter, and the winners of the primary and general elections.

4.1 Legislative and Electoral Equilibria

Solving from the back forwards, the legislative game is straightforward. Baron and Ferejohn (1989) show that in equilibrium, the legislator chosen to make the first offer constructs a random coalition of $\frac{K-1}{2}$ other legislators and keeps the remainder for herself. Let l be the legislator who makes the offer, C be the legislators selected to be in the coalition, and D be the remaining legislators. Then equilibrium offers are:

$$B_k = \begin{cases} \frac{(2-\delta)K-\delta}{2} & \text{if } k = l; \\ \delta & \text{if } k \in C; \\ 0 & \text{if } k \in D. \end{cases}$$

Since the game is symmetric, each legislator has an expected return of 1 from the legislative bargaining session. This in turn means that if a group is promised T_{ijk} in transfers from a given candidate's platform, then this is also their expected total legislative payout if that candidate is elected to office.

Candidates then adopt platforms to maximize their votes, balancing their offers to the various groups. Dixit and Londregan (1996), building on Lindbeck and Weibull (1987), show that in equilibrium the candidates adopt identical redistributive platforms: $T_{i1k} = T_{i2k}$ for each group i in a given district k . Consequently, voters cast their ballots for the candidate for whom they have the higher ideological affinity to start with.

Furthermore, the share of the benefits offered to group i in equilibrium is

$$T_i = \frac{\pi_i n_i}{\sum_j \pi_j n_j}, \quad (3)$$

where

$$\pi_i = [\kappa_i \phi_i(0)]^{1/\epsilon}$$

and $\phi(\cdot) = \Phi'(\cdot)$. The π_i parameters can be thought of as each group's political power. The value of π_i increases for groups with larger values of κ_i , so groups get a bigger share of the legislative pie the more they care about distributive as opposed to ideological issues. A group's power also grows with $\phi_i(0)$, which is the density of their distribution function at the point where voters are indifferent between the two candidates running for office. This term captures the “swinginess” or “pivotality” of a group: the greater the percentage of a group's members who are indifferent between the candidates, or close to it, the more benefits they get.

The intuition behind this result is straightforward. First, in equilibrium, the candidates offer the same platform to voters, so this will make no difference in voters' decisions.⁴ Since the offers T_i cancel out, those voters who are indifferent between the parties in equilibrium are those for whom $X_i^j = 0$ in the first place. When deciding whether to transfer funds from one group to another, then, it is these marginal voters who will gain or lose; hence the candidates pay off the groups in ratios proportional to their $\phi(0)$ values.

4.2 Minority Power and the Distribution of Voters

We pause to examine the implications of Equation 3 for the distributive benefits T_i . Ignoring for the moment the ideological benefits of electing different types of representatives, minorities will prefer to spread their voters across districts so as to maximize their total distributive return.

⁴Indeed, the most remarkable aspect of the equilibrium is that there exists a pure strategy in platforms that is adopted by both candidates. See Dixit and Londregan (1996, pp. 1149-50) for the details.

The types of districting schemes that provide the most benefits to minorities depend on the shape—concave or convex—of the function in Equation 3. Since we are interested in its behavior on the surface

$$N_{BD} + N_{WD} + N_R = P,$$

where P is the total district population, we rewrite Equation 3 as

$$\begin{aligned} f(N_{BD}, N_{WD}) &= \frac{\pi_{BD}N_{BD}}{\pi_{BD}N_{BD} + \pi_{WD}N_{WD} + \pi_R N_R} \\ &= \frac{\pi_{BD}N_{BD}}{\pi_{BD}N_{BD} + \pi_{WD}N_{WD} + \pi_R(P - N_{BD} - N_{WD})} \\ &= \frac{\pi_{BD}N_{BD}}{(\pi_{BD} - \pi_R)N_{BD} + (\pi_{WD} - \pi_R)N_{WD} + P\pi_R} \end{aligned} \quad (4)$$

Denoting the denominator in 4 as Σ , we have:

$$\frac{\partial f}{\partial N_{BD}} = \frac{\pi_{BD}[\pi_R(P - N_{WD}) + \pi_{WD}N_{WD}]}{\Sigma^2} \quad (5)$$

$$\frac{\partial f}{\partial N_{WD}} = \frac{\pi_{BD}N_{WD}(\pi_R - \pi_{WD})}{\Sigma^2}. \quad (6)$$

Equation 5 is always positive, as one would expect; adding more minority voters to a district increases their share of distributive benefits. Equation 6 is positive if $\pi_R > \pi_{WD}$, so blacks benefit if voters from the more powerful non-minority group are replaced with voters from the less powerful group.

