TFP Declines: Misallocation or Mismeasurement?

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[Link to the latest version]

Abstract

This paper documents the sectoral growth paths of measured total factor productivity (TFP) in southern Europe during the boom that proceeded the great contraction (1996 to 2007). Using both aggregate and firm-level panel data, I show that TFP in sectors that displayed fast expansion, such as construction, dropped significantly, while in non-expanding sectors, such as manufacturing, it stayed stable. I evaluate the relevance of two alternative explanations of this phenomenon: capital misallocation (the increase in capital was directed to less productive firms) and labor quality mismeasurement (lower quality of incoming labor was not fully captured in the TFP calculation). I find that the misallocation channel is almost negligible. Moreover, worker-firm matched data shows that labor quality did deteriorate in the expanding sectors but not in the others, giving credence to the labor-quality mismeasurement hypothesis. A model featuring both the misallocation and the mismeasurement channels and calibrated to match the micro-level productivity distribution and labor quality distribution predicts that the drop in true TFP was small if labor quality is measured properly.

JEL: D22, D24, E22, F41, O16, O47

Keywords: Labor Quality Mismeasurement, Capital Misallocation, Total Factor Productivity, Eurozone Integration

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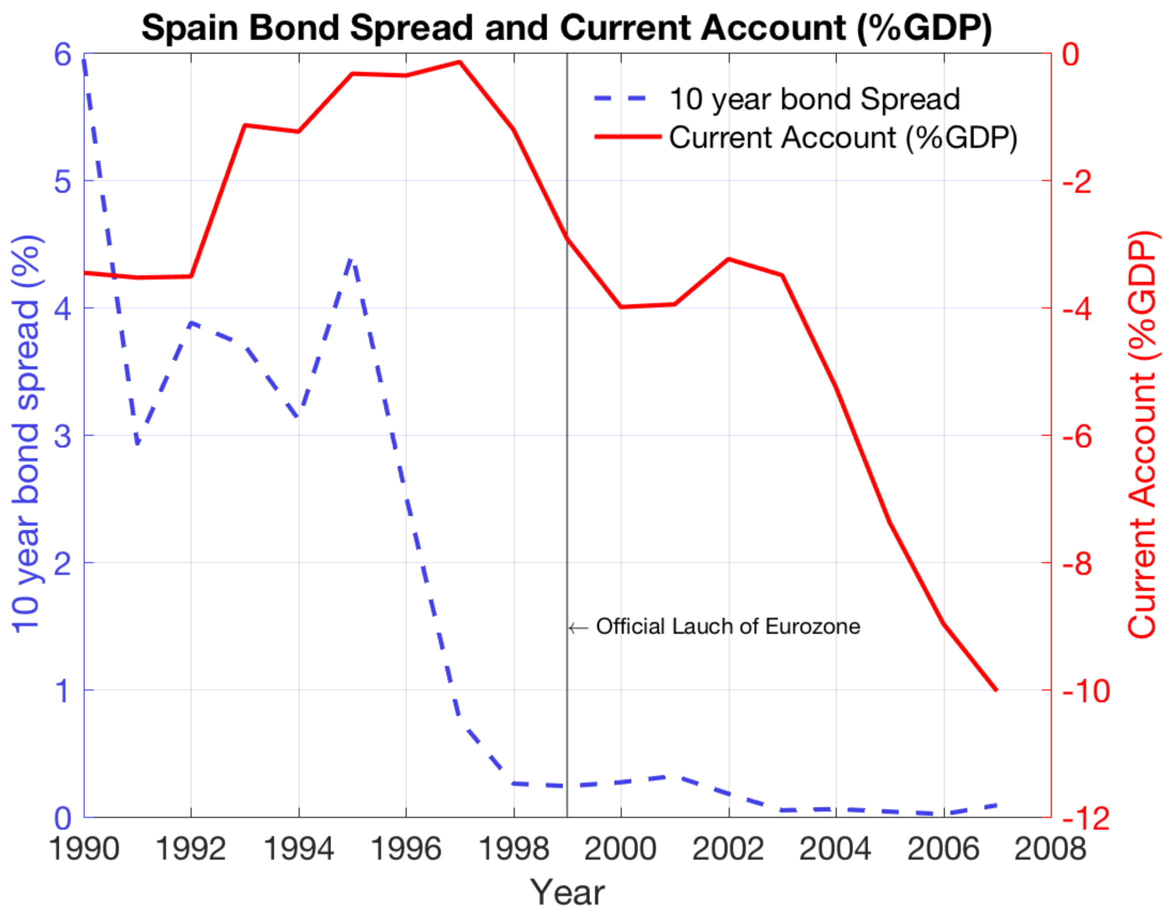
1 Introduction

The Eurozone integration (officially started in 1999, expectation started in 1996) was accompanied by plummeting of borrowing cost and continuing deterioration of the current accounts in Spain and other southern European countries, as shown in Figure 1.1. The economy was booming, but its measured total factor productivity (TFP) was decreasing. Researchers and policy makers often blame this negative correlation between the expansion of the economy and TFP growth on capital/resource misallocation. The idea is that the cheap credit flowed more into less productive firms and the compositional change of the economy brought down the average productivity.

Figure 1.1: Interest Rate Spread and Current Account



Raw data: WDI
The definition of spread: the difference between the bond yield of Spain and Germany

This paper challenges the view that the misallocation channel is the main explanation for the TFP drop, and it proposes that the labor-quality mismeasurement channel is a more reasonable explanation. The labor-quality mismeasurement channel ascribes the measured TFP drop to the lower efficiency of the incoming labor compared to the existing labor force in expanding sectors. The idea is that the TFP calculation does not fully capture labor quality, which is automatically translated to the TFP measured as a residual. Although the labor quality can be measured to some extent with limited observable characteristics, such as education, age, and gender, other dimensions such as tenure are usually not widely observed.

The argument for labor-quality mismeasurement channel is developed in five steps. First, I show that TFP decline is much more severe in expanding sectors (such construction and real sector) than in relatively stable sectors (such as manufacturing), using both aggregate data and firm-level data. The aggregate TFP data is from Klems, calculated under the assumption of the constant return to scale. The firm-level TFP is calculated using Amadeus data obtained from vintage discs. The firm-level TFP measurement methodologies are built on Levinsohn and Petrin (2003) and De Loecker (2011).[1] Although constant return to scale is not imposed in the firm-level TFP estimation, the result is very close to it.

Second, the growth of dispersion of the marginal revenue product of capital (MRPK) suggests there is more misallocation of capital in non-expanding sectors than in expanding sectors. Using firm-level data, I demonstrate that the growth of dispersion of the MRPK is more significant in non-expanding sectors than in expanding sectors. The dispersion of the MRPK is usually considered as an indication of the capital misallocation following Chang-Tai Hsieh (2009), and it serves as the main evidence for the papers (such as Gopinath et al. (Forthcoming), and Garcia-Santana et al. (2016)) arguing that capital misallocation caused the TFP stagnation problem in southern Europe. If capital misallocation is the real

[1] I only discuss the mismeasurement of the labor quality but not the mismeasurement of the capital quality is because studies like Sakellaris and Wilson (2004), show that newly invested capital has, on average, higher quality than the existing one. Therefore, if we take into account capital quality mismeasurement, then the TFP growth paths of the expanding sectors and non-expanding sectors would be even more divergent.

explanation of the TFP drop, we should observe more growth of the dispersion of the MRPK in expanding sectors than in non-expanding sectors. [2]

Third, decomposition of the TFP growth indicates that the between-firm TFP growth is negligible compared to the within-firm growth in the sub-sample of stayers. Compared to the dispersion growth of the MRPK, this is a more direct piece of evidence showing that the capital misallocation channel cannot be the main explanation, but the labor-quality mismeasurement channel can be. If capital misallocation is the main channel, we should observe that resources move more into less productive firms. In other words, the decomposition result should reveal that between-firm TFP change accounts for the lion's share of the TFP drop. The data shows otherwise: both in expanding sectors and in non-expanding sectors, the between-firm change of TFP is miniscule. The within-firm TFP change accounts for one-third of the total TFP drop in expanding sectors but increased slightly in expanding sectors. This observation is consistent with the labor-quality mismeasurement channel.

Fourth, using the worker-firm matched data, I establish that the limited observable characteristics of workers are not sufficient to control for labor quality and that labor quality beyond education, age and gender deteriorates in expanding sectors but not in non-expanding sectors. The worker-firm matched dataset is from the Eurostat Structure of Earnings Survey. The characteristics that KLEMS dataset employs to control for labor quality are education, age and gender, which only account for one-third of the wage variation in the manufacturing sector and less than 20 percent of the wage variation in the construction sector. Worker's tenure (which can be thought as an imperfect proxy for experience), especially that of firm managers, has decreased significantly in expanding sectors but has increased in non-expanding sectors. Moreover, the distribution of unobserved labor quality is backed out by taking out firm fixed effects and observed labor characteristics from the real hourly wage, which can be fed into the model later. More specifically, by running the regression of hourly

[2]There are papers that both support Chang-Tai Hsieh (2009) and argue against it. Whether dispersion of the MRPK is a good measure of the capital misallocation or not, the misallocation channel as the main explanation for the TFP drop is inconsistent with data.

wage on firm characteristics, the distribution of the residual is taken as the distribution of labor quality.

Last but not least, I build a model featuring both the misallocation channel and the mismeasurement channel and calibrate it using the micro-level data. The model shows that once the labor quality mismeasurement is properly measured, the TFP drop is small. The model works as follows: a negative interest rate shock (Eurozone integration) enables low-productivity firms in the non-tradable sector to enter the production by borrowing. This brings down the average productivity of this sector. Moreover, borrowing costs for non-tradable firms are also lowered and allows them to borrow more; thus, the sector expands. The tradable sector is not affected by the shock, since it is assumed that the tradable firms are far less financially constrained. Therefore, there is no expansion in this sector. The expansion in the non-tradable sector increases base wage and attracts the labor from the tradable sector. The marginal worker entering the non-tradable sector is less efficient compared to the average existing workers, while the way the TFP is calculated treats incoming workers the same as the existing ones. So, the lower efficiency of the worker is translated into lower measured TFP. The existence of the mismeasurement channel makes the true TFP drop much less acute than the measured TFP suggests.

Related Literature. This paper contributes to a body of work that studies the measured TFP drop during the Eurozone integration period. Compared to the papers that argue capital misallocation is the main channel for measured TFP drop, this paper studies an additional labor quality mismeasurement channel and finds it to be more important. Reis (2013), Calligaris (2015), Dias, Marques and Richmond (2016), Garcia-Santana et al. (2016), Cette, Fernald and Mojon (2016) and Gopinath et al. (Forthcoming) all argue that capital misallocation is the main reason that TFP has declined in southern Europe. Cette, Fernald and Mojon (2016) provides aggregate evidence based on VAR analysis that interest rate drop triggers the productivity decline. But this correlation between negative interest rate shock and productivity change is consistent with the mechanism in my model as well. Calligaris

(2015) and Gopinath et al. (Forthcoming) both use firm-level data and provide evidence that removing the heterogeneity of productivity can substantially increase the aggregate productivity substantially. The analysis, however, is restricted to the manufacturing sector and thus ignores the significant difference between expanding sectors and non-expanding sectors. Dias, Marques and Richmond (2016) and Garcia-Santana et al. (2016) extend the misallocation to multiple sectors with administrative data. However Dias, Marques and Richmond (2016) provides only the dispersion of the productivity, which is not necessarily due to capital misallocation. Garcia-Santana et al. (2016) posits that more government influence is associated with more misallocation. Reis (2013) argues that capital misallocation in the non-tradable sector is more severe than that in the tradable sector based on the inference from aggregate data.

There are alternative theories explaining the TFP drop during the Eurozone integration period. Benigno and Fornaro (2014) assumes that labor employed by the tradable sector has a learning-by-exporting effect which depends on the size of employment due to the positive externality. This theory is based on the assumption that the tradable sector has to shrink in the absolute term, while in the data we observe only the relative shrinkage of the tradable sector. Antonia Diaz (2016) shows the correlation between the governmental subsidy to residential structure purchase and the TFP drop. It takes the measured TFP drop as given and estimates the subsidy has to be 50 percent of the price of residential structure to generate the observed TFP dynamics. Challe, Lopez and Mengus (2016) argues the decline of the quality of institution due to the capital inflow can explain the dismal TFP performance. However, it does not distinguish the institutional quality among different sectors.

The labor quality mismeasurement channel in my model can be linked to two bodies of work. Theoretically, I incorporate the partial equilibrium model of Young (2014) into a general equilibrium model. The economic narrative that labor quality in an expanding sector could deteriorate dates back to Roy (1951). The empirical part in this paper is very related to the analysis pioneered by Abowd and Kramarz (1999) using worker-firm matched data. Card et al. (2016) is a recent paper in the same literature. These papers argue that wage variation can be decomposed to firm characteristics and worker characteristics.

Other papers address the connection between the business cycles and the quality of the labor force. My paper discusses the labor quality deterioration during a boom of the economy. Solon, Barsky and Parker (1994) finds that true procyclicality of real wage is obscured in aggregate time series because of a composition bias due to more low-skill workers during expansions. Mulligan (2011) studies the higher labor productivity during the recession of 2008-9, and part of the reason is that the remaining labor force had higher quality relative to that before the recession. Mueller (2017) shows that in recessions the pool of unemployed tends to have workers with higher quality.

Another strand of literature to which my paper connects to is the literature of firm-level TFP measure. My estimation of firm-level TFP follows methodologies in Levinsohn and Petrin (2003), De Loecker (2011) and Ackerberg, Caves and Frazer (2015). The origin of the literature can be traced back to Olley and Pakes (1996). The firm-level TFP measure in my paper confirms the widely studied phenomenon of the very dispersed firm-level TFP measure discussed thoroughly in Syverson (2011).

My paper also links to the literature studying whether static dispersion of the MRPK is a good measure of capital misallocation. Chang-Tai Hsieh (2009) argues that it is a very good indicator of capital misallocation and that reducing dispersion could lead to a productivity increase and an output boost. However, there are papers arguing that other reasons could lead to the dispersion of the MRPK. For example Asker, Collard-Wexler and Loecker (2014) finds that capital adjustment cost can explain 80-90 percent of the cross-industry and cross-country variation in the dispersion of the MRPK. Bils, Klenow and Ruane (2017) finds that measurement error plays a big role in explaining measured misallocation. My paper does not take a stand in this debate. Whether the dispersion of the MRPK is a good measure of capital misallocation or not, I instead show that the misallocation channel cannot explain the TFP drop during the Eurozone integration period.

The remainder of the paper is organized as follows. Section 2 shows stylized facts using aggregate data and calculates the dual measure of the TFP to argue against a possible explanation that measured TFP growth reflects the markup growth. Section 3 employs the Spainish firm-level data to show that capital misallocation is not the right explanation and that labor-quality mismeasurement might be the dominant channel. Section 4 provides evidence that labor quality does deteriorate in expanding sectors but not in non-expanding sectors. Section 5 presents the model. Section 6 shows the calibration and the numerical result. Section 7 concludes.

2 Aggregate Time Series Evidence on TFP Decline

In this section, I show using aggregate data that measured TFP declined or stagnated, especially for expanding sectors .

The aggregate time series data presented in this section are from KLEMS. The KLEMS dataset estimates the TFP mainly for European countries on the two-digit sectoral level. It has different release dates, the one used here is the 2009 release[3]. There are two reasons why I use the 2009 release instead of releases of other years. First, the 2009 release provides the best combination of the temporal coverage and geographical coverage. Since this paper primarily studies the booming period before the 2008 great recession, the 2009 release, with observations until 2007, fits the purpose very well. Moreover, the 2009 release has a lot more geographical coverage, as it includes countries such as Japan and Korea. The two countries experienced booms in certain sectors during the 1990s. Then we can see if what we observe in Europe can be observed elsewhere in a different time period. Second, the 2009 release is the latest release that divides the sector based on the ISIC Rev.3 or NACE 1.1 standard, which is in perfect consistency with the firm-level data that I will show in the next section.

[3]The newest release is 2016. Before that, there were the 2012 release, the 2009 release, the 2008 release and the 2007 release.

2.1 Primal Measure of TFP

The TFP estimation method used in the KLEMS dataset is the primal measure. It assumes that the production function is a constant return to scale Cobb-Douglas function. Thus, the TFP growth will be measured as the growth rate of the Solow-residual:

$\Delta \ln A_{st} = \Delta \ln Y_{st} - \bar{s}^K_{st} \Delta \ln K_{st} - \bar{s}^L_{st} \Delta \ln L_{st}$,

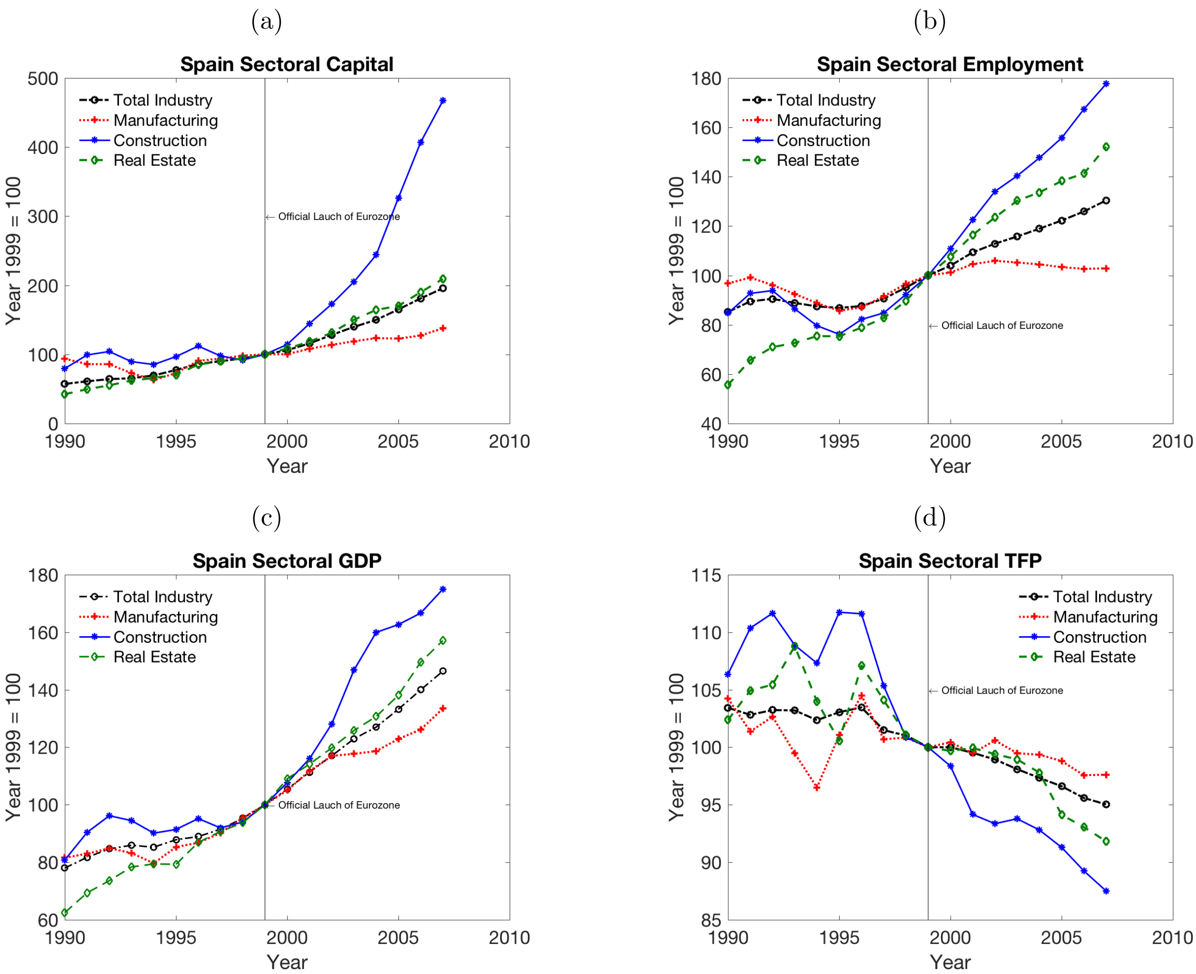
where $s^L_{st} = \frac{\text{Labor Income}}{\text{Nominal GDP}}$, indicating the labor share of sector s at time t, and $\bar{s}^L_{st}$ is the two-period average of s^L_{st}. s^K_{st} is the capital share.

