Columbia University
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Office Hours: Tuesdays 11am to 12pm

Economics G6222
Advanced Macroeconomic Analysis II
Economic Fluctuations
Second Half

Homework 5

Due 26-Nov-2008

Comparing Two Common Filters:

Two filters commonly used to characterize business cycles in macroeconmics are growth rates and the Hodrick-Prescott filter. The purpose of this homework is to compare theoretically and empirically these two filters.

1. With this homework, you received a MAT file containing four time series: real output (y), real consumption (c), real gross investment (ivv), and hours (h). The data are for the United States (seasonally adjusted) and the sample is 1955:Q1 to 2006:Q4. Construct a 3×4 table displaying the standard deviation, correlation with output growth—defined as $\ln(y_t/y_{t-1})$—and serial correlation of the variables $\ln(y_t/y_{t-1})$, $\ln(c_t/c_{t-1})$, $\ln(ivv_t/ivv_{t-1})$, and $\ln(h_t)$.

2. Now consider the variables $\ln(y_t)$, $\ln(c_t)$, $\ln(ivv_t)$, and $\ln(h_t)$]. HP filter each of these time series individually using a smoothing parameter value of 1600 . Denote the HP filtered data as y_t^c, c_t^c, i_t^c, and h_t^c. Construct a 3×4 table displaying the standard deviation, correlation with output (y_t^c), and serial correlation of the four HP filtered time series.

3. Consider the following RBC model in which households solve the problem

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \frac{[C_t(1-h_t)^\chi]^{1-\sigma}-1}{1-\sigma}$$

subject to

$$K_{t+1} = (1-\delta)K_t + I_t$$
$$Y_t = K_t^\alpha (X_t h_t)^{1-\alpha}.$$
$$C_t + I_t = Y_t.$$

$\mu = 1.016$. Fix the parameter χ to ensure that households allocate 20 percent of their time to work ($h = 0.2$). Report the implied value of χ.

Using a first-order approximation of the equilibrium conditions of the model. construct a 3×4 table displaying the unconditional population standard deviation, correlation with output growth and serial correlation of the variables $\ln(Y_t/Y_{t-1})$, $\ln(C_t/C_{t-1})$, $\ln(I_t/I_{t-1})$, and $\ln(h_t)$. Compare these theoretical second moments with their empirical counterparts. Identify the dimension along which the model is least successful in mimicking the data.

4. Now consider the population Hodrick-Prescott filtered version of the variables $\ln(Y_t)$, $\ln(C_t)$, $\ln(I_t$, and $\ln(h_t)$ implied by the model. The fact that you are computing the population Hodrick-Prescott filter means that you need not perform any simulations with artificial data. Instead, define the HP filter as follows. Consider any of the four variables in question, say Y_t. Let $y_t \equiv \ln Y_t$. Then, express y_t as the sum of its trend component, y_t^τ, and its cyclical component, y_t^c.

$$y_t = y_t^\tau + y_t^c. \tag{1}$$

The population Hodrick-Prescott Filter is defined as the solution to the following problem

$$\min_{y_t^c} E_0 \sum_{t=0}^{\infty} \left\{ \left(y_t^c\right)^2 + \lambda \left[\left(y_t^\tau - y_{t-1}^\tau\right) - \left(y_{t-1}^\tau - y_{t-2}^\tau\right) \right]^2 \right\}$$

subject to (1). In the above expression, λ is the smoothing parameter.

Derive the first-order conditions associated with this problem in terms of current, past, and future expected values of y_t^c and $g_t^y \equiv \ln(Y_t/Y_{t-1})$ alone (i.e., get rid of present, past, and future expected values of y_t^τ).

5. For the purpose of computing population second moments of HP filtered variables, you can add, for each of the four variables, the optimality condition obtained in the previous item to the set of equilibrium conditions of the model. This procedure will require the definition of additional state and control variables. Construct a 3⋈4 table displaying the unconditional population standard deviation, correlation with output (y_t^c), and serial correlation of the variables y_t^c, c_t^c, i_t^c, and h_t^c. Use a smoothing parameter value of 1600. Compare these theoretical second moments with their empirical counterparts. Identify the dimension along which the model is least successful in mimicking the data. Relate the answer to this question to that given in the case in which business cycles were characterized using growth rates and the unfiltered level of hours.