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Y2K*

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This paper studies, within a general equilibrium model, the dynamics of Year 2000 (Y2K)-type shocks: anticipated, permanent losses in output whose magnitude can be lessened by investing resources in advance. The implied dynamics replicate three observed characteristics of those triggered by the Y2K bug. (1) Precautionary investment: Investment in solving the Y2K problem begins before the year 2000. (2) Investment delay: Although economic agents have been aware of the Y2K problem since the 1960s, investment did not begin until recently. (3) Investment acceleration: As the new millennium approaches, the amount of resources allocated to solving the Y2K problem increases. In addition, the model predicts that Y2K investment peaks at the end of 1999. *Journal of Economic Literature* Classification Numbers: E22, E32

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1. INTRODUCTION

A number of professional economists have compared the expected recession associated with the Year 2000 (Y2K) computer date problem with that caused by the oil price shock in 1973–1974 (Yardeni, 1998). In anticipation of this problem, as the millennium comes to an end, both the government and the private sector have been allocating significant amounts

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of resources to its remediation. Federal Reserve Governor Edward W. Kelley, Jr., estimates that resources allocated to solving the Y2K problem will cost the U.S. economy 1/10 of 1% of GDP in 1998 (see his testimony before the U.S. Senate Committee on Commerce, Science, and Transportation on April 28, 1998). Although at a slower pace, similar efforts are under way in the rest of the world. The Gartner Group has estimated that worldwide the cost associated with solving the Y2K problem will total 300 to 600 billion U.S. dollars.

The macroeconomic dynamics triggered by the Y2K problem are characterized by the following three facts: First, the millennium bug has induced *precautionary investment* in the sense that the allocation of resources aimed at solving the Y2K problem began before the year 2000. Second, there has been *investment delay*. Although economic agents have been well aware of the Y2K problem since the 1960s, the allocation of real resources devoted to its solution did not begin until the 1990s. Third, investment in the Y2K problem has been *accelerating*, particularly sinced 1997.

In this paper, we embed the Y2K problem into a simple dynamic general equilibrium framework. We model the Y2K problem as a situation in which before the year 2000 agents learn that in the year 2000 output will experience a permanent decline. Agents can lessen the output decline by investing resources in advance. The fact that resources allocated to solving the Y2K problem become productive only in the year 2000 is the key element driving the dynamics of the model.

We study the Y2K problem within the context of a standard optimizing growth model featuring an Arrow–Kurz (A-K) technology for the production of goods. This technology allows agents to shift resources across time at a constant rate of return. In spite of its simplicity, the model can account for the three main facts associated with the Y2K problem: precautionary investment, investment delay, and acceleration. In addition, the model predicts that Y2K investment will peak at the end of 1999.

2. THE MODEL

Consider an economy populated by a large number of identical, infinitely lived consumers with preferences described by the utility function

$$\int_0^\infty e^{-\rho t} u(c_t)\, dt, \tag{1}$$

where c_t denotes consumption in period t and $\rho > 0$ denotes the subjective discount factor. The instant utility function $u(\cdot)$ is assumed to be twice continuously differentiable, strictly increasing, and strictly concave. The representative consumer is endowed with an initial stock of capital k_0.

Output, y_t, is assumed to be perishable and to be produced with the linear technology $y_t = Ak_t$, where A, the marginal product of capital net of depreciation, is assumed to be strictly positive.

In period zero, agents learn that beginning in period $T > 0$ they will experience a loss in income given by g_t. Here T is meant to represent the year 2000 and g_t the Y2K problem. Agents can invest resources in advance to reduce the magnitude of the Y2K problem. Specifically, we assume that agents can build "Y2K capital," which we denote by I_t, and that the Y2K problem is a decreasing function of I_t, $g_t = g(I_t)$, where g: $R^+ \rightarrow R^+$ is twice continuously differentiable and satisfies $\lim_{x \to \infty} g'(x) = 0$. Let i_t denote Y2K investment. Then the law of motion of the stock of Y2K capital is assumed to take the form

$$\dot{I}_t = v(i_t), \qquad I_0 = 0, \tag{2}$$

where v: $R^+ \rightarrow R^+$ is assumed to be twice continuously differentiable and to satisfy $v(0) = 0$ and $\lim_{x \to \infty} v'(x) = 0$. Y2K investment is assumed to be irreversible; that is,

$$i_t \geq 0. \tag{3}$$

To ensure that agents will always find it optimal to invest in solving the Y2K problem, we impose the following assumption:

Assumption 1. $-v'(0)g'(0) > A$.

This assumption says that if the household chooses not to invest any resources in solving the Y2K problem until the year 2000 ($i_t = 0\ \forall t \leq T$), then at the beginning of the new millennium, the rate of return on Y2K investment, $-v'(0)g'(0)$, exceeds the rate of return on physical capital, A.