Constant return to scale implies $s^L_{st} + s^K_{st} = 1$. Capital stock is measured using the perpetual inventory model, and labor is measured by the limited quality adjusted labor index. We will discuss more how the labor measured is problematic in terms of labor quality control. O'Mahony and Timmer (2009) provides more detailed information about how KLEMS measures aggregate sectoral level TFP.

Figure 2.1 plots the sectoral level GDP, capital stock, labor employment and TFP. The black dash-dot line represents for the total industry, the red dotted line the manufacturing sector, the blue solid line the construction sector and the green dashed line the real estate sector.

Subfigures (a) and (b) show that the expansion of the manufacturing sector was mild in any measure compared to that of the construction sector and the real estate sector. In 2007, the real capital stock in the construction sector was almost five times higher than that of 1999; labor employment was almost two times higher. Contrastingly, the manufacturing sector stayed stable. The real capital stock increased nearly 40 percent, and labor employment barely 3 percent. Now turning to the GDP growth in subfigure (c), the difference between the construction sector and the manufacturing sector is much smaller, a 75 percent increase for the former, and a 30 percent increase for the latter. The trends in the subfigures (a), (b) and (c) give rise to the measured TFP trend in subfigure (d). The TFP of the construction sector dropped by more than 10 percent in less than 10 years, while that of the manufacturing sector declined by very little.

Figure 2.1: Factor Inputs, Output and TFP



Raw data: KLEMS
1999 TFP is normalized to 100

Evidence from more countries shows that the same phenomenon is not only observed in Spain, but also in other southern European countries as well. In all the four countries[4] presented in Table 1, there exists a negative correlation between the sectoral expansion and the TFP growth. On the left panel of Table 1, I list the three most expanded sectors on the one-digit level in terms of the relative labor growth; on the right panel, I list the three least expanded sectors for Portugal, Spain, Ireland and Italy. One striking difference is that the measured TFP declines much more for the sectors experiencing relatively greater expansion.

Searching in the KLEMS dataset results in a finding of three other countries/periods experiencing a huge drop of TFP in fast expansionary sector: Finland from 1984 to 1990, Japan from 1986 to 1991, and Korea from 1988 to 1997. I choose the period systematically: the stopping point is the year before the documented year of the crisis, and the starting point is the year when current account trend reverses. In Table 2, I list the two most expanded sectors on the left panel, and the manufacturing sector on the right panel. Sandal (2004) documents the Nordic banking crisis in the early 1990s in Finland, Norway, and Sweden. It attributes the cause to the strong credit and asset price booming before that. There is a huge expansion of real estate sector such that the relative employment increases 4.0% annually between 1984 and 1990 in Finland,[5] with an annual TFP drop at 2.64 percent. The annualized growth rate of relative employment of manufacturing sector is -2.68 percent, while the TFP increases at 3.13 percent per year. Shiratsuka (2005) documents the asset price bubble in Japan in the 1980s. The same observation appears again, as a fast-expanding real estate sector coexists with negative measured TFP growth. Radelet and Sachs (1998) analyzes the East Asia financial crisis and its prelude, in which an expanding real estate sector and hotel/restaurant sector have experience a continuous measured TFP decline.

[4] They are four countries in the GIIPS group. Greece is not presented due to data availability.

[5] Norway is not in the KLEMS 2009 release and the data of Sweden does not date back to the early 1980s.

Table 1: Expansion and TFP Growth (1999 - 2007 Annualized)

	Most expanding			Least expanding		
Countries	**Sectors**	$\Delta \ln \frac{L_{st}}{L_t}$ (%)	$\Delta \ln TFP_{st}$ (%)	**Sectors**	$\Delta \ln \frac{L_{st}}{L_t}$ (%)	$\Delta \ln TFP_{st}$ (%)
Portugal	Real estate	2.57	-4.81	Utility	-5.51	-0.29
	Hotels & Restaurants	1.66	-2.43	Finance	-3.12	4.32
	Wholesale & Retail	1.39	-2.38	Manufacturing	-2.59	-0.72
Spain	Construction	3.36	-1.7	Mining & Quarrying	-4.29	0.52
	Real estate	2.18	-1.22	Manufacturing	-2.98	-0.14
	Hotels & Restaurant	1.94	-2.63	Utility	-2.21	0.19
Ireland	Construction	4.82	-2.74	Agriculture	-7.76	2.25
	Community service	1.45	-1.84	Utility	-4.41	-0.42
	Mining & Quarrying*	0.93	-0.87	Manufacturing	1.31	4.93
Italy	Real estate	3.35	-0.71	Utility	-2.59	-0.13
	Construction	2.67	-1.21	Agriculture	-1.55	-0.55
	Hotels & Restaurant	2.56	-2.27	Manufacturing*	-1.48	-0.13

Raw data: KLEMS
All the numbers are in percentages. The growth rate of Portugal is calculated between 1999 and 2005, others 1999 - 2007
* For Ireland, Mining & Quarrying is the fourth most expanded sector. For Italy, Manufacturing is the fifth least expanded sector.

Table 2: Other Booming Periods (Annualized)

	Most expanding			Least expanding		
Countries	**Sectors**	$\Delta \ln \frac{L_{st}}{L_t}$ (%)	$\Delta \ln TFP_{st}$ (%)	**Sectors**	$\Delta \ln \frac{L_{st}}{L_t}$ (%)	$\Delta \ln TFP_{st}$ (%)
Finland	Real estate	4.0	-2.64	Manufacturing	-2.68	3.13
(1984-1990)	Community services	1.75	-0.7			
Japan	Real estate	4.48	-2.14	Manufacturing	-0.41	3.82
(1986-1991)	Hotels & Restaurants	1.51	-1.27			
Korea	Real estate	11.72	-2.09	Manufacturing	-4.26	4.20
(1988-1997)	Hotels & Restaurants	9.89	-2.86			

Raw data: KLEMS
All the numbers are in percentages.

2.2 Dual Measure of TFP

In this subsection, I show that the dual measure of TFP tracks the primal measure well, which is consistent with the perfect competition assumption.

In the previous subsection, it is assumed that $s^L_{st} + s^K_{st} = 1$. One implication of this assumption is that the market is perfectly competitive: the labor share and capital share adds up to one, so that there is no profit. The concern then is that this assumption is too strong. A valid suspicion is that the decline in the measured TFP in expanding sectors may not reflect the drop of the productivity, but merely a drop of the markup if the market is not perfectly competitive.

According to Hsieh (2002),[6] with the assumption that the market is perfectly competitive,

[6]Hsieh (2002) shows that in the cases of Singapore and Taiwan, the dual measure does not matches the

there exists an identity:[7]

$$\underbrace{\hat{Y}_{st} - s^K_{st}\hat{K} - s^L_{st}\hat{L}_{st}}_{\text{Primal: } \hat{A}^P_{st}} = \underbrace{s^K_{st}\hat{r}_{st} + s^L_{st}\hat{w}_{st}}_{\text{Dual: } \hat{A}^D_{st}} \tag{1}$$

The left-hand side of equation 1 is the primal measure, which in principle is the measure used in KLEMS. The right-hand side of equation 1 is the dual measure, $\hat{r}_{st}$ is the growth rate of the rental price of capital, and $\hat{w}_{st}$ is the growth rate of wage. s^K_{st} and s^L_{st} are respectively the capital share and the labor share.

If the market is not perfectly competitive, the output should be divided into factor shares and profit:

$$Y_{st} = r_{st}K_{st} + w_{st}L_{st} + \pi_{st} \tag{2}$$

where π_{st} is the profit of sector s at time t.

Then we have a similar expression as in equation 1:

$$\underbrace{\hat{Y}_{st} - (1 - s^L_{st})\hat{K}_{st} - s^L_{st}\hat{L}_{st}}_{\text{Primal: } \hat{A}^P_{st}} = \underbrace{(1 - s^L_{st})\hat{r}_{st} + s^L_{st}\hat{w}_{st}}_{\text{Dual: } \hat{A}^D_{st}} + s^\pi_{st}(\hat{s}^\pi_{st} - \hat{s}^K_{st}) \tag{3}$$

Equation 3 shows that if we still mistakenly assume that the labor share and the capital share sums up to one if the truth is not, the primal measure would exceed the dual measure by $s^\pi_{st}(\hat{s}^\pi_{st} - \hat{s}^K_{st})$.

In Figure 2(a), I show that the dual measure and the primal measure of the TFP of Spain are indeed very close to each other. In Figure 2(b), I present the scatter plot of the primal measure versus the dual measure for the seven countries/periods explored in the previous subsection. Every dot represents the annualized TFP growth for that country during the period associated. The red solid line is the linearly fitted line of the scatter plot, and the

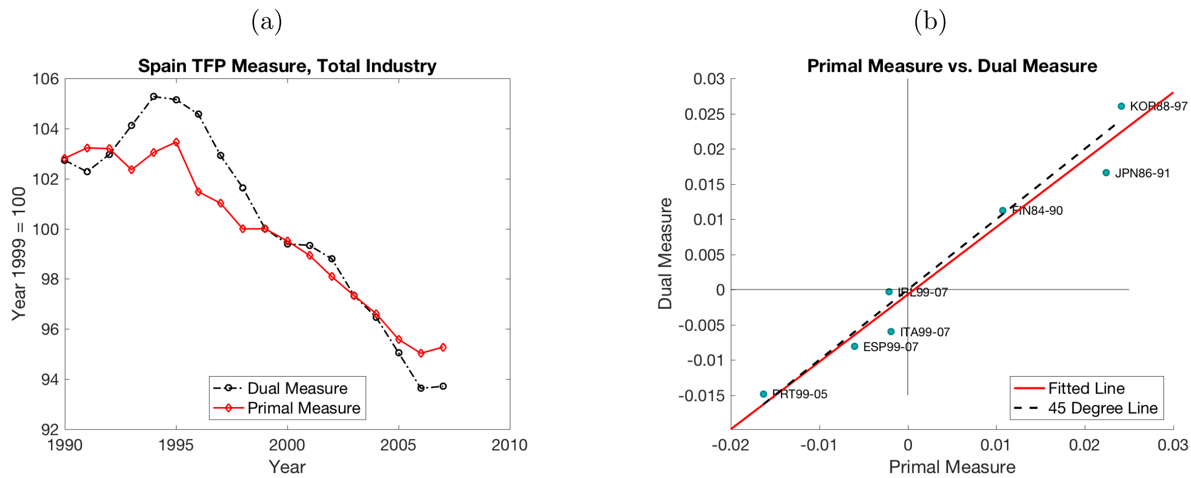
primal measure well. He does not question the validity of the specification of the assumption of market condition, but instead questions the quality of national account. In the case of European data, however, the data quality is much less of a concern.

[7] The derivation of the equation can be found in Appendix A.

black dashed line is the 45-degree line. The closeness of the two lines indicates that the two measures matches each other very well.

A more specific way to read Figure 2.2 is that the term $s^{\pi}_{st}(\hat{s}^{\pi}_{st} - \hat{s}^{K}_{st})$ in equation 3 is small. And Figure 2.3 proves it is because the profit share s^{π}_{st} is small, which further implies that the competitive market assumption is a reasonable one. Indeed, if $s^{\pi}_{st}(\hat{s}^{\pi}_{st} - \hat{s}^{K}_{st})$ is small is due to the fact that the difference between the growth rate of capital share and that of profit share $(\hat{s}^{\pi}_{st} - \hat{s}^{K}_{st})$ is small, then we would expect the labor share to increase. The reason is that attributing the measured TFP decline to markup drop would require profit share to decrease as well. Then the capital share would need to decline at about the same rate. The combination of the decline in capital share and the decline in profit share requires the labor share to increase. However, Figure 2.3 shows that the labor share is actually decreasing in Spain.

Figure 2.2: Primal Measure and Dual Measure



Raw data: KLEMS
1999 TFP is normalized to 100
Next to every dot there are three letters and numbers; the three letters stands for countries, and the numbers are the starting and end years: PRT99-05 (Portugal 1999 -2005), ESP99-07(Spain 1999 - 2007), ITA99-07 (Italy 1999 - 2007), IRL99-07(Ireland 1999 - 2007), FIN84-90 (Finland 1984 - 1990), JPN86-91 (Japan 1986 - 1991), KOR88-97 (Korea 1988 - 1997)
Portugal data is from 1999 - 2005 is because of data availability in the the release of the 2009 version of KLEMS.

Figure 2.3: Labor Share of Spain

Raw data: KLEMS

3 Firm-level Evidence on TFP Decline

In this section, I show that the trend of TFP observed in aggregate data also exists in the firm-level data using very different estimation methods. Moreover, the firm-level data suggests that the capital misallocation channel cannot explain the TFP drop, but the labor-quality mismeasurement channel can.

Firm-level data used in this section are the AMADEUS firm-level panel data of Spain from 1999 to 2007. I describe how I construct the dataset in Appendix B.

Then I merge the price data to the firm-level data. The ideal price data would be firm-level producer price. However, it is not available. The price data used in the paper are the two-digit sectoral level data of nominal value added and intermediate inputs from KLEMS

ISIC 3. 2009 release. The price data of fixed assets are quasi two-digit sectoral level data from the same source. Some two-digit sectors share the same price index of fixed assets. For example, food/beverage and tobacco are two different two-digit sectors, but they have the identical price index of fixed assets.

3.1 Capital Misallocation

In this subsection, I prove that the capital misallocation channel is not the channel of the first-order importance to explain the TFP drop. More specifically, I show that the growth of the dispersion of the marginal revenue product of capital (MRPK) is higher in non-expanding sectors relative to expanding sectors.

The calculation of the MRPK follows Chang-Tai Hsieh (2009) and Gopinath et al. (Forthcoming). The production function is assumed to be Cobb-Douglas: $Y_{ist} = A_{ist} K_{ist}^{\alpha_s} L_{ist}^{\beta_s}$. Here it is not necessary that the production function is constant return to scale, but it is assumed to be time invariant. The MRPK is defined as follows:

$$\text{MRPK}_{ist} :\equiv \alpha_s \mu_s \tfrac{P_{ist} Y_{ist}}{K_{ist}} \tag{4}$$

where μ_s is the time invariant mark-up of sector s, P_{ist} is the price of the output of firm i in sector s at time t. If it is perfect competition, then $\mu_s = 1$. If it is monopolistic competition with a CES aggregator, then $\mu_s = \frac{\sigma_s}{\sigma_s - 1}$, where σ_s is the time invariant elasticity of substitution.

$P_{ist}Y_{ist}$ is the nominal value-added of the firm calculated as the difference between the operational income and the material cost. K_{ist} here is defined as the fixed asset deflated by the capital price from KLEMS, and L_{ist} is the number of people employed.

The dispersion of the MRPK of sector s is defined as the standard deviation of the log MRPK:

$$\text{Dispersion of MRPK}_{st} :\equiv std(\ln(\text{MRPK}_{ist})) \tag{5}$$

Here I do not have to assign the values to α_s, β_s and μ_s since after taking the log value of the MRPK, the constant term across all firms within a sector becomes additive. So it does not add to the variation of the log value of the MRPK.

Figure 3.1 plots the evolution of the dispersion of the MRPK in both the manufacturing sector and the construction sector. Both curves have an upward trend, but the manufacturing sector clearly has a higher growth of the dispersion of the MRPK than that of the construction sector.

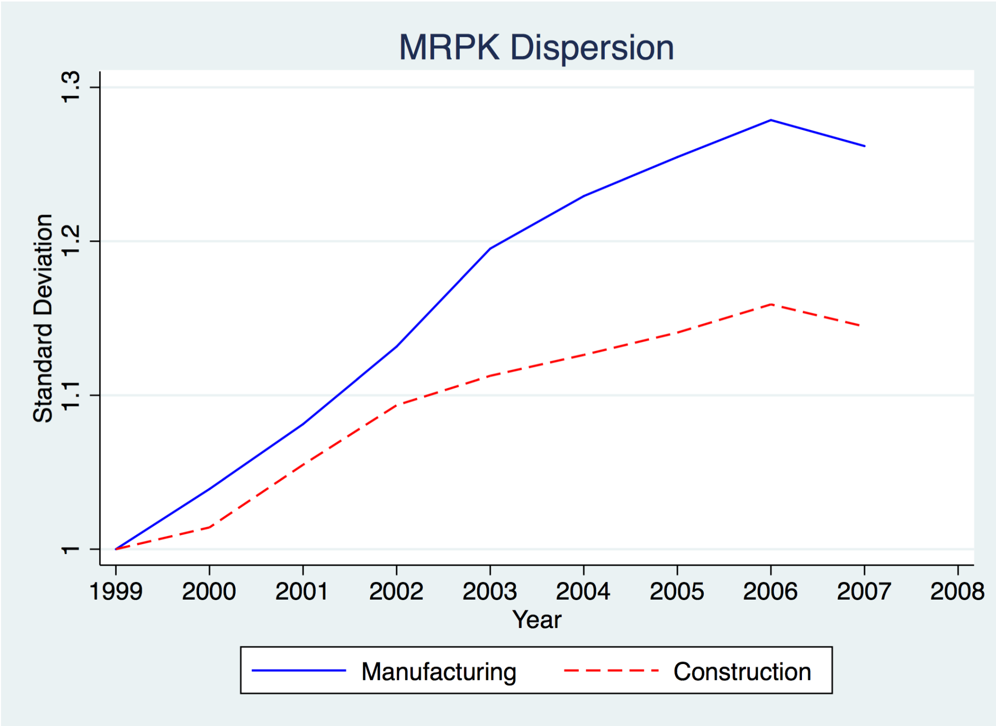
According to Chang-Tai Hsieh (2009), the increasing dispersion of the MRPK is an indicator of the worsening situation of capital misallocation. The idea is that without distortion on capital allocation the marginal productivity of all firms should be equalized, and there would be no capital misallocation. Thus, the dispersion of the MRPK should always be zero.[8]

If we focus on just one sector, it is tempting to draw the conclusion that the capital misallocation is increasing within that sector. However, if the capital misallocation channel is really the main reason TFP drops, we should expect the sector with more TFP drop to experience a higher growth of the dispersion of the MRPK. However, we observe the opposite in Figure 3.1.

The main message in this subsection is that the capital misallocation cannot be the main driver of the TFP drop.

[8]Even if the dispersion of the MRPK is not necessarily a good measure of capital misallocation, as argued by Asker, Collard-Wexler and Loecker (2014) and Bils, Klenow and Ruane (2017), one still needs to reject capital misallocation as the main explanation for the TFP decline, since almost all the papers favoring the argument of capital misallocation follow Chang-Tai Hsieh (2009).

Figure 3.1: MRPK Dispersion Comparison



Raw data: Amadeus Spain

3.2 Time Series Trend of TFP

In this subsection, I present the time series trend TFP with firm-level data. It also shows that average TFP declines much more in expanding sectors than in non-expanding sectors.

This paper estimates the firm-level TFP using different methodologies, which gives very similar results. More specifically, I employed the Ackerberg, Caves and Frazer (2015) extension of Levinsohn and Petrin (2003) and De Loecker (2011) methodologies. Olley and Pakes (1996) is often cited side by side with Levinsohn and Petrin (2003). Both papers try to solve the potential endogeneity problem caused by the correlation between the unobserved productivity and factor inputs. Olley and Pakes (1996) assumes the investment contains the information on productivity, while Levinsohn and Petrin (2003) assumes the intermediate inputs contain information on productivity. Intermediate inputs could be better than investment as a proxy for productivity due to the lumpiness of the investment. As pointed out by Ackerberg, Caves and Frazer (2015), treating labor as a free variable in the first stage of estimation is problematic because productivity under some data generating processes. To deal with the concern that the difference between the firm-level price and sectoral-level price might bias the estimation result, I also include De Loecker (2011) methodology.

