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Information Acquisition, Noise Trading and Speculation in Double Auction Markets*

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Abstract

This paper analyzes how the size of a double auction market affects information acquisition and trading behavior as well as allocative and informational efficiency. As the main result, this paper shows that with endogenous information if the number of traders and the units a trader is allowed to trade are sufficiently large an efficient equilibrium allocation fails to exist. This inefficiency result is driven by a novel strategic effect that is neither present in an auction where the role of buyers and sellers are assigned exogenously nor in a double auction with exogenous private information. This paper formalizes the notion of how a hedging market becomes a speculative market when the market becomes centralized and large and discusses the link between optimal security design and optimal market design.

Journal of Economic Literature Classification Numbers: G14, D82, D83

Key words: double auction, endogenous lemons problem, information acquisition, noise trading, speculation

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1. Introduction

The introduction of a financial innovation or a new hedging instrument is socially desirable if it can improve risk sharing and facilitate efficient trades in the economy. For example, the trading of commodity futures and options allows producers to hedge the risk of input and output price fluctuations. Financial investors can buy credit default swaps (CDS) to insure against the default risk of a bond or a mortgage-backed security (MBS). The trading of money market mutual funds, asset-backed commercial paper or Agency MBS expands the class of money-like securities that firms and financial institutions can use to manage their short-term cash and liabilities.

A new financial instrument is typically a client-tailored product which is traded in decentralized over-the-counter markets. But as the demand increases these instruments tend to become standardized which may reinforce more demand. The trading of commodity futures are now conducted on organized futures exchanges. When the demand for CDS on MBS increased, CDS were standardized and also written on MBS indices. In a large market agents can scale up the amount of trade because there are potentially more counterparties to trade with.

This paper analyzes a model of how the nature of trading and market outcomes may change when the size of the market increases from bilateral trade to large centralized trades. Does a financial innovation which was introduced as a hedging instrument become a speculative instrument when the market becomes large and anonymous? This paper provides a formal notion of hedging and speculation and shows how centralizing and enlarging the market affects information acquisition and trading behavior as well as allocative and informational efficiency.

A canonical mode of bilateral trade is simultaneous offer bargaining which is a special case of a double auction with two agents. On the other hand a double auction with a large number of agents is a prototype of a centralized market. This paper connects bilateral bargaining with centralized trading and analyzes information acquisition in small and large double auction markets. As the main result, this paper shows that if the number of traders and the units a trader is allowed to trade are sufficiently large then an efficient equilibrium allocation fails to exist. Equilibria with positive volume of trade are in mixed strategies and have the following properties. Traders endogenously become informed speculators, uninformed defensive traders, and noise traders in a sense precisely defined in the paper. Because of defensive trading the equilibrium allocation is inefficient. Because of endogenous noise trading the equilibrium price is not fully revealing of the traders' aggregate information.

Formally, this paper considers a market with $2N$ traders and an asset with uncertain (common) value v which is either v_L or v_H with equal probability. It is common knowledge that the asset is worth $v+\Delta$ to N traders (natural buyers), and v to the other N traders (natural sellers). $\Delta>0$ is a constant and captures the difference in private valuations and a shortcut for trading gains due hedging needs.[1] Trade is conducted in a double auction market where all traders submit buy or sell orders simultaneously. Before the trading stage a trader can acquire costly information about the payoff v of the asset. Each trader maximizes his expected payoff.

The benchmark case is a setting with two traders. In the N=1 pair case a natural buyer and seller play a simultaneous offer bargaining game. This can be interpreted as a model of over-the-counter (OTC) trading.[2] In this small double auction if the information cost is high, OTC trading yields an efficient allocation. If the information cost is low, an efficient allocation also exists but both agents acquire (socially wasteful) information. If the information cost is intermediate then no pure strategy equilibrium with trade exists although the traders maintain symmetric information in equilibrium and the gains from trade are common knowledge. The mere concern about information acquisition by the other trader and an endogenous lemons problem can render efficient trade unattractive.[3] In a mixed strategy equilibrium depending on the outcome of the randomization a trader becomes a defensive trader, a noise trader or an informed speculator.

The paper then analyzes double auctions with many natural buyers and sellers and derives two main results. (1) In a large double auction the potential lemons problem an uninformed trader faces exists not only for low and intermediate information costs but also for large information costs. As the number of traders increases, the potential speculative profit of an informed trader increases because there are potentially more uninformed traders to exploit. If the information cost is large, the trading equilibria in a large double auction are in mixed strategies in which a set of traders becomes informed speculators, a set of traders stays uninformed and behaves like noise traders and a set of traders stays uninformed and does not

[1] A natural buyer is a trader who wants to buy the asset to store his excess cash and thus has a higher private valuation for the asset than a trader who wants to raise cash and sell this asset.

[2] Bargaining is a standard feature in many decentralized debt markets, such as those for asset-backed securities, collateralized debt obligations, syndicated loans, corporate, municipal and government bonds. See Duffie et al. (2005) for search based model of OTC trading.

[3] This no trade result is different from Myerson and Satterthwaite (1983) because the gains from trade are common knowledge in the present model. It is also different from Akerlof (1970) and Gresik (1991) since there is no asymmetric information about the common valuation in equilibrium.

trade so that the equilibrium allocation is inefficient. Because of endogenous noise trading the price is not fully revealing.

(2) In a bilateral double auction if the information cost is low, there exist pure strategy equilibria where trade occurs with probability one. In contrast, in a double auction with more than two traders, there exists no equilibrium in pure strategies. The reason is free-riding of uninformed traders. If trade occurs in both states and the price is fully revealing, the best response is to not acquire information and submit non-defensive orders. But if the fraction of noise traders is large, informed traders can move prices and make speculative profits.

This paper formalizes the notion of how a hedging market becomes a speculative market when the size of the market increases and shows that equilibrium allocations are not efficient because a fraction of uninformed traders stay away from trading. For a range of information costs, these large market equilibria are also not asymptotically and approximately efficient. In addition, this paper provides a strategic foundation for the Grossman and Stiglitz (1980) impossibility result of informationally efficient markets and a strategic foundation for the noise trading assumption that is commonly employed in the market microstructure literature in finance. In equilibrium a fraction of rational liquidity traders submit orders that are prone to adverse selection. This behavior can be interpreted as noise trading in a market microstructure model.

From a conceptual point of view, this paper shows that the inefficiency results are driven by a novel strategic effect in double auction markets with information acquisition that is neither present in a standard auction where the role of buyers and sellers are assigned exogenously nor in a double auction with exogenous private information as in Reny and Perry (2006) who show that a large double auction is both allocative and informationally efficient.

The next section relates this paper to the literature. Section 3 introduces the model. Section 4 analyzes information acquisition in a small double auction. Section 5 analyses information acquisition in large double auctions. Section 6 concludes with a discussion of some market microstructure implications and how security design is linked to the organization of markets which is yet an unexplored topic.

2. Relation to the Literature

This paper is most closely related to Reny and Perry (2006) who analyze a large limit double auction market where traders have exogenous private information and show that such a market is allocative and informationally efficient. The present paper assumes that there are gains from trade and ex ante all traders have identical information about the value of the asset and analyzes the implications of information acquisition and endogenous private information for allocative and informational efficiency in small and large double auction markets. This paper shows that if the number of traders and the units a trader is allowed to trade are sufficiently large, then a double auction market is neither allocative nor informationally efficient.

Reny and Perry (2006) employ a more general information and valuation structure. Yet the key economic reason why their result does not hold in this setting is the following. In the present model private information is endogenous and an informed trader has to cover his information cost. If the price is fully revealing, then some traders have profitable deviations. (i) An informed trader chooses not to acquire costly information since there is no speculative profit to make. (ii) Since there is no lemons problem, no uninformed trader submits defensive offers. Consequently, some of these traders deviate to noise traders. On the other hand, if there are too many noise traders and very few informed, then an informed trader can move prices and make speculative profits. In a mixed strategy equilibrium the price is not fully revealing. Because of the potential lemons problem some traders behave defensively and the allocation is not efficient.

