Department of Economics
Columbia University

Instructors: Stephanie Schmitt-Grohé
Martín Uribe

Problem Set 2
Economics G6222
Advanced Macroeconomic Analysis II: Economic Fluctuations
Due September 25, 2008

This homework asks you to use Matlab to find a log-linear approximation to the policy function of simple RBC models. Following the steps outlined below requires having access to a version of Matlab with the Symbolic Math Toolbox. For example, the Matlab version on the cluster has Symbolic Math toolbox. (You can use the programs neoclassical_model_ss.m, neoclassical_model.m, and neoclassical_model_run.m posted on our Second-Order Approximation Website as a guide.)

1. **Replicating King, Plosser, Rebelo I**

   Create a Matlab function (and name it kpr_ss.m) that computes the non-stochastic steady state of the KPR model developed in class. Assume, as in class, that $F(k,h) = k^{\alpha}h^{(1-\alpha)}$ and $U = \frac{\left[c(1-h)^{\theta_l}\right]^{(1-\sigma)}}{1-\sigma}$ The output arguments of this function should be the deterministic steady state values of hours (h), consumption ($c = C/Z$), capital ($k = K/Z$), total factor productivity (a), and the values taken by the deep structural parameters of the model: $\alpha, \sigma, \theta_l, \delta, \gamma_z$, and β. Calibrate the model as described in section 3.2.2 of KPR I. Assume that the serial correlation of total factor productivity, A_t, is $\rho_A = 0.9$. Report the steady state values of c, h, k, and A. Find the consumption share in output and the capital output ratio.

   (a) Create a Matlab function (and name it kpr_model.m) that returns symbolic expressions for the Jacobian of the function f describing the equilibrium conditions with respect to the natural logaritm of hours, consumption, capital, and total factor productivity. In terms of the notation used in class kpr_model.m should return symbolic expressions for f_x, $f_{x'}$, f_y, $f_{y'}$. These matrices should be expressed only in terms of the steady state values of the variables, that is, in terms of a, k, c, h and the six deep structural parameters. (Use findsym.m to check whether this is so.)

   (b) Using the Matlab function gx_hx.m or gxhx.m posted on my website find the transition matrix hx for the evolution of the state variables. Is the equilibrium locally unique? Report the numerical eigenvalues of the matrix hx.

2. **Can Hours Decline on Impact in Response to a Positive TFP shock?**

   (a) Discuss at an intuitive level how the initial response of hours to an increase in total factor productivity is influenced by the deep structural parameters of the model. In particular, which parameterization would emphasize the wealth effect of a positive TFP shock. Provide some numerical results to illustrate your argument. Can you find a parameterization for which hours fail to increase much (relative to the baseline calibration) on impact or even decline?

   (b) Now modify the model to allow for adjustment cost in consumption. In particular, assume that the period utility function takes the form:

$$U(C_t - bC_{t-1}, h_t)$$

where $b \in [0,1)$ is a parameter, governing the degree of habit formation. Assume that agents internalize that their current consumption choice, C_t, will affect their utility not only in period t but also in period $t+1$. Show how the equilibrium conditions are changed by this introduction by internal habit formation. Provide an intuitive explanation how the presence of habits should affect the initial response of hours to a positive productivity shock. Present the complete set of equilibrium conditions in stationary form. Solve for the non-stochastic steady state. Assume that the habit parameter takes the value 0.5. Find a log-linear approximation to the solution to the model and present parameterizations of the model for which the initial response of hours to a technology shock is either negative or very close to zero.