The estimation results in Tables 3 demonstrates two things: (1) different estimation methods reveal similar results, and (2) constant return to scale may be a good approximation. Table 3 shows the coefficient of production function. The upper panel represents the point estimations, while the lower panel shows the corresponding standard errors. The left panel is the estimation with the full sample, while the middle panel is the estimation of the half sample. In the half sample, I exclude the firms with less than or equal to five observations. So, in the half sample, all firms exist in at least two of the vintage discs. The right panel is the estimation with the subsample of only stayers. Comparison across samples shows that the full sample has a higher labor share and a lower capital share relative to the half sample and subsample of stayers. The production function in the full sample is closer to constant return to scale.

Figure 3.2 shows a strikingly difference between the trends of the manufacturing sector and the construction sector: the mean of the log value of TFP of the former declines little compared to that of the latter. More specifically, Figure 3.2 shows the weighted and unweighted log value of TFP in the manufacturing and construction sectors, aggregated from the firm-level TFP measured by the methodologies in Levinsohn and Petrin (2003) and De Loecker (2011) with the full sample of firms. Horizontally, the first row plots the unweighted mean, valued-added-weighted mean, and labor-weighted mean of the log value TFP of the manufacturing sector using the De Loecker and the Levinsohn-Petrin estimators. The second row plots the same trends in the construction sector. To make the comparison more clearer, the third row puts the valued-weighted mean of the log value of TFP of the two sectors in the same scale. Vertically, the left column and the right shows almost identical aggregate trends, although there is a slight difference in the point estimation of the coefficients of the production function. From 1999 to 2007, the average TFP of the manufacturing sector drops from 0.08 to 0.12 log points, depending on the aggregation weights. Although the decline trend seems similar in the construction sector, the magnitude is much bigger: the TFP drop is from 0.45 to 0.5 log points. Figure 3.2 further reveals that the value-weighted mean of the log value of TFP is always above the unweighted mean, which implies that the higher value-added firms have higher TFP.

One interesting point in Figure 3.2 is the discrepancy between the firm-level TFP trend and the KLEMS measure. From the firm-level TFP measure, we observe a more profound TFP drop. This is because the KLEMS measure partially controls the labor quality, while the firm-level TFP measure does not control for it at all. I will discuss this issue in detail in the next section.

Figure 3.3 and Figure 3.4 are copies of Figure 3.2 with different subsamples instead of the full sample. The divergence of the trends for TFP between the manufacturing sector and the construction sector is still there, with a sharp drop in the latter and even a slight increase in the former. The magnitude of the drop in the construction sector is much smaller though in Figure 3.3 and Figure 3.4 compared to that in Figure 3.2, about 0.25 and 0.15, respectively, against about 0.5 measured in log points. This comparison reflects that younger firms may contributes significantly to the measured TFP drop.

Figure 3.5 is a zoom-in graph of Figures 3.2, 3.3 and 3.4, in the sense that the distributions of the log value of TFP of two end years are plotted. The mean of the 1999 distribution of the log value of TFP is normalized to zero. Horizontally, the first row shows the result of the full sample, the second row that of the half sample, and the third row that of the subsample of only stayers. Vertically, the first column is the result of distribution of manufacturing sector and the second column that of the construction sector.

The pattern that has been observed in Figures 3.2, 3.3 and 3.4 can also be observed in Figure 3.5 by how much the 2007 log TFP distribution changes relative to the 1999 one. In the left column, the two distribution overlaps with each other quite well, meaning the aggregate TFP is not that different between 1999 and 2007 in the manufacturing sector. In the right column, there is a clear move of the distribution to the left, implying a significant decline of TFP in the construction sector during the same time period.

A new stylized facts that cannot be observed in the aggregate time series is the dispersion of the TFP. The first row is the distribution based on the full sample. The right tail of the construction sector extends to the right only slightly, while it stays almost the same for the manufacturing sector. However, in the construction sector, the left extension of the left tail is much more pronounced compared to that of the manufacturing sector. The extension of the left tail can be interpreted as the the entry of the new firms that could not enter the production procedure without the sector expansion. There are proportionally more firms like this in the construction sector than in the manufacturing sector because the expansion scale is very different in the two sectors, as shown in the first section. The second row and the third row are the distribution based on the half sample and sub sample of only stayers. The tails of the distributions do not seem too different between 1999 and 2007.

Table 3: Production Function Coefficients

	Full Sample				**Half Sample**				**Subsample(Stayers)**			
Methods	**DL**		**LP**		**DL**		**LP**		**DL**		**LP**	
Sectors	α_l	α_k	α_l	α_k	α_l	α_k	α_l	α_k	α_l	α_k	α_l	α_k
Manufacturing	0.794	0.200	0.821	0.208	0.688	0.225	0.691	0.226	0.678	0.237	0.680	0.239
Construction	0.801	0.195	0.782	0.183	0.618	0.179	0.617	0.172	0.622	0.185	0.615	0.171
Real Estate	0.832	0.203	0.745	0.179	0.575	0.187	0.550	0.175	0.575	0.175	0.541	0.185

Standard Errors

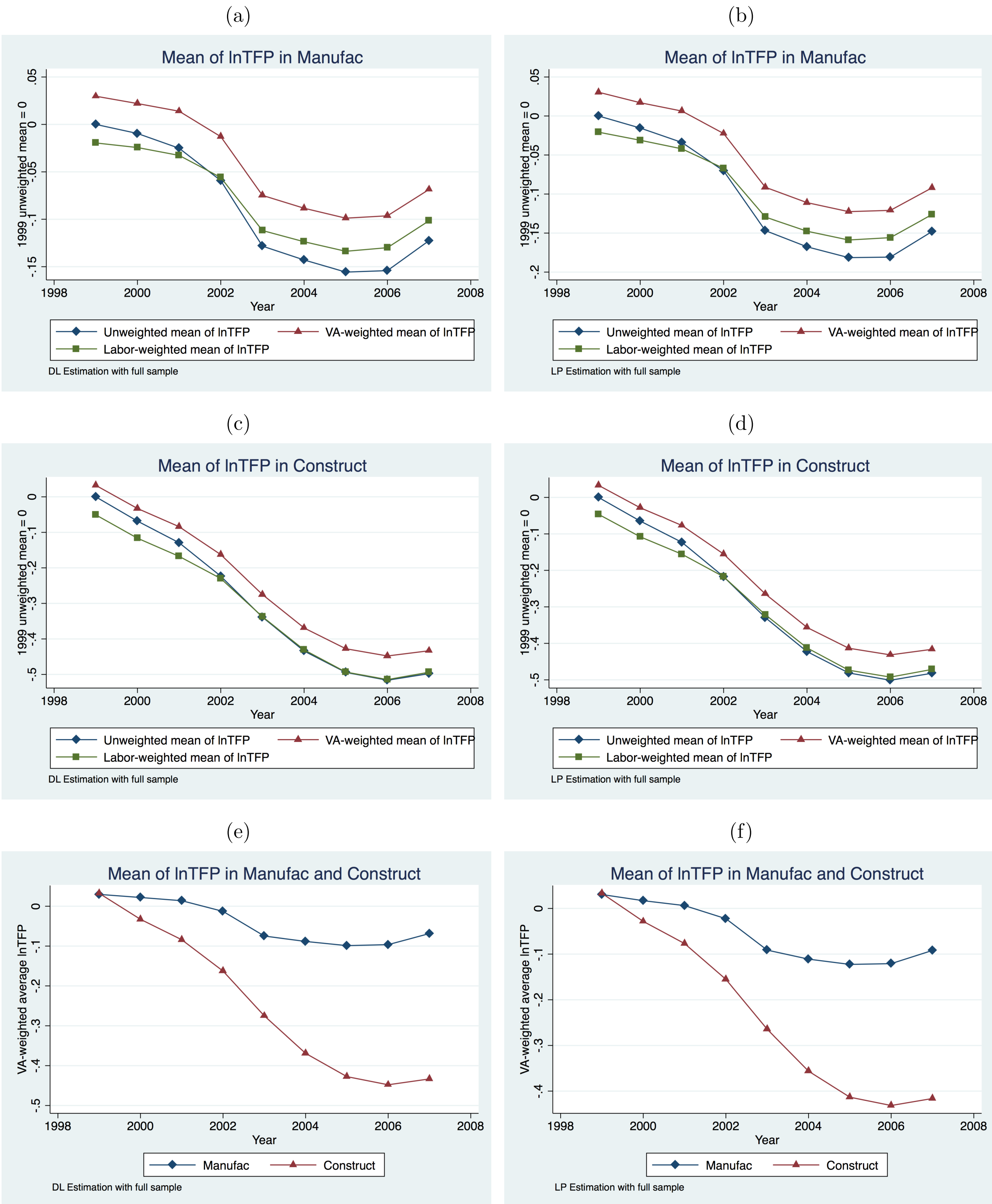
	SE α_l	SE α_k	SE α_l	SE α_k	SE α_l	SE α_k	SE α_l	SE α_k	SE α_l	SE α_k	SE α_l	SE α_k
Manufacturing	0.00027	0.00002	0.00006	0.00004	0.00009	0.00002	0.00007	0.00001	0.00359	0.00980	0.00013	0.00001
Construction	0.00157	0.00036	0.00083	0.00272	0.00021	0.00018	0.00048	0.00015	0.00216	0.00576	0.00185	0.00242
Real Estate	0.00052	0.00002	0.00033	0.00023	0.01248	0.00717	0.00003	0.00004	0.01927	0.00715	0.00045	0.00028

Raw data: Amadeus Spain

DL means the estimation method is De Loecker (2011); LP means the estimation method is Levinsohn and Petrin (2003)

Full sample is the sample with all firms; Half sample is the sub sample with observations more than 5; Sub sample (stayers) is the sub sample with firms that have observations every year.

Figure 3.2: Mean lnTFP Trend Full Sample



Raw data: Amadeus Spain, Full Sample

The left three graphs use the De Loecker estimator, the right three graphs use the Levinsohn and Petrin estimator.

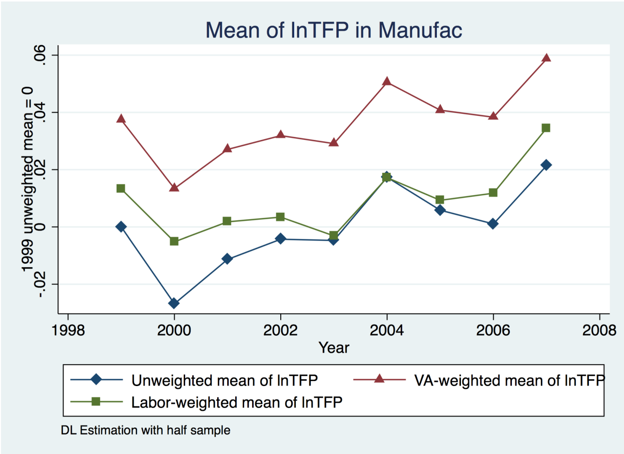
The mean is normalized such that in year 1999, unweighted mean of lnTFP is zero.

The trend of graph is comparable across all subgraphs. But the levels of lnTFP is comparable only within the same sector and with same estimation.

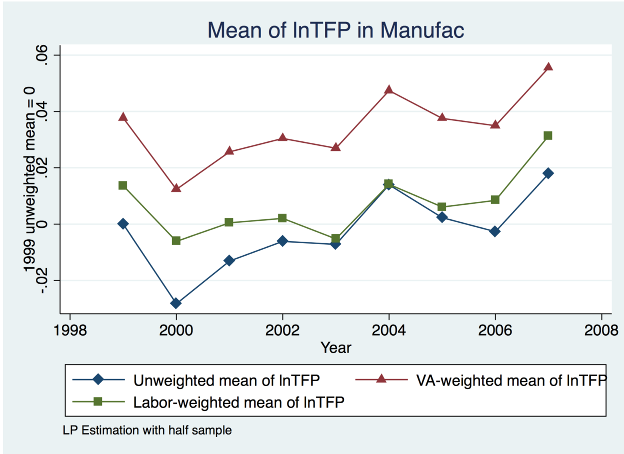
In sub-graph 3(e) and 3(f), I put the va-weighted mean of lnTFP of both manufacturing sector and construction sector together for a more direct comparison.

Figure 3.3: Mean lnTFP Trend Half Permanent Sample

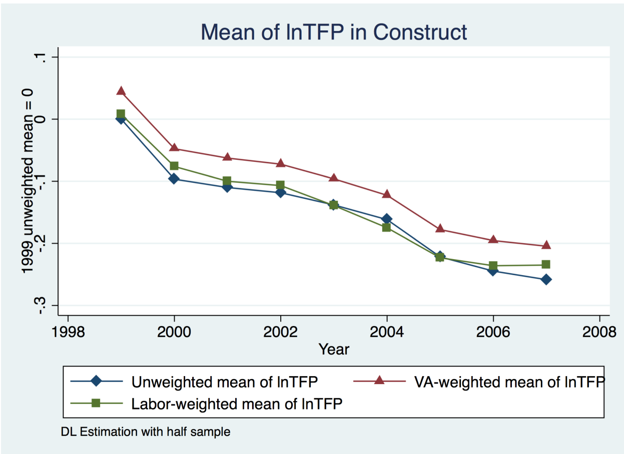
(g)



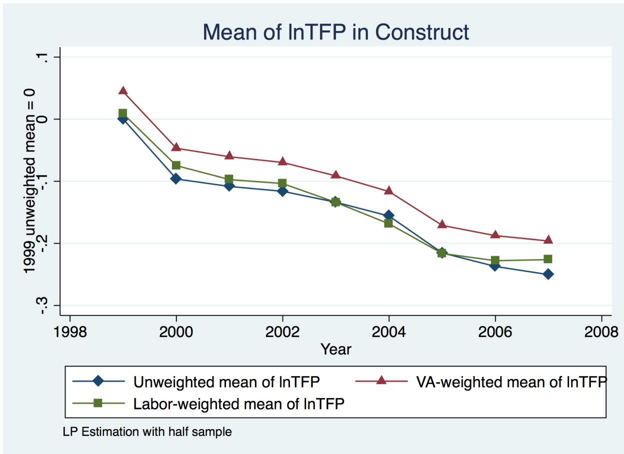
(h)



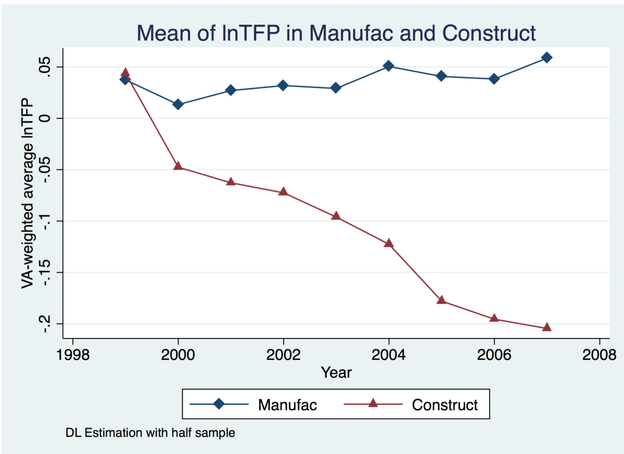
(i)



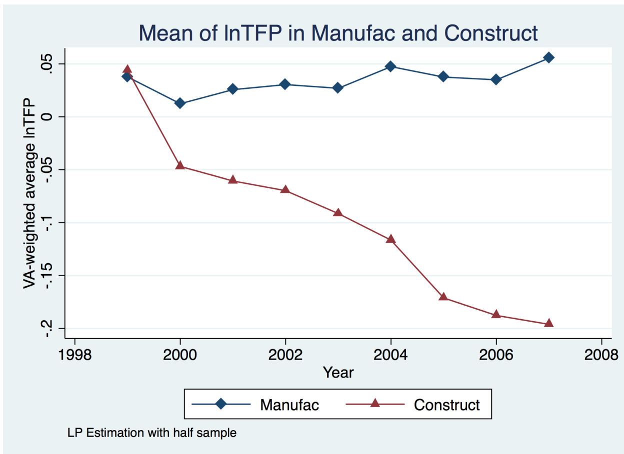
(j)



(k)



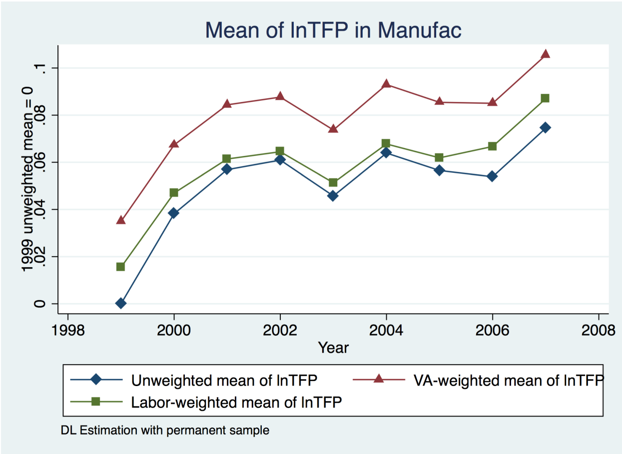
(l)



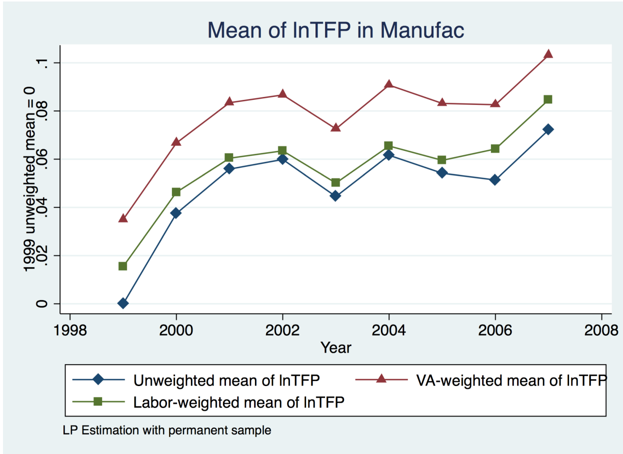
Raw data: Amadeus Spain, Half Sample (obs more than 5)
The left three graphs use the De Loecker estimator, the right three graphs use the Levinsohn and Petrin estimator.
The mean is normalized such that in year 1999, unweighted mean of lnTFP is zero.
The trend of graph is comparable across all subgraphs. But the levels of lnTFP is comparable only within the same sector and with same estimation.
In sub-graph 3(k) and 3(l), I put the va-weighted mean of lnTFP of both manufacturing sector and construction sector together for a more direct comparison.