Output can be allocated to consumption, investment in physical capital, or investment in solving the Y2K problem. Thus, the flow resource constraint of the household is given by

$$c_t = \begin{cases} Ak_t - \dot{k}_t - i_t & t < T \\ Ak_t - \dot{k}_t - i_t - g(I_t) & t \geq T \end{cases}, \tag{4}$$

where $\dot{k}_t$ denotes net investment in physical capital at time t. The household chooses sequences $\{i_t, I_t, c_t, k_t\}$ to maximize (1) subject to (2), (3), and

(4), given k_0 and I_0. The first-order conditions of the consumer's problem are (Arrow and Kurz, 1970) (2), (3), (4), and

$$\phi_t^k = u'(c_t) \tag{5}$$

$$\dot{\phi}_t^k = \phi_t^k(\rho - A) \tag{6}$$

$$\left[\phi_t^k - \phi_t^i v'(i_t)\right] i_t = 0 \tag{7}$$

$$\phi_t^k - \phi_t^i v'(i_t) \geq 0 \tag{8}$$

$$\dot{\phi}_t^i = \begin{cases} \rho\phi_t^i & t < T \\ \rho\phi_t^i + \phi_t^k g'(I_t) & t \geq T \end{cases} \tag{9}$$

$$\lim_{t\to\infty} e^{-\rho t}\phi_t^k \geq 0 \tag{10}$$

$$\lim_{t\to\infty} e^{-\rho t}\phi_t^i \geq 0 \tag{11}$$

$$\lim_{t\to\infty} e^{-\rho t}\phi_t^k k_t = 0 \tag{12}$$

$$\lim_{t\to\infty} e^{-\rho t}\phi_t^i I_t = 0, \tag{13}$$

where ϕ_t^i and ϕ_t^k are time-differentiable Lagrange multipliers associated with (2) and (4), respectively.

3. Y2K INVESTMENT DYNAMICS

Our first result is that once Y2K investment becomes positive, it must continue to be positive until the year 2000. To see this, suppose that $i_t > 0$ for some $t < T$ and that $i_{t'} = 0$ for some $t < t' \leq T$. Then, by (7), $\phi_t^k = \phi_t^i v'(i_t)$. Because $A > 0$, Eqs. (6) and (9) together with the assumed concavity of v imply that $\phi_{t'}^k < \phi_{t'}^i v'(0)$, which violates Condition (8). Furthermore, if investment is positive at any time before the year 2000, then investment increases over time until the beginning of the millennium. To see this, note first that if $i_{t'} > 0$ for $t' < T$, then, as shown above, $i_t > 0\ \forall t' < t \leq T$. Thus, by Eq. (7), $\phi_t^k = \phi_t^i v'(i_t)\ \forall t' \leq t \leq T$. Because ϕ_t^k and ϕ_t^i are differentiable, this equation implies that i_t is also differentiable for $t' < t < T$. From differentiating this expression and using Eqs. (6) and (9), it follows that $\dot{i}_t = -Av'(i_t)/v''(i_t) > 0\ \forall t' < t < T$. We summarize these results in the following proposition.

PROPOSITION 1 (Accelerating Y2K Investment). *If* $i_t > 0$ *for some* $t < T$, *then* $i_{t'} > 0$ *for all* $t \leq t' \leq T$. *Furthermore*, $i_{t'} < i_{t''}$ *for all* $t \leq t' < t'' < T$.

Next, we establish that the model exhibits precautionary investment, in the sense that agents find it optimal to begin to allocate resources to solving the Y2K problem before the arrival of the year 2000.

PROPOSITION 2 (Precautionary Y2K Investment). *If Assumption* 1 *is satisfied*, *then there exists a* $t < T$ *such that* $i_t > 0$.

