Racial Gerrymandering and Minority Representation

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Question: How to design institutions that afford political minorities policy benefits?

- Stable political systems require that all major groups "buy in," see outcomes as legitimate.
- Otherwise, result is oppression and/or violence, loss of productive resources.
- Analogy: the protection of minority investors as a key component of smoothly working capital markets.

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Institutions from the Minority's Point of View

- In a majoritarian system, political minorities must form coalitions to exert influence over policy.
- These coalitions can be made at the electoral or legislative stages.
- Institutions can favor one type of coalition formation over the other; this may involve tradeoffs.
  - Consociational Democracy
  - Two-party vs. Multi-party systems
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Why is redistricting so studied? It has a little bit of everything...

1. Rich historical tradition
2. Connection with real politics
3. Theoretically interesting, yet lot to do
4. Good data for testing theories
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Georgia v. Ashcroft Details

- Georgia was controlled by Democrats, spread black voters out across districts for a partisan gerrymander.
- Question was whether Georgia's State Senate redistricting plan was "retrogressive" under §5 of the VRA.
  - This section requires Southern states to pre-clear new laws that might affect minorities' voting rights.
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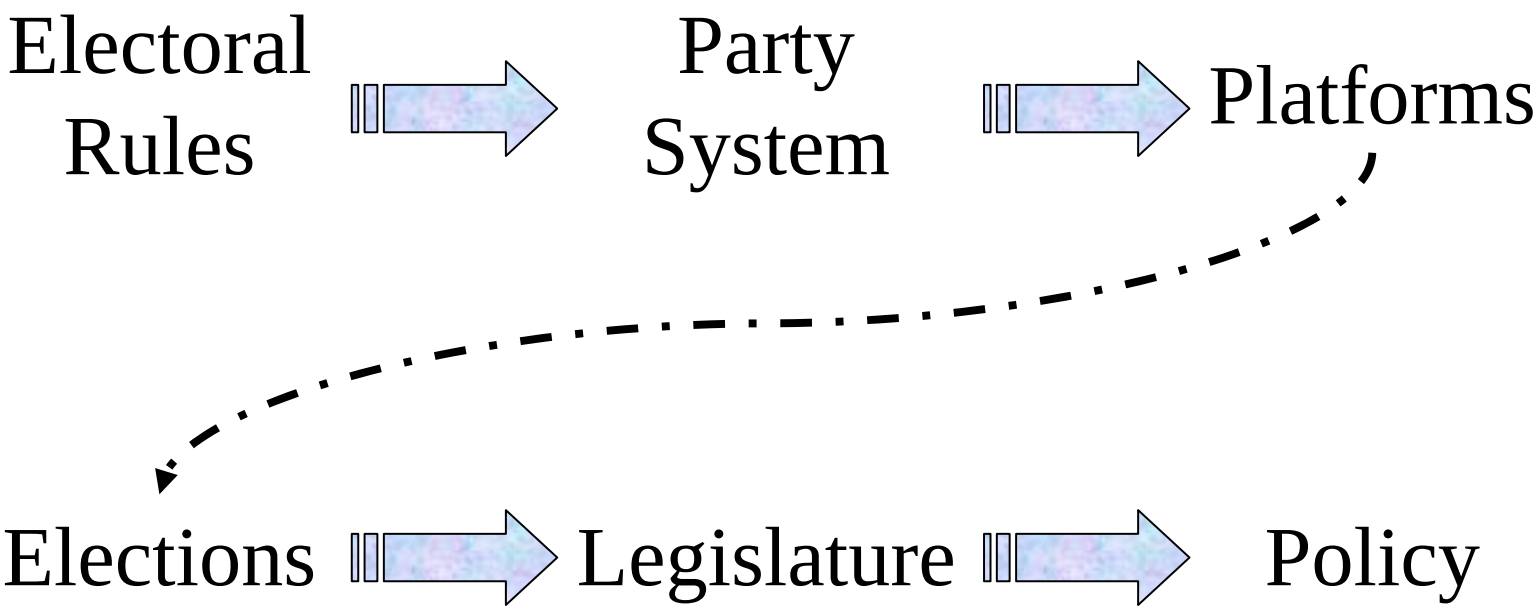
Protecting Minorities: The Big View



Previous Literature on Redistricting

- Theory on partisan redistricting: Owen and Grofman 1988; Cox and Katz 1999; Gilligan and Matsusaka 1999
- Empirical literature on racial redistricting: Cameron, Epstein & O'Halloran 1994; Lublin 1997; Epstein and O'Halloran 2000
- Theory on racial redistricting: Shotts 2001; 2002
  1. Single policy dimension; minority group's extreme
  2. Competing gerrymanderers in each state
  3. Majority-minority mandate can only move policy in liberal (minority-preferred) direction

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Our Take on the Problem

- We want to capture a possible tradeoff between coalition formation at the electoral and legislative stages.
- We therefore want our model to include:
  1. Both electoral and legislative stages, with strategic platforms and far-sighted voting
  2. Multidimensional setting, for coalition formation at each stage of the process
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Triangles: Districting Made Simple(x)

- Groups $\Theta = \{BD, WD, R\}$ to be split across K districts
- Statewide populations $\mathbf{N}_{BD}$, $\mathbf{N}_{WD}$, and $\mathbf{N}_{R}$
- Then the districting matrix will look like:

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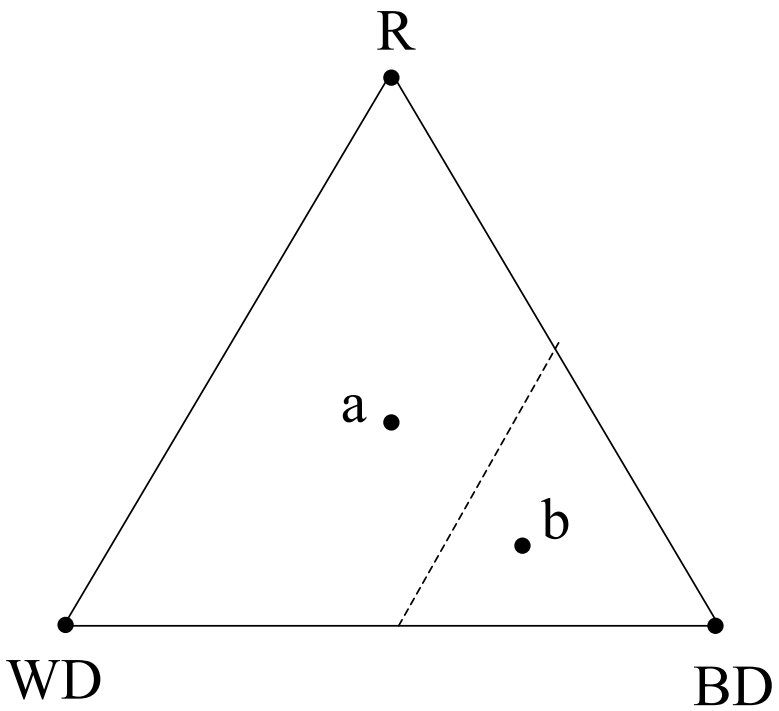
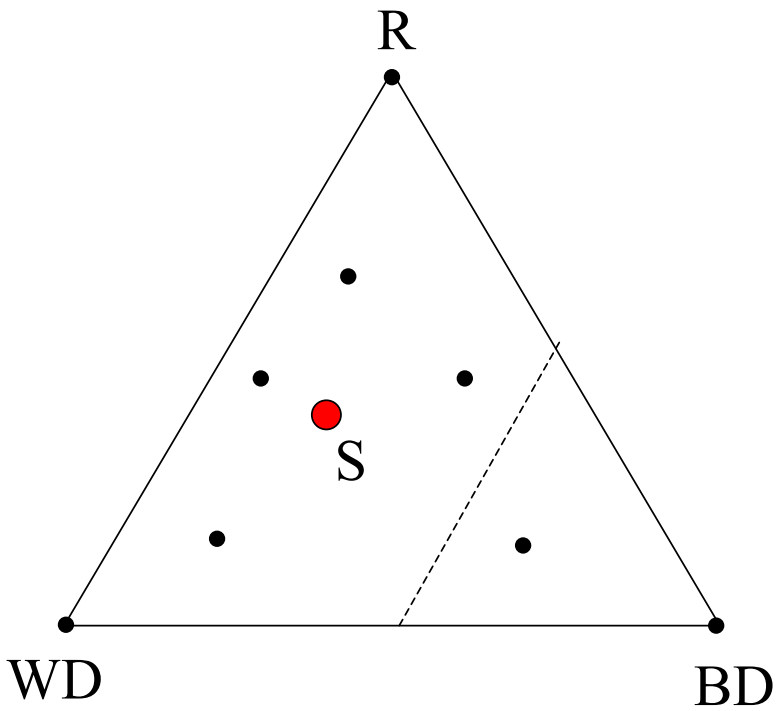
Simplex Representation

Can represent the problem as a set of points in the two-dimensional simplex S^2:



Simplex Representation

Then districts have to average to the overall statewide proportions:



Elections

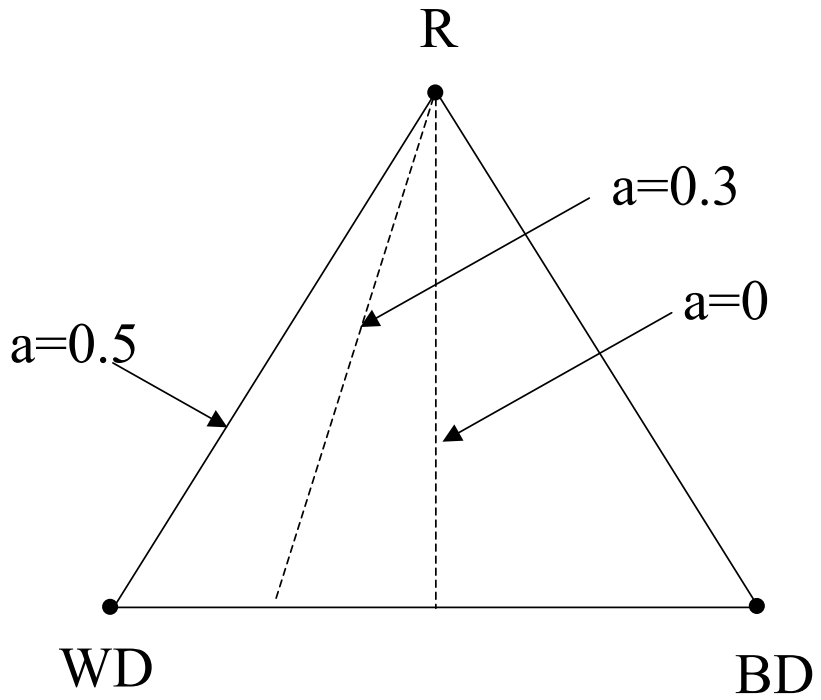
First, WD and BD candidate run in a primary, with the winner facing the R candidate. Assumptions:

1. No BD or R crossover;
2. In the primary, a fraction a of WD voters cross over to cast their ballots for the BD candidate, with $0 \leq a \leq \frac{1}{2}$;
3. In a BD vs. R general, a fraction b of WD voters cast their ballots for the BD candidate, with $a \leq b \leq 1$; and
4. In a BD vs. WD general election, a fraction c of WD voters cast their ballots for the WD candidate, with $b \leq c \leq 1$.