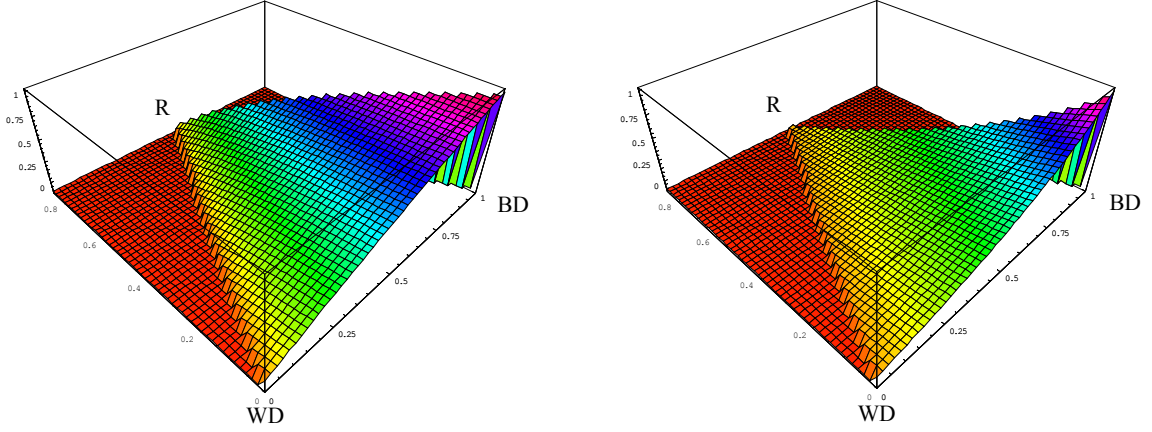
The derivatives of Equation 4 with respect to the groups' relative powers are straightforward:

$$\begin{aligned} \frac{\partial f}{\partial \pi_{BD}} &= \frac{N_{BD}\pi_R(P - N_{BD} - N_{WD}) + N_{BD}N_{WD}\pi_{WD}}{\Sigma^2} > 0; \\ \frac{\partial f}{\partial \pi_{WD}} &= -\frac{N_{BD}N_{WD}\pi_{BD}}{\Sigma^2} < 0; \\ \frac{\partial f}{\partial \pi_R} &= -\frac{N_{BD}\pi_{BD}(P - N_{BD} - N_{WD})}{\Sigma^2} < 0. \end{aligned}$$

Thus increases in the minority group's power are beneficial, while increases in the power of either other group decreases the minority's utility.

To determine the concavity/convexity of the payoff function, we calculate the determinants of the principal minors of the Hessian matrix:

$$H = \begin{bmatrix} \frac{\partial^2 f}{\partial N_{BD}^2} & \frac{\partial^2 f}{\partial N_{BD} \partial N_{WD}} \\ \frac{\partial^2 f}{\partial N_{WD} \partial N_{BD}} & \frac{\partial^2 f}{\partial N_{WD}^2} \end{bmatrix}.$$



(a) Concave: $\pi_{BD} = 7, \pi_{WD} = 6, \pi_R = 3$ (b) Convex: $\pi_{BD} = 3, \pi_{WD} = 6, \pi_R = 7$

Figure 8: Concave and convex payoff functions

We then have:

$$\begin{aligned}
 \frac{\partial^2 f}{\partial N_{BD}^2} &= \frac{-2(\pi_{BD} - \pi_R)[\pi_R(P - N_{WD}) + \pi_{WD}N_{WD}]}{\Sigma^3} \\
 \frac{\partial^2 f}{\partial N_{BD} \partial N_{WD}} &= \frac{\pi_{BD}(\pi_R - \pi_{WD})[P\pi_R + N_{BD}(\pi_R - \pi_{BD}) + N_{WD}(\pi_R - \pi_{WD})]}{\Sigma^3} \\
 \frac{\partial^2 f}{\partial N_{WD}^2} &= \frac{2N_{BD}\pi_{BD}(\pi_{WD} - \pi_R)^2}{\Sigma^3} \\
 \det(H) &= \frac{\partial^2 f}{\partial N_{BD}^2} * \frac{\partial^2 f}{\partial N_{WD}^2} - \left(\frac{\partial^2 f}{\partial N_{BD} \partial N_{WD}} \right)^2 = \frac{\pi_{BD}^2(\pi_R - \pi_{WD})^2}{\Sigma^4}. \quad (7)
 \end{aligned}$$