A second and more subtle reason why their result in the double auction stage does not apply to this setting is that they assume that all traders are endowed with private signals of the same precision while in the present model the traders who do not acquire information have information with strictly lower precisions and there is a fraction of such traders. The need to cover information costs and the existence of a fraction of uninformed traders give rise to trading behavior that is not present in Reny and Perry (2006). As a best response (in the auction stage) the uninformed traders randomize over placing defensive and noise-type orders while informed traders always speculate.

This paper is also related to the auction literature. Milgrom (1981), Matthews (1984), Hausch and Li (1993), Persico (2000), Jackson (2003) and Bergemann and Pesendorfer (2007) analyze information acquisition in auctions where only the buyers' side considers information acquisition. The seller is non-strategic and just wants to sell the asset. In contrast, the present model assumes that all traders behave strategically and can acquire information. Because of the endogenous lemons problem a strategic buyer (seller) may not want to buy (sell) and forgoes the trading gain.[4]

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[4] The two-sided strategic behavior (even with exogenous private information) gives rise to some technical difficulties since the random variables are not affiliated. Jackson and Swinkels (2005) prove the existence of a mixed strategy equilibrium with positive volume of trade in double auctions with exogenous private information.

The present paper derives a novel strategic effect that is not present in an auction where the role of buyers and sellers are assigned exogenously. This paper shows that the best response of an informed natural buyer might be to submit a sell order while the best response of an informed natural seller might be to submit a buy order. In a standard auction bidders cannot place a sell order and the auctioneer cannot buy. Furthermore, this paper shows that an informed natural seller cannot exploit uninformed buyers but he can potentially exploit uninformed sellers. Therefore, it is the ability to submit both orders that can cause the equilibrium allocation to be inefficient and the price to be not fully revealing. This phenomenon is not present in a standard auction (e.g. as in Swinkels and Pesendorfer (2000)) where the role of buyers and sellers are assigned exogenously.

Complementary to the auction literature in Economics is the market microstructure literature in Finance. The present paper provides a strategic foundation for the behavior assumption in the noisy rational expectations equilibrium (REE) framework which constitutes a workhorse model in financial economics. This literature typically assumes three exogenous types of agents: (i) informed traders (speculators), (ii) uninformed traders without real trading motives (market makers), and (iii) uninformed traders with real trading motives or different private valuations of the asset (liquidity traders). An assumption in REE models with exogenous noise is that the trading behavior of liquidity traders is inelastic. These agents do not care about adverse selection and they just want to trade some exogenous amount of the asset irrespective of prices. See e.g. Grossman and Stiglitz (1980), Hellwig (1980), Kyle (1985, 1989), and Glosten and Milgrom (1985).

Verrecchia (1982), Jackson (1991), Barlevy and Veronesi (2000), Mendelson and Tunca (2004), Veldkamp (2006), and Muendler (2007) analyze information acquisition in a financial market setting and assume that a subset of traders (liquidity traders) is either not maximizing or not allowed to acquire information. In contrast, the present paper assumes that all traders can acquire information and are maximizing their expected utility and provides a strategic foundation for the noise trading assumption in the market microstructure literature.[5]

A second important difference between this paper and many market microstructure models concerns the trading environment. In the present model there is no market maker who observes the order flow and determines the price.[6] The inefficiency results of the present

[5] Trueman (1988) and Dow and Gorton (1997) provide a theory of noise trading based on agency considerations in a delegated portfolio management setting. In the present model, all agents trade on their own behalf.

[6] In noisy REE models market makers are typically needed so as to prevent informed traders from making infinite profits.

paper raise the question of whether market makers who have some private information by observing order flows but are forbidden to speculate, are needed to facilitate efficient trades in centralized markets. Section 5 discusses this question and further market microstructure implications.

3. The Model

There are *2N* risk neutral traders in a market for an asset with uncertain (common) value v which is either v_L or v_H with equal probability. The asset is worth $v+\Delta$ to the first N traders. These traders have a high *private* valuation of the asset and are "natural" buyers (B-agents). The asset is worth v to traders $N+1$ to $2N$ ("natural" sellers or S-agents). Δ is a constant and captures the gains from trade when a pair of such agents trade with each other. To focus on common values uncertainties, this paper assumes that Δ is common knowledge.[7]

If a B-agent has bought one unit, then he also has the (marginal) valuation of v. Thus the net payoff of a B-agent when buying the first unit is $v+\Delta-p$ and $v-p$ for further units where p denotes the transaction price. The net payoff of selling any unit is $p-(v+\Delta)$. The net payoff of an S-agent for selling (or buying) any unit is $p-v$ (or $v-p$).[8] The efficient allocation is for each natural buyer to buy one unit and each natural seller to sell one unit. The total gains from trade are $N\Delta$.

Assumption A

$0<\Delta<\frac{1}{8}(v_H-v_L)$.[9]

A trader has two types of actions, an information acquisition decision and a trading decision. (i) First, a trader decides whether to obtain a perfect signal about the true value of the asset by incurring the cost $c>0$. Information acquisition is denoted with $n_i \in \{0,1\}$ and is *not*

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[7] The allocative consequences of private information about private values Δ have been analyzed in Chatterjee and Samuelson (1983) and Myerson and Satterthwaite (1983).

[8] The specific valuation of $v+\Delta$ can be interpreted as a shortcut for the marginal valuation of a risk averse trader with a low endowment of the risky asset. Having hedged their positions, traders have the same marginal valuation. See Footnote 1 for further interpretations of this utility function. If rational agents have the same private or marginal valuation of the asset and there are no gains from trade and the No-speculative trade Theorem applies. See Milgrom and Stokey (1982).

[9] This assumption makes the analysis interesting. If Δ is large, agents are not concerned about lemons problems. Suppose v_L=1, v_H=2, and Δ=100. A B-trader is willing to buy at the price, say p=90.

observable by the other traders.[10] (ii) The trading decision consists of choosing *either* a limit bid price $b_i \in R_+$ to buy $u_i \in \{0,1,..,M\}$ units of the asset *or* a limit ask price $s_i \in R_+$ to sell $u_i \in \{0,1,..,M\}$ units.[11] This paper assumes that there is no wealth or short selling constraint. So any trader can be a buyer or seller. The term natural buyer refers to a trader who has a high private valuation of the asset and thus should buy one unit from the welfare perspective.

Formally, a pure strategy of trader i is denoted with $a_i = (n_i, d_i)$ where $d_i \in \{d_B, d_S\}$ and $d_B=(b_i,u_i)$ and $d_S=(s_i,u_i)$. If $n_i=1$, a trading strategy of an informed trader i specifies his order d_{iL} and d_{iH} when the true value of the asset is v_L and v_H, respectively. A mixed strategy is a probability distribution over pure strategies and denoted with σ_i.

The following examples illustrate this notation. (i) ($n_i=0$, $d_{iB}=(v_L,1)$) is a pure strategy where trader i does not acquire information and submits a bid price of v_L to buy one unit. (ii) ($n_i=1$, $d_{iSL}=(\mathrm{E}[v],M)$, $d_{iBH}=(v_H,1)$) is a pure strategy where trader i acquires information, submits an ask price of E[v] to sell M units if $v=v_L$ and a bid price of v_H to buy one unit if $v=v_H$. (iii) A mixed strategy is e.g. a randomization that puts probability 0.4 on the pure strategy (i), probability 0.6 on the pure strategy (ii), and zero probability on any other pure strategy.

The exact allocation and pricing rule are specified in the subsequent sections. The solution concept is Bayesian Nash equilibrium (BNE). A BNE in pure strategies in this game is a profile $\{a_i^*\}_{i=1}^{2N}$, such that $EU_i(a_i^*, a_{-i}^*) \geq EU_i(a_i, a_{-i}^*)$, for all potential pure strategies a_i of trader i where i=1,..,2N. A BNE in mixed strategies is a profile $\{\sigma_i^*\}_{i=1}^{2N}$ of probability distributions over pure strategies, such that $EU^i(\sigma_i^*, \sigma_{-i}^*) \geq EU^i(\sigma_i, \sigma_{-i}^*)$, for all potential probability distributions σ_i of trader i where i=1,..,2N. Equilibrium always refers to a BNE.