Proof. We establish the proposition in three steps, by showing that (a) if $i_t = 0$ $\forall t < T$, then $i_T = 0$; (b) if $i_t = 0$ $\forall t < T$, then $i_t = 0$ $\forall t > T$; and (c) $i_t = 0$ $\forall t$ is impossible. (a) Suppose that $i_t = 0$ $\forall t < T$ and that $i_T > 0$. Then from (7), it follows that $\phi_T^k = \phi_T^i v'(i_T)$. On the other hand, (8) implies that $\lim_{t \to T} \phi_t^k \geq \lim_{t \to T} \phi_t^i v'(0)$, or, because ϕ^k and ϕ^i are continuous, $\phi_T^k \geq \phi_T^i v'(0)$. But this contradicts $\phi_T^k = \phi_T^i v'(i_T)$ because v is strictly concave. (b) Suppose that $i_t = 0$ $\forall t < T$. By (a), $i_T = 0$. Suppose that $i_t > 0$ for some $t > T$. Let $\underline{t} \equiv \inf\{t : i_t > 0\}$. Then in any interval around $\underline{t}$, $\exists t' \leq \underline{t}$ and $t'' > \underline{t}$ such that $i_{t'} = 0$ and $i_{t''} > 0$. It follows from (8) that $\phi_{t'}^k \geq \phi_{t'}^i v'(0)$. Equations (6) and (9) imply that $\dot{\phi}_t^k$ and $\dot{\phi}_t^i$ are continuous. It follows from Assumption 1 that $\dot{\phi}_{t'}^i / \phi_{t'}^i < \dot{\phi}_{t'}^k / \phi_{t'}^k$. Therefore, $\phi_{t''}^k \geq \phi_{t''}^i v'(0)$. Thus, the concavity of v and the fact that $i_{t''} > 0$ imply that $\phi_{t''}^k > \phi_{t''}^i v'(i_{t''})$, contradicting (7). (c) Suppose $i_t = 0$ $\forall t$. Then for $t \geq T$, $\dot{\phi}_t^k = (\rho - A)\phi_t^k$ and $\dot{\phi}_t^i = \rho \phi_t^i + g'(0)\phi_t^k$. Thus, $\phi_{T+t}^i = [\phi_T^i + g'(0)\phi_T^k / A]\, e^{\rho t} - [g'(0)\phi_T^k / A] e^{(\rho - A)t}$ $\forall t \geq 0$. If $[\phi_T^i + g'(0)\phi_T^k / A] > 0$, then $\phi_t^i \to \infty$ at the rate ρ, violating (8) because ϕ_t^k grows at the rate $\rho - A$. If $[\phi_T^i + g'(0)\phi_T^k / A] < 0$, then $e^{-\rho t}\phi_t^i \to \phi_T^i + g'(0)\phi_T^k / A$, violating (11). Finally, if $[\phi_T^i + g'(0)\phi_T^k / A] = 0$, then $\phi_T^k / (\phi_T^i v'(0)) = A / (-v'(0) g'(0))$. The right-hand side of this expression is less than one by Assumption 1; thus the left-hand side violates (8). ∎

In the following proposition, we show that if at any time in the new millennium Y2K investment is positive, then its rate of return must be at least as large as the rate of return on physical capital.

PROPOSITION 3. *If* $i_t > 0$ *for* $t \geq T$, *then* $-v'(i_t) g'(I_t) \geq A$.

Proof. Suppose that $i_{t'} > 0$. Then, because $I_t \geq I_{t'}$ for $t \geq t'$, (9) implies that $\dot{\phi}_t^i \geq \rho \phi_t^i + \phi_t^k g'(I_{t'})$ $\forall t \geq t'$. Using (6) and integrating this expression yield

$$\phi_t^i \geq \left[\phi_{t'}^i + \frac{g'(I_{t'})\, \phi_{t'}^k}{A} \right] e^{\rho(t - t')} - \frac{g'(I_{t'})\, \phi_{t'}^k}{A} e^{(\rho - A)(t - t')},$$

which violates (13) unless $\phi^i_{t'} + g'(I_{t'})\phi^k_{t'}/A \leq 0$. The result follows immediately from the fact that $\phi^k_{t'} = \phi^i_{t'} v'(i_{t'}) > 0$. ▮

We are now ready to show a key prediction of the model, namely that agents may optimally choose to delay Y2K investment.

PROPOSITION 4 (Y2K Investment Delay). *For T sufficiently large, there exsits a $t' \in (0, T)$ such that $i_t = 0$ for all $t < t'$.*

Proof. Let $t' \in (0, T)$ and suppose that $i_{t'} > 0$ $\forall T$. By Proposition 1, $i_t > 0$ $\forall t' \leq t \leq T$. From (7), (9), and (6), it then follows that $\phi^k_{t'} e^{(\rho - A)(T - t')} = v'(i_T)\phi^i_{t'} e^{\rho(T - t')}$, which, using (5) and simplifying, becomes $v'(i_{t'})e^{-A(T-t')} = v'(i_T)$. Since $i_{t'} > 0$, $0 < v'(0) < \infty$, and $v'' < 0$, this expression implies that $v'(0)e^{-A(T-t')} > v'(i_T)$. Thus, $\lim_{T \to \infty} v'(i_T) = 0$, violating the condition $-v'(i_T)g'(I_T) \geq A$ derived in Proposition 3. ▮

In the case that agents choose to delay Y2K investment, the transition from no investment to positive investment will be smooth. In fact, as the next proposition shows investment is continuous everywhere.