The determinant of the entire H matrix is clearly positive, but the value of $\frac{\partial^2 f}{\partial N_{BD}^2}$ is indeterminate, meaning that the H matrix can be either positive definite or negative definite, depending on the parameter values. For the purposes of optimization, this means that the surface could be concave or convex. Examples of each are given in Figure 8. The left-hand figure, drawn with $\pi_{BD} = 7, \pi_{WD} = 6$, and $\pi_R = 3$, shows a concave version of the payoffs, while the right-hand figure, drawn with $\pi_{BD} = 3, \pi_{WD} = 6$, and $\pi_R = 7$, is convex.

The importance of this difference is clear. If we wish to maximize the overall return to minorities, then in the concave case a social planner would divide them evenly across districts, since concentrating them up can only decrease utility. But with a convex payoff function, concentrating minority voters in a few districts will be superior.

Note that the difference between the two curves lies in the relative power of minorities compared with the other groups: concave for more powerful minorities, and convex for less powerful. This intuition is supported by ex-

aminating the line connecting any given point in the triangle (WD, BD, R) to the corner where the district is entirely composed of BD voters, $(0, 1, 0)$, keeping the ratio of WD to R voters constant throughout. This defines a parameterized path:

$$\begin{aligned} g(t) &= \frac{\pi_{BD}t}{\pi_{BD}t + \pi_{WD}\alpha(1-t) + \pi_R(1-\alpha)(1-t)} \\ &= \frac{\pi_{BD}t}{[\pi_{BD} - \alpha\pi_{WD} - (1-\alpha)\pi_R]t + \alpha\pi_{WD} + (1-\alpha)\pi_R} \end{aligned} \quad (8)$$

$$g'(t) = \frac{\pi_{BD}[\alpha\pi_{WD} + (1-\alpha)\pi_R]}{\{[\pi_{BD} - \alpha\pi_{WD} - (1-\alpha)\pi_R]t + \alpha\pi_{WD} + (1-\alpha)\pi_R\}^2} > 0 \quad (9)$$

$$g''(t) = -\frac{2\pi_{BD}[\pi_{BD} - \alpha\pi_{WD} - (1-\alpha)\pi_R][\alpha\pi_{WD} + (1-\alpha)\pi_R]}{\{[\pi_{BD} - \alpha\pi_{WD} - (1-\alpha)\pi_R]t + \alpha\pi_{WD} + (1-\alpha)\pi_R\}^3}. \quad (10)$$

The expression in Equation 10 is negative (positive) if $\pi_{BD} > (<) \alpha\pi_{WD} - (1-\alpha)\pi_R$; that is, when blacks' power is greater (less than) the weighted average of the other groups'. Since a negative second derivative denotes a concave function, we have the result that as minority voters gain power, all else being equal, optimal gerrymanders divide them more equally across districts.

Proposition 1 *Let*

$$V(\mathbf{D}) = \max_{\mathbf{d}_i, \mathbf{d}_j \in \mathbf{D}^*} N_{BD_i} - N_{BD_j}$$

be the maximum difference between the BD population of any two districts in an optimal districting scheme. Then $\frac{\partial V}{\partial \pi_{BD}} \leq 0$, so that minority voters are spread out less as their power increases.

Furthermore, combining this with Equation 6, we have the result that optimal districting schemes will concentrate black voters in a few districts when their power is low, spread them out when their power is high, and as much as possible combine them in districts with the less powerful of the other two groups. In fact, this is somewhat descriptive of the course of majority-minority districting over time; concentrated minority districts had higher percentages of black voters in previous decades than they do now.

Table 2 illustrates optimal districts for varying levels of groups' power, done for a state with three districts in which the population proportions of BD , WD , and R voters are 25%, 40%, and 35%, respectively. The WD voters' power is fixed at $\pi_{WD} = 3$, while the other two groups' power varies between 1 and 5.⁵ Note that, as predicted, the variance $V(\mathbf{D})$ declines and blacks'

⁵In this formulation of the model, absent incentives to elect a candidate from one group as opposed to another, White Democrats and Republicans are indistinguishable.

utility rises within each set of observations as π_{BD} increases, and that, where possible, black voters are put into districts with more voters from the less powerful of the other groups.