This paper discusses *three* notions of efficiencies. (1) *Allocative efficiency*: An allocation is efficient if the realized gains G from trade is $G=N\Delta$. (2) *Social Efficiency (Welfare)*: The outcome is socially efficient or welfare W maximizing if $G=N\Delta$ *and* no risk neutral agent acquires (socially useless) information, i.e. $W=G-\Sigma c=N\Delta$. (3) *Informational efficiency*: The price is informationally efficient or fully revealing if it reflects the joint information of the traders. The focus of the paper is on allocative (i.e. G notion of) efficiency.

[10] An earlier version of the paper shows that if information acquisitions are observable prior to the trading stage, then there always exist efficient trading equilibria in which no trader acquires information. But this assumption is not realistic, especially in a large market.

[11] The restriction on the bidding strategy where a trader is only allowed to *either* submit a buy *or* a sell order simplifies the notation but is not crucial for the qualitative results.

Observation 1

A (no trade) equilibrium always exists. A set of such equilibria is given by the following strategies: No trader acquires information and all B-agents only choose to buy at very low bid prices (e.g. $b \leq v_L$), while all S-agents only choose to sell at very high ask prices (e.g. $s \geq v_H + \Delta$).

4. Information Acquisition in a Small Double Auction

This section analyzes the two trader (N=1 pair) case which can be interpreted as simultaneous offer bargaining in an OTC market. The efficient allocation is for the (natural) buyer to buy one unit from the (natural) seller. It is without loss of generality to focus on the case where agents submit to trade one unit of the asset. The allocation and pricing rule in this small double auction is as follows. Trade occurs if the bid price of the buyer is higher than the ask price of the seller, i.e. $b \geq s$. The transaction price is $p=(b+s)/2$ and the net payoffs are $U^B = v + \Delta - p$ and $U^S = p - v$. Otherwise no trade occurs and the net payoffs are zero.[12] If information is acquired, the information cost c is subtracted from the payoff.

This section derives two benchmark results. Proposition 1 characterizes the set of payoff maximizing equilibria in pure strategies and shows when efficient equilibrium outcomes are attainable in a decentralized market. Proposition 2 gives a characterization of the properties of mixed strategy equilibria which play an important role in the characterization of welfare and equilibria in a large double auction.

Proposition 1 (*Pure Strategy BNE*)

(a) If $c \geq \frac{1}{4}(v_H - v_L - \Delta)$, there exists a continuum of socially efficient BNE ($W=G=\Delta$).

(b) If $\frac{1}{2}\Delta < c < \frac{1}{4}(v_H - v_L - \Delta)$, there exist no pure strategy BNE with trade (G=0). In any pure strategy BNE no agent acquires information.

(c) If $c \leq \frac{1}{2}\Delta$, there exists a continuum of allocative efficient BNE (G=Δ). In any such BNE both traders acquire information and $W=\Delta-2c$. The price is fully revealing (to a third party).

Proof: See Appendix.

[12] If the buyer does not buy he can consume the unspent amount p. If the seller does not sell, he has a utility v from owning the asset. There is no utility change, i.e. the net payoff is zero.

Proposition 1 (c) shows that if the information cost is low, there exist efficient equilibrium allocations but both traders acquire socially wasteful information.[13] Part (b) shows that if the information cost is intermediate, there exists no pure strategy BNE with trade at all.

The intuition for the no trade result is as follows. (i) If both agents acquire information, then total information costs exceed the trading surplus. (ii) If one agent acquires information the best response of the other agent is to account for the lemons problem and submits a defensive offer price. Trade occurs with probability 0.5 and the informed agent cannot cover his information cost since $c>\frac{1}{2}\Delta$. (iii) Suppose both agents remain uninformed and trade at the price p=E[v]. The buyers expected payoff is Δ. Yet he has an incentive to acquire information and only buys if $v=v_H$. This strategy yields $EU^B=\frac{1}{2}[(v_H+\Delta)-E[v]]-c=\frac{1}{4}(v_H-v_L)+\frac{1}{2}\Delta-c>\Delta$ since $c<\frac{1}{4}(v_H-v_L-\Delta)$. In other words, if the information cost intermediate, the mere concern about information acquisition by the other trader renders efficient trade unattractive. The uninformed buyer submits a low bid price and uninformed seller submits a high ask price so as to account for a potential lemons problem.

Proposition 1 shows that welfare W is not monotonic in information cost. This result is similar to Dang (2008) who analyzes information acquisition in take-it-or-leave-it offer bargaining. But the equilibrium behavior in Proposition 1 is different from Dang (2008) who shows that if information cost is low, there is no pure strategy equilibrium with trade. If the information cost is intermediate, there is no pure *and* mixed strategy equilibrium with trade. In the present paper a mixed strategy BNE with trade exists.

Definition

(i) A trader plays a *defensive strategy*, if he chooses (0,b) with $b\leq v_L+\Delta$ or (0,s) with $s\geq v_h$. Such a trader is called a *defensive trader*.

(ii) A trader plays a *noise-type strategy*, if he chooses (0,b) or (0,s) with $b,s\in$[E[v],E[v]+Δ]. Such a trader is called a *noise trader*.

(iii) A trader plays a *speculative strategy*, if he chooses $(1,b_L,b_H)$ with $b_L\leq v_L+\Delta$ and $b_H\in$[E[v],E[v]+Δ] or $(1,s_L,s_H)$ with $s_L\in$[E[v],E[v]+Δ] and $s_H\geq v_H$. Such a trader is called an *informed speculator*.

[13] If $c\leq 0.5\Delta$, there also exist (asymmetric) pure strategy BNE in which trade occurs with probability 0.5. For example, the buyer chooses n_B=1 and b=(v_L,v_H) and the seller chooses n_S=0 and s=v_H. But this BNE is Pareto dominated by equilibria in Proposition 1(c). Note, there exists no BNE with trade if both agents are uninformed.

In other words, a trader is called a *defensive trader* if he is uninformed and his offer accounts for the potential lemons problem. A trader is called a *noise trader* if he is uninformed and proposes a price around the expected value of the asset. A trader is called an *informed speculator* if he only buys (sells) at a price around E[v] when the true state is v_H (v_L).

Proposition 2 (*Mixed Strategy BNE*)

Suppose $\frac{1}{2}\Delta<c<\frac{1}{4}(v_H-v_L-\Delta)$.

(a) In any mixed strategy BNE with positive probability of trade (i.e. $E[G]>0$), the traders put strictly positive probability on the defensive, noise-type and speculative strategies.

(b) The outcome in any such BNE has the following properties. (i) Trade does not occur if both traders are informed or at least one trader is a defensive trader. (ii) The price is not fully revealing. (iii) Both traders have zero net payoff (i.e. $E[W]=0$).

Proof: See Appendix.

Proposition 2 shows that depending on the outcome of the equilibrium randomization, a trader may become a noise trader, a defensive trader, or an informed speculator and any equilibrium with positive probability of trade has this property. In particular, the expected net payoff of all traders is zero which will play a crucial role for the welfare implications of a large market. The following example highlights the intuition behind Proposition 2.

Suppose the traders are only allowed to choose three offer prices $b,s\in\{l,m,h\}$ where $l=v_L+\frac{1}{2}\Delta$, $m=E[v]+\frac{1}{2}\Delta$, and $h=v_H+\frac{1}{2}\Delta$. Appendix shows that in the unique (non-degenerated) mixed strategy equilibrium the buyer randomizes over the strategies $(0,l)$, $(0,m)$ and $(1,(l,m))$. The seller randomizes over the strategies $(0,h)$, $(0,m)$ and $(1,(m,h))$. Trade only occurs in the following three events: (i) Both traders choose $(0,m)$. (ii) The seller chooses $(0,m)$ and the buyer chooses $(1,(l,m))$ *and* the true state is v_H. (iii) The buyer chooses $(0,m)$ and the seller chooses $(1,(m,h))$ *and* the true state is v_L. The probability of trade is $\frac{16c^2}{(v_H-v_L)^2-4\Delta^2}$, and if trade occurs the price is $p=m=E[v]+\frac{1}{2}\Delta$ and not fully revealing. The following remarks highlight the economic incentives of a buyer in a mixed strategy equilibrium.