Elections

The BD candidate will win the primary if

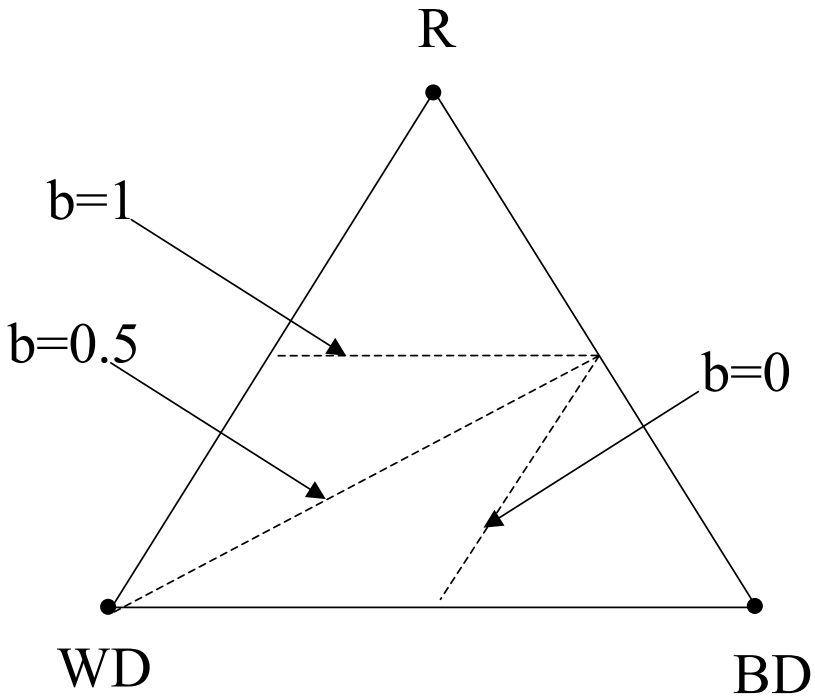
$$N_{BD} \geq (1 - 2a)\, N_{WD}.$$



Elections

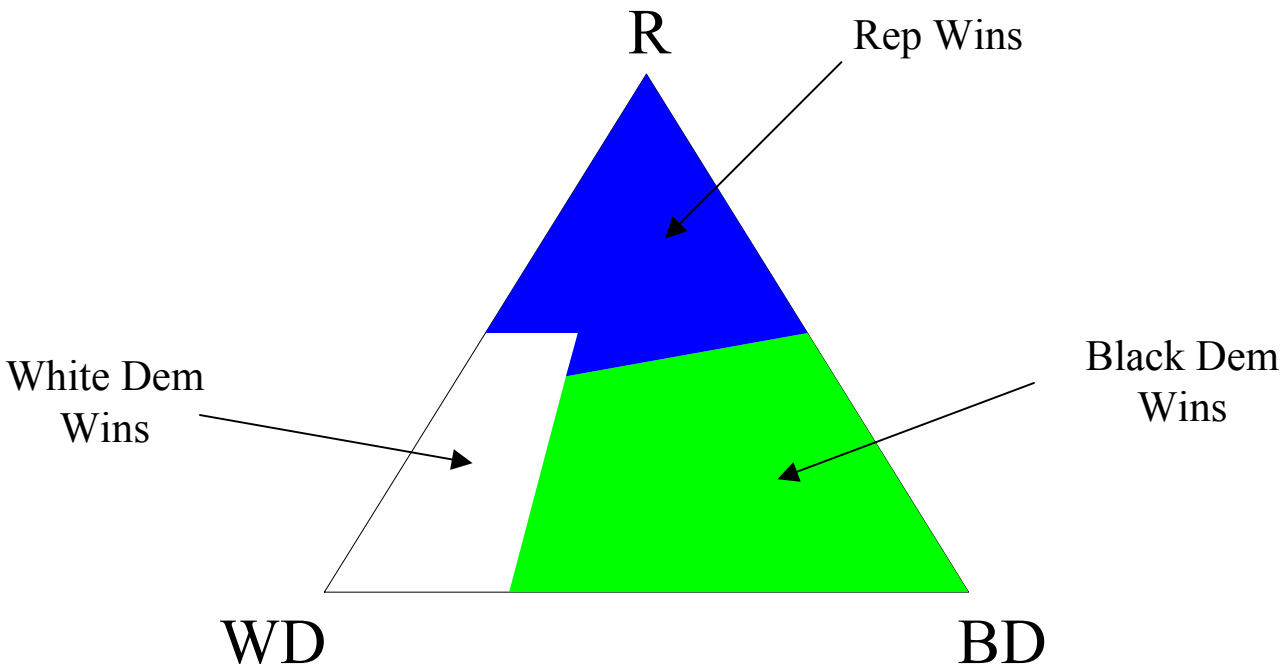
The BD candidate will go on to win the general if:

$$N_{BD} + bN_{WD} \geq 1/2.$$



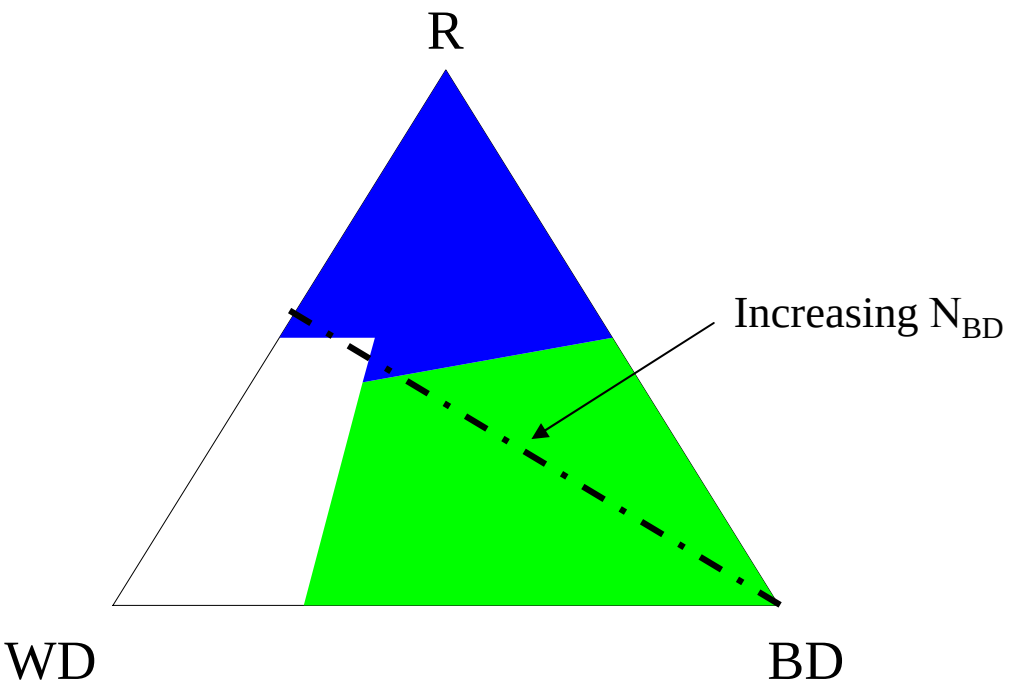
Elections

Overall, the winners in each region are:



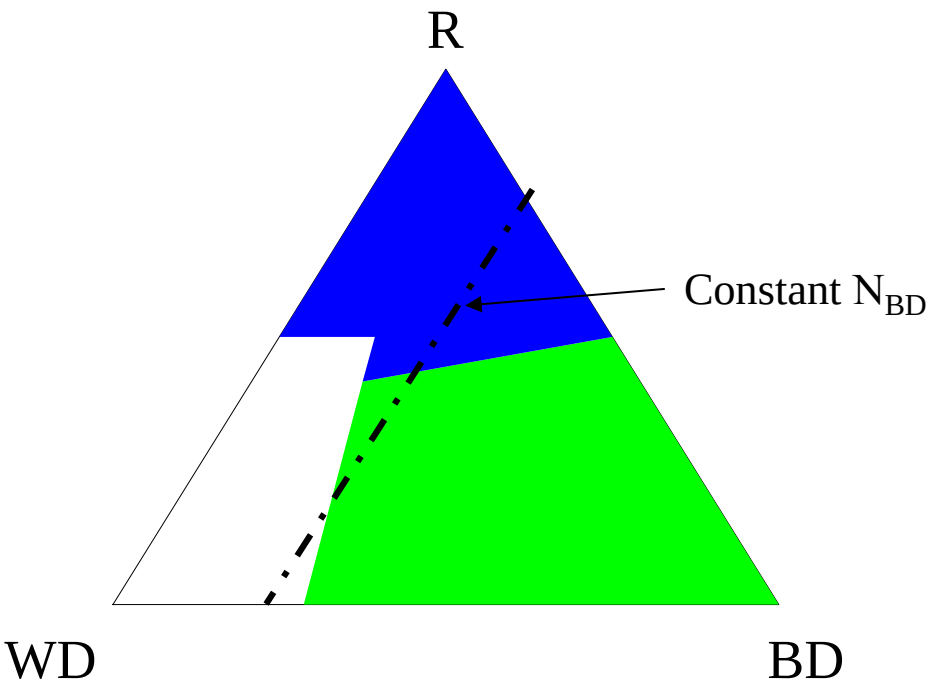
Some Preliminary Results

Increasing the number of black democrats, keeping the ratio of white democrats and republicans constant:



Some Preliminary Results

Replacing white democrats with republicans, keeping the ratio of black democrats constant:



Model: Groups and Districts

- Voter types $\Theta = \{BD, WD, R\}$, with statewide populations $\mathbf{N}_{BD}$, $\mathbf{N}_{WD}$, and $\mathbf{N}_R$.
- A district is a vector $\mathbf{d} = (N_{BD}, N_{WD}, N_R)$ of voters, with $\mathcal{D}$ the set of all possible districts.
- Then a *valid districting scheme* is a function $\mathbf{D} : S^2 \to \mathcal{D}^K$ such that in any given district, $\sum_i N_{ik} = \mathbf{N}/K$, and across districts $\sum_k N_{ik} = \mathbf{N}_i$.

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Candidates and Elections

- Three candidates in each district, BD, WD, & R, with a BD vs. WD primary, winner against R.
- Candidate j in district k adopts platform offering proportion T_{ijk} of the district's redistributive benefits to group i, $\sum_i T_{ijk} = 1$.
- These can be thought of as promises to different groups for a certain share of the legislative pie:
  - Effort across different issue areas
  - Staff positions, campaign workers
  - Type and location of pork barrel projects

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- A candidate is a vector $c = (\theta, T_{BD}, T_{WD}, T_R)$; let $\mathcal{C}$ be the set of all possible candidates.
- An election is a mapping $\mathbf{L} : \mathcal{D}^K \times \mathcal{C}^{3K} \rightarrow \mathcal{C}^K$.
- Voters are farsighted: they vote for the candidate who will give greatest expected utility, given voting strategies in all other districts.
- Assume probabilistic voting to smooth out response curves, so that the probability type j wins, Ψ_j, is equal to their vote share.