4.3 Optimal Districts

To the analysis in the previous section, we now add the ideological utility that members of a group gain from their representatives. We begin by defining the average utility per voter of a given type i for a j type representative:

$$\bar{X}_i^j = \int_{-\infty}^{\infty} X_i^j d[\Phi(X_i)].$$

Then the total utility to voters in a district for electing a type j representative is $n_i \bar{X}_i^j$.

We must also define the type of the winning candidate in any given district, thus generalizing the triangle analysis from Section 2. First, notice that the crossover rates are given by the $\Phi_i^e(0)$ functions for group i in an election of type e , where for convenience we label the primary as election $e = 1$, a BD vs. R general as type $e = 2$, and a WD vs. R general as $e = 3$. For instance, in a BD vs. WD primary, a proportion $\Phi_{BD}^1(0)$ of black voters will vote for the WD candidate, and the remaining $1 - \Phi_{BD}^1(0)$ will vote for the BD candidate. We redefine these quantities as *crossover* rates χ_i^e , in accordance with the usual standard for voting studies. Thus, a proportion χ_{BD}^1 of black voters cross over to vote for the WD candidate in the primary, while $1 - \chi_{BD}^1$ vote for the black candidate. Conversely, χ_{WD}^1 of white voters cross over to vote for the BD candidate, and $1 - \chi_{WD}^1$ vote for the WD. For reference, a table of these crossover rates is given in Table 3.

Then the BD candidate will win the primary if:

$$\begin{aligned} n_{BD}(1 - \chi_{BD}^1) + n_{WD}\chi_{WD}^1 &\geq n_{BD}\chi_{BD}^1 + n_{WD}(1 - \chi_{WD}^1) \\ n_{BD}(1 - 2\chi_{BD}^1) &\geq n_{WD}(1 - 2\chi_{WD}^1) \\ \frac{n_{BD}}{n_{WD}} &\geq \frac{1 - 2\chi_{WD}^1}{1 - 2\chi_{BD}^1}, \end{aligned}$$

similar to Equation 1 above. Similar calculations show that a BD candidate will win the general election against a Republican if:

$$(1 - \chi_{BD}^2 - \chi_R^2)n_{BD} + (1 - \chi_{WD}^2 - \chi_R^2)n_{WD} + \chi_R^2 \geq \frac{1}{2},$$

and a WD candidate can win the general if:

$$(1 - \chi_{BD}^3 - \chi_R^3)n_{BD} + (1 - \chi_{WD}^3 - \chi_R^3)n_{WD} + \chi_R^3 \geq \frac{1}{2}.$$

π_{BD}	π_{WD}	π_R	n_{BD}^1	n_{WD}^1	n_R^1	n_{BD}^2	n_{WD}^2	n_R^2	n_{BD}^3	n_{WD}^3	n_R^3	$V(D)$	Util.
1	3	1	75%	0%	25%	0%	76%	24%	0%	44%	56%	75%	0.25
2	3	1	44%	0%	56%	31%	20%	49%	0%	100%	0%	44%	0.32
3	3	1	0%	100%	0%	39%	0%	61%	36%	20%	44%	39%	0.39
4	3	1	0%	100%	0%	38%	0%	62%	37%	20%	43%	38%	0.43
5	3	1	30%	0%	70%	15%	85%	0%	30%	35%	35%	15%	0.48
1	3	2	0%	20%	80%	75%	0%	25%	0%	100%	0%	75%	0.20
2	3	2	75%	0%	25%	0%	37%	63%	0%	83%	17%	75%	0.25
3	3	2	42%	0%	58%	33%	20%	47%	0%	100%	0%	42%	0.30
4	3	2	35%	0%	65%	11%	89%	0%	29%	31%	40%	24%	0.35
5	3	2	30%	0%	70%	19%	81%	0%	26%	39%	35%	11%	0.40
1	3	3	0%	20%	80%	0%	100%	0%	75%	0%	25%	75%	0.17
2	3	3	0%	40%	60%	75%	0%	25%	0%	80%	20%	75%	0.22
3	3	3	15%	1%	85%	30%	61%	9%	30%	58%	12%	15%	0.25
4	3	3	25%	69%	6%	25%	48%	27%	25%	3%	72%	0%	0.31
5	3	3	25%	55%	20%	25%	33%	42%	25%	32%	43%	0%	0.36
1	3	4	0%	45%	55%	75%	25%	0%	0%	50%	50%	75%	0.17
2	3	4	0%	76%	24%	75%	25%	0%	0%	19%	81%	75%	0.22
3	3	4	0%	0%	100%	0%	95%	5%	75%	25%	0%	75%	0.25
4	3	4	39%	61%	0%	36%	59%	5%	0%	0%	100%	39%	0.29
5	3	4	7%	0%	93%	33%	55%	12%	35%	65%	0%	28%	0.33
1	3	5	0%	11%	89%	0%	84%	16%	75%	25%	0%	75%	0.17
2	3	5	0%	46%	54%	0%	49%	51%	75%	25%	0%	75%	0.22
3	3	5	0%	70%	30%	0%	25%	75%	75%	25%	0%	75%	0.25
4	3	5	0%	0%	100%	40%	60%	0%	35%	60%	5%	40%	0.29
5	3	5	38%	62%	0%	0%	0%	100%	37%	58%	5%	38%	0.33