(a) The “honest” strategy $(1,(l,h))$ is chosen with zero probability by both traders. In other words, an informed buyer always speculates and bids $b=m$ in state v_H. If he is honest and chooses $b=h$ in state v_H then trade also occurs in the event when the seller chooses $(0,h)$ or $(1,(m,h))$. He does not do it, because if he is randomizing between the speculative strategy

(1,(l,m)) and the "honest" strategy (1,(l,h)) and thus is indifferent between them, then both strategies with information acquisition is strictly dominated by the defensive strategy (0,l). In other words, if a trader acquires information, being honest is a strictly dominated strategy. Thus there is no trade if both traders are informed.[14]

(b) Although the minimum price the seller demands is $s=E[v]+\frac{1}{2}\Delta$, the buyer chooses (0,l) with positive probability, i.e. he does not trade with positive probability. In order to make the seller indifferent between acquiring and not acquiring information, equilibrium randomization requires an uninformed buyer to bids $v_L+\frac{1}{2}\Delta$ frequently enough so as to discourage too frequent information acquisition by the seller. Since (0,l) is played with positive probability and $U^B(0,l)=0$ and the buyer is indifferent between this and other strategies, his expected payoff is zero in any mixed equilibrium.

(c) In the mixed strategy equilibrium an uninformed trader proposes the offer price $E[v]+\frac{1}{2}\Delta$ with positive probability so that he may suffer adverse selection. Yet his equilibrium payoff is non-negative since he meets an uninformed trader with positive probability. In such a case he realizes the trading gain without suffering a speculative loss.

(d) There are three interrelated reasons why the equilibrium price is not fully revealing. (i) There is no trade between two informed traders. (ii) There is no trade if one trader plays a defensive strategy. (iii) Suppose the buyer does not acquire information and observes trade at $p=E[v]+\frac{1}{2}\Delta$. In this case he does not know whether the seller has chosen (0,m) or (1,(m,h)). Although his posterior belief for $v=v_L$ increases, it is strictly below one. Otherwise he would know for sure that he has made a bad deal and this cannot be an equilibrium outcome.

5. Information Acquisition in Large Double Auctions

This section analyzes the $2N$ trader case where $N>1$ and establishes the main results. Traders $i=1,..,N$ are natural buyers (B-agents) denoted with $\{B_1,..,B_N\}$. Trader $i=N+1,..,2N$ are natural sellers (S-agent) denoted with $\{S_{N+1},..S_{2N}\}$. Each trader can submit *either* one bid price b to buy up to M units *or* one ask price s to sell up to M units. The buy and sell orders of the traders are ranked according to the bid and ask prices, respectively. This generates an aggregate demand and supply schedule. The market price is set to equalize aggregate demand and supply. (i) If there are multiple-market clearing prices, the price is determined as $p=\frac{1}{2}(b'+s')$ where b' is the lowest bid price and s' the higher ask price such that $b'\geq s'$. (ii) If

[14] This is in contrast to Proposition 1(c) where trade only occurs with probability 1, if both traders are informed.

there is excess demand (supply) at the market clearing price, the orders with the highest bid prices (lowest ask price) are executed first. The remaining units are allocated with equal probability to the traders who propose the same offer price. These trading rules are adopted from Reny and Perry (2006, section 4.1).

Observation 2

If traders cannot acquire information ($c=\infty$), for any M and N there exist socially efficient equilibria ($W=G=N\Delta$). (i) If $M=1$, then an equilibrium price is $p\in[\mathrm{E}[v],\mathrm{E}[v]+\Delta]$. For example, an equilibrium strategy profile is where all natural buyers submit a bid price of $b=\mathrm{E}[v]+k$ to buy one unit and all natural sellers submit an ask price of $s=\mathrm{E}[v]+k$ to sell one unit where $k\in[0,\Delta]$. (ii) If $M>1$, then the equilibrium price is $p=\mathrm{E}[v]$ and unique. Note if $p>\mathrm{E}[v]$, then a seller wants to sell more units. There is underbidding until $p=\mathrm{E}[v]$.

Observation 3

If information cost $c=0$, for any M and N there exist BNE with $W=G=N\Delta$. (i) If $M=1$, then an equilibrium price is $p\in[v,v+\Delta]$. For example, one equilibrium strategy profile is where all traders acquire information and all B-agents and S-agents submit a price $b=s=v+k$ to trade one unit where $k\in[0,\Delta]$. (ii) If $M>1$, then the equilibrium price is $p=v$ and unique.

The rest of the section derives the main results of the paper. (i) For any finite information cost, if the number N of traders and the units M a trader can trade are sufficiently large so as the informed agent can potentially cover the information cost, then a BNE with $G=G^*=N\Delta$ fails to exist and G does not converges to G^* for any N. (ii) If the information cost is large and the market is sufficiently large, then any BNE with positive volume of trade is in mixed strategies and $W=0$.

A. Traders can trade *M*=1 units

This subsection analyses the case where information cost is positive and finite. In order to facilitate comparison with the small ($N=1$) double auction, this section assumes that a trader is only allow to trade $M=1$ unit but there are $N>1$ pairs of traders submitting orders to the double auction.

Lemma 1

Suppose N>1, M=1 and 0<c≤$\frac{1}{2}\Delta$. There exists no pure strategy BNE with $G=N\Delta$. Even if $N\rightarrow\infty$, G is strictly bounded away from $N\Delta$ in a pure strategy BNE.

Proof

Proposition 1 shows that for $c\leq\frac{1}{2}\Delta$, there exists no pure strategy BNE with trade at $p\in[E[v],E[v]+\Delta]$. So in a BNE with positive volume of trade some traders acquire information.

Claim 1: There exists no pure strategy BNE with $G=N\Delta$ in which more than one B-agent and more than one S-agent acquire information.

Proof Claim 1: Suppose traders B_1, B_2, S_{N+1} acquire information and $G=N\Delta$. Ex post individual rationality implies that the transaction or market clearing price $p\in[v, v+\Delta]$. Otherwise an informed B_1 (S_{N+1}) does not buy (sell). So the price is fully revealing. (Note, it is assumed that $v_L+\Delta<v_H$.) But given p is fully revealing, a best response of B_2 is not to acquire information and submit $b_2>v_H+\Delta$ (noise-type offer) and gets one unit without costly information acquisition. Provided a pure strategy BNE with $G=N\Delta$ exists, at most one pair of B-and S-agent acquires information; and all traders trade one unit.

Claim 2: There exists no pure strategy BNE with $G=N\Delta$ in which one B-agent and one S-agent acquire information.

Proof Claim 2: Consider the following strategy profile where all traders trade one unit and the informed traders cover their information costs. Traders B_1 and S_{N+1} acquire information and they submit $b_{1L}=s_{N+1,L}=v_L+\frac{1}{2}\Delta$ at v_L and $b_{1H}=s_{N+1,H}=v_H+\frac{1}{2}\Delta$ at v_H. To be able to trade one unit in both states, uninformed traders submit non-defensive orders, i.e. B_i submits $b_i\geq v_H+\frac{1}{2}\Delta$ and S_i submit $s_i\leq v_L+\frac{1}{2}\Delta$.

(i) Without loss of generality, consider the following aggregate demand and supply schedules in state v_L,:

$$B \qquad b_{1L}=v_L+\tfrac{1}{2}\Delta \;\leq b_2=v_H+\tfrac{1}{2}\Delta \;\leq\; b_3 \;\leq\; b_4 \;\leq\ldots\leq b_N$$

$$S \qquad s_{N+1,L}=v_L+\tfrac{1}{2}\Delta\geq\; s_{N+2}=v_L+\tfrac{1}{2}\Delta\geq s_{N+3}\geq s_{N+4}\geq\ldots\geq s_{2N}.$$

Given this offer profile, the market clearing price is $p=v_L+\frac{1}{2}\Delta$. Does a trader have an unilateral profitable deviation?[15]

[15] It is easy to see that uninformed trades have no profitable deviations.