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- An election is a mapping $\mathbf{L} : \mathcal{D}^K \times \mathcal{C}^{3K} \rightarrow \mathcal{C}^K$.
- Voters are farsighted: they vote for the candidate who will give greatest expected utility, given voting strategies in all other districts.
- Assume probabilistic voting to smooth out response curves, so that the probability type j wins, Ψ_j, is equal to their vote share.

Legislature and Policy Outcomes

- The winners of the K district elections then go to a legislature $\mathbf{L} \in \mathcal{C}^K$, so that $\mathbf{L} = \mathbf{L}(\mathbf{D}(S^2))$.
- They have K dollars to divide across districts, which they do via (open rule) legislative bargaining.
- Yields a vector $(B_1, B_2, \ldots, B_K)$ of district-specific benefits, with $B_k \geq 0$ and $\sum_k B_k \leq K$.

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- Voters in group i of district k, with representative type j, will receive $T_{ijk} * B_k$ in total benefits.
- Individual benefits are then $b_{ij} = (T_{ijk} * B_k)/N_{ik}$.
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Voters

- Each voter receives utility $U(\cdot)$ from redistribution, and ideological benefit X^j for a candidate of type j.
- The voter will therefore prefer prefer candidate 1 over 2 if:

$$E[U_i(b_{i1})] - E[U_i(b_{i2})] > X^2 - X^1.$$

- Define the cutpoint X_i for group i in an election between candidates of types 1 and 2 by:

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- Then the marginal utility of an extra dollar of consumption is

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The Social Planner's Problem

- Evaluate districting plans according to their impact on minority voters, assuming that a social planner wishes to maximize minority voters' overall welfare.
- Then the social planner selects:

$$\mathbf{D}^* \in \underset{\mathbf{D}\in\mathcal{D}^K}{\operatorname{argmax}} \sum_{i=1}^{\mathbf{N}_{BD}} X_i^{\theta[L_k(\mathbf{D})]} + E\left[U_i(b_i) \mid \mathbf{P}\left(\mathbf{L}\left(\mathbf{D}\right)\right)\right].$$

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- Say legislator l is selected; she forms a random coalition C of others, remainder D excluded. Then:

$$B_k = \begin{cases} \frac{(2-\delta)K-\delta}{2} & \text{if } k = l; \\ \delta & \text{if } k \in C; \\ 0 & \text{if } k \in D. \end{cases}$$

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Analysis: Maximizing Distributive Benefits

Note that $N_{BD} + N_{WD} + N_R = P$, and rewrite the share function as:

$$f\left(N_{BD}, N_{WD}\right) = \frac{\pi_{BD} N_{BD}}{\left(\pi_{BD} - \pi_R\right) N_{BD} + \left(\pi_{WD} - \pi_R\right) N_{WD} + P\pi_R}.$$

Then

$$\frac{\partial f}{\partial N_{BD}} = \frac{\pi_{BD}[\pi_R(P - N_{WD}) + \pi_{WD} N_{WD}]}{\Sigma^2} > 0$$

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Proposition

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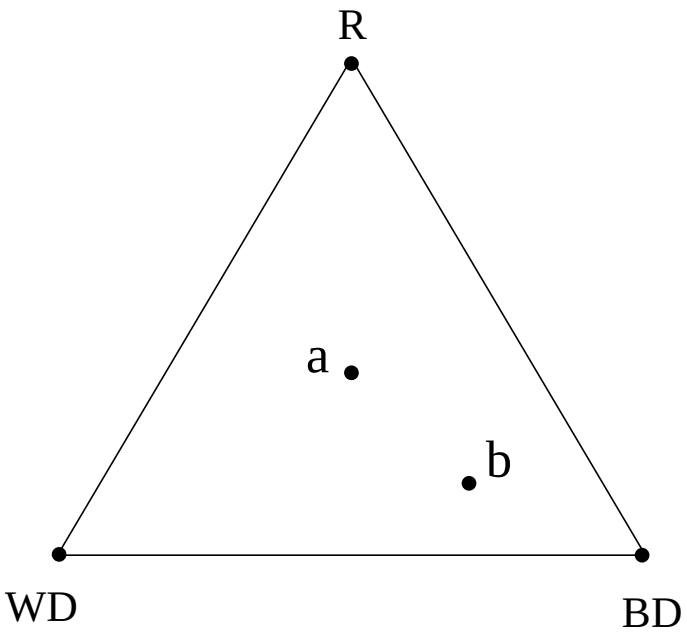

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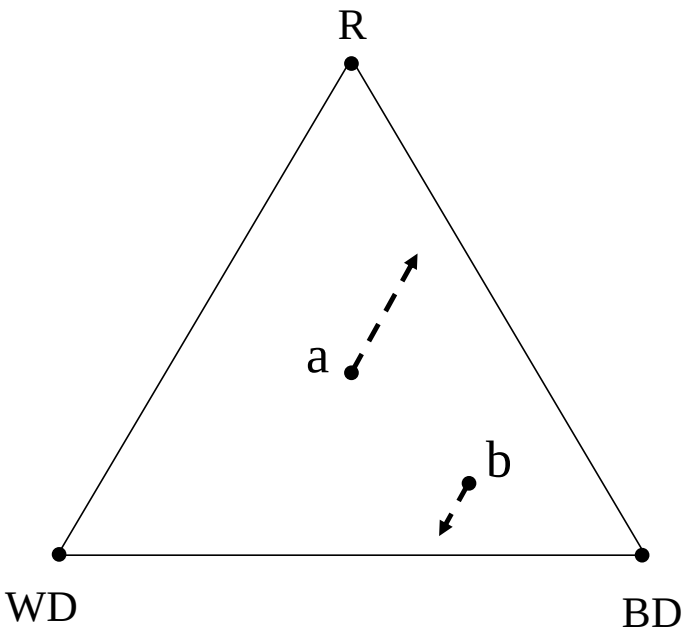