Figure 3.4: Mean lnTFP Trend Permanent Sample

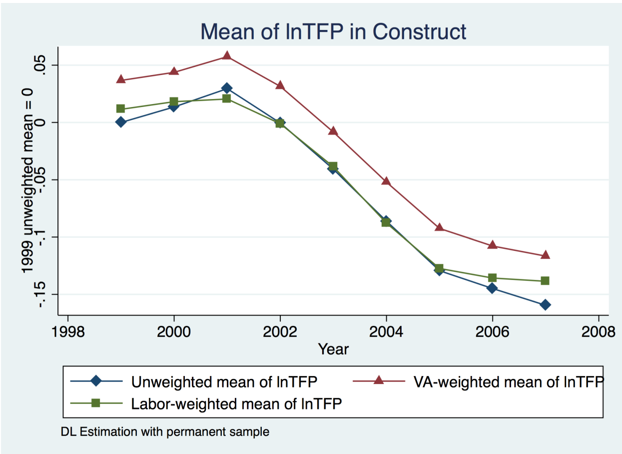
(m)



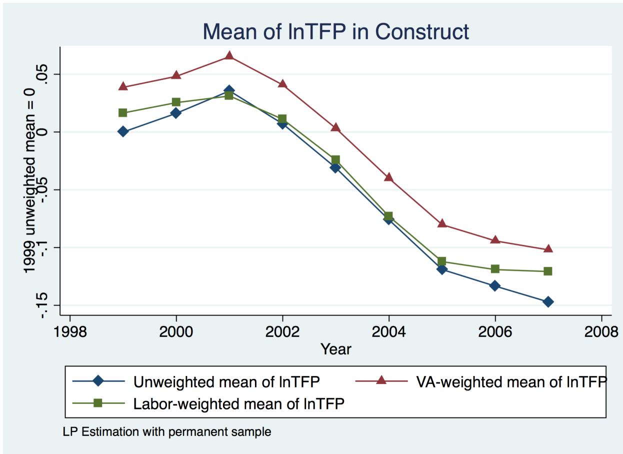
(n)



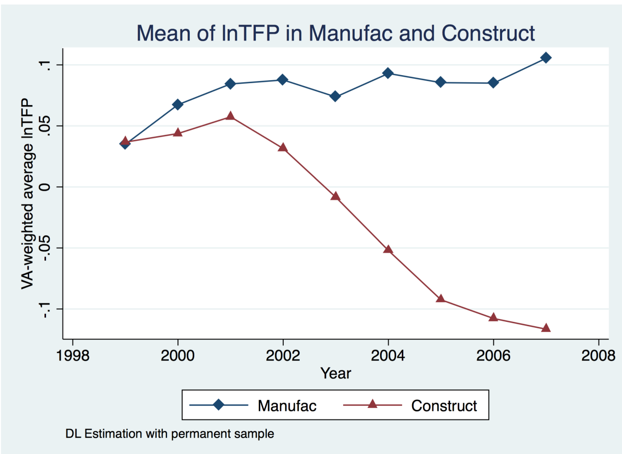
(o)



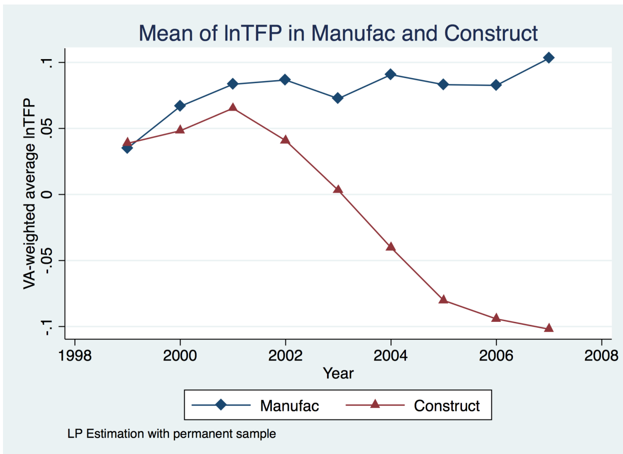
(p)



(q)



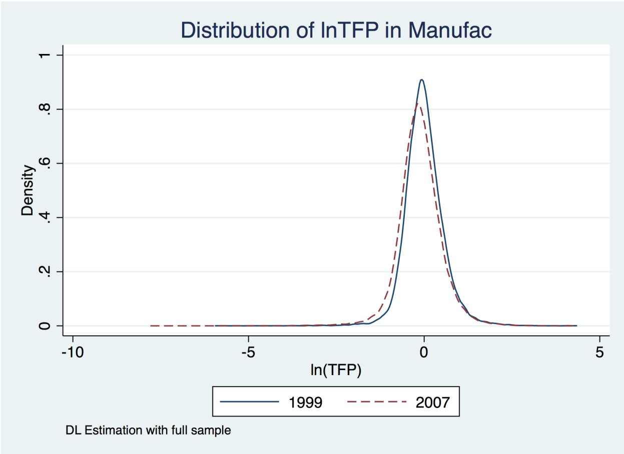
(r)



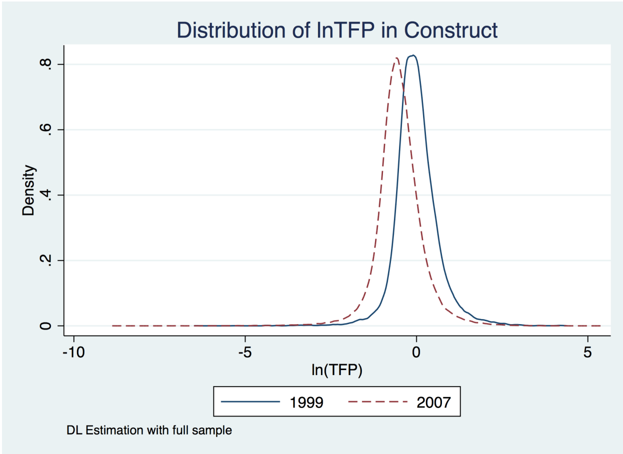
Raw data: Amadeus Spain, Permanent Sample (has obs every year)
The left three graphs use the De Loecker estimator, the right three graphs use the Levinsohn and Petrin estimator.
The mean is normalized such that in year 1999, unweighted mean of lnTFP is zero.
The trend of graph is comparable across all subgraphs. But the levels of lnTFP is comparable only within the same sector and with same estimation.
In sub-graph 3(q) and 3(r), I put the va-weighted mean of lnTFP of both manufacturing sector and construction sector together for a more direct comparison.

Figure 3.5: lnTFP Distribution

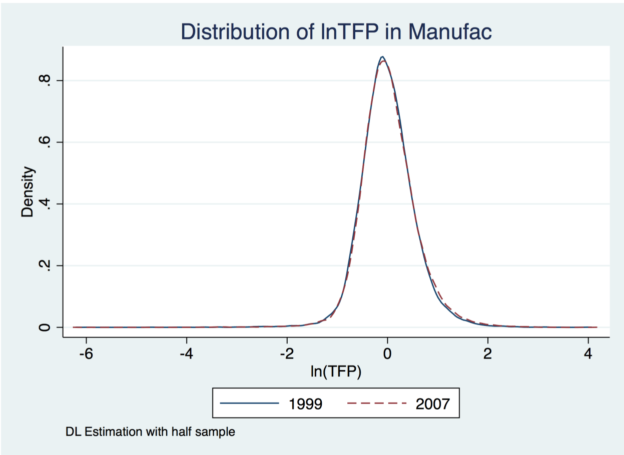
(s)



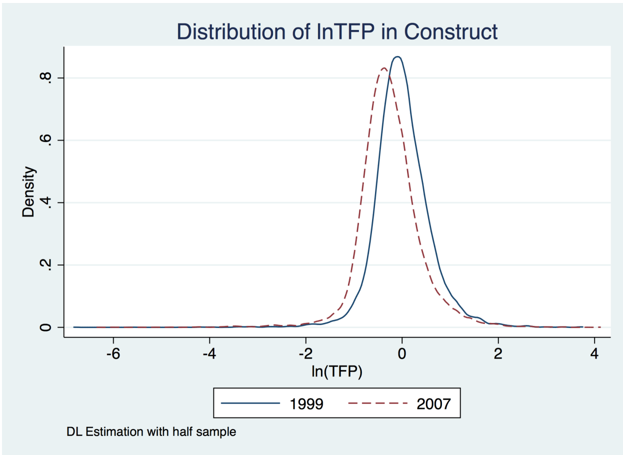
(t)



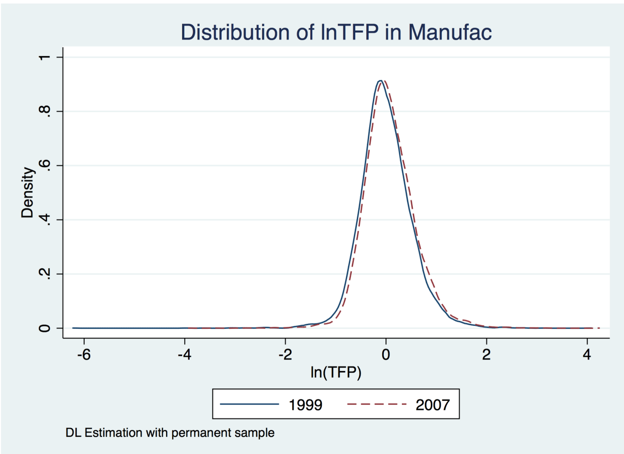
(u)



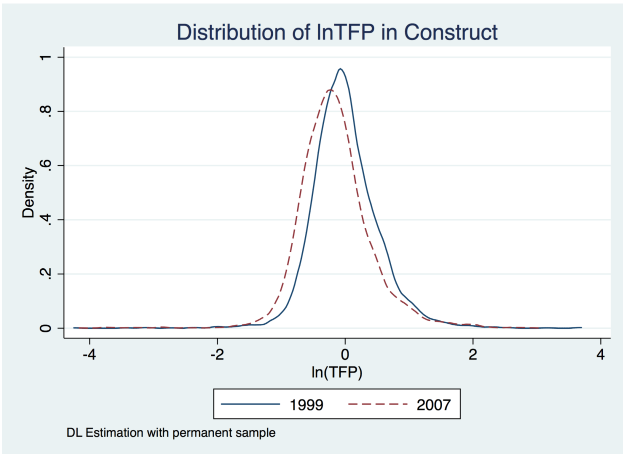
(v)



(w)



(x)



Source: Amadeus Spain
First row: full sample; second row: half sample; third row: subsample with stayers
TFP is the De Loecker estimator.
The mean is normalized such that in the year 1999, unweighted mean of lnTFP is zero.

3.3 TFP Growth Decomposition

In this subsection, I present another piece of evidence that the capital misallocation channle cannot be the main reason TFP drops, but th labor-quality mismeasurement channel can be.

The sectoral average TFP growth can be decomposed into five components: within-firm term, between-firm term, cross term, entry term and exit term. The first three components come from the firms that always stay in the sample (sub sample of stayers), the entry term is from the newly incoming firms, and the exit term is from the firms that exit the sample. The decomposition result shows that the between-firm component is almost negligible. This means that capital misallocation is not important in explaining the TFP drop within the subsample of stayers. The importance of the within-firm component implies consistency with the mismeasurement channel.

Following Alvarez, Chen and Li (2017), the change of the weighted average of the log value of TFP can be decomposed to five terms, as follows:

$$
\begin{aligned}
&\Delta a_{st}^{total} \equiv \bar{a}_{st} - \bar{a}_{sr} \\
&\equiv \sum_{i=1}^{N_{st}} \frac{Y_{ist}}{\mathrm{Y}_{st}} a_{ist} - \sum_{i=1}^{N_{sr}} \frac{Y_{isr}}{\mathrm{Y}_{sr}} a_{isr} \\
&= \underbrace{\frac{\mathrm{Y}_{sr}^{stay}}{\mathrm{Y}_{sr}} \sum_{i \in stay} \frac{Y_{isr}}{\mathrm{Y}_{sr}^{stay}} (a_{ist} - a_{isr})}_{\Delta a_{st}^{Within}} + \underbrace{\frac{\mathrm{Y}_{sr}^{stay}}{\mathrm{Y}_{sr}} \sum_{i \in stay} \left[\left(\frac{\mathrm{Y}_{st}^{stay}}{\mathrm{Y}_{st}} / \frac{\mathrm{Y}_{sr}^{stay}}{\mathrm{Y}_{sr}} \right) \frac{Y_{ist}}{\mathrm{Y}_{st}^{stay}} - \frac{Y_{isr}}{\mathrm{Y}_{sr}^{stay}} \right] (a_{isr} - \bar{a}_{sr})}_{\Delta a_{st}^{Between}} \qquad (6) \\
&+ \underbrace{\sum_{i \in stay} \left(\frac{Y_{ist}}{\mathrm{Y}_{st}} - \frac{Y_{isr}}{\mathrm{Y}_{sr}} \right) (a_{ist} - a_{isr})}_{\Delta a_{st}^{Cross}} + \underbrace{\sum_{i \in enter} \frac{Y_{ist}}{\mathrm{Y}_{st}} (a_{ist} - \bar{a}_{sr})}_{\Delta a_{st}^{Entry}} - \underbrace{\sum_{i \in exit} \frac{Y_{isr}}{\mathrm{Y}_{sr}} (a_{isr} - \bar{a}_{sr})}_{\Delta a_{st}^{Exit}}
\end{aligned}
$$

where

$$\mathrm{Y}_{st} = \sum_{i=1}^{N_{st}} Y_{ist};$$

$$\mathrm{Y}_{sr} = \sum_{i=1}^{N_{sr}} Y_{isr};$$

$$\mathrm{Y}_{st}^{stay} = \sum_{i \in stay} Y_{ist};$$

$$\mathrm{Y}_{sr}^{stay} = \sum_{i \in stay} Y_{isr};$$

where $a = \ln A$, Y_{st} is the total real value added in year t of sector s, and Y_{sr} is its counterpart in reference year r. N_{st} is the total number of firms. *stay* is the subset of firms that exist both in year t and year r. *exit* is the subset of firms that exist in the reference year r but do not in year t, (i.e., firms that exit the sample). *enter* is the subset of firms that do not exist in the reference year r but do in year t, (i.e., firms that enter the sample). Y^{stay} is the total real value added of the subset of firms in *stay*. Technically speaking, equation 6 has one more term: $\bar{a}_{sr}\big[\sum_{i \in stay}(\frac{Y_{ist}}{\mathrm{Y}_{st}} - \frac{Y_{isr}}{\mathrm{Y}_{sr}}) + \sum_{i \in enter}\frac{Y_{ist}}{\mathrm{Y}_{st}} + \sum_{i \in exit}\frac{Y_{isr}}{\mathrm{Y}_{sr}}\big]$, but since we can normalize $\bar{a}_{sr}$ to be zero, it is ignored.

Then the change of the weighted average of log TFP of year t in sector s relative to the reference year r can be decomposed into five parts: "within," "between," "cross," "entry" and "exit." The "within" term keeps the weight of the reference year unchanged but varies the TFP of individual firms, so it indeed measures the contribution of the log TFP change within the same firms that exist both in year t and in reference year r. If we further assume that on average a firm does not have a TFP drop, then the "within" term measures the TFP change stemming from the mismeasurement of the labor quality of the firms that survive.

The "between" term keeps the TFP of firms unchanged but varies the weight of individual firms. So it indeed measures the relative firm size change due to the reallocation of the resource. If $\Delta a_{st}^{Between} > 0$, it means that high productive firms become larger in size. If $\Delta a_{st}^{Between} < 0$, it means that low productive firms expand more, which means the allocation efficiency worsens. If $\Delta a_{st}^{Between} \approx 0$, it implies that the misallocation channel may not be important, at least in the subset of *stay* firms.

The "cross" term captures the correlation between the change of TFP and change of the size. If $\Delta a_{st}^{Crosss} > 0$, it means that when a firms grows in size, it also grows in productivity. If $\Delta a_{st}^{Crosss} < 0$, it means that a firm expands in size but decreases in productivity.

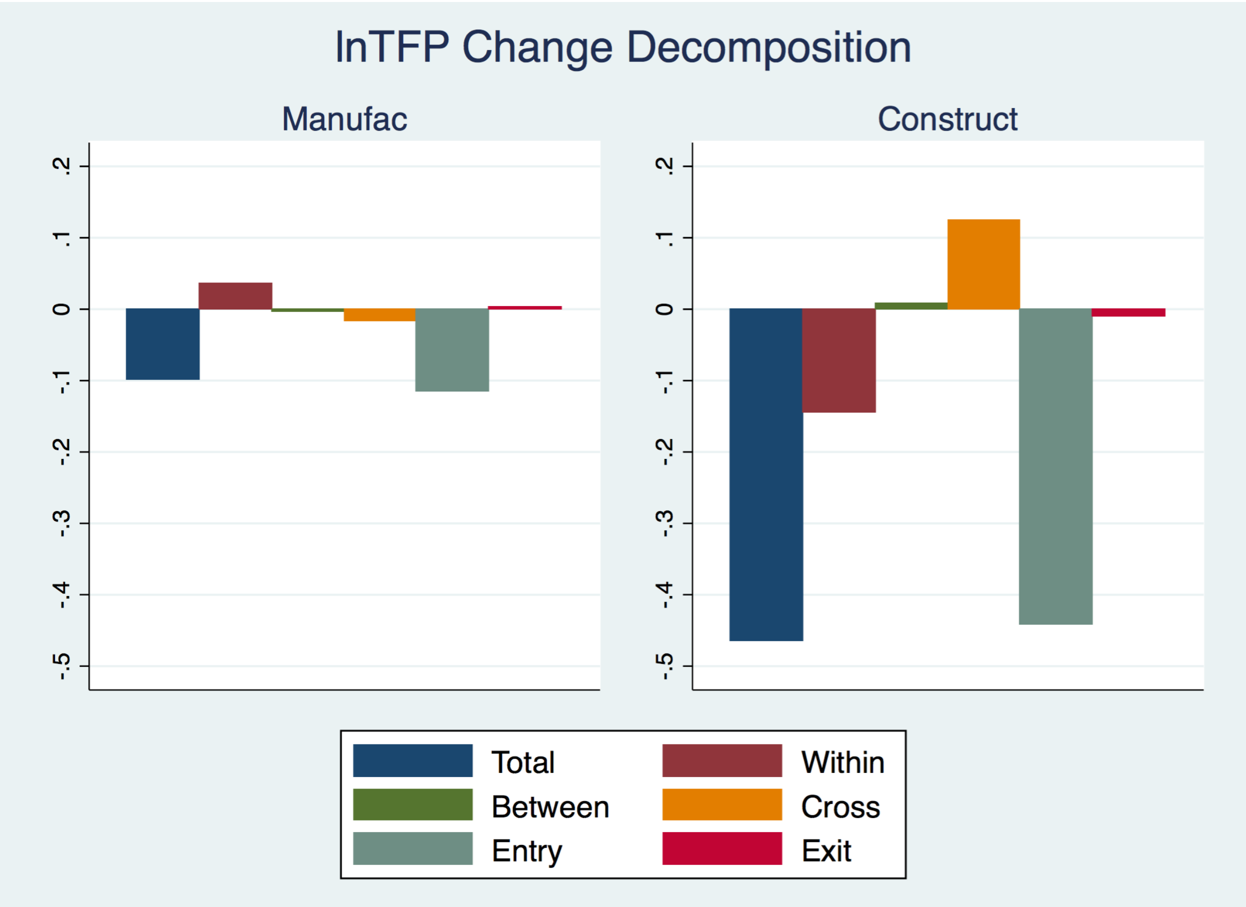
The "entry" term measures the weighted average of the log TFP of firms that newly enter the market in year t but do not exit in reference year r. So this term contains the change both from the misallocation channel and from the mismeasurement channel.

The "exit" term measures the weighted average of the log TFP of firms that exist in reference year r but do not exist anymore in year t.

Figure 3.6 plots graphically the decomposition based on equation 6 between year 2007 and reference year 1999, for both the manufacturing sector and the construction sector. The navy bars, standing for the total log TFP change in both sectors, echoes the observation in Figure 3.2: a small TFP drop is observed in the construction sector while a big TFP drop is observed in the construction sector. The "within" part is strikingly different in the two subfigures; while it is slightly positive in the manufacturing sector, it accounts for almost one-third of the TFP drop in the construction sector. The "between" term is small in both sectors. This stark contrast between the two sectors implies that at least in the subsample of stayers, the misallocation cannot be the dominant channel.

It is also observed that the "entry" bar is as important as the "total" bar. This means that the group of newly entering firms has a measured TFP that is much lower than the weighted average of the reference year. However, this bar contains both the misallocation channel and the mismeasurement channel. Therefore, we need a model to tear apart these two channels within the "entry" group.

Figure 3.6: Decomposition



Raw data: Amadeus Spain, full sample
$\Delta a_{st}^{total} = \Delta a_{st}^{Within} + \Delta a_{st}^{Between} + \Delta a_{st}^{Cross} + \Delta a_{st}^{Entry} + + \Delta a_{st}^{Exit}$ The full definition of the decomposition is in equation 6.