(a) Informed S_{N+1} cannot push up the price. If he raises his ask price, the market clearing price is still $p=v_L+\frac{1}{2}\Delta$. (b) Informed B_1 is not able to reduce the market clearing price either. (c) But an informed B_1 is able to move the market clearing price up and then sell rather than buy one unit although he is a natural buyer. Given the (above) profile, a best response of informed B_1 is to place an ask price s_1 smaller than s_{2N} to sell one unit. There are N–1 units of buy orders and N+1 units of sell orders and the new profile is:

$$B' \qquad b_2= v_H+\tfrac{1}{2}\Delta\leq b_3\leq b_4\leq\ldots\leq b_N$$

$$S' \qquad s_{N+1,L}=v_L+\tfrac{1}{2}\Delta\geq\ s_{N+2}=v_L+\tfrac{1}{2}\Delta\geq s_{N+3}\geq s_{N+4}\geq\ldots\geq s_{2N}\geq s_{1L}$$

The market clearing price is $p=\frac{1}{2}(b_2+s_{N+1})=\frac{1}{2}(v_H+\frac{1}{2}\Delta+v_L+\frac{1}{2}\Delta)=\mathrm{E}[v]+\frac{1}{2}\Delta$. Informed B_1 sells one unit and his expected payoff is larger than $\frac{1}{2}\Delta-c$. But this strategy profile is not a BNE, since the expected payoffs of all uninformed B-agents are negative.

(ii) In state v_H, without loss of generality consider the aggregate demand and supply schedule:

$$B \qquad b_{1H}=v_H+\tfrac{1}{2}\Delta\ \leq b_2=v_H+\tfrac{1}{2}\Delta\quad\leq b_3\quad\leq b_4\ \leq\ldots\leq b_N$$

$$S \qquad s_{N+1,H}=v_H+\tfrac{1}{2}\Delta\geq s_{N+2}= v_L+\tfrac{1}{2}\Delta\ \geq\ s_{N+3}\geq s_{N+4}\geq\ldots\geq s_{2N}.$$

A best response of informed S_{N+1} is to choose not to sell but to place a bid price $b_{N+1,H}$ to buy one unit where $b_{N+1,H}> b_N$. There are N–1 units of sell orders and N+1 units of buy orders.

$$B' \qquad b_{1H}=v_H+\tfrac{1}{2}\Delta\leq b_2= v_H+\tfrac{1}{2}\Delta\leq b_3\leq b_4\leq\ldots\leq b_N<b_{N+1}$$

$$S' \qquad s_2=v_L+\tfrac{1}{2}\Delta\geq s_3\geq s_4\geq\ldots\geq s_N.$$

This offer profile yields the market clearing price $p=\frac{1}{2}(b_{1H}+s_2)=\frac{1}{2}(v_L+\frac{1}{2}\Delta+v_H+\frac{1}{2}\Delta)$ $=\mathrm{E}[v]+\frac{1}{2}\Delta$. Informed trader S_{N+1} buys one unit. This response yields a strictly higher expected payoff to him than $\frac{1}{2}\Delta-c$. But this strategy profile is not a BNE, since the expected payoffs of all uninformed S-agents are negative. These deviation strategies hold given $b_{1L}=s_{N+1,L}=v_L+k_L$ at v_L and $b_{1H}=s_{N+1,H}=v_H+k_H$ at v_H where $k_L,k_H\in[0,\Delta]$.

Consequently, there exists no BNE where traders on both sides of the markets acquire information. In a pure strategy BNE with trade only traders on one side of the market acquire information. Since there is a lemons problem, uninformed traders on the other side of the market submit defensive orders so that at most $G=\frac{1}{2}N\Delta$. **QED**

The proof of Lemma 1 highlights a novel strategic effect why a pure strategy efficient BNE in a large double auction does not exist if information is endogenous and the information cost is low. An efficient equilibrium requires that all B-agents buy and all S-agents sell. Suppose there is one informed S-agent and one informed B-agent and the market clearing price is $p=v$. What causes this efficient BNE fail to exist? One might think that uninformed B-agents may be concerned about being exploited by the informed S-agent. This is not the case since uninformed buyers are protected by the informed B-agent.

Interestingly, the set of agents an informed S-agent can exploit are uninformed sellers. In state v_H the uninformed S-agent (natural seller) becomes a buyer and is able to "manipulate" the market clearing price so that uninformed sellers receive a price lower than v_H and make a loss. Anticipating this potential lemons problem, uninformed S-agents submit a high ask price to sell so that they only trade in state v_H which makes the allocation inefficient. This observation shows that a standard auction where the informed seller cannot submit an order to buy the object does not have this feature of inefficiencies.

Lemma 2

Suppose $N>1$, $M=1$ and $0<c\leq\frac{1}{2}\Delta$. The welfare maximizing pure strategy BNE yields $G=\frac{1}{2}N\Delta$ and $W=\frac{1}{2}N(\Delta-2c)$.

Proof

There exists a pure strategy BNE with $G=0.5N\Delta$. In such a BNE *all* B-agents acquire information and *no* S-agent acquires information (or vice versa). The following is a BNE. *All* B-agents acquire information and submit an ask price $s_{iL}=v_H$ to sell one unit in state v_L and a bid price $b_{iH}=v_H$ to buy one unit in state v_H . *No* S-agent acquires information and they all submit an ask price $s_j=v_H$ to sell one unit. There is no trade in state v_L. It is easy to see that no trader has a profitable deviation. In particular, no B-agent has an incentive to stay uninformed and bid $b_i=v_H$. In state v_L he buys at the price v_H. Since all B-agents acquire costly information and N units are traded in state v_H, $G=\frac{1}{2}N\Delta$ and $W=\frac{1}{2}N\Delta-Nc$.[16] Analogously it is a BNE if all S-agents acquire information.[17] **QED**

Lemma 2 shows that if information cost is low, in the most allocative efficient pure strategy BNE all N traders on one side of the market and no trader on the other side of the market

[16] Any market clearing price $p\in[v_H, v_H+\Delta-c]$ can be sustained as a BNE.

[17] The least efficient equibria of this type is where one informed B-agent trade with one uninformed S-agent in state v_H. All other traders do not acquire information and do not trade.

acquire information and this yields $G=0.5N\Delta$. So the welfare is $W=0.5N\Delta-Nc=0.5N(\Delta-2c)$ and strictly smaller than the welfare that is obtained when trade is conducted in N separate bilateral small double auctions where all traders acquire information and $W=N(\Delta-2c)$.

There is a free-riding problem in a large double auction. Proposition 1(c) shows that if the information cost is low, there exists an equilibrium where trade occurs with probability one and both traders acquire information and the price is fully revealing. If there are $N>1$ pair of traders and the price is fully revealing, in both states, i.e. $p\in[v,v+\Delta]$, the best response of some traders is not to acquire information. On the other hand if the number of uninformed traders is large, an informed trader can move prices and make speculative profits while uninformed traders suffer a loss. Consequently, there exists no BNE where there is trade in both states and price is fully revealing. Thus there exists no pure strategy BNE with $G=N\Delta$ even for N arbitrary large.[18]

Lemma 3

Suppose $N>1$, $M=1$ and $\frac{1}{2}\Delta<c<\frac{1}{4}(v_H-v_L-\Delta)$. (i) Any BNE with positive volume of trade is in mixed strategies where $E[G]<N\Delta$ and $W=0$.

Proof

The proof is very similar to the proof of Proposition 1(b). Since $2c>\Delta$, no pair of informed natural buyer and informed natural seller can jointly cover their information costs. If only agents on one-side of the market are informed, trade occurs with probability 0.5 and informed agents have a negative payoff. If no trader is informed there is no trade because no uninformed trader is willing to trade at any price $p\in[E[v],E[v]+\Delta]$ since information cost is smaller than the speculative profit (i.e. $c<\frac{1}{4}(v_H-v_L-\Delta)$). Thus if the information cost is in some intermediate range, there is no pure strategy equilibrium with positive volume of trade for any N (arbitrary large). As in the small double auction, a strictly positive fraction of traders does not acquire information and does not trade.

B. Traders can trade $M>1$ units

This subsection assumes that traders can trade $M>1$ units and derives the main result of the paper.

[18] Though interesting and insightful, the analysis of a BNE in mixed strategies is very complicated. The main result of the paper focuses on information costs $c>\frac{1}{2}\Delta$.