Table 2: Districting plans that maximize minority voters' utility, under the assumptions: $N_{BD} = 25\%$, $N_{WD} = 40\%$, $N_R = 35\%$, $\bar{X}_{BD}^{BD} = 0.1$, $\bar{X}_{BD}^{WD} = 0.3$, $\bar{X}_{BD}^R = 0$, $\chi_{WD}^1 = 20\%$, $\chi_{WD}^2 = \chi_{WD}^3 = 50\%$

Group	Election	BD	WD	R
BD	Primary	$1 - \chi_{BD}^1$	χ_{BD}^1	
WD	Primary	χ_{WD}^1	$1 - \chi_{WD}^1$	
BD	BD/R	$1 - \chi_{BD}^2$		χ_{BD}^2
WD	BD/R	$1 - \chi_{WD}^2$		χ_{WD}^2
R	BD/R	χ_R^2		$1 - \chi_R^2$
BD	WD/R		$1 - \chi_{BD}^3$	χ_{BD}^3
WD	WD/R		$1 - \chi_{WD}^3$	χ_{WD}^3
R	WD/R		χ_R^3	$1 - \chi_R^3$

Table 3: Crossover rates

These equations define regions in the triangle similar to those in Figure 5, but with more complicated expressions for the slopes of the lines.

We can modify our previous plots to include the extra utility voters receive for different types of representatives, as for example in Figure 9. This is just the left-hand (concave) graph from Figure 8 combined with the areas in which different candidates will win the general election: White Democrats in the left foreground, Black Democrats in the right foreground, and Republicans above, similar to Figure 5. Thus, we have Figure 8 with “cliffs,” showing black voters’ extra utility from electing a White Democrat or Black Democrat, using Republicans as the baseline.

This extra dimension (literally and figuratively) to the analysis can modify the results in the previous section by adding an extra incentive to create concentrated minority districts, to the point where BD candidates can get elected, or at least having a WD elected rather than a Republican. These districts are of the type in which BD candidates can just eke out a win; that is, they sit near the border of the region in which BD candidates win, so as to not waste extra votes that could be more fruitfully used elsewhere.

Since the objective function is now discontinuous, it does not admit of an analytic solution. We therefore provide 500-draw Monte Carlo simulation results in Table 4, done for the same values of BD, WD, and R power as in Table 2, and using the same overall population proportions.⁶ The extra

⁶The Visual Basic code for these simulations is available upon request.

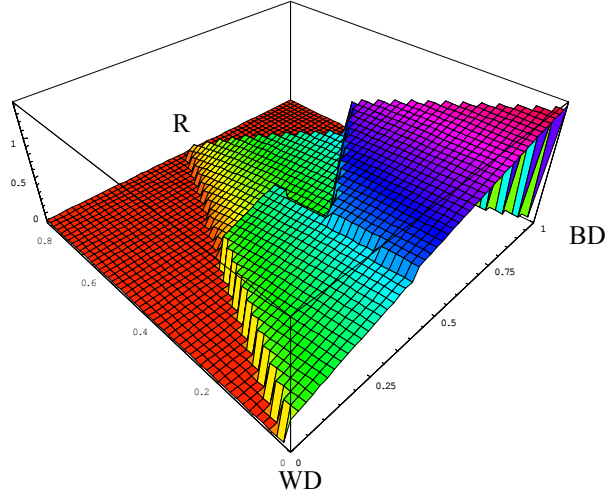


Figure 9: Minority voters' total expected payoffs, including ideological benefits

utility of electing a WD is 0.1 and of electing a BD is 0.3, relative to a baseline of 0 for a Republican. And the WD primary crossover rate is 20%, while the general election crossovers are 50%.