4 Evidence of Labor Quality from Worker-Firm Matched Data

In this section, I present two results using worker-firm matched data from the Structure of Earnings Survey (SES) of Eurostat: first, the labor quality control with only limited observable characteristics fails to capture a big share of the wage variation; second, the labor quality deteriorates in expanding sectors but not in non-expanding sectors.

The SES data are obtained by a two-stage random sampling approach of enterprises or local units (first-stage) and employees (second stage). The frequency of the survey is every four years. The data used in this paper are from the surveys of 2002 and 2006.

There are a few technical complications. First, although the anonymization procedure used to protect the privacy of firms and workers might change the precision of the survey, the statistics of the data shows that such modification has a statistically insignificant effect on the information of the survey. The natural step of anonymization is to replace names of firms and workers by codes which are not identifiable. This step does not change the real content of the survey. However, even after this step, firms and workers are still subject to the risk of "spontaneous identification" due to the information revealed by their characteristics. So a further anonymization procedure is to make the characteristics of firms or workers a bit vaguer if there exists such a risk. For example, if in a certain area there is only one firm that employs more than 250 employees, then the size of that firm may be modified to more than 49 employees. Such changes only affect a very small group of observations.

Another technical complication is the consistency of the survey across years. The 2002 survey of Spain does not include local units of enterprises with fewer than 10 employees, but the 2006 survey does include those small local units. Therefore, to make the data comparable across years, I delete the workers working in the local units with fewer than 10 employees. This may cause an upward bias of the labor quality change in the expanding sector, and I will discuss it in the subsection 4.2.

Third, the randomization in selecting firms and workers and the anonymization procedure render the SES data to be cross-sectional for each survey. Alternatively speaking, the SES data has no panel feature, which leaves it inappropriate to run the two-way fixed-effect model as in Abowd and Kramarz (1999) and Card et al. (2016). However, I can still back out the distribution of the unobserved labor quality by running the regression of wage on firm fixed effect and observed labor quality. The potential assortative matching between firms and workers will result in a less dispersed residual wage compared to the dispersion of the unobserved labor quality. I will discuss in subsection 4.3 that this actually underestimates the importance of the unobserved labor quality. [9]

4.1 Insufficiency of the KLEMS' Control of Labor Quality

In this subsection, I argue that the labor quality control in the KLEMS dataset is limited and not sufficient to capture a big portion of the wage variation.

The evidence to support this argument comes from examining how much wage variation can be explained by the observed labor quality characteristics in the KLEMS dataset. More specifically, I investigate the R-square statistics of the regression of log wage on the observed labor quality characteristics in the KLEMS dataset. According to O'Mahony and Timmer (2009), the KLEMS dataset cross-classifies the labor force by gender, educational attainment and age into 18 categories (respectively, $2 \times 3 \times 3$ types). The SES worker-firm matched data have more detailed categorization of educational attainment and age (respectively, 6 types).

More specifically, I run the following regression for each year on the sectoral level and for the entire economy.

$$\ln(w_{jst}) = \alpha_{0st} + \text{gender}_{jst} + \text{education}_{jst} + \text{age}_{jst} + \varepsilon_{jst}, \tag{7}$$

[9] Abowd and Kramarz (1999) and other following papers such as Card et al. (2016) usually show that there is very little correlation between the worker fixed effect and the firm fixed effect.

where w_{jst} is the deflated wage bill of worker j in sector s at time t. The coefficients of the regression are omitted for the sake of simplicity.

Workers can be divided into two categories by gender, six by education, and six by age.

The result of the regression run in equation 7 is shown in Table 4. In the whole economy and in the manufacturing sector, about one-third of the wage variation can be explained by the observed labor quality characteristics used in KLEMS. In the construction sector, however, the same characteristics only account for about 20 percent of the wage variation.

The R-square statistics reveal two messages. First, generally speaking a majority of wage variation cannot be explained by the variation of the relatively easily observed labor characteristics such as gender, education attainment and age. Second, this unexplained wage variation problem is much worse in the construction sector. In order to capture the labor quality more precisely, more variables are needed.

4.2 Observed Labor Quality beyond KLEMS

In this subsection, I present evidence that labor quality deteriorates in expanding sectors compared to stable sectors beyond the dimensions controlled by KLEMS, (i.e., gender, age and education). One important dimension of workers' quality is the tenure, which depicts the length of the service in enterprise. The worker-firm matched data shows a significant difference of tenure length change between the construction sector and the manufacturing sector, both in average terms and for firm managers.

The average tenure in the construction sector has decreased by 2.5 percent, while that of the manufacturing sector has increased by 2.5 percent. One possible scenario is that people with low experience moved into the expanding construction sector, while no such labor movement into the non-expanding manufacturing sector.

A 5 percent difference in tenure growth might not seem large, but we have to take into account the following issues. First, it is just the growth difference from 2002 to 2006. Under a simplistic assumption that the growth rate is constant from 1999 to 2007, there would be

Table 4: Regression of ln(wage) on Observed Individual Characteristics

	(1)	(2)	(3)	(4)	(5)	(6)
	2002			2006		
	All Sectors	Manufac.	Construc.	All Sectors	Manufac.	Construc.
	b/se	b/se	b/se	b/se	b/se	b/se
Gender F	0.000	0.000	0.000	0.000	0.000	0.000
	(.)	(.)	(.)	(.)	(.)	(.)
Gender M	0.243***	0.270***	0.199***	0.222***	0.263***	0.173***
	(0.00)	(0.00)	(0.01)	(0.00)	(0.00)	(0.01)
educ 1	0.000	0.000	0.000	0.000	0.000	0.000
	(.)	(.)	(.)	(.)	(.)	(.)
educ 2	0.058***	0.038***	0.034***	0.034***	0.026***	0.015*
	(0.00)	(0.00)	(0.01)	(0.00)	(0.00)	(0.01)
educ 3	0.274***	0.258***	0.163***	0.221***	0.214***	0.143***
	(0.00)	(0.00)	(0.01)	(0.00)	(0.00)	(0.01)
educ 4	0.356***	0.335***	0.176***	0.281***	0.286***	0.160***
	(0.00)	(0.00)	(0.01)	(0.00)	(0.00)	(0.01)
educ 5	0.666***	0.658***	0.556***	0.599***	0.561***	0.482***
	(0.00)	(0.00)	(0.01)	(0.00)	(0.00)	(0.01)
educ 6	0.775***	1.000***		0.650***	0.716***	0.285
	(0.02)	(0.04)		(0.01)	(0.04)	(0.17)
age 14-19	0.000	0.000	0.000	0.000	0.000	0.000
	(.)	(.)	(.)	(.)	(.)	(.)
age 20-29	0.123***	0.159***	0.108***	0.096***	0.164***	0.076***
	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)	(0.02)
age 30-39	0.319***	0.322***	0.218***	0.250***	0.298***	0.159***
	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)	(0.02)
age 40-49	0.478***	0.494***	0.293***	0.382***	0.431***	0.221***
	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)	(0.02)
age 50-59	0.573***	0.636***	0.363***	0.480***	0.557***	0.293***
	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)	(0.02)
age 60+	0.531***	0.613***	0.380***	0.501***	0.578***	0.367***
	(0.01)	(0.01)	(0.03)	(0.01)	(0.02)	(0.02)
Constant	1.270***	1.271***	1.392***	1.387***	1.350***	1.514***
	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)	(0.02)
Adj.R-sqr	0.344	0.365	0.198	0.328	0.334	0.198
Obs	216400	83808	15548	219723	77381	16641

Raw data from Structure of Earnings Survey - Eurostat

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

a 10 percent difference in tenure growth between two sectors. Second, deletion of the local units with fewer than 10 employees may contribute to the underestimation of the difference of the growth rate of the tenure. The exclusion of of those small firms also excluds newly incoming labor. Since the expanding sectors have more such small firms and probably more unexperienced incoming workers, the difference in the growth rate of tenure could potentially be bigger.

The tenure of certain important occupations, such as managers, is arguably more important than just the average tenure, as it may reflect how much they know about managing the firm. The average tenure of the firm managers[10] in the construction sector has dropped from 8.94 years to 6.42 years from 2002 to 2006, which is a 28 percent decrease. However, the average tenure of the firm managers in the manufacturing sector has increased from 11.69 to 14.11 years during the same period of time, which is a 21 percent increase.

One concern of the evidence drawn from tenure length would be the age variation has almost captured all the tenure variation. However, the correlation between the age groups and the tenure groups shows that the age group variation does not capture all the variation in tenure group variation. Using the tenure group definition in the first column in Table 5, the SES data show that correlation between age and tenure is only 0.52 in all sectors, 0.32 in the construction sector and 0.61 in the manufacturing sector. This correlation is not due to the ad hoc definition of the tenure group. Using a different tenure group definition shown in the second column of Table 5, the correlation between age group and tenure group is 0.53 in all sectors, 0.34 in the construction sector and 0.62 in the manufacturing sector. If we consider yet another definition of tenure group as in column 3 of 5, the correlation between age group and tenure group is 0.53 in all sectors, 0.34 in the construction sector and 0.61 in the manufacturing sector. The message is that although there is positive correlation between tenure and age, the correlation is not 1. It means the tenure variable does contain information that the age variable does not.

[10]In the occupation classification, the firm managers are coded as 12 "corporate managers" and 13 "managers of small enterprises."

Table 5: Tenure Group and Age Group

Tenure Group 1	Tenure Group 2	Tenure Group 2	Age Group
< 10	< 5	< 3	< 20
$[10, 20)$	$[5, 15)$	$[3, 13)$	$[20, 30)$
$[20, 30)$	$[15, 25)$	$[13, 23)$	$[30, 40)$
$[30, 40)$	$[25, 35)$	$[23, 33)$	$[40, 50)$
$[40, 50)$	$[35, 45)$	$[33, 43)$	$[50, 60)$
≥ 50	≥ 45	≥ 43	≥ 60

The age group is from the SES-Eurostat data
Tenure group 1, tenure group 2 and tenure group 3 are by the author's definition.

4.3 Unobserved Labor Quality

In this subsection, I show how I back out the distribution of the unobserved labor quality using worker-firm matched data. Although the worker-firm matched data have more information than KLEMS to characterize labor quality such as tenure length, it is impossible to exhaust all the labor characteristics that are related to labor quality. Other labor quality dimensions, such as diligence, communication skills, etc., are hard to measure by the data.

To back out the total unobserved labor quality, I assume that the wage variation has three sources: the firm characteristics, the observed labor characteristics used by KLEMS and other dimensions of labor quality beyond KLEMS. The idea is that after running the regression of the wage on gender, education, age and firm fixed effect, the residual wage variation can be attributed to other dimensions of labor quality beyond KLEMS.

The specification of the regression is shown in equation 8.

$$\ln(w_{jst}) = \alpha_{i(j)st} + \text{gender}_{jst} + \text{education}_{jst} + \text{age}_{jst} + \varepsilon_{jst}, \quad (8)$$

where w_{jst} is the deflated wage bill of worker j in sector s at time t, $i(j)$ indicates the firm's identifier where worker j works; and $\alpha_{i(j)st}$ is the firm fixed effect.[11] The coefficients of the regression are omitted for the sake of simplicity.

The result of the regression is shown in Table 6. Compared to the result of the regression without firm fixed effect in Table 4, the adjusted R-square has increased from 20-40 percent to 60-70 percent.

The unexplained wage variation in equation 8 can be interpreted as the unobserved labor quality, but I have to deal with the following concerns. First, there is potential assortative matching between firms and workers. While assortative matching itself is an open question,[12] this problem only makes firm fixed effects capture some labor characteristics. Alternatively speaking, the unobserved labor characteristics should probably capture wage variation larger than just 30 - 40 percent. Second, some labor search models predict that even without labor heterogeneity there should be wage variation because of the search friction, but more evidence suggests the frictional wage dispersion only explains a small part of the wage variation.

The distribution of the residual wage distribution can then be used in the calibration of the model.

[11] Firm fixed effect can be identified since there are more than one workers in each firm. On average, there are 8.6 observations in each firm.

[12] Abowd and Kramarz (1999) and Card et al. (2016) find there is very limited assortative matching, while Borovicková and Shimer (2017) argues significant assortative matching.

Table 6: Regression of ln(wage) on Observed Individual Characteristics and Firm Fixed Effect

	(1) 2002 All Sectors b/se	(2) Manufac. b/se	(3) Construc. b/se	(4) 2006 All Sectors b/se	(5) Manufac. b/se	(6) Construc. b/se
Gender F	0.000	0.000	0.000	0.000	0.000	0.000
	(.)	(.)	(.)	(.)	(.)	(.)
Gender M	0.182***	0.186***	0.197***	0.170***	0.190***	0.186***
	(0.00)	(0.00)	(0.01)	(0.00)	(0.00)	(0.01)
age 14-19	0.000	0.000	0.000	0.000	0.000	0.000
	(.)	(.)	(.)	(.)	(.)	(.)
age 20-29	0.070***	0.071***	0.068***	0.063***	0.112***	0.057**
	(0.01)	(0.01)	(0.02)	(0.01)	(0.02)	(0.02)
age 30-39	0.221***	0.207***	0.156***	0.184***	0.218***	0.144***
	(0.01)	(0.01)	(0.02)	(0.01)	(0.02)	(0.02)
age 40-49	0.333***	0.332***	0.212***	0.278***	0.337***	0.183***
	(0.01)	(0.01)	(0.02)	(0.01)	(0.02)	(0.02)
age 50-59	0.383***	0.400***	0.263***	0.331***	0.401***	0.234***
	(0.01)	(0.01)	(0.02)	(0.01)	(0.02)	(0.02)
age 60+	0.391***	0.435***	0.276***	0.369***	0.431***	0.296***
	(0.01)	(0.01)	(0.02)	(0.01)	(0.02)	(0.03)
educ 1	0.000	0.000	0.000	0.000	0.000	0.000
	(.)	(.)	(.)	(.)	(.)	(.)
educ 2	0.045***	0.042***	0.055***	0.033***	0.044***	0.062***
	(0.00)	(0.00)	(0.01)	(0.00)	(0.01)	(0.01)
educ 3	0.136***	0.148***	0.112***	0.107***	0.131***	0.115***
	(0.00)	(0.01)	(0.01)	(0.00)	(0.01)	(0.01)
educ 4	0.174***	0.181***	0.107***	0.134***	0.163***	0.136***
	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)	(0.02)
educ 5	0.435***	0.472***	0.402***	0.368***	0.409***	0.333***
	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)	(0.01)
educ 6	0.600***	0.664***		0.501***	0.554***	0.034
	(0.03)	(0.06)		(0.02)	(0.07)	(0.23)
Constant	1.510***	1.531***	1.474***	1.587***	1.543***	1.536***
	(0.01)	(0.01)	(0.02)	(0.01)	(0.02)	(0.03)
Adj.R-sqr	0.687	0.714	0.639	0.629	0.626	0.593
Obs	216400	83808	15548	219723	77381	16641

Raw data from Structure of Earnings Survey - Eurostat

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

5 Model

This is an infinite horizon model that features both the misallocation and the mismeasurement channels. With the key distributions calibrated to micro data, the model predicts a much milder true TFP drop compared to the measured one.

The misallocation channel is built on Reis (2013), and the mismeasurement channel is built on Young (2014). The model has four types of agents: a household with heterogeneous workers, a tradable sector with a representative firm, a non-tradable sector with heterogeneous entrepreneurs, and a representative bank.

The mechanism of the model is as follows: a negative interest rate shock (Eurozone integration) enables the low-productivity firms in the expanding sector (the non-tradable sector) to enter the production by borrowing. This brings down the average productivity of this sector. Moreover, the borrowing cost for existing non-tradable firms is also lowered and allows them to borrow more; thus, the sector expands. The tradable sector is not affected by the shock since it is assumed that the tradable firms are far less financially constrained. Therefore, there is no expansion in this sector. The expansion in the non-tradable sector increases base wage and attracts the labor from the tradable sector. The marginal worker entering the non-tradable sector is less efficient compared to the average existing workers, while the way the TFP is calculated treats new workers the same as the existing ones. The lower efficiency of the worker is translated to the lower imputed TFP. The existence of the mismeasurement channel makes the true TFP drop much less acutely than the measured TFP suggests.

5.1 Household

The household is one big decision maker for consumption choice. Although there are different types of workers in the household, the household only cares about the income on the aggregate level and maximizes the aggregate utility. This technique has been used in Gertler, Kiyotaki et al. (2010). This assumption implies that there is perfect consumption insurance within the household.

The source of the income for the household is the labor income in both sectors. To make the model more tractable and intuitive, I assume the household in the economy is hand-to-mouth. So the consumption of the household is

$$C_t^H = \frac{w_t^T L_t^T + w_t^N L_t^N}{p_t}, \tag{9}$$

where the efficiency labor supply in the tradable sector is defined as

$L_t^T \equiv \int_0^\infty z_T g_T(z_T) G_{N|T}(\frac{w_t^T}{w_t^N} z_T | z_T) dz_T$; and the efficiency labor supply in the non tradable sector is defined as

$L_t^N \equiv \int_0^\infty z_N g_N(z_N) G_{T|N}(\frac{w_t^N}{w_t^T} z_N | z_N) dz_N$;

p_t is the price index, which is defined as

$p_t \equiv \frac{p_t^T C_t^T + p_t^N C_t^N}{C_t} = [\gamma^\xi (p_t^T)^{1-\xi} + (1-\gamma)^\xi (p_t^N)^{1-\xi}]^{\frac{1}{1-\xi}}$,

w_t^T and w_t^N are respectively the base wage of each sector.

Conceptually, it is not difficult to give the household access to the bond market.

The efficiency labor L_t^T and L_t^N are different from the numbers of workers employed, but take into the consideration of the labor productivity.

Each individual within the household is otherwise identical except for the productivity in each sector. The productivity in tradable sector is z^T, and that of the non-tradable sector is z^N, and the pair of the productivity (z^T, z^N) is drawn from some joint cumulative distribution $G(z^T, z^N)$ independently. A worker provides 1 unit of inelastic labor, so his/her efficiency labor is z^T in the tradable sector and z^N in the non-tradable sector. A worker chooses to enter the tradable sector if he/she can earn higher income there, that is, $w_t^N z^N < w_t^T z^T$; or alternatively $z^N < z_T/\omega_t$, where $\omega_t = \frac{w_t^N}{w_t^T}$. If $z_T \geq z_N \omega_t$, the household chooses to enter the non-tradable sector.

5.2 Firms

In the model, the tradable sector and the non-tradable sector are modeled very differently in terms of financial constraints. The non-tradable sector has both a collateral constraint and a working capital constraint. This modeling technique is abstracted from the fact that the non-tradable firms are on average smaller than the tradable firms, and thus they are more financially constrained. Using the US firm-level data, Chodorow-Reich (2014) shows that the employment of smaller firms is more affected by the negative credit supply shock to their banks. This is because the sticky bank-borrower relationships make it harder for smaller firms to switch from affected banks to good banks. Moreover, the small firms lack other sorts of financing rather than borrowing from banks. The paper also claims that the findings of "Small vs Big" are consistent with the existing literature, such as Duygan-Bump, Levkov and Montoriol-Garriga (2015), explaining this by lower level of transparency within smaller firms. Beck, Demirgüç-Kunt and Maksimovic (2005) employs unique, cross-country firm-level survey data to prove that being small in size is correlated to facing more financial obstacles. Some may suspect the correlation between size and financial constraint is only sensible within a sector. However, the result of both papers are across sectors.

5.2.1 Tradable Sector

There is one representative firm in the tradable sector. The firm borrows at the foreign interest rate, and hires the efficiency labor in the labor market. It is assumed that the technology and the capital stock are active in the next period.

The production function is Cobb-Douglas:

$$Y_t^T = A_{t-1}^T (K_{t-1}^T)^{\alpha_T} (L_t^T)^{1-\alpha_T}, \tag{10}$$

where Y_t^T is the real output of the tradable sector of the current period, and it is produced with the technology and capital stock of the previous period, A_{t-1}^T and K_{t-1}^T, as well as with the labor employment of the current period, L_t^T.