Lemma 4

Suppose $M,N>1$ and $0<c\leq\frac{1}{2}\Delta$. The maximum welfare in a pure strategy BNE is $W=\frac{1}{2}N(\Delta-2c)$.

Proof

This follows from Lemmas 1 and 2. The strategy profile where no S-agent acquires information but all B-agents acquire information, is a BNE here even if agents can trade $M>1$ units. In this BNE trade occurs only in state v_H and the market clearing price is $p=v_H$ and unique. Since the (marginal) valuation of buying a second unit is v_H, an informed H-trader has no incentive to buy more than one unit so he has no profitable deviations.[19]

Lemma 5

Suppose $M,N>1$. Define $Q=\min[M,N]$ and $c^{\mathrm{L}}=\frac{1}{4}(2Q-1)(v_H-v_L)$. If $c<c^{\mathrm{L}}$, then there exists no pure strategy BNE with trade in which no agent acquires information acquisition.

Proof

The proof is based on three arguments. (a) If a pure strategy BNE exists where trade occurs and no agent acquires information, then $G=N\Delta$. (b) If a pure strategy BNE exists where trade occurs and no agent acquires information acquisition, then trade is executed at the price $p=E[v]$. (c) No pure strategy trading BNE without information acquisition exists where trade is executed at the price $p=E[v]$.

The following arguments prove claim (a). Suppose no trader acquires information, and the offer price profiles $B=(b_1,\ldots,b_N)$ and $S=(s_{N+1},\ldots,s_{2N})$ yield a market clearing price, $p\in(E[v], E[v]+\Delta)$. Suppose $b_i<p$ and B_i does not get to buy the asset. Given (B,S), B_i can do better by choosing $b_i\geq p$ and gets one unit with positive probability and $EU>0$. Any B-agent or S-agent who does not get to buy or sell one unit of the asset at the resulting price, has not played a best response. At $p=E[v]$, an "unsatisfied" B-agent will deviate. At $p=E[v]+\Delta$, an "unsatisfied" S-agent will deviate. This reasoning implies that if a trading equilibrium without information acquisition exists, then all traders are "satisfied", i.e. N units are traded.

[19] But in contrast to Lemma 1, it is not a BNE anymore if all S-agents acquire information. In such a case trade only occurs in state v_L and the market clearing price is at least $p=v_L+c$. When traders can trade M>1 unit, informed S-agent has an incentive to sell more than one unit. There is an incentive to undercut other ask prices. So there is no pure strategy BNE where S-agents acquire information.

Therefore, a candidate offer profile (B,S) for being part of a pure strategy BNE must have $b_i \geq p$ and $s_j \leq p$ for $i=1,..,N$ and $j=1+N,..,2N$, where p is the resulting market price given (B,S).

The proof of claim (b) is as follows. Suppose each trader trades one unit and the bid ask profile (B,S) gives rise to the price $p>E[v]$. An S-agent who sells one unit has not played a best response. There is incentive to sell more and underbid the other sellers. Consequently, only if $p=E[v]$, then all uninformed trader who trades one unit has no profitable deviation.

The proof of claim (c) is similar to the proof of Proposition 1. Suppose that no trader acquires information and the bid ask profile (B,S) yields the market price $p=E[v]$ and all traders trade one unit each. Then there exist unilateral profitable deviations. For example, natural seller S_i acquires information. In state v_H, he chooses $b_i=E[v]+\gamma_b$ to buy $(Q-1)$ units $Q=\min[M,N]$ and γ_b is chosen such that b_i is larger than the Q-*th* highest bid prices given $B=(b_1,..,b_N)$.[20] (Note, his offer does not affect the market clearing price.) S_i gets to buy Q units. His payoff in this state is $(Q-1)(v_H-p)-c=\frac{1}{2}(Q-1)(v_H-v_L)-c$. In state v_L, he chooses $s_i=E[v]-\gamma_s$ to sell (short) Q units where γ_s is chosen such that s_i is smaller than the ask prices of the $(Q-1)$ low valuation traders. His payoff in this state is $\frac{1}{2}Q(v_H-v_L)-c$.

Therefore, the expected payoff of S_i with information acquisition is $EU_i=\frac{1}{4}(2Q-1)(v_H-v_L)-c$. Consequently, if (B,S) gives rise to $p=E[v]$ and $c<\frac{1}{4}(2Q-1)(v_H-v_L)$ a natural seller acquires information and speculates. So there exists no pure strategy trading equilibrium without information acquisition.[21] **QED**

In a small double auction, if $c \geq \frac{1}{4}(v_H-v_L-\Delta)$, there exists a BNE with trade in which no trader acquire information. Lemma 5 states that even for large information cost, no such efficient BNE exists if the number N of traders and the units M they can trade are sufficiently large. The reason is that information acquisition is worthwhile even if the cost is large since there are potentially more uninformed traders to exploit. This speculative threat "destroys" an equilibrium with an efficient allocation.

[20] The maximum unit a trader is allowed to trade is M. If $N<M$, a trader can trade at most with N traders. In the proposed strategy profile, the maximum units he can effectively trade is $Q=\min[M,N]$.

[21] Analogously for a B-trader, if $c<\frac{1}{4}(2Q-1)(v_H-v_L-\Delta)$, then a B-trader speculates.

Proposition 3

Suppose $c \neq \frac{1}{2} N\Delta$. For any cost $c > \frac{1}{2}\Delta$, there exists an integer Q^*, such that if $M,N \geq Q^*$, then any BNE with positive volume of trade is in mixed strategies and any symmetric BNE has W=0.

<u>Proof:</u> See Appendix.

The following example highlights the intuition for the non-existence of a pure strategy BNE with positive volume of trade in a large double auction if the information cost is large. Suppose $(v_H - v_L)=4$. Then $c^L = \frac{1}{4}(2Q-1)(v_H - v_L) = 2Q-1$ in Lemma 5 and so $Q^* = \frac{1}{2}c+1$. For any (large) information cost c, if $M,N > \frac{1}{2}c+1$, then there is no pure strategy BNE with $G=N\Delta$.

Suppose k S-agents and k B-agents submit $E[v]$ to sell and buy one unit, respectively, where k is such that $k \leq \frac{1}{2}c+1 < k+1$. Suppose all other S-agents submit a high ask price and all other B-agents submit a low bid price. Given this strategy profile, no trader has an incentive to acquire information. Note, in order to cover the information an informed trader must "exploit" at least *k+1* uninformed trader but there is only k of them.

But given this profile, a best response of a B-agent who does not trade is to submit a sufficiently high bid price so as to obtain one unit of the asset. But given there are k+1 B-agent willing to buy at the price $E[v]$, a S-agent will acquire information. But given a S-agent is informed, no uninformed trader will submit $E[v]$ as a bid or ask price. So there is no pure strategy equilibrium with positive volume of trade.

In particular, if c is large (e.g. $c > N\Delta$) then it is easy to see that the price is not fully revealing. An informed trader cannot cover his cost even if he sells N units to the all B-agents at the price is $v+\Delta$. Since the price in any pure strategy equilibrium is fully revealing, there exists no pure strategy equilibrium with positive volume of trade. Since the price is not fully revealing a strictly positive fraction of uninformed traders submits defensive orders and does not trade.

Furthermore, a trader who submits a defensive order and does not trade has zero net payoffs. In a symmetric BNE, all traders chose the same randomization over pure strategies, i.e. all traders play the defensive strategy with positive probability. In order to be indifferent between behaving defensively and any other strategy, including a strategy of information acquisition and speculation, the expected payoff of all pure strategies must be the same and must yield zero expected net payoffs. Therefore, in any symmetric mixed strategy equilibrium the net payoff of all traders is zero. This result holds even if N converges to infinity.

Proposition 3 derives the inefficiency result for information costs that are not too small. It would be interesting to study approximate efficiency in mixed strategy BNE when information costs converges to zero and the number of traders is large. Such an analysis is technically demanding and left for future research.