Note that the variances are generally higher in this table as compared with Table 2, due to the increased desire to concentrate minorities up to the point where a BD candidate can be elected in some districts. Note also that the rule stating that $V(D)$ weakly decreases within each subgroup of five simulations no longer need hold, and indeed in the last group it fails.

5 The Impact of Changing Preferences

We now use the framework developed in the previous sections to examine the impact of various changes that have taken place in the South over the past three decades: increasing black voter registration, the defection of many White Democrats to the Republican party, and decreasing white racism. For each development, we analyze its impact on minority electoral success, policy benefits, and optimal redistricting plans.

5.1 Increasing Black Registration

Prior to the passage of the 1965 Voting Rights Act, many southern states enacted laws to, *de facto*, disenfranchise blacks. Such devices as the grandfather clause, poll taxes, and white-only primaries, not to mention direct

π_{BD}	π_{WD}	π_R	n_{BD}^1	n_{WD}^1	n_R^1	n_{BD}^2	n_{WD}^2	n_R^2	n_{BD}^3	n_{WD}^3	n_R^3	$V(D)$	Util.
1	3	1	0%	61%	39%	75%	0%	25%	0%	59%	41%	75%	0.42
2	3	1	0%	61%	39%	46%	11%	43%	29%	48%	23%	46%	0.50
3	3	1	46%	8%	46%	29%	48%	23%	0%	65%	35%	46%	0.57
4	3	1	28%	47%	26%	31%	38%	31%	16%	35%	49%	15%	0.62
5	3	1	17%	36%	47%	28%	43%	28%	29%	41%	29%	12%	0.67
1	3	2	47%	13%	41%	0%	64%	36%	28%	44%	28%	47%	0.37
2	3	2	29%	41%	29%	46%	13%	41%	0%	65%	35%	46%	0.46
3	3	2	28%	43%	28%	44%	14%	42%	3%	63%	34%	41%	0.52
4	3	2	30%	41%	29%	32%	37%	32%	13%	42%	45%	18%	0.57
5	3	2	29%	42%	29%	17%	35%	48%	29%	43%	28%	12%	0.63
1	3	3	28%	47%	25%	0%	63%	37%	47%	11%	42%	47%	0.35
2	3	3	48%	21%	31%	0%	53%	47%	27%	46%	27%	48%	0.43
3	3	3	5%	54%	41%	30%	45%	25%	40%	21%	39%	35%	0.48
4	3	3	10%	40%	50%	30%	49%	21%	35%	31%	35%	24%	0.54
5	3	3	12%	45%	42%	28%	44%	28%	35%	31%	35%	22%	0.58
1	3	4	0%	50%	50%	27%	46%	27%	48%	24%	28%	48%	0.34
2	3	4	0%	50%	50%	27%	46%	27%	48%	24%	28%	48%	0.41
3	3	4	0%	50%	50%	43%	33%	23%	32%	37%	32%	43%	0.46
4	3	4	14%	36%	50%	30%	45%	24%	31%	39%	31%	17%	0.51
5	3	4	28%	46%	25%	30%	40%	30%	17%	33%	50%	13%	0.56
1	3	5	35%	34%	31%	0%	50%	50%	40%	36%	24%	40%	0.32
2	3	5	0%	50%	50%	36%	29%	34%	39%	41%	21%	39%	0.39
3	3	5	34%	31%	34%	0%	50%	50%	41%	39%	21%	41%	0.45
4	3	5	0%	0%	100%	39%	61%	0%	36%	59%	5%	39%	0.49
5	3	5	38%	62%	0%	0%	0%	100%	37%	58%	5%	38%	0.53

Table 4: Simulation results. Assumptions: $N_{BD} = 25\%$, $N_{WD} = 40\%$, $N_R = 35\%$, $\bar{X}_{BD}^{BD} = 0.1$, $\bar{X}_{BD}^{WD} = 0.3$, $\bar{X}_{BD}^R = 0$, $\chi_{WD}^1 = 20\%$, $\chi_{WD}^2 = \chi_{WD}^3 = 50\%$

