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Take-home Midterm Examination
Economics G6222
Advanced Macroeconomic Analysis II: Economic Fluctuations
Hand in your answer at the beginning of class on November 11, 2008

Investment Adjustment Costs, Decreasing Returns, and the Value of the Firm over the Business Cycle

Consider an economy populated by a large number of identical agents with preferences described over consumption, C_t and leisure, ℓ_t

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, \ell_t), \tag{1}$$

where β denotes the subjective discount factor. The total time endowment per period is assumed to be unity so that hours worked, denoted h_t, are given by

$$h_t = 1 - \ell_t. \tag{2}$$

Assume that $U(C, \ell) = (C\ell^6)^{1-\sigma}/(1-\sigma)$

Output, denoted Y_t, is produced with a homogeneous-of-degree-one production function that takes as inputs capital, K_t, labor, h_t, and land, L. Land is assumed to be a fixed factor owned by households that is not accummulable. Owners of physical capital can control the intensity with which the capital stock is utilized. Let u_t measure capacity utilization in period t. The effective amount of capital services households supply to firms in period t is given by $u_t K_t$. Increasing the intensity of capital utilization entails a cost in the form of a faster rate of depreciation. That is, the depreciation rate is an increasing and convex function $\delta(u_t)$ of the rate of capacity utilization, u_t.

The production technology is buffeted by transitory productivity shocks denoted z_t and by permanent productivity shocks denoted X_t. Assume that the law of motion of these technology shocks is $\ln z_{t+1} = 0.95 \ln z_t + \epsilon^1_{z,t}$ and $\ln \mu^x_{t+1}/\mu^x = \epsilon^1_{x,t}$; $\quad \mu^x_t \equiv \frac{X_t}{X_{t-1}}$.

The production function is given by

$$Y_t = z_t F(u_t K_t, X_t h_t, X_t L) = z_t (u_t K_t)^{\alpha_K} (X_t h_t)^{1-\alpha_K-\alpha_L} (X_t L)^{\alpha_L}, \tag{3}$$

where the production function parameters α_K and α_L satisfy $0 \leq \alpha_K$, α_L and $\alpha_K + \alpha_L < 1$.

The resource constraint of the economy then takes the form:

$$C_t + A_t I_t = Y_t, \tag{4}$$

where the variable A_t denotes the technical rate of transformation between consumption and investment goods and I_t denotes gross investment in period t. The relative price of investment is exogenous and stochastic and evolves over time according to $\ln \mu^a_{t+1}/\mu^a = \epsilon^1_{a,t}$; $\quad \mu^a_t \equiv \frac{A_t}{A_{t-1}}$.

The evolution of physical capital over time is subject to investment adjustment costs of the form:

$$K_{t+1} = (1 - \delta(u_t))K_t + I_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right]. \tag{5}$$

The function S introduces investment adjustment costs. In the deterministic steady state, the function S is assumed to satisfy $S = S' = 0$ and $S'' > 0$.

1. Find the complete set of equilibrium conditions.
2. Find a stationarity inducing transformation of the equilibrium conditions.
3. Characterize the non-stochastic steady state analytically.
4. Calibrate the economy as follows. The time unit is a quarter. Assume that the capital share is 30 percent, that the intertemporal elasticity of substitution is 1/2 ($\sigma = 2$), that output growth is 0.45 percent per quarter, that investment price declines at the rate 0.43 percent per quarter. Choose a quarterly subjective discount factor of 0.98, an annual depreciation rate of 10 percent, and assume that in the steady state capacity utilization is equal to unity, $u = 1$. Further, assume $\delta(u) = \delta_0 + \delta_1(u-1) + \frac{\delta_2}{2}(u-1)^2$, with $\delta_2 = 0.1$ and an investment adjustment cost function $S(x) = \frac{5}{2}(x - \mu^i)^2$, where μ^i is the steady state growth rate of investment, I_t.

   Report the numerical values of hours, the investment share, the consumption share, and the value of the firm relative to output in the non-stochastic steady state for the case $\alpha_L = 0$ and $\alpha_L = 0.1$. In calculating the value of the firm assume that the capital stock and land is owned by firms and that the value of the firm is calculated at the end of the period, that is, not taking into account the current returns to land and capital. Provide intuition regarding how the presence of the fixed factor affects the steady state.

5. Find (log-linear approximation to) the impulse response of the level of output, consumption, investment, hours, and stock prices. Approximate stock prices by the end-of-period value of the firm. Consider impulse response to:

   (a) a one-precent increase in z_1 that is learned in period 0

   (b) a one-percent increase in X_1 that is learned in period 0

   (c) a one-percent increase in A_1 that is learned in period 0

   Report results for the case that $\alpha_L = 0$ (a model with constant returns in capital and labor) and for the case that $\alpha_L = 0.1$ (a model with decreasing returns in capital and labor). (To facilitate grading you should report for each case numerical values and plots for the impulse responses, and eigenvalues of your hx matrix. Indicate which variables are states and which ones are co-states.) Evaluate how well/poorly each model, that is, the one with the fixed factor and the one without it, explains actual observed business cycle comovements. Which of the three sources of uncertainty considered generates comovement that makes it compelling as a major source of business cycles in the sense that it produces comovement of the type observed in U.S. data. Provide a verbal explanation/intuition for your findings. No credit will be given for answers without a verbal explanation/discussion.