Another interesting extension is to analyze the exogenous presence of market makers in this model and identify the set of conditions under which such players would improve efficiency. Stocks are traded in centralized markets with many designated market makers. For example, 59 market makers were supporting the stock trading of Apple Inc. on NASDAQ in April 1994 (see Christie and Schultz (1994)). Market makers observe order flows and attempt to earn profits from exclusively facilitating and bridging trade between buyers and sellers but they are not allowed to speculate.[22]

6. Conclusion

This paper analyzes how the size of a double auction market affects information acquisition and trading behavior of agents as well as allocative and informational efficiency. As the main result, this paper shows that with endogenous information an efficient equilibrium allocation fails to exist if the number of traders and the units a trader is allowed to trade are sufficiently large. This paper formalizes the notion of how a hedging market can become a speculative market as the market becomes centralized and large. This inefficiency result is driven by a novel strategic effect that is neither present in a standard auction where the role of buyers and sellers are assigned exogenously nor in a double auction with exogenous private information.

A puzzling question is why stocks are traded in centralized markets but debt instruments such as corporate bonds, commercial papers, syndicated loans, asset-backed securities, sales and repurchase agreements and other money market instruments are traded in over-the-counter (OTC) markets where a buyer and seller negotiate about prices and quantities in a non-anonymous fashion. There is a very large theoretical and empirical

[22] Madhavan and Panchapagesan (2000) analyze the role of market makers for price discovery in the double auction type overnight market on the NYSE, and state that "*there is strong evidence that the NYSE's designated dealer (specialist) sets a more efficient price than the price that would prevail in a pure call market using only public orders*." (p.656)

literature on stock trading but a relatively small literature on the trading of debt instruments although the latter class of assets is much larger in terms of issuance and trading volumes. [23]

Dang, Gorton and Holmstrom (2012) distinguish equity and debt instruments in terms of their so-called information sensitivity and show that when agents trade debt they have the least incentive to acquire costly information about the payoff of the security while equity is very information sensitive. Empirically, institutional stock investors produce much more information than institutional bond investors who mainly rely on rating information provider by credit rating agencies. An interesting question is why low information sensitive securities tend to be traded in decentralized markets but high information sensitive securities are traded in centralized markets.

Trading in decentralized short term debt (funding) markets works completely differently then stock trading in centralized markets. Dang, Gorton and Holmstrom (2012) argue that the share and common understanding of asset value and where no agent has an incentive to acquire information is essential for trade in funding markets where the delay of trading can cause the bankruptcy of market participants. In contrast, stock trading seems to be driven by differences of opinions and where agents have a high incentive to acquire information because stocks are intrinsically more information sensitive and private information is revealed through continuous trading.

In funding market very large volume of trade is conducted without due diligence while the volume per trade is small in stock markets. The trading of a large block of stocks is conducted on non-anonymous upstairs market or in dark pools or executed by high frequency traders who split blocks of stocks into many small units so as to avoid negative price impact in centralized markets since market makers adjust prices when seeing an anonymous order with large volume.

The present paper provides a tractable framework for linking decentralized and centralized trading and highlights a potential explanation for why debt instruments are traded in OTC markets.[24] By being able to leverage up the order size in a large anonymous market,

[23] For example, in 2007, the total issuance of equities in the U.S. amounted to $246 billion and was much smaller than the issuance of Treasuries ($752 billion), corporate bonds ($1.204 trillion), or mortgage related instruments ($2.047 trillion). Also, in terms of daily trading volume in 2007, $87.1 billion ($68.6 billion) of stocks were traded on the NYSE (NASDAQ), while Agency MBS ($320 billion), Treasury ($546 billion conducted by 19 primary dealers), or repo (5.81 trillion) had a much higher daily trading volume. See SIFMA Research Quarterly, May 2008.

[24] Market microstructure models of stock trading in centralized markets might not be an appropriate model for studying the decentralized bond trading among institutional traders in OTC markets.

agents may have an incentive to acquire information even if the per unit information sensitivity of the asset is low. This can cause endogenous lemons problems and not all gains from trade are realized. In contrast, the non-anonymity of counterparties is an important feature of trade in funding markets.

In the finance literature there are different proxies for "liquidity". Trading volume is a measure of liquidity. Low adverse selection concern is another notion of liquidity. With respect to these two notions, this paper argues that some debt instruments (e.g. AAA rated corporate bonds), are liquid in the sense of low adverse selection concerns exactly because it is difficult to leverage up trade and thus agents have less incentive to acquire information. Trading volume alone might be a misleading proxy for liquidity.[25]

The analysis of how security design and the information sensitivity of a security are linked to the organization of markets is yet an unexplored topic. Empirically, low (high) information sensitive securities tend to be traded in non-anonymous decentralized (anonymous centralized) markets. Existing literature on market design takes the form of securities as given while the security design literature takes the security market as given. Therefore, the research on the joint design of optimal securities and optimal markets can provide important insights for the regulation of the financial system.

[25] From an accounting perspective, commercial papers and AAA rated bonds are considered as cash-equivalent and liquid securities but stocks are not. Cash like securities are supposed to have a stable value as well as easy to sell with little price impact.

Appendix

Proof of Proposition 1

The efficient allocation is when the natural buyer and seller trade one unit. It is without loss of generality to focus on trading strategies where traders submit to trade one unit. So the order size is omitted. A pure strategy of buyer and seller is denoted with $t_B=(n_B,b)$ and $t_S=(n_S,s)$, respectively.

Part (a): Suppose both agents do not acquire information. The set of mutually acceptable prices is $p\in[E[v], E[v]+\Delta]$. So the set of potentially best responses with trade and without information acquisition is $(0,b)$ and $(0,s)$ with $b=s=E[v]+k$ and $k\in[0, \Delta]$. In such a (*k-sharing*) outcome the buyer gets $EU^B=\Delta-k$, and the seller gets $EU^S=k$.

For $c\geq\frac{1}{4}(v_H-v_L-\Delta)$, the following arguments show that no agent has a profitable deviation. Suppose the buyer acquires information and speculates. In state v_L he chooses a bid price $b_L<s$ and no trade occurs. In state v_H he chooses $b_H=s$ and makes some speculative profits since he pays less than the true value of the asset.[26] This response yields $EU^B=\frac{1}{2}[(v_H+\Delta)-(E[v]+k)]-c=\frac{1}{4}(v_H-v_L)+\frac{1}{2}(\Delta-k)-c$ which is smaller than $\Delta-k$. Analogously, given (0,b), speculation is not a profitable deviation for the seller.

Part (b): From above, if $c<\frac{1}{4}(v_H-v_L-\Delta)$, the best response of buyer to (0,s) is to choose $(1,b_L,b_H)$ with $b_L>$s and $b_H=s$. In this case, the seller suffers an endogenous lemons problem since $EU^S=\frac{1}{2}(\Delta+k)-\frac{1}{4}(v_H-v_L)<0$. (Note, $\Delta<\frac{1}{8}(v_H-v_L)$.) Analogously, if $\frac{1}{4}(v_H-v_L)+\frac{1}{2}k-c>k$, the seller's best response to $(0,b)$ with $b=E[v]+k$ is to choose $(1,s_L,s_H)$ with $s_L=b$ and $s_H>b$. Consequently, a k-sharing trading outcome without information acquisition cannot be established as a BNE in pure strategies, if $c<max\{\pi-\frac{1}{2}(\Delta-k), \pi-\frac{1}{2}k\}$ where $\pi\equiv\frac{1}{4}(v_H-v_L)$.[27]

It remains to show that there is also no pure strategy BNE with one-sided or two-sided information acquisition. It is easy to see that if $c>\frac{1}{2}\Delta$, then no pure strategy equilibrium exists in which both traders acquire information. Suppose only the seller acquires information. The

[26] Assumption A implies that in state v_L, $v_L+\Delta<E[v]$. Thus the best response of an informed buyer is not to trade.