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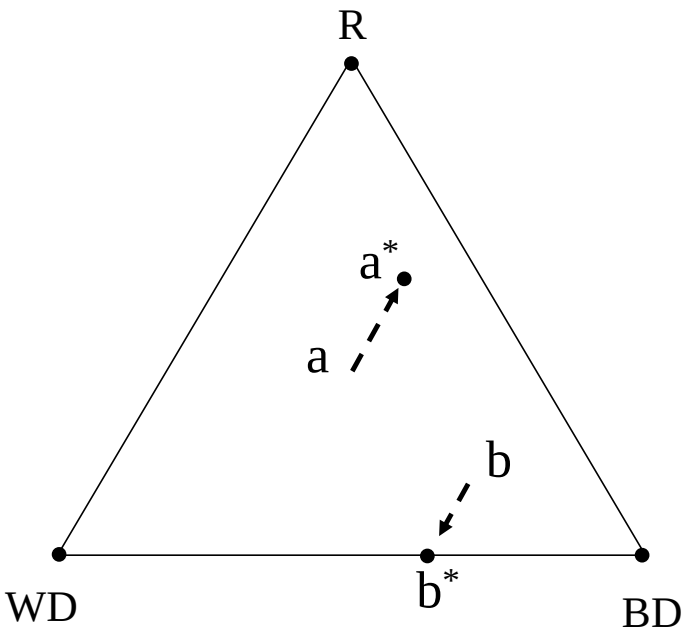

Maximizing Distributive Benefits

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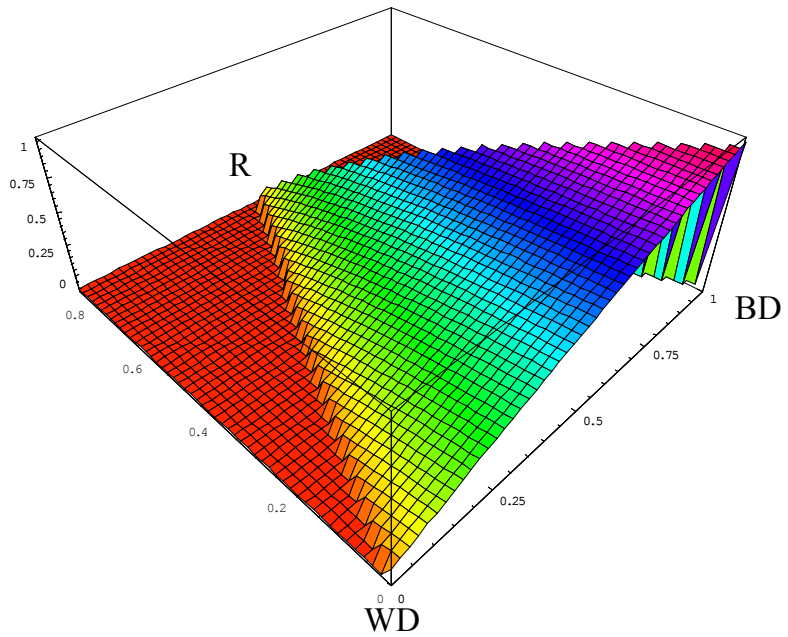

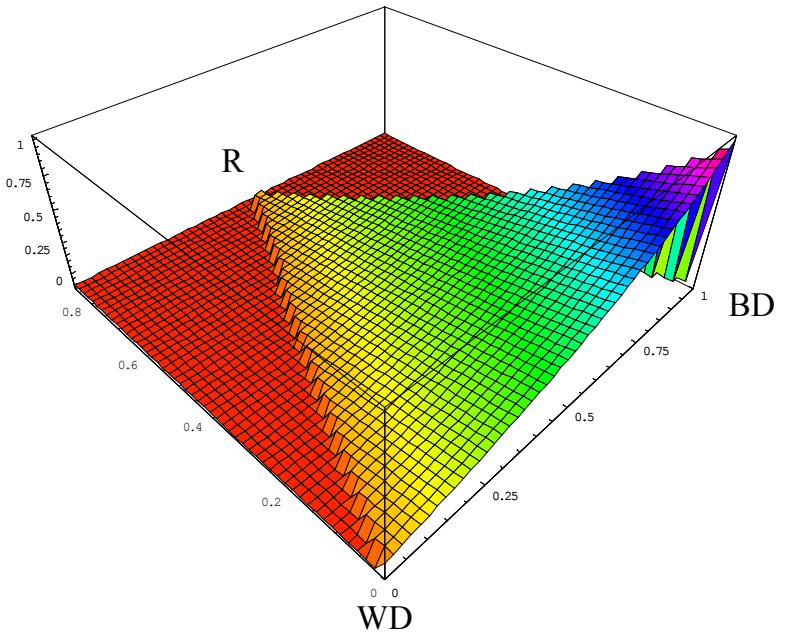

Examine the line connecting any given point in S^2 to (0,1,0), keeping the ratio of WD to R voters constant.
Yields a parameterized path

$$g(t) = \frac{\pi_{BD} t}{\pi_{BD} t + \pi_{WD} \alpha (1-t) + \pi_R (1-\alpha)(1-t)}.$$

Then

$$g''(t) = -\frac{2\pi_{BD}[\pi_{BD} - \alpha\pi_{WD} - (1-\alpha)\pi_R]}{\Lambda^3[\alpha\pi_{WD} + (1-\alpha)\pi_R]^{-1}}.$$

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This means that as minority groups gain power, districting schemes that maximize their distributive benefits spread them out more evenly across districts.