The factors market are assumed to be competitive.

The profit maximization gives two first order conditions:

$$\alpha_T A_{t-1}^T (K_{t-1}^T)^{\alpha_T - 1} (L_t^T)^{1-\alpha_T} = 1 + r_t^f \tag{11}$$

$$(1-\alpha_T) A_{t-1}^T (K_{t-1}^T)^{\alpha_T} (L_t^T)^{-\alpha_T} = w_t^T \tag{12}$$

From equation 11 we can see that the interest rate at which the tradable firm borrows is r_f , which is the foreign interest rate.

Equations 11 and 12 pin down the base wage of the tradable sector:

$$w_t^T = (1-\alpha_T)(\alpha_T)^{\frac{\alpha_T}{1-\alpha_T}} A_{t-1}^{\frac{1}{1-\alpha_T}} (1+r_t^f)^{-\frac{\alpha_T}{1-\alpha_T}} \tag{13}$$

Here, we can see that there is a one-to-one map from the true TFP of the tradable sector to the wage, so we will not consider the mismeasurement in the tradable sector.

5.2.2 Non-tradable Sector

The non-tradable sector has a distribution of entrepreneurs with the CDF of TFP $H(a)$, and $a \in [\underline{a}, \bar{a}]$. The entrepreneurs maximize their lifetime discounted utility. By achieving this goal, the entrepreneurs first solve a static profit maximization problem and then a dynamic optimal wealth allocation problem. In other words, in period t an entrepreneur has to decide first whether to enter the production process and then how much to spend on consumption.

It is also assumed that the technology and the capital stock are active in the next period as in the tradable sector. The static profit maximization problem can be solved using backwards induction. The entrepreneur has to choose how much to invest in the capital stock if she enters the production process. If she opts to stay out of the production, she puts the wealth less consumption in the domestic bank.

We will see in the following paragraphs that once the entrepreneur decides to enter the production process, she will invest all her wealth into the capital stock. Moreover, she faces a borrowing constraint: the debt she has to pay back in the next period has to be smaller than a fraction of the potential output less the wage bill. Furthermore, the entrepreneurs face a working capital constraint.[13] The idea of working capital constraint is that the firm must hold η units of a non-interest-bearing asset (cash) for each unit of wage payments. This constraint increases the marginal cost of labor hiring for the firm by $w_N \frac{\eta r}{1+r}$.

Let us first solve the problem of the entrepreneur decides to enter the production process in time t. She needs to solve the profit maximization problem as follows:

$$\pi_t^N = \max_{\{l_t, b_t\}} \quad p_t^N a_{t-1} k_{t-1}^{\alpha_N} l_t^{1-\alpha_N} - \tilde{w}_t^N l_t - b_t$$

$$b_t \leq \theta(p_t^N a_{t-1} k_{t-1}^{\alpha_N} l_t^{1-\alpha_N} - \tilde{w}_t^N l_t) \tag{14}$$

$$k_{t-1} = \hat{v}_{t-1} + \frac{b_t}{1+r_t^b}$$

where θ is a collateral constraint ratio, r_t^b is the loan rate, and $\tilde{w}_t^N = w_t^N(1 + \frac{\eta r_t^b}{1+r_t^b})$, and $\hat{v}_{t-1}$ is the wealth of the period $t-1$ that has not been consumed, which is defined as $\hat{v}_{t-1} = v_{t-1} - p_{t-1}c_{t-1}$ The capital stock used in the production is the sum of her own wealth and the borrowed money. The reason why the borrowed money is discounted by $1 + r_t^b$ is that the borrowing happens at the beginning of the period and the repayment happens at the end of the same period.

The reason to add the working-capital constraint is because the increase of foreign borrowing ϕ in the model is a supply shock: the lower borrowing cost induces the non-tradable firms to borrow more, employ more and produce more. However, more non-tradable goods

[13]This working capital constraint is a model technique widely used in the international macroeconomics field, such as Neumeyer and Perri (2005), Uribe and Yue (2006), CHANG and FERNNDEZ (2013) and Uribe and Schmitt-Grohé (2017). The main reason to introduce the working capital constraint is to provide a supply-side channel through which the interest rate shock matters more.

push down the price, which reduces the profitability of the non-tradable sector, and hence decreases the employment. Therefore, these two forces counteract one another. The introduction of the working-capital constraint will make the increase in ϕ another positive supply shock, thus increasing the employment in the non-tradable sector. This force will drive up the base wage ratio between the non-tradable sector and the tradable sector, attracting labor flow into the non-tradable sector. The quantitative effect of the working-capital constraint is very low though.

Taking the first-order condition of the profit with respect to l_t, we get

$$l_t = [\frac{(1-\alpha_N)p_t^N a_{t-1}}{\tilde{w}_t^N}]^{1/\alpha_N} k_{t-1} \tag{15}$$

Using equation 15 to replace l_t in problem 14, the problem can be written as:

$$\pi_t^N = \max_{\{k_{t-1}\}} \quad x_t(a_{t-1})k_{t-1} - (1+r_t^b)(k_{t-1} - \hat{v}_{t-1})$$

$$(1+r_t^b)(k_{t-1} - \hat{v}_{t-1}) \leq \theta x_t(a_{t-1})k_{t-1}$$

where $x_t(a_{t-1})$ is the return on capital k_{-1}, and is defined as follows:

$$x_t(a_{t-1}) = \alpha_N(1-\alpha_N)^{\frac{1-\alpha_N}{\alpha_N}}[\frac{p_t^N a_{t-1}}{(\tilde{w}_t^N)^{1-\alpha_N}}]^{1/\alpha_N} \tag{16}$$

Since now the profit maximization problem becomes a linear problem in k_{t-1}, the result depends on the sign of the coefficient of k_{t-1}: $x_t(a_{t-1}) - (1+r_t^b)$.

Here we use a "guess-and-verify" strategy to solve for the problem. Since here we already assume that the entrepreneur enters the production in period t, it means $k_{t-1} > 0$. Thus, we do not have to consider the equilibrium where $x_t(a_{t-1}) < (1+r_t^b)$.

Guess that $x_t(a_{t-1}) \geq (1+r_t^b)$. Then the borrowing constraint is binding; that is,

$$k_{t-1}(a_{t-1}) = \frac{\hat{v}_{t-1}}{1 - \frac{\theta x_t(a_{t-1})}{1+r_t^b}} \tag{17}$$

Profit is:

$$\pi_t^N(a_{t-1}) = x_t(a_{t-1})k_{t-1} - (1+r_t^b)(k_{t-1} - \hat{v}_{t-1}) = \frac{(1-\theta)x_t(a_{t-1})\hat{v}_{t-1}}{1-\frac{\theta x_t(a_{t-1})}{1+r_t^b}}.$$

The return on the wealth of entrepreneurs who produce is:

$$R_t(a_{t-1}) = \frac{(1-\theta)x_t(a_{t-1})}{1-\frac{\theta x_t(a_{t-1})}{1+r_t^b}}.$$

The entrepreneurs compare the return on wealth and the deposit rate to determine whether she wants to enter the production, so there is a cutoff productivity a^* under which an entrepreneur opt to save in domestic deposit.

$$R_t(a_{t-1}^*) = \frac{(1-\theta)x_t(a_{t-1}^*)}{1-\frac{\theta x_t(a_{t-1}^*)}{1+r_t^b}} = 1 + r_t^d \tag{18}$$

where r_t^d is the domestic deposit rate.

In the bank's problem, we will see that $r_t^d > r_t^b$. Moreover $x_t(a_{t-1})$ is an increasing function on a_{t-1}. Thus, for all the producing entrepreneurs, we have $R_t(a_t) > 1+r_t^d > 1+r_t^b$. From this inequality, we know that once in the production function, $x_t(a_{t-1}) > 1+r_t^b$. Then our initial *guess* is verified.

Moreover, the output of the non-tradable sector in period t is:

$$y_t^N = a_{t-1}k_{t-1}^{\alpha_N}l_t^{1-\alpha_N} = [\frac{(1-\alpha_N)p_t^N a_{t-1}}{\tilde{w}_t^N}]^{(1-\alpha_N)/\alpha_N}\frac{a_{t-1}\hat{v}_t}{1-\frac{\theta x_t(a_{t-1})}{1+r_t^b}}$$

The gross revenue of the non-tradable firm is

$$p_t^N y_t^N = x_t(a_{t-1})k_{t-1} + \tilde{w}_t^N l_t = \frac{\hat{v}_t}{\alpha_N}\frac{x_t(a_{t-1})}{1-\frac{\theta x_t(a_{t-1})}{1+r_t^b}} \tag{19}$$

After solving the static profit maximizing problem, we can solve for the dynamic problem of the entrepreneurs.

Let $I(a_t > a_t^*)$ be the indicator of producing. a_t^* is determined by equation 18. Since $R_t(a_{t-1})$ is an increasing function, an entrepreneur produces if she has a TFP higher than the the cutoff TFP, or she puts her wealth in the domestic bank as deposit.

Thus, the dynamic of the individual wealth is $v_{t+1} = [I(a_{t-1} > a_{t-1}^*)R_t(a_{t-1}) + (1 - I(a_{t-1} > a_{t-1}^*))(1 + r_t^d)](v_t - p_t c_t)$.

And the entrepreneur has to solve the following dynamic problem:

$$\max_{c_t} \mathbb{E}\, \beta^t \ln(c_t)$$

$$\text{s.t. } v_{t+1} = [I(a_{t-1} > a_{t-1}^*)R_t(a_{t-1}) + (1 - I(a_{t-1} > a_{t-1}^*))(1 + r_t)](v_t - p_t c_t)$$

The solution for this problem is that

$$\begin{aligned} c_t &= (1-\beta)\tfrac{v_t}{p_t} \\ v_{t+1} &= \beta[I(a_{t-1} > a_{t-1}^*)R_t(a_{t-1}) + (1 - I(a_{t-1} > a_{t-1}^*))(1 + r_t)]v_t \end{aligned} \tag{20}$$

A entrepreneur's consumption is always a constant fraction of her wealth.

5.3 Bank

The bank maximizes its profit subjects to a budget constraint:

$$\max B_t - F_t - D_t$$

$$\text{s.t. } \tfrac{B_t}{1+r_t^b} = \tfrac{D_t}{1+r_t^d} + \tfrac{F_t}{1+r_t^f}$$

$$F_t \leq \phi B_t$$

where B_t is the face value (FV) of the loan to the non-tradable sector, F_t is the FV of the borrowing from the foreign countries, and D_t is the FV of the deposit from the non-tradable sector. ϕ, which controls how much foreign borrowing the bank can get, is the most important parameter of the model. Also, we assume here that the bank has no equity and it finances all its loans by borrowing from abroad and deposits.

In the equilibrium, we will consider the case $r_t^f < r_t^d$, so the bank would borrow from abroad to the maximum: $F_t = \phi B_t$. Moreover, because of the linear technology of the bank, it has to attain zero profits in the equilibrium, so $B_t = F_t + D_t$.

Hence, in the equilibrium, the deposit has to be a constant fraction of the total lending:

$$D_t = (1 - \phi) B_t \tag{21}$$

Therefore, we have a relationship among the three interest rates:

$$\frac{1}{1 + r_t^b} = \frac{1 - \phi}{1 + r_t^d} + \frac{\phi}{1 + r_t^f} \tag{22}$$

since $\phi \in [0, 1]$, $r_t^f \leq r_t^b \leq r_t^d$.

5.4 Market Clearing Conditions

5.4.1 Non-tradable Goods

The demand for the non-tradable goods has to be equal to the supply.

First, let us pin down the demand for the non-tradable goods C_t^N, which comes from two sources – the demand from non-tradable sector entrepreneurs and that from the household. The consumption aggregator is a constant elasticity substitution (CES) aggregator of tradable and non-tradable goods, that is the total consumption $C_t = [\gamma (C_t^T)^{1-\frac{1}{\xi}} + (1 - \gamma)(C_t^N)^{1-\frac{1}{\xi}}]^{\frac{\xi}{\xi-1}}$, where $\gamma \in (0, 1)$, and ξ is the elasticity of substitution between tradable goods and non-tradable goods.

We can then derive the price index:

$p_t \equiv \frac{p_t^T C_t^T + p_t^N C_t^N}{C_t} = [\gamma^\xi (p_t^T)^{1-\xi} + (1-\gamma)^\xi (p_t^N)^{1-\xi}]^{\frac{1}{1-\xi}}$;

and express the non-tradable consumption in terms of total consumption:

$$C_t^N = C_t(\frac{p_t^N}{p_t(1-\gamma)})^{-\xi}. \tag{23}$$

The derivation of the price index and the demand of non-tradable good can be found in Appendix C.

Using the fact of the equilibrium condition that the demand for total consumption comes from two sources: the consumption of the entrepreneurs and consumption of the household, so $C_t = C_t^H + \int c_t dG(t)$.

Combining equations 9, 20, 23 and the above condition , we have

$C_t^N = (C_t^H + \int c_t dH(a_{t-1}))(\frac{p_t^N}{p_t(1-\gamma)})^{-\xi} = (\frac{w_t^T L_t^T + w_t^N L_t^N}{p_t} + \int (1-\beta)\frac{v_t}{p_t} dH(a_{t-1})(\frac{p_t^N}{p_t(1-\gamma)})^{-\xi} = (\frac{w_t^T L_t^T + w_t^N L_t^N}{p_t} + (1-\beta)\frac{V_t}{p_t})(\frac{p_t^N}{p_t(1-\gamma)})^{-\xi}$,

where V_t is the aggregate wealth of entrepreneurs, defined as $V_t = \int v_d dH(a_{t-1})$

Supply of the non-tradable good is as follows:

$Y_t^N = \int_{a_{t-1}^*}^{\bar{a}} y_t dH(a_{t-1}) = \int_{a_{t-1}^*}^{\bar{a}} \frac{\tilde{w}_t l_t}{(1-\alpha_N)p_t^N} dH(a_{t-1}) = \frac{\tilde{w}_t L_t^N}{(1-\alpha_N)p_t^N}$

Since $C_t^N = Y_t^N$, we have

$$\frac{\tilde{w}_t^N L_t^N}{(1-\alpha_N)p_t^N} = (\frac{w_t^T L_t^T + w_t^N L_t^N}{p_t} + (1-\beta)\frac{W_t}{p_t})(\frac{p_t^N}{p_t(1-\gamma)})^{-\xi} \tag{24}$$

5.4.2 Loan and Deposit Market Clearing

By integration of the debt of producing entrepreneurs, we can get the aggregate loan. And by integration of the deposit of non-producing entrepreneurs, we get the aggregate deposit. Then we can link them using equation 21 and get the following equation:

$$(1+r_t^d)H(a_{t-1}^*) = \frac{\theta(1-\phi)}{1-\theta}\int_{a_{t-1}^*}^{\bar{a}} R_t(a_{t-1})dH(a_{t-1}) \tag{25}$$

By simple manipulation, we can get the expression of the integration:

$$\int_{a_{t-1}^*}^{\bar{a}} R_t(a_{t-1})dH(a_{t-1}) = \frac{(1+r_t^d)(1-\theta)H(a_{t-1}^*)}{\theta(1-\phi)} \tag{26}$$

The details of the derivation can be found in Appendix D.

5.4.3 Non-tradable Sector Labor Market Clearing

By equating labor supply and labor demand in the non-tradable sector, with the law of motion of aggregate wealth of entrepreneurs in the Non-tradable sector, we can get the labor market clearing condition in the following equation:

$$\int_0^{\infty} z_N g_N(z_N) G_{T|N}(\frac{w_t^N}{w_t^T} z_N|z_N) dz_N = \frac{1-\alpha_N}{(1-\theta\phi)\alpha_N \tilde{w}_t^N} V_t \tag{27}$$

The details of the derivation of the law of motion of the aggregate wealth of entrepreneurs and the labor market clearing condition of the non-tradable sector can be found in Appendix E. All the equilibrium conditions can be found in Appendix G.

5.5 The Mismeasurement of TFP

Suppose the non-tradable sector has a Cobb-Douglas production function:

$$Y_t^N = A_{t-1}^N (K_{t-1}^N)^{\alpha_N} (L_t^N)^{1-\alpha_N},$$

L^N here is the efficiency labor in the non-tradable sector, and it can be expressed as $L^N = q^N \bar{z}^N$, where q^N is the share of labor that works in the non-tradable sector.

The mathematical definition is $q^N = \int_0^{\infty} g_N(z^N) G_{T|N}(\frac{w^N}{w^T} z^N|z^N) dz^N$, and $L^N = \int_0^\infty z^N g_N(z^N) G_{T|N}(\frac{w^N}{w^T} z^N | z^N) dz^N$.

It follows that $\bar{z}^N = \frac{L^N}{q^N}$

.

We measure TFP in macroeconomics as the Solow residual, so the true TFP growth rate is

$$\hat{A}^N(true) = \hat{Y}^N - \alpha_N \hat{K}^N - (1 - \alpha_N)(\hat{q}^N + \hat{\bar{z}}^N) \tag{28}$$

However, for a macro-econometrician, the labor efficiency change is unobservable; therefore, when calculating the Solow residual, what is actually estimated is following:

$$\hat{A}^N(est) = \hat{Y}^N - \alpha_N \hat{K}^N - (1 - \alpha_N)\hat{q}^N = \hat{A}^N(true) + (1 - \alpha_N)\hat{\bar{z}}^N \tag{29}$$

We can define the elasticity of average sectoral labor efficiency with respect to the size of the sector:

$$\zeta = \frac{d\bar{z}^x}{dq^N} \frac{q^N}{\bar{z}^N} \tag{30}$$

Now, the estimated TFP in equation 31 can be transformed to:

$$\hat{A}^N(est) = \hat{A}^N(true) + (1 - \alpha_N)\zeta\hat{q}^N \tag{31}$$

As long as this elasticity ζ is not zero, there will be mis-measurement.

It is easy to see that $\xi > -1$. In the definition of $\bar{z}^N$, if the increase of q^N does not change L^N, ζ would be -1, but the amount of efficiency labor L^N also increases when there are more people enter the sector. Therefore ζ should be bigger than -1.

The interesting point is to determine the sign of the elasticity. If $\xi < 0$, then when the sector expands, the average sectoral labor efficiency decreases, which leads to an underestimation of sectoral TFP.

According to Young (2014), we can have the following theorem:

Theorem: If the following two conditions are satisfied:

1.the distributions of z_T and z_N are independent: $G(z_T, z_N) = G_T(z_T)G_N(z_N)$;

2.$\frac{g_x(z_x)z_x}{G_x(z_x)}$ (where $x = T, N$) are decreasing functions,

then $\zeta \leq 0$.

The first condition is basically saying that if a person is born to be a good chef, he/she may or may not be an efficient engineer, or a good micro theory professor may struggle in the field of macro economics. The second condition is more like a technical requirement, and all widely used distributions satisfy this property. The proof of the theorem can be found in Appendix F.

Let's see how it works out in an analytical example.

Besides the assumption that z^T and z^N are independent, let us assume furthermore that cdfs G_T and G_N are exponentially distributed over $[0, \infty]$; that is, $G_T(z^T) = 1 - e^{-\lambda_T z^T}$, and $G_N(z^N) = 1 - e^{-\lambda_N z^N}$. Accordingly, pdfs are $g_T(z^T) = \lambda_T e^{-\lambda_T z^T}$ and $g_N(z^N) = \lambda_N e^{-\lambda_N z^N}$.

With the assumption on the distribution of the labor quality in the two sectors, we can compute the closed form efficiency labor in both sectors as follows:

$$L^T = \frac{1}{\lambda_T} - \frac{\lambda_T}{(\lambda_T + \lambda_N/\omega)^2} \tag{32}$$

$$L^N = \frac{1}{\lambda_N} - \frac{\lambda_N}{(\lambda_N + \lambda_T\omega)^2} \tag{33}$$

Moreover, we can compute the quantity of labor as follows:

$$q^T = \frac{\lambda_N}{\lambda_N + \lambda_T\omega} \tag{34}$$

$$q^N = \frac{\lambda_T\omega}{\lambda_N + \lambda_T\omega} \tag{35}$$

The sum of the quantity of labor in two sectors equal to the total quantity of labor which is normalized to 1, that is, $q_T + q_N = 1$.