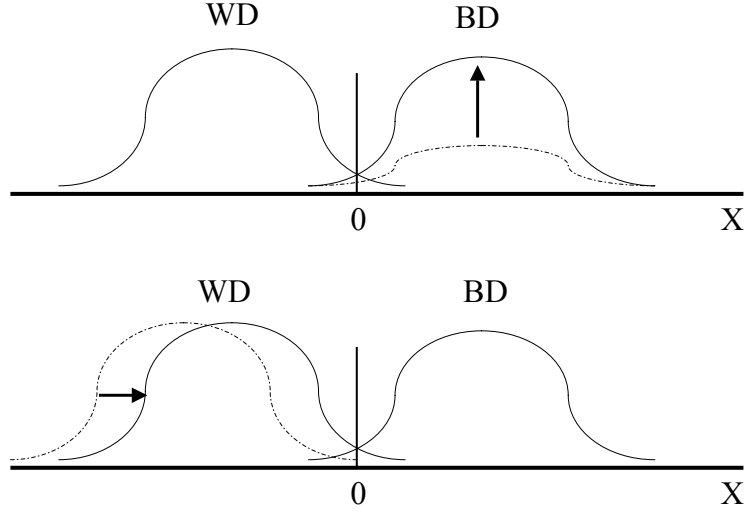


Figure 10: Impact on relative power of WD and BD voters of increase in black registration (top) and decreasing white racism (bottom).

intimidation, minimized blacks' participation in politics. When one form of discrimination was outlawed, the states would switch to another.

This continued until the VRA swept away all such “tests and devices,” and its Section 5 preclearance provisions required covered states to obtain the permission of the federal government before adopting any new law that might impact minorities' ability to vote. The most direct result of passing the VRA was thus to greatly increase blacks' participation, to the point where now, in most areas of the South, minorities register and vote at rates at or above those of white voters.

In the model above, the impact of such an increase in statewide N_{BD} is (usually) unambiguous: it acts just like an increase in power π_{BD} , and so both increases the flow of benefits to minority constituents and makes it easier to elect minorities to office, thus increasing their ideological benefits. The electoral benefits were illustrated in Figure 7: increasing the percent of black voters while keeping the ratio of WD and R voters constant nearly always makes it easier to elect black candidates. We say “nearly always” because, as pointed out in the earlier discussion of the figure, there are some exceptional regions where, under sincere voting, Republicans get elected rather than Democrats. Other than this, though, the overall effect should be to increase descriptive black representation.

The shift in legislative pork barrel benefits is illustrated in the top half of Figure 10, where the horizontal axis shows the ideological distributions of the WD and BD voters, with the 0, or indifference, point in the middle. The increase in the size of the black electorate increases their power by raising $\phi_{BD}(0)$ while also increasing the number of districts that could elect a BD candidate.

Further, according to the model, the first response of district-drawers as the number of blacks registered and voting increases should be to create concentrated minority districts, and indeed this is what happened in the 1970's and 1980's, with one rule of thumb stating that districts had to be at least 65% black to be “effectively” majority-minority.

As black participation continues to increase, the response should be to concentrate minority voters less, spreading them out more evenly across districts. Again, this is happening, but not without considerable resistance, with divided opinions both within the black community and without. Some worry that reducing the black majorities in these districts will dilute their influence over policy and reduce the number of blacks in office, thereby giving back some of the hard-won gains of the civil rights movement. Others see it as a natural progression of blacks into mainstream politics, and a way to spread their influence over greater areas. This debate continues to be fought, and the recent Supreme Court case *Georgia v. Ashcroft* mentioned in the introduction will hardly put it to rest.

5.2 The Rebirth of Southern Republicanism

The second notable development concerns the breakup of the formerly “Solid South” democratic party. Since Reconstruction, southerners had identified the Republicans as the party of Lincoln and the North, and thus voted nearly unanimously for Democratic candidates. But Democratic support of the VRA and other civil rights measures in the 1960's led inexorably to the defection of many southerners to the Republicans, who after all had a solid conservative message that appealed to many voters.

The electoral impact of this shift has been investigated in Figure 7: as we move from a situation where WD voters dominate the political landscape to one where R voters are in evidence as well, it becomes easier to elect BD candidates to office. Thus the electoral and hence ideological impact on black voters is positive.

In addition, Equation 6 above shows that blacks will benefit in terms of policy benefits if the politically stronger group of non-minority voters is replaced by the weaker group, where again power is measured in terms of $\phi_i(0)$. Given that WD voters are more centrist than Republicans, the change is indeed in the desired direction, allowing blacks to compete more equally for their share of the legislative pie. Thus the switch from White Democrats to Republicans is also in blacks' favor.