And whenever possible, combine minority voters with voters from the less powerful non-minority group.

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Ideological Utility

First, see how districting schemes affect Ψ_{BD}, the probability of electing a minority candidate to office:

Proposition

1. $\partial \Psi_{BD} / \partial N_{BD} > 0$, so increasing the number of black democrats in a district always increases the probability of electing a black democrat.
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Overall Ideological Utility

Define the average utility per voter of a given type i for a j type representative:

$$\overline{X}_i^j = \int_{-\infty}^{\infty} X_i^j d[\Phi(X_i)].$$

Overall expected utility for minority voters includes both the type elected and their average attachment to representatives of that type:

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There exists $\widehat{\beta} > 0$ such that $\beta < \widehat{\beta} \Rightarrow E(X)$ is convex on S^2.

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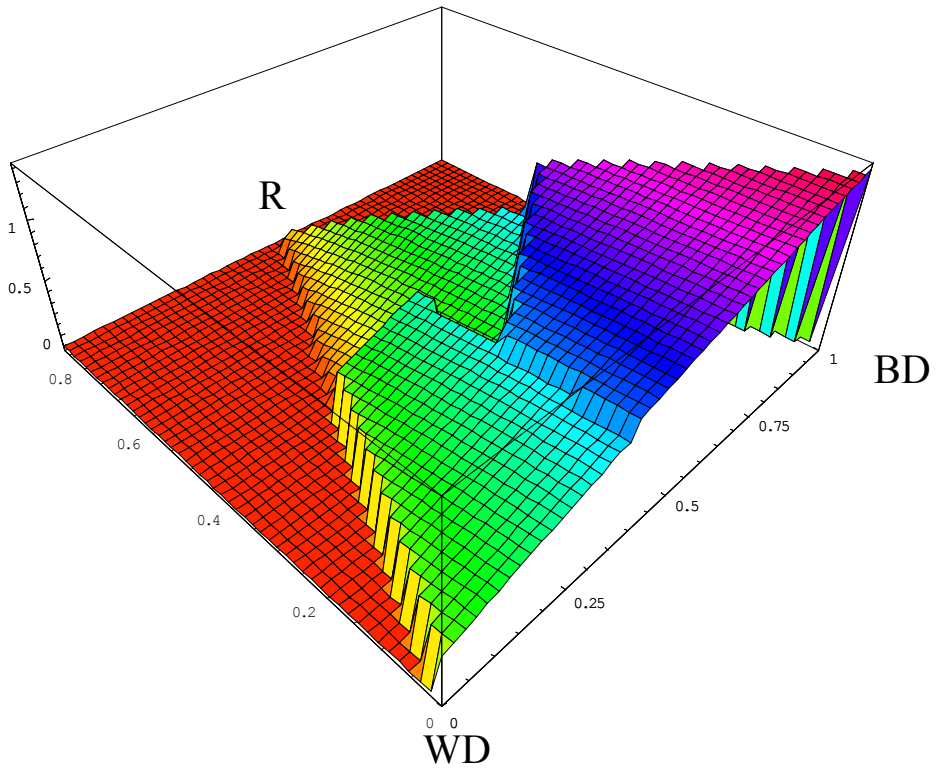
Optimal Districting Schemes

How to maximize overall utility?



Optimal Districting Schemes

- How do the two elements discussed above—distributive and ideological utility—combine?
- In particular, is it still true that as minorities gain power, optimal districting schemes spread them out less?
- Key to the solution is that the two effects are *linked*:
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Recall that $\pi_i = [\kappa_i \phi_i(0)]^{1/\epsilon}$, so there are two ways a group's power can increase:

1. The group becomes more pivotal ($\phi_i(0)$ increases)
   - Then voters cross over to vote for more non-minority candidates.
   - This is equivalent to an increase in β.
   - So the ideological utility function becomes more concave.

Optimal Districting Schemes

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2. The group cares more about distributive as opposed to ideological benefits (κ_i increases)
   - Then the relative weight on distributive benefits increases.
   - So as this becomes more concave, the overall utility function becomes more concave as well.

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Either way, the end result is that increased group power means optimal schemes spread minority voters out more across districts.

QED!

Application: Increasing Minority Voters

- The biggest impact of the VRA was to increase black participation in Southern politics.
- Led many conservative Southern Democrats (Thurmond, Wallace) to court the black vote through services and pork.

