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Problem Set 4
Economics G6222
Advanced Macroeconomic Analysis II: Economic Fluctuations
Due October 30, 2008

Consider a second-order accurate approximation to the level of welfare in the business-cycle model of King, Plosser, Rebelo I that we outlined in class. Assume that $U(c,h) = c^{1-\sigma}(1-h)^{\theta_l(1-\sigma)}/(1-\sigma)$. Calibrate the economy as follows. Assume that the time unit is one quarter and that the real interest rate in the deterministic steady state is 4 percent per year. Assume that the intertemporal elasticity of substitution is 1/0.95, the annual depreciation rate is 10 percent, and that the capital share is 0.42. Agents devote 20 percent of their time to market work in the nonstochastic steady state. Abstract from long run growth. Assume that the stationary technology shock, z_t, follows a first-order autoregressive process of the form $\ln z_{t+1} = \rho_z \ln z_t + \sigma_\epsilon \eta \epsilon_{t+1}$. Assume that $\rho_z = 0.9$. Set η such that the standard deviation of output is 1.72 percent. Report the value of η you find. Report the eigenvalues of the matrix hx.

1. State the equilibrium conditions you are using. (This could be just copied from an earlier homework.)

2. Show your calibration and find the non-stochastic steady state values of output, hours, consumption, capital, investment, and $V_t \equiv E_t \sum_{j=0}^{\infty} \beta^j U(c_{t+j}, h_{t+j})$.

3. Find a second order accurate approximation to the unconditional expectation of the level of welfare. Discuss how expected welfare changes with the presence of uncertainty, that is, with the parameter σ_ϵ. Furthermore, compute a second-order accurate approximation to the mean of (the level of) output, capital, and consumption and discuss how those means depend on the uncertainty parameter σ_ϵ. Provide an intuitive explanation for your findings.

4. Now perform a welfare cost of business cycle calculation in the spirit of Lucas 1987. Specifically, find which percentage of their deterministic steady-state consumption stream agents would be willing to give up to be as well off as in a world with stochastic variations in the level of total factor productivity. (Here you have to find the second-order accurate approximation to the unconditional mean of V_t first). Compare your results to those of Lucas' welfare cost of business cycle and provide a verbal explanation for the differences.

5. Suppose the government, in an effort to stabilize business cycles, introduces a cyclical income tax. In particular, assume that it levies proportional income taxes in the amount of τ_t each period on labor income, $w_t h_t$, and on capital income granting a depreciation allowance. Tax revenue is rebated to households lump sum each period (government expenditure continue to be zero at all times). Assume that the government sets the tax rate to minimize the variance of output. Show the complete set of equilibrium conditions. Report the values takes by the endogenous variables in the non-stochastic steady state. Report the eigenvalues of hx. Calculate the (second-order accurate) welfare gain/cost of this stabilization policy taking as the point of comparison a policy of no stabilization. Provide an intuitive explanation of your results.