Then the average quality of labor in each sector can be computed as follows:

$$\bar{z}^T \equiv \frac{L^T}{q^T} = \frac{1}{\lambda_T} + \frac{\omega}{\lambda_N + \lambda_T \omega} \tag{36}$$

$$\bar{z}^N \equiv \frac{L^N}{q^N} = \frac{1}{\lambda_N} + \frac{1}{\lambda_N + \lambda_T \omega} \tag{37}$$

Then it is easy to see that when the wage ratio ω increases, the labor moves out from the tradable sector into the non-tradable sector, and the average labor quality of the tradable sector increases while that of the non-tradable sector decreases.

6 Numerical Result

In this section, I present a calibrated version of the model and show that the prediction of this calibrated model matches the data. It also shows that the mismeasreument channel contributes much more than the misallocation channel to the TFP drop in the model.

6.1 Calibration and Impulse Response Functions

Following Reis (2013), one period of the model is set to be four years to justify the absence of nominal rigidities and the assumption of the i.i.d firm-level productivity shock. The risk-free rate r_f is set to be 0.08 and $\beta = 0.84$ is picked in order to make sure that the average steady-state capital return is around 0.16.

According to Table 3, setting $\alpha_N = 0.3$ and $\alpha_T = 0.3$ is a good approximation and close to the convention of the calibration of the Cobb-Douglas production function.

The productivity level of the tradable sector A_T is set to be the average productivity of the non-tradable sector: $A_T = \exp(0) = 1$.

The coefficient θ measures the percentage of the finance that comes from the bank. The BIS data shows that the credit from the bank to a non-financial corporation should be around 0.3. Since it is even more difficult for the non-tradable firms to borrow from the banks, I set $\theta = 0.2$.

The elasticity of substitution $\xi = 2$ is a very conventional number. The coefficient that governs the share of the non-tradable consumption $\gamma = 0.5$, which is also a conventional number.

The distribution of the productivity of the non-tradable firms is log-normal, which matches the full sample TFP distribution of the construction sector of Spain.

The working capital constraint parameter η is set to be 0.5. According to Uribe and Schmitt-Grohé (2017), this parameters means that the firm needs to hold half of the wage bill in advance, which means two years of wage bill in this model. However, to modify the parameter to a smaller number does not affect the prediction of the model.

The distribution of labor quality in the tradable sector and non-tradable sector matches the distribution of the non-observable individual characteristics from the analysis of the SES of Eurostat for Spain.

The overview of the calibration is listed in Table 7.

Table 7: **Calibration**

Parameters	β	r_f	α_N	α_T	A^T	η	θ	ξ	γ
	0.84	0.08	0.3	0.3	1	0.5	0.2	2	0.5
Distribution of a			Lower Bound (a_1)			Upper Bound (a_2)		μ	σ
Log-normal			exp(-8)			exp(4)		0	0.45
Distribution of z^N			Lower Bound (z_1^N)			Upper Bound (z_2^N)		μ	σ
Log-normal			exp(-8)			exp(3)		-3	1.98
Distribution of z^T			Lower Bound (z_1^T)			Upper Bound (z_1^T)		μ	σ
Log-normal			exp(-7)			exp(5)		-2.2	1.2

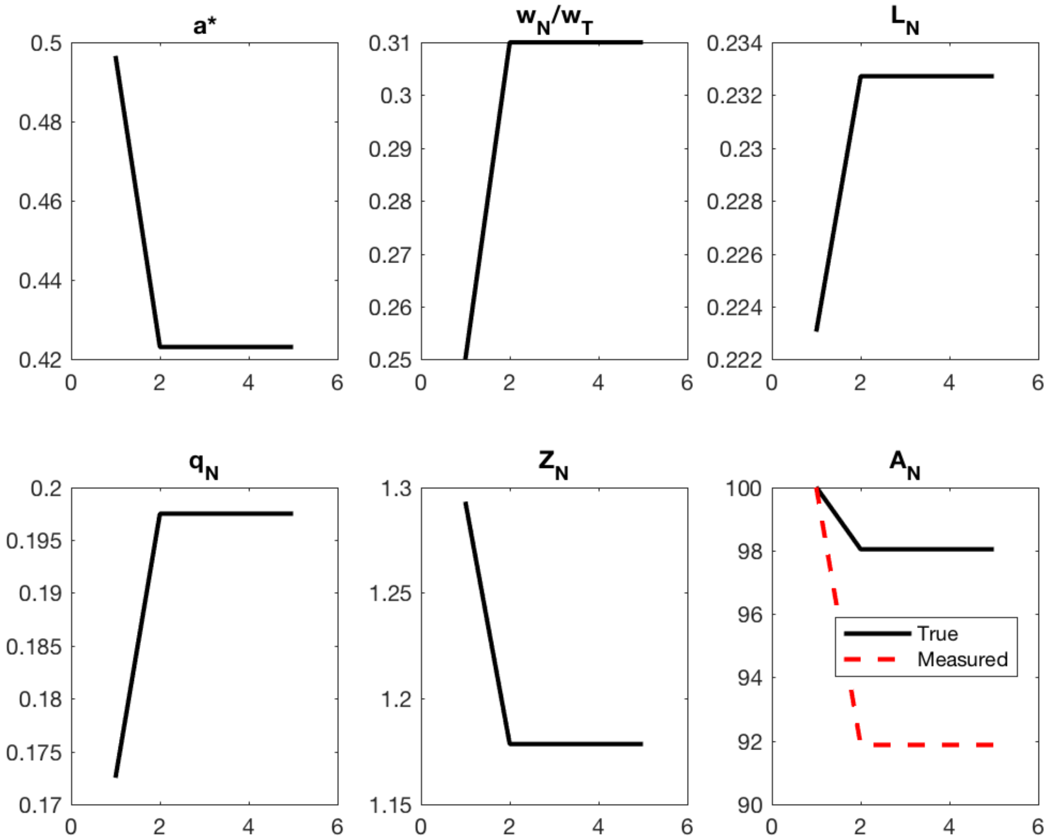
Figure 6.1 shows the impulse responses of the key variables in the model to explain the

TFP drop in the non-tradable sector, and thus the entire economy. The shock here is that ϕ rises from 0 to 0.6 in the first period. The model jumps from one steady state to the new steady state very quickly and stays there. According to equation 18, the lowered deposit rate will induce a lower cutoff productivity a^*, at which an entrepreneur enters the market and produces. The lowered cutoff a^* triggers the average productivity to drop, but only to a limited amount. That is the black solid line in the "A_N" graph of Figure 6.1.

The shock also changes the borrowing cost of the firms in the non-tradable sector to a much lower level, which can be seen from equation 22. This leads to an expansion of the non-tradable sector. Therefore, the wage ratio between the non-tradable sector and tradable sector $\omega = \frac{w_N}{w_T}$ increases. The relative increase of wage in the non-tradable sector attracts people to move into this sector, which explains the increase of the number of people in graph "q_N", and also the total efficiency labor increase in graph "L_N." However, the average quality of the non-tradable sector, as in graph "Z_N," decreases due to the Theorem in subsection 5.5. The lowered average quality explains the measured TFP drop, as shown by the red dashed line in graph "A_N".

Therefore, the key graph in Figure 6.1 is graph "A_N." The most important message from this graph is that the measured TFP drop is much worse than the one without any mismeasurement of the labor quality. Alternatively speaking, this means that if we can measure the labor quality correctly and take it into the consideration in TFP estimation, then corrected TFP drop will be much more mild compared to the TFP data we see now. This prediction of the TFP drop is in line with the TFP decomposition that I perform in Figure 3.6, showing that the mismeasurement channel dominates the misallocation channel in explaining the TFP drop of the non-tradable sector.

Figure 6.1: Impulse Responses/Trasitional Paths



The calibration of the model is listed in Table 7.

7 Conclusion

This paper has documented differentiated TFP growth paths between expanding sectors and non-expanding sectors for southern European countries between 1996 and 2007. Careful analysis of aggregate data and micro-level data shows that capital misallocation cannot explain this phenomenon, but labor quality mismeasurement can. If labor quality is treated properly, the true TFP drop of the expanding sector would be much smaller. Therefore, the true TFP drop of the total economy would be smaller as well. One policy implication we draw from this paper is that we should review the policy targeting the misallocation problem. Also this paper calls for a revision of TFP calculation that incorporates more labor quality than the state-of-the-art research such as KLEMS does.

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Appendices

A Primal Measure and Dual Measure of TFP

The derivation in this subsection is an extension of Hsieh (2002).

Assume the market is perfectly competitive. The sector s's output at time t Y_{st}, should be equal to the payment to the factors of the production, say capital and labor for the purpose of illustration:

$$Y_{st} = r_{st}K_{st} + w_{st}L_{st} \tag{38}$$

where K_{st} and L_{st} are respectively the capital stock and labor employment, and r_{st} and w_{st} are the rental price of capital and the wage.

Take the total derivative of the equation above with respect to time and divide it by Y_{st}:

$$\frac{dY_{st}}{Y_{st}} = \frac{K_{st}dr_{st}}{Y_{st}} + \frac{dK_{st}r_{st}}{Y_{st}} + \frac{L_{st}dw_{st}}{Y_{st}} + \frac{dL_{st}w_{st}}{Y_{st}} \tag{39}$$

Change the previous equation by making the labor share and the capital share appear:

$$\frac{dY_{st}}{Y_{st}} = \frac{K_{st}r_{st}}{Y_{st}}\frac{dr_{st}}{r_{st}} + \frac{K_{st}r_{st}}{Y_{st}}\frac{dK_{st}}{K_{st}} + \frac{L_{st}w_{st}}{Y_{st}}\frac{dw_{st}}{w_{st}} + \frac{L_{st}w_{st}}{Y_{st}}\frac{dL_{st}}{L_{st}} \tag{40}$$

The discrete time counterpart is to replace operator d by Δ:

$$\frac{\Delta Y_{st}}{Y_{st}} = \frac{K_{st}r_{st}}{Y_{st}}\frac{\Delta r_{st}}{r_{st}} + \frac{K_{st}r_{st}}{Y_{st}}\frac{\Delta K_{st}}{K_{st}} + \frac{L_{st}w_{st}}{Y_{st}}\frac{\Delta w_{st}}{w_{st}} + \frac{L_{st}w_{st}}{Y_{st}}\frac{\Delta L_{st}}{L_{st}}.$$

Let s_{st}^K denote the capital share and s_{st}^L the labor share. By definition, $s_{st}^K = \frac{K_{st}r_{st}}{Y_{st}}$ and $s_{st}^L = \frac{L_{st}w_{st}}{Y_{st}}$. Moreover, $\frac{dx}{x}$ is the growth rate of variable x, denoted as $\hat{x}$.

Then, the previous equation can be written as follows after rearrangement of terms:

$$\underbrace{\hat{Y}_{st} - s_{st}^K\hat{K} - s_{st}^L\hat{L}_{st}}_{\text{Primal: } \hat{A}_{st}^P} = \underbrace{s_{st}^K\hat{r}_{st} + s_{st}^L\hat{w}_{st}}_{\text{Dual: } \hat{A}_{st}^D} \tag{41}$$

The left-hand side is the primal measure used in KLEMS data, and the right-hand side is the dual measure. The derivation only depends on one single assumption of market competitiveness without any other assumption such as the form of the production function.

If the production is Cobb-Douglas $Y_{st} = A_{st}K_{st}^{\alpha}L_{st}^{1-\alpha}$, then $s^K = \alpha$, $s^L = 1 - \alpha$.

It is very straightforward to extend the dual measure to more than two input factors: $\hat{A}_{st}^D = \Sigma_{j=1}^n s_{st}^j\hat{r}_{st}^j$, where $\hat{r}_{st}^j$ is the growth rate of the price of input j and s_{st}^j is its share.

It should be noticed that the theoretical equivalence between the primal measure and is also true with more general CES production function.

Suppose $Y_{st} = A_{st}(\alpha K_{st}^{\frac{\rho-1}{\rho}} + (1-\alpha)L_{st}^{\frac{\rho-1}{\rho}})^{\frac{\rho}{\rho-1}}$, where ρ is the elasticity of substitution. The Cobb-Douglas function is a special case, where the elasticity of substitution is $\rho = 1$. This can be shown by using the l'Hopitale's rule.

Still under the assumption that the market is competitive:

$$\begin{aligned} r_{st} &= \frac{\partial Y_{st}}{\partial K_{st}} = A_{st}\alpha K_{st}^{\frac{\rho-1}{\rho}-1}(\alpha K_{st}^{\frac{\rho-1}{\rho}} + (1-\alpha)L_{st}^{\frac{\rho-1}{\rho}})^{\frac{\rho}{\rho-1}-1} \\ w_{st} &= \frac{\partial Y_{st}}{\partial L_{st}} = A_{st}(1-\alpha)L_{st}^{\frac{\rho-1}{\rho}-1}(\alpha K_{st}^{\frac{\rho-1}{\rho}} + (1-\alpha)L_{st}^{\frac{\rho-1}{\rho}})^{\frac{\rho}{\rho-1}-1} \end{aligned} \tag{42}$$

Denote $s_{st}^K = \frac{r_{st}K_{st}}{Y_{st}} = \frac{\alpha K_{st}^{\frac{\rho-1}{\rho}}}{\alpha K_{st}^{\frac{\rho-1}{\rho}} + (1-\alpha)L_{st}^{\frac{\rho-1}{\rho}}}$,
and $s_{st}^L = \frac{r_{st}K_{st}}{Y_{st}} = \frac{(1-\alpha)L_{st}^{\frac{\rho-1}{\rho}}}{\alpha K_{st}^{\frac{\rho-1}{\rho}} + (1-\alpha)L_{st}^{\frac{\rho-1}{\rho}}}$.

The difference between the general CES function and Cobb-Douglas function is that the labor share and capital share now depend on capital stock and employment.

Without the the assumption of perfect competition, then the output is divided into three parts, labor share, capital share and profit:

$$Y_{st} = r_{st}K_{st} + w_{st}L_{st} + \pi_{st} \tag{43}$$

where π_{st} is the profit of sector s at time t.

Performing a similar operation on the previous equation, we get:

$$\hat{Y}_{st} - s_{st}^K \hat{K}_{st} - s_{st}^L \hat{L}_{st} = s_{st}^K \hat{r}_{st} + s_{st}^L \hat{w}_{st} + s_{st}^\pi \hat{\pi}_{st} \tag{44}$$

Replace $s_{st}^K = 1 - s_{st}^L - s_{st}^\pi$ in the previous equation:

$$\hat{Y}_{st} - (1 - s_{st}^L - s_{st}^\pi)\hat{K}_{st} - s_{st}^L \hat{L}_{st} = (1 - s_{st}^L - s_{st}^\pi)\hat{r}_{st} + s_{st}^L \hat{w}_{st} + s_{st}^\pi \hat{\pi}_{st} \tag{45}$$

After rearranging the terms,

$$\hat{Y}_{st} - (1 - s_{st}^L)\hat{K}_{st} - s_{st}^L \hat{L}_{st} = (1 - s_{st}^L)\hat{r}_{st} + s_{st}^L \hat{w}_{st} + s_{st}^\pi (\hat{\pi}_{st} - \hat{K}_{st} - \hat{r}_{st}) \tag{46}$$

Since $\hat{\pi}_{st} - \hat{K}_{st} - \hat{r}_{st} = \hat{\pi}_{st} - \widehat{r_{st}K_{st}} = \hat{\pi}_{st} - \hat{Y}_{st} - (\widehat{r_{st}K_{st}} - \hat{Y}_{st}) = \hat{s}_{st}^\pi - \hat{s}_{st}^K$,

the previous equation can be rewritten as:

$$\underbrace{\hat{Y}_{st} - (1 - s_{st}^L)\hat{K}_{st} - s_{st}^L \hat{L}_{st}}_{\text{Primal: } \hat{A}_{st}^P} = \underbrace{(1 - s_{st}^L)\hat{r}_{st} + s_{st}^L \hat{w}_{st}}_{\text{Dual: } \hat{A}_{st}^D} + s_{st}^\pi (\hat{s}_{st}^\pi - \hat{s}_{st}^K) \tag{47}$$

Even with the true condition not being perfect competition, we can still calculate the primal measure and dual measure of TFP growth. However, the previous equation shows that it is no longer true that the primal measure equals the dual measure: the former exceeds the latter by $s_{st}^\pi (\hat{s}_{st}^\pi - \hat{s}_{st}^K)$.

How is the dual measure computed in the data? Here are the steps:

- Compensation of Labor $Comp_L$: directly observed
- Labor L: directly observed, total hours or total employees
- Labor share $s_L = \frac{Comp_L}{Y}$
- Nominal wage $w^n = \frac{Comp_L}{L}$
- Real wage growth $\hat{w} = \hat{w}^n - \pi$, π GDP-deflator inflation (Source: WDI)
- Compensation of Capital $Comp_K = Y - Comp_L$
- Capital share: $s_K = \frac{Comp_K}{Y}$
- Capital Stock K: Estimated by perpetual inventory model
- Nominal rental price: r^n from KLEMS data, and real rental price $\hat{r} = \hat{r}^n - \pi$.
- Real rental price growth $\hat{r} = \hat{r}^n - \pi$

B Amadeus Spain Summary Statistics

Following Kalemli-Ozcan et al. (2015), the data are downloaded from four vintage discs of AMADEUS[14] (June 2000, June 2003, June 2006, and December 2009) to deal with the issues of download cap and missing records.[15] From each disk, last five observations are downloaded, which are not necessarily the last five years. For example, in 2006 dataset, the last five observations could be 2005, 2004, 2003, 2002, and 2000. The missing 2001 data may be due to the facto that there is no report. Before merging the data from different vintage disks, I update the BVD ID of the firms that had BVD ID change between 1999 and 2009 to their BVD ID number in 2009, following the information downloaded from website idchanges.bvdinfo.com.[16] The updated BVD ID number then serves as the unique identifier to merge the firms. After merging the data using the BVD ID number, I then drop all the duplicates and drop all the consolidated firms. Some summary statistics are listed here.

[14] AMADEUS is a product by Bureau van Dijk. Unlike the ORBIS dataset, which provides firm-level data for companies around the world, AMADEUS focuses on European countries.

[15] Kalemli-Ozcan et al. (2015) documents that if someone tries to download a lot of data at one time, the download cap will translate into missing information of the downloaded data. It is also documented in their paper that if a firms does not report anything in the last 5 years, it would be excluded even if it is still in operation.

[16] This is the website that stores the history of BVD ID changes for all firms in products of Bureau Van Dijk.

Table 8: Size Distribution of Spanish Firms in All Sectors

	YEAR	1 - 19	20 - 249	>=250	Total
Full Sample	1999	64539	34951	1595	101085
	2000	93653	41902	1869	137424
	2001	124955	47166	2109	174230
	2002	170876	51343	2134	224353
	2003	285163	56363	2239	343765
	2004	406743	60387	2290	469420
	2005	461818	63904	2339	528061
	2006	496480	67471	2442	566393
	2007	457458	65859	2484	525801
	Total	2561685	489346	19501	3070532
Half Permanent Sample	1999	52540	29828	1206	83574
	2000	77971	36700	1495	116166
	2001	89330	42004	1739	133073
	2002	94877	46531	1895	143303
	2003	100290	50257	2041	152588
	2004	103077	52674	2131	157882
	2005	102434	53470	2145	158049
	2006	100743	54002	2180	156925
	2007	94579	51535	2198	148312
	Total	815841	417001	17030	1249872
Permanent Sample	1999	19775	11048	433	31256
	2000	20502	12803	505	33810
	2001	20858	13939	565	35362
	2002	21223	14972	604	36799
	2003	21433	15572	632	37637
	2004	21666	15973	647	38286
	2005	22000	16232	697	38929
	2006	22077	16602	733	39412
	2007	22109	16715	780	39604
	Total	191643	133856	5596	331095

Source: Amadeus Spain

In the permanent sample, the change of the numbers of firms in each category is mainly because of data availability, meaning in 2007 more labor data of firms are observed compared to 1999.