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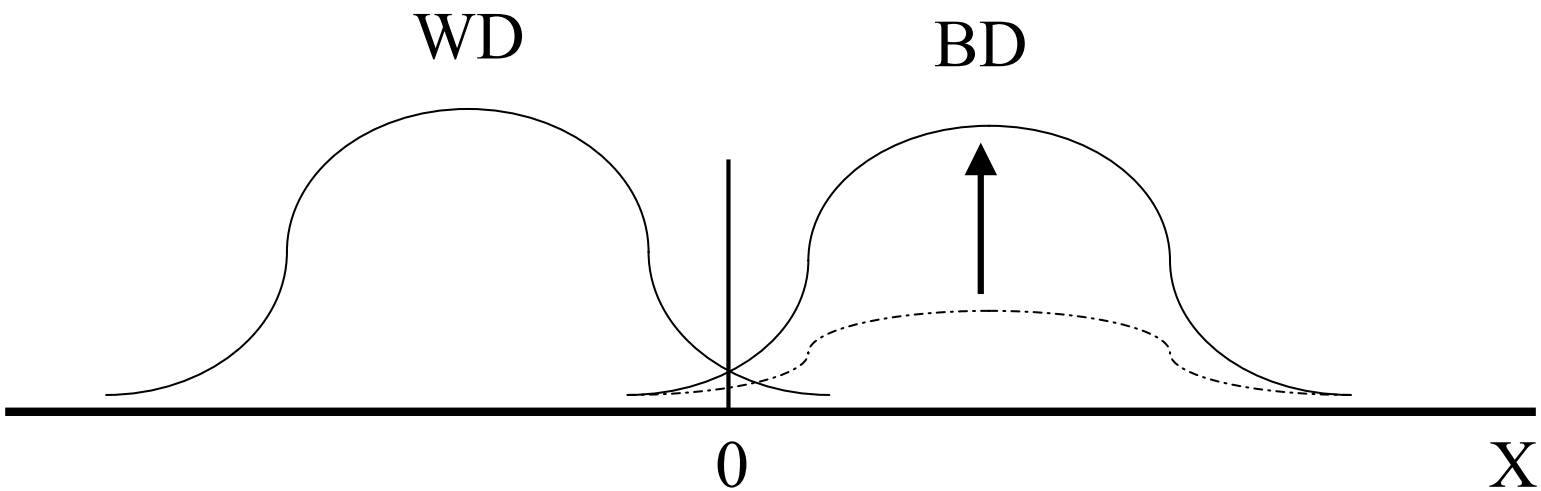

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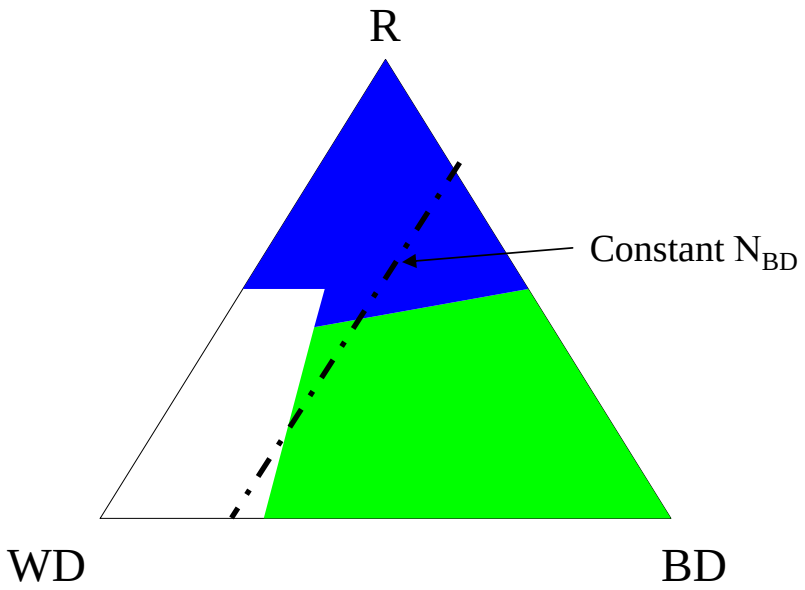

Changes in Ideological Attachments

- White democrats more willing to vote for black candidates.
  - Makes it easier to elect blacks to office
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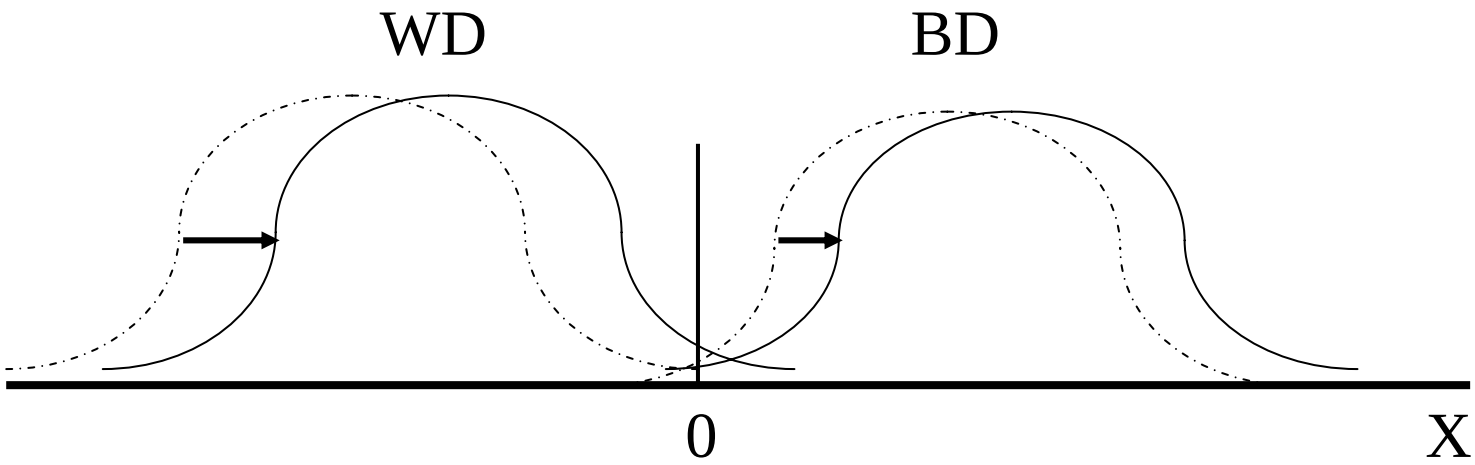

Epilogue

Supreme Court ruled in favor of Georgia in its redistricting case

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Sets up interesting questions for upcoming VRA §5 renewal.