5.3 Decreasing White Racism

Finally, we come to the increased willingness of white voters of all stripes to vote for minority candidates, due to steadily decreasing racism in the South. Figures 3 and 4 above show the impact of such changes in the values of a and b , both of which expand the region in which a BD candidate can win the general election. Thus decreasing racism does help blacks win office, and indeed, the number of elected blacks in the South has skyrocketed since the adoption of the VRA.⁷

But the impact on distributive benefits, illustrated in the bottom half of Figure 10, is not straightforward. For white voters to become less racist means that they have less ideological aversion to blacks' holding office, which means that their distribution of X values will shift to the right, as in the figure. This in turn will, at first, increase white voters' density at $X = 0$, meaning that they will enjoy more legislative benefits as they become more pivotal. This will continue until the central hump of the distribution passes the 0 point, past which decreased racism also leads to less of a share of the legislative pie.

Since less than 50% of white voters reliably support black candidates, though, we may assume that we are still on the upward slope of the distribution function. Hence white voters may be gaining increasing benefits from candidates' platforms at black voters' expense. Of course the tradeoff in terms of increased descriptive representation may well be worthwhile, but it is still interesting to note that decreased racism is not an unalloyed good for minority voters. In fact, there have long been rumblings that Democrats in office, white and black alike, take their black constituents for granted and give them less than their fair share of insiders' benefits. This may be true, and if so, the model given here provides a plausible rationale for why it would happen.

6 Conclusion

To conclude we return to our motivating example—the recent *Georgia v. Ashcroft* decision—to see how it relates to the analysis above. The courts have interpreted Section 5 of the VRA to mean that the federal government should preclear a proposed change in state laws, including redistricting, if and only if that change does not retrogress from the existing status quo. If we interpret retrogression to mean that the change will decrease the minority's expected utility, then our model predicts that, in general, a rise in minority power should dictate in favor of spreading minority voters out across districts,

⁷See the essays Davidson and Grofman (1994) for detailed state-by-state analyses attributing the rise in black office holding directly to the VRA.

just as the challenged Georgia redistricting plan did in *Ashcroft*. Indeed, keeping the previous districting scheme, which concentrated black voters into relatively few districts, may well decrease black voters' utility, wasting their votes and helping to elect more Republicans to the legislature.

Of course, it may be that the plan passed went too far in spreading out black voters; after all, such a change might trade off descriptive representation—electing blacks to office, which we term ideological benefits—for substantive representation—passing policies preferred by the minority community, which we term distributive benefits. If voters value highly the ideological returns to having many minority office-holders, then they may be unwilling to make such a tradeoff. However, the biggest outcry against the plan came not from blacks voters or legislators (43 out of the 46 black legislators in the State Assembly voted in favor of the plan), but from Republicans, against whom the plan was mainly directed.

Comparing our results with those of others, Shotts (2001) presents a model in which majority-minority district requirements have no effect on liberal gerrymanderers, whereas they can limit the options of conservative districters. This finding contrasts with our result that the optimal districting schemes for minorities often have no majority-minority districts; hence such a requirement would decrease blacks' utility.

However, the models are not really comparable, so the conflict is only apparent. In the Shotts model, district-drawers from different states compete to influence the median voter in a national legislature, whereas the plans in our model maximize the policy favorability of a state legislature or a state's congressional contingent. Furthermore, the Shotts result really only holds for concentrated minority requirements up to 50% black; over that, liberal gerrymanderers can be restricted just as can conservative ones. So to say that his is a result pertaining to majority-minority districts—usually understood to mean a requirement of concentration over 50%—is really a misnomer.

Finally, a number of possible extensions to our model are apparent. First, our current legislative model is simple, so as to focus on the logic of changing preferences and group powers. Hence there are no real legislative parties or permanent coalitions. But if one wanted to investigate the impact of changing legislative rules, committee powers, or party leadership on districting, then these elements could be incorporated into the legislative model.

One could also examine the impact of other electoral rules. Here, minorities must win first a primary and then a general election via plurality votes to gain office. One could just as well use single transferable votes, multi-member districts, approval voting, or any other popular election method and see how that changes the results, both in getting minorities elected and in the policy favors paid by candidates of one type to their supporters of another type. As the saying goes, we leave these for future investigation.

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