Table 9: Number of Firms in All Sectors in Spain

	YEAR	SectorA	SectorB	SectorC	SectorD	SectorE	SectorF	SectorG	SectorH	SectorI	SectorJ	SectorK	Sector r	Total
Full Sample	1999	2158	346	915	33692	681	17696	50444	4625	8157	741	21060	5177	145692
	2000	2676	417	1094	38072	894	24178	59586	6171	10023	966	30375	6772	181224
	2001	3979	531	1236	42958	1062	30725	70497	9123	11857	1445	43488	9419	226320
	2002	6187	678	1364	49336	1385	40089	84810	13820	14695	2504	71215	14671	300754
	2003	10837	938	1843	70607	2034	66222	122065	22648	21282	5034	129891	24259	477660
	2004	14945	1236	2307	92508	2698	92571	160212	31728	28283	6561	180876	33380	647305
	2005	16675	1304	2461	98099	3346	106416	175378	36529	30976	7642	211653	37804	728283
	2006	17092	1402	2486	100231	5385	117408	181437	39693	31974	8205	232591	40198	778102
	2007	14218	1250	2218	92457	5921	109082	164805	34955	28981	6760	196745	33950	691342
	Total	88767	8102	15924	617960	23406	604387	1069234	199292	186228	39858	1117894	205630	4176682
Full Sample	1999	1623	297	802	28545	538	14547	42552	3697	6604	379	14024	3752	117360
	2000	1948	344	971	32739	708	20230	50918	4808	8271	492	20711	4932	147072
	2001	2242	380	1027	34416	776	22840	55303	5537	8955	527	23454	5502	160959
	2002	2366	388	1037	35010	823	23942	57078	5880	9349	531	24565	5845	166814
	2003	2760	402	1092	37305	872	25924	59765	6259	9890	564	26920	6096	177849
	2004	3049	453	1116	39164	916	26940	61019	6418	10240	574	28588	6193	184670
	2005	3020	448	1103	38463	893	26719	60396	6350	10055	541	28247	6143	182378
	2006	2979	431	1081	37788	872	26315	59415	6270	9876	535	27492	6024	179078
	2007	2758	394	1007	35467	797	24665	55771	5844	9173	502	25209	5521	167108
	Total	22745	3537	9236	318897	7195	212122	502217	51063	82413	4645	219210	50008	1483288
Full Sample	1999	541	117	285	11202	162	5103	16462	1183	2339	57	3611	1161	42223
	2000	465	98	271	10518	158	4873	15565	1156	2243	53	3453	1135	39988
	2001	465	98	271	10518	159	4875	15550	1156	2241	52	3454	1135	39974
	2002	465	98	271	10518	159	4875	15550	1156	2241	52	3454	1135	39974
	2003	465	98	271	10518	159	4875	15550	1156	2241	52	3454	1135	39974
	2004	593	120	298	11445	182	5147	15914	1219	2395	55	3832	1143	42343
	2005	593	120	298	11446	182	5147	15913	1219	2395	55	3833	1143	42344
	2006	593	120	298	11446	182	5147	15913	1219	2395	55	3833	1143	42344
	2007	593	120	298	11446	182	5147	15913	1219	2395	55	3833	1143	42344
	Total	4773	989	2561	99057	1525	45189	142330	10683	20885	486	32757	10273	371508

Raw data: Amadeus Spain

The main three sectors used in the paper are: SectorD, SectorF and SectorK, which are respectively the manufacturing sector, the construction sector and the real estate sector. The other sectors are: SectorA, agriculture; SectorB, fishing; SectorC, mining; SectorE, utility; SectorG, wholesale and retail; SectorH, hotels and restaurants; SectorI, transport; SectorJ, financial intermediation; Sector r, others including education, community social service, public administration etc.

C Derivation of Price Index and Demand

Using the cost minimization method, we can back out the price of consumption goods in terms of the prices of tradable and non-tradable goods:

$$\min_{C_t^T, C_t^N} p_t^T C_t^T + p_t^N C_t^N$$

$$\text{s.t. } [\gamma (C_t^T)^{1-\frac{1}{\xi}} + (1-\gamma)(C_t^N)^{1-\frac{1}{\xi}}]^{\frac{\xi}{\xi-1}} = C_t$$

By solving this problem, we get $C_t^N = \frac{C_t(\frac{p_t^N}{1-\gamma})^{-\xi}}{[\gamma^\xi (p_t^T)^{1-\xi} + (1-\gamma)^\xi (p_t^N)^{1-\xi}]^{\frac{\xi}{\xi-1}}}$

and $C_t^T = \frac{C_t(\frac{p_t^T}{\gamma})^{-\xi}}{[\gamma^\xi (p_t^T)^{1-\xi} + (1-\gamma)^\xi (p_t^N)^{1-\xi}]^{\frac{\xi}{\xi-1}}}$.

Moreover, the price index of the consumption goods

$p_t \equiv \frac{p_t^T C_t^T + p_t^N C_t^N}{C_t} = [\gamma^\xi (p_t^T)^{1-\xi} + (1-\gamma)^\xi (p_t^N)^{1-\xi}]^{\frac{1}{1-\xi}}$.

Therefore, we get $C_t^N = C_t(\frac{p_t^N}{p_t(1-\gamma)})^{-\xi}$, $C_t^T = C_t(\frac{p_t^T}{p_t \gamma})^{-\xi}$.

A special case is that when $\xi = 1$, the CES aggregator degenerates to a Cobb-Douglas aggregator, $C_t = (C_t^T)^\gamma (C_t^N)^{1-\gamma}$, and $p_t = \frac{(p_t^T)^\gamma ((p_t^N)^{(1-\gamma)}}{\gamma^\gamma (1-\gamma)^{(1-\gamma)}}$, and the ratio of the non-tradable expenditure on total expenditure is constant, $C_t^N p_t^N = (1-\gamma) C_t p_t$.

D Derivation of Loan and Deposit Equilibrium Condition

By definition, the loan from the bank to non-tradable firms is:

$B_t = \int_{a_t^*}^{\bar{a}} b_t dH(a_{t-1})$

$= \int_{a^*_{t-1}}^{\bar{a}} \theta x_t(a_{t-1}) k_{t-1} dH(a_{t-1})$, Since $b_t = \theta x_t(a_{t-1})k_{t-1}$

$= \int_{a^*_{t-1}}^{\bar{a}} \theta x_t(a_{t-1}) \frac{\beta v_{t-1}}{1-\frac{\theta x_t(a_{t-1})}{1+r^b_t}} dH(a_{t-1})$, Since $k_{t-1} = \frac{\beta v_{t-1}}{1-\frac{\theta x_t(a_{t-1})}{1+r^b_t}}$

$= \frac{\theta\beta}{1-\theta} \int_{a^*_{t-1}}^{\bar{a}} v_{t-1} R_t(a_{t-1}) dH(a_{t-1})$, Since $R_t(a_{t-1}) = \frac{(1-\theta)x_t(a_{t-1})}{1-\frac{\theta x_t(a_{t-1})}{1+r^b_t}}$

$= \frac{\theta\beta V_{t-1}}{1-\theta} \int_{a^*_{t-1}}^{\bar{a}} R_t(a_{t-1}) dH(a_{t-1})$, Since a_t is i.i.d

.

To see why a_t i.i.d can lead to the separation of the integration of the product of v_{t-1} and $R_t(a_{t-1})$, note that

$\int_{a^*_{t-1}}^{\bar{a}} v_{t-1} R_t(a_{t-1}) dH(a_{t-1}) = \underbrace{[\int_{\underline{a}}^{\bar{a}} dH(a_{t-1})]}_{=1} \int_{a^*_{t-1}}^{\bar{a}} v_{t-1} R_t(a_{t-1}) dH(a_t)$

$= \underbrace{[\int_{\underline{a}}^{\bar{a}} v_{t-1} dH(a_{t-1})]}_{V_{t-1}} \int_{a^*_{t-1}}^{\bar{a}} R_t(a_{t-1}) dH(a_{t-1}) = V_{t-1} \int_{a^*_{t-1}}^{\bar{a}} R_t(a_t) dH(a_t).$

By definition, the deposit from the non-producing entrepreneurs is:

$D_t = \int_{\underline{a}}^{a^*_{t-1}} d_t dH(a_{t-1})$

$= \int_{\underline{a}}^{a^*_{t-1}} (1+r^d_t)(v_t - p_t c_t) dH(a_{t-1})$, Since non-producing entrepreneurs only consume and save

$= \int_{\underline{a}}^{a^*_{t-1}} \beta(1+r^d_t) v_{t-1} dH(a_{t-1})$

$= \beta(1+r^d_t) V_{t-1} H(a^*_{t-1})$.

Combining the aggregate loan and aggregate deposit and equation 21, we can get rid of the aggregate wealth V_t and get:

$$(1+r^d_t)H(a^*_{t-1}) = \frac{\theta(1-\phi)}{1-\theta} \int_{a^*_{t-1}}^{\bar{a}} R_t(a_{t-1}) dH(a_{t-1}) \tag{48}$$

Then we can get the expression of the integration:

$$\int_{a^*_{t-1}}^{\bar{a}} R_t(a_{t-1}) dH(a_{t-1}) = \frac{(1+r^d_t)(1-\theta)H(a^*_{t-1})}{\theta(1-\phi)} \tag{49}$$

E Derivation of Labor Market Equilibrium Condition in the Non-tradable Sector

The law of motion of the aggregate wealth comes from the law of motion of the individual wealth, (i.e., equation 20).

By definition: $V_t = \int_{\underline{a}}^{\bar{a}} v_t dH(a_t)$

$= \int_{\underline{a}}^{\bar{a}} \beta[I(a_t > a_t^*)R_t(a_{t-1}) + (1 - I(a_{t-1} > a_{t-1}^*))(1 + r_t^d)]v_{t-1}dH(a_{t-1})$

$= \beta \int_{a_{t-1}^*}^{\bar{a}} v_{t-1}R_t(a_{t-1})dH(a_{t-1}) + \beta \int_{\underline{a}}^{a_{t-1}^*}(1 + r_t)v_{t-1}dH(a_{t-1})$

$$= \beta V_{t-1} \underbrace{\int_{a_{t-1}^*}^{\bar{a}} R_t(a_{t-1})dH(a_{t-1})}_{=\frac{(1+r_t^d)(1-\theta)H(a_{t-1}^*)}{\theta(1-\phi)}} + \beta(1 + r_t^d)V_{t-1}H(a_{t-1}^*), \text{ Since } a_t \text{ is i.i.d}$$

$= \beta(1 + r_t^d)V_{t-1}H(a_{t-1}^*)\frac{1-\theta\phi}{\theta(1-\phi)}$

Labor demand in the non-tradable sector should be aggregated from the heterogeneous firms in this sector:

$L_t^N = \int_{a_{t-1}^*}^{\bar{a}} l_t dH(a_{t-1}) = \int_{a_{t-1}^*}^{\bar{a}} [\frac{(1-\alpha_N)p_t^N a_{t-1}}{\tilde{w}_t^N}]^{1/\alpha_N} k_{t-1}dH(a_{t-1})$

$= \int_{a_{t-1}^*}^{\bar{a}} [\frac{(1-\alpha_N)p_t^N a_{t-1}}{\tilde{w}_t^N}]^{1/\alpha_N} \frac{\beta v_{t-1}}{1-\frac{\theta x_t(a_{t-1})}{1+r_t^b}} dH(a_{t-1})$, Since $k_{t-1} = \frac{\beta v_{t-1}}{1-\frac{\theta x_t(a_{t-1})}{1+r_t^b}}$

$= \int_{a_{t-1}^*}^{\bar{a}} \frac{(1-\alpha_N)x_t(a_{t-1})}{\alpha_N \tilde{w}_t^N} \frac{\beta v_{t-1}}{1-\frac{\theta x_t(a_{t-1})}{1+r_t^b}} dH(a_{t-1})$, Since $x_t(a_{t-1}) = \alpha_N(1 - \alpha_N)^{\frac{1-\alpha_N}{\alpha_N}} [\frac{p_t^N a_{t-1}}{(\tilde{w}_t^N)^{1-\alpha_N}}]^{1/\alpha_N}$

$= \frac{\beta(1-\alpha_N)}{(1-\theta)\alpha_N \tilde{w}_t^N} \int_{a_{t-1}^*}^{\bar{a}} R_t(a_{t-1})v_{t-1}dH(a_{t-1})$, Since $R_t(a_{t-1}) = \frac{(1-\theta)x_t(a_{t-1})}{1-\frac{\theta x_t(a_{t-1})}{1+r_t^b}}$

$$= \frac{\beta(1-\alpha_N)V_{t-1}}{(1-\theta)\alpha_N \tilde{w}_t^N} \underbrace{\int_{a_{t-1}^*}^{\bar{a}} R_t(a_{t-1})dH(a_{t-1})}_{=\frac{(1+r_t^d)(1-\theta)H(a_{t-1}^*)}{\theta(1-\phi)}}, \text{ Since } a_t \text{ is i.i.d}$$

$$= \frac{1-\alpha_N}{(1-\theta\phi)\alpha_N \tilde{w}_t^N} \underbrace{\beta(1 + r_t^d)V_{t-1}H(a_{t-1}^*)\frac{1 - \theta\phi}{\theta(1 - \phi)}}_{=V_t}$$

$= \frac{1-\alpha_N}{(1-\theta\phi)\alpha_N \tilde{w}_t^N} V_t$

By equating the labor supply and labor demand in the non-tradable sector, with the law of motion of aggregate wealth, we can get the labor market clearing condition in the following equation:

$$\int_0^\infty z_N g_N(z_N) G_{T|N}(\frac{w_t^N}{w_t^T} z_N | z_N) dz_N = \frac{1-\alpha_N}{(1-\theta\phi)\alpha_N \tilde{w}_t^N} V_t \quad (50)$$

F Proof of $\zeta < 0$

Theorem: If the following two conditions are satisfied:

1.the distributions of z_T and z_N are independent: $G(z_T, z_N) = G_T(z_T) G_N(z_N)$;

2.$\frac{g_x(z_x) z_x}{G_x(z_x)}$ (where $x = T, N$) are decreasing functions,

then $\zeta \leq 0$.

Proof: To prove $\zeta \leq 0$, we just have to prove $\frac{d\bar{z}_x}{dq_x} \leq 0$.

$$\frac{d\bar{z}_x}{dq_x} = \frac{1}{q_x}(\frac{dL_x/d\omega}{dq_x/d\omega} - \bar{z}_x) = \frac{1}{q_x}(\frac{dL_x}{dq_x} - \bar{z}_x) \quad (51)$$

where $\omega = \frac{w_x}{w_y}$. So $\zeta \leq 0$ really means that the marginal worker who enters the sector has a lower efficiency comparing to the sectoral average.

Now use the first equality in 52 and plug in the value defined in the beginning of this section:

$$\begin{aligned} \frac{d\bar{z}_x}{dq_x} &= \frac{1}{q_x}(\frac{\int_0^\infty z_x^2 g_x(z_x) g_{y|x}(\omega z_x|z_x) dz_x}{\int_0^\infty z_x g_x(z_x) g_{y|x}(\omega z_x|z_x) dz_x} - \frac{\int_0^\infty z_x g_x(z_x) G_{y|x}(\omega z_x|z_x) dz_x}{\int_0^\infty g_x(z_x) G_{y|x}(\omega z_x|z_x) dz_x}) \\ &= \frac{1}{q_x}(\mathbb{E}(a) - \mathbb{E}(b)) \end{aligned} \quad (52)$$

where $F_a(t) = \frac{\int_0^t \eta(\omega z_x) g_x(z_x) G_{y|x}(\omega z_x|z_x) dz_x}{\int_0^\infty \eta(\omega z_x) g_x(z_x) G_{y|x}(\omega z_x|z_x) dz_x}$, $\eta(\omega z_x) = \frac{w z_x g_{y|x}(\omega z_x|z_x)}{G_{y|x}(\omega z_x|z_x)}$,

and $F_b(t) = \frac{\int_0^t g_x(z_x) G_{y|x}(\omega z_x|z_x) dz_x}{\int_0^\infty g_x(z_x) G_{y|x}(\omega z_x|z_x) dz_x}$.

Now when z_T and z_N are independent, $\eta(z_x) = \frac{g_x(z_x) z_x}{G_x(z_x)}$.

Moreover, when it is a decreasing function, $F_a(t) \geq F_b(t)$, which gives the First Stochastic Dominance. Thus $\mathbb{E}(a) \leq \mathbb{E}(b)$.

Therefore, $\zeta \leq 0$.

G The Definition of Equilibrium

A set of variables: $\{a^*_{t-1}, r^d_t, r^b_t, p^N_t, p_t, w^N_t, \tilde{w}^N_t, w^T_t, \omega_t, V_t, L^T_t, L^N_t\}$ that satisfies the following equations ($p^T_t = 1$):

$$
\begin{aligned}
& w^T_t = (1-\alpha_T)(\alpha_T)^{\frac{\alpha_T}{1-\alpha_T}} A_{t-1}^{\frac{1}{1-\alpha_T}} (1+r^f_t)^{-\frac{\alpha_T}{1-\alpha_T}} \\
& w^N_t = \omega_t w^T_t \\
& \tilde{w}^N_t = w^N_t (1 + \tfrac{\eta r^b_t}{1+r^b_t}) \\
& L^T_t = \textstyle\int_0^\infty z^T g_T(z^T) G_{N|T}(\tfrac{w^T_t}{w^N_t} z^T | z^T) dz^T \\
& L^N_t = \textstyle\int_0^\infty z^N g_N(z^N) G_{T|N}(\tfrac{w^N_t}{w^T_t} z^N | z^N) dz^N \\
& \tfrac{\tilde{w}^N_t L^N_t}{(1-\alpha_N) p^N_t} = (\tfrac{w^T_t L^T_t + w^N_t L^N_t}{p_t} + (1-\beta)\tfrac{V_t}{p_t})(\tfrac{p^N_t}{p_t(1-\gamma)})^{-\xi} \\
& p_t = [\gamma^\xi (p^T_t)^{1-\xi} + (1-\gamma)^\xi (p^N_t)^{1-\xi}]^{\frac{1}{1-\xi}} \\
& L^N_t = \tfrac{1-\alpha_N}{(1-\theta\phi)\alpha_N \tilde{w}^N_t} V_t \\
& \textstyle\int_{a^*_{t-1}}^{\bar{a}} R_t(a_{t-1}) dH(a_{t-1}) = \tfrac{(1+r^d_t)(1-\theta)H(a^*_{t-1})}{\theta(1-\phi)} \\
& V_t = \beta(1+r^d_t)V_{t-1}H(a^*_{t-1})\tfrac{1-\theta\phi}{\theta(1-\phi)} \\
& \tfrac{1}{1+r^b_t} = \tfrac{1-\phi}{1+r^d_t} + \tfrac{\phi}{1+r^f_t} \\
& R_t(a^*_{t-1}) = \tfrac{(1-\theta)x_t(a^*_{t-1})}{1-\frac{\theta x_t(a^*_{t-1})}{1+r^b_t}} = 1 + r^d_t \Leftrightarrow p^N_t = \tfrac{(\tilde{w}^N_t)^{1-\alpha_N}}{a^*_{t-1}\alpha_N^{\alpha_N}(1-\alpha_N)^{(1-\alpha_N)}} (\tfrac{1}{\frac{1-\theta\phi}{1+r^d_t} + \frac{\theta}{1+r^f_t}})^{\alpha_N}
\end{aligned}
\tag{53}
$$