PROPOSITION 5 (No Jumps in Y2K Investment). *i_t is continuous.*

Proof. Suppose investment is discontinuous at t'. Then, because v' is continuous, $\exists \epsilon > 0$ such that for any $\delta > 0$ one can find a point $t''(\delta)$ satisfying $|t' - t''(\delta)| < \delta$ and $|v(i_{t'}) - v(i_{t''(\delta)})| > \epsilon$. Suppose $i_{t'} > 0$. Then, by (7), $\phi^k_{t'} = v'(i_{t'})\phi^i_{t'}$. Because ϕ^k_t and ϕ^i_t are continuous, as $\delta \to 0$, $\phi^k_{t''(\delta)} \to \phi^k_{t'}$ and $\phi^i_{t''(\delta)} \to \phi^i_{t'}$. Thus, for δ sufficiently small, $\phi^k_{t''(\delta)} \neq v'(i_{t''(\delta)})\phi^i_{t''(\delta)}$, which, by (7), implies that $i_{t''(\delta)} = 0$. But if $i_{t''(\delta)} = 0$, then $v'(i_{t''(\delta)}) - v'(i_{t'}) > \epsilon > 0$; thus, for δ sufficiently small we have $\phi^k_{t''(\delta)} < v'(i_{t''(\delta)})\phi^i_{t''(\delta)}$, which violates (8). If $i_{t'} = 0$, then $\phi^k_{t'} \geq \phi^i_{t'} v'(i_{t'})$. Also, $i_{t''(\delta)}$ must be positive. Thus, the continuity of ϕ^k_t and ϕ^i_t and the fact that $v'(i_{t'}) - v'(i_{t''(\delta)}) > \epsilon$ imply that for δ sufficiently small, $\phi^k_{t''(\delta)} > v'(i_{t''(\delta)})\phi^i_{t''(\delta)}$, which violates (8). ▮

The presence of physical capital is crucial in generating continuity in Y2K investment. One can show that in an endowment economy Y2K investment displays a discrete decline in the year 2000 (Schmitt-Grohé and Uribe, 1998).

We complete the characterization of equilibrium by studying the dynamics of Y2K investment in the new millennium. The following proposition shows that Y2K investment peaks with the arrival of the new millennium, is monotonically decreasing thereafter, and converges to zero in a continuous fashion.

PROPOSITION 6 (Y2K Investment Dynamics in the New Millennium). (a) *Deceleration*: *If $i_t > 0$ for $t > T$, then $\dot{i}_{t'} < 0$ for all $T < t' < t$.* (b) *Investment converges to zero*: $\lim_{t \to \infty} i_t = 0$. (c) *Investment peaks at the arrival of the year* 2000: *$i_T > i_t$ for all t.*

Proof. (a) We first prove that if $i_{t*} = 0$ for some $t* \geq T$, then $i_t = 0$ $\forall t > t*$. Suppose, contrary to the claim, that $i_t > 0$ for some $t > t*$. Let $\underline{t} \equiv \inf\{t > t* : i_t > 0\}$. Then in any interval around $\underline{t}\, \exists t' \leq \underline{t}$ and $t'' \geq \underline{t}$ such that $i_{t'} = 0$ and $i_{t''} > 0$. It follows from (8) that $\phi^k_{t'} \geq \phi^i_{t'} v'(0)$. By Proposition 3 and Conditions (8) and (9) $\dot{\phi}^i_{t'}/\phi^i_{t'}$ is continuous and less than $\dot{\phi}^k_{t'}/\phi^k_{t'}$, which is also continuous. Therefore, $\phi^k_{t''} \geq \phi^i_{t''} v'(0)$. Thus, the concavity of v and $i_{t''} > 0$ imply that $\phi^k_{t''} > \phi^i_{t''} v'(i_{t''})$, contradicting (7). We now show deceleration. Suppose $i_{t'} > 0$ for some $t' > T$. It then follows immediately from the above argument that $i_t > 0$ $\forall T \leq t < t'$. Therefore, $\phi^k_t = v'(i_t)\phi^i_t$ $\forall T \leq t \leq t'$. Differentiating this expression and using (6) and (9) yield $\dot{i}_t = -v'(i_t)/v''(i_t)[v'(i_t)g'(I_t) + A]$. From the facts that v is increasing and concave and that the expression in square brackets is nonpositive (Proposition 3) it follows that Y2K investment is nonincreasing. Assume that $\dot{i}_t = 0$ for some $t < t'$. Then differentiating $\dot{i}_t$ yields $\ddot{i}_t = -v'(i_t)^2/v''(i_t)g''(I_t)i_t > 0$, which implies that $\dot{i}_{t''} > 0$ for some $t'' > t$, violating the fact that investment is nonincreasing. (b) It follows from (a) that i_t is nonincreasing for $t \geq T$. Thus, because i_t is bounded below by zero, it must converge. Suppose i_t converges to some positive constant $i*$. Then $I_t \to \infty$ and $-v'(i_t)g'(I_t) \to 0$, violating Proposition 3. (c) The result follows directly from (a), (b), and Propositions 1 and 2. ∎

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