[27] This condition has a simple economic interpretation. If the information cost is smaller than the speculative profit, π, net the opportunity cost of speculation, then trade at a price $p\in$[E[v], E[v]+Δ] is not a equilibrium outcome. If the buyer acquires information and speculates, he does not trade in state v_L and ex ante he forgoes the surplus $(\Delta-k)$ with probability 0.5. If the seller speculates, his opportunity cost of speculation is $\frac{1}{2}k$. For $k=\frac{1}{2}\Delta$, the opportunity cost of speculation for both traders is $\frac{1}{4}\Delta$. The set of efficient equilibria 'shrinks" with information cost. If $c=\frac{1}{4}(v_H-v_L-\Delta)$, only the equal-split ($k=\frac{1}{2}\Delta$) outcome is attainable as an efficient BNE.

assumption $\Delta<\frac{1}{8}(v_H-v_L)$ implies that $v_H>E[v]+\Delta$ and $E[v]>v_L+\Delta$. A standard lemons argument shows that given the seller is informed, the best response of an uninformed buyer is to bid at most $v_L+\Delta$. Trade only occurs in state v_L, and the seller's payoff is at most $EU^S=\frac{1}{2}\Delta-c<0$. In such a case, no trader acquires too expensive and non-exploitable information, but because of the endogenous lemons problem the buyer proposes $b\leq v_L+\Delta$ and the seller proposes $s\geq v_H$. So no pure strategy BNE with trade exists.

Part (c): There are two statements. (i) The set of (symmetric and asymmetric) equilibria where trade occurs with probability 1 (called *full* trade BNE) is given by $t_B=t_S=(1, v_L+\Delta-r, v_H+z)$ where $r,z\in[0,\frac{1}{2}\Delta-c]$.[28] (ii) In any full trade BNE both traders acquire information and the price is fully revealing. (Note, offer strategies can be asymmetric.)

Proof: (i) Consider the strategy pair $t_B=t_S=(1, v_L+\Delta-r, v_H+z)$ with $r,z\in[0, \Delta]$. In this case $EU^B=\frac{1}{2}[v_L+\Delta-(v_L+\Delta-r)]+\frac{1}{2}[v_H+\Delta-(v_H+z)]-c=\frac{1}{2}\Delta-\frac{1}{2}(z-r)-c$ and $EU^S=\frac{1}{2}\Delta+\frac{1}{2}(z-r)-c$.

If the buyer chooses $(0,b)$ with $b=v_H+z$ then $EU^B=\frac{1}{2}[\mathrm{v_L}+\Delta-\frac{1}{2}(v_L+\Delta-r+v_H+z)]+\frac{1}{2}[v_H+\Delta-(v_H+z)]=\frac{3}{4}\Delta-\frac{1}{4}(v_H-v_L)-\frac{1}{4}(3z-r)$. For $r=\Delta$ and $z=0$, the buyers' payoff is maximal and yet $EU^B=-\frac{1}{4}(v_H-v_L)+\Delta<0$. If the buyer chooses $(0,b)$ with $b=v_L+\Delta-r$ then $EU^B=\frac{1}{2}r$.[29] If the seller chooses $(0,s)$ with $s=v_L+\Delta-r$ then $EU^S=\frac{1}{4}\Delta+\frac{1}{4}(z-r)-\frac{1}{4}(v_H-v_L)$. For $r=0$ and $z=\Delta$, the seller's payoff is maximal and yet $EU^S=\frac{1}{4}\Delta-\frac{1}{4}(v_H-v_L)<0$. If the seller chooses $(0,s)$ with $s=v_H+z$ then $EU^S=\frac{1}{2}z$.

Consequently, $t_B^*=t_S^*=(1, v_L+\Delta-r, v_H+z)$ are best responses if the following two conditions hold: $EU^B=\frac{1}{2}\Delta-\frac{1}{2}(z-r)-c>\frac{1}{2}r$ and $EU^S=\frac{1}{2}\Delta+\frac{1}{2}(z-r)-c>\frac{1}{2}z$. Define $\kappa=\frac{1}{2}\Delta-c$. Then for any $r,z\in[0,\kappa]$, (t_B^*,t_S^*) constitutes a BNE.

Proof: (ii) If $c\leq\frac{1}{2}\Delta$, no full trade equilibrium exists in which (a) no trader acquires information or only one trader acquires information. The assumption $\Delta<\frac{1}{8}(v_H-v_L)$ and $c\leq\frac{1}{2}\Delta$ imply that $c<\frac{1}{4}(v_H-v_L-\Delta)$. So there is no trading equilibrium without information acquisition. Suppose that only the buyer acquires information and he chooses (b_L,b_H) with $b_L=v_L+\Delta-r$ and $b_H=v_H+z$. If there is to be full trade the uninformed seller must choose $(0,s)$ with $s=b_L$. For any $r,z\in[0, \Delta]$, $EU^S<0$. Analogously for $n_B=0$ and $n_S=1$. So no full trade occurs if only one trader acquires information. **QED**

[28] If $c=\frac{1}{2}\Delta$, then $t_B=t_S=(1, v_L+\Delta, v_H)$ is the only full trade BNE.

[29] It is easy to see that the strategies $(0,b)$ where $b<s_L$, $b\in(s_L,s_H)$ or $b>s_H$ are weakly dominated.

Proof of Proposition 2

The set of mixed strategies BNE is large. The proof proceeds as follows. First, a mixed strategy BNE is derived under the assumption that the traders can only choose three offer prices. Then the equilibrium properties are shown to hold even if this restriction is relaxed.

Assumption A

The traders can only choose offer prices from the set $b,s\in\{l,m,h\}$ where $l=v_L+\frac{1}{2}\Delta$, $m=E[v]+\frac{1}{2}\Delta$, and $h=v_H+\frac{1}{2}\Delta$. (These trading strategies lead to an equal-split of surplus.)

Observation A

An informed buyer does not choose $b>v+\Delta$ at v, while an informed seller does not choose $s<v$ at v. An uninformed buyer does not bid more than his expected valuation, i.e. $b>E[v]+\Delta$, and an uniformed seller does not choose $s<E[v]$. These actions are (weakly) dominated choices.

Step 1

Given Assumption A and Observation A, one can focus on the following pure strategies that are not dominated. For the buyer, these are $(0,l)$, $(0,m)$, $(1,l,m)$, $(1,l,h)$, and $(1,l,l)$. For the seller, these are $(0,m)$, $(0,h)$, $(1,l,h)$, $(1,m,h)$, and $(1,h,h)$. The buyer puts probability σ_{B1} on $(0,l)$, σ_{B2} on $(0,m)$, σ_{B3} on $(1,l,m)$, σ_{B4} on $(1,l,h)$, and σ_{B5} on $(1,l,l)$. The seller puts probability σ_{S1} on $(0,h)$, σ_{S2} on $(0,m)$, σ_{S3} on $(1,m,h)$, σ_{S4} on $(1,l,h)$, and σ_{S5} on $(1,h,h)$.

The expected payoffs of the buyer are given as follows.

$$EU^B(0,l) = \tfrac{1}{2}\sigma_{S4}\tfrac{1}{2}\Delta$$

$$EU^B(0,m) = \tfrac{1}{2}\sigma_{S4}(\tfrac{1}{2}\Delta-\tfrac{1}{4}(v_H-v_L))+\tfrac{1}{2}\sigma_{S3}(\tfrac{1}{2}\Delta-\tfrac{1}{2}(v_H-v_L))+\sigma_{S2}\tfrac{1}{2}\Delta$$

$$EU^B(1,l,m) = \tfrac{1}{2}\sigma_{S4}\tfrac{1}{2}\Delta+\tfrac{1}{2}\sigma_{S2}(\tfrac{1}{2}\Delta+\tfrac{1}{2}(v_H-v_L))-c$$

$$EU^B(1,l,h) = \tfrac{1}{2}\sigma_{S4}\tfrac{1}{2}\Delta+\tfrac{1}{2}(\sigma_{S3}+\sigma_{S4})\ \tfrac{1}{2}\Delta+\tfrac{1}{2}\sigma_{S1}\tfrac{1}{2}\Delta+\tfrac{1}{2}\sigma_{S2}(\tfrac{1}{2}\Delta+\tfrac{1}{4}(v_H-v_L))-c$$

$$EU^B(1,l,l) = \tfrac{1}{2}\sigma_{S4}\tfrac{1}{2}\Delta-c$$

The expected payoffs of the seller are given as follows.

$$EU^S(0,h) = \tfrac{1}{2}\sigma_{B4}\tfrac{1}{2}\Delta$$

$$EU^S(0,m) = \tfrac{1}{2}\sigma_{B4}(\tfrac{1}{2}\Delta-\tfrac{1}{4}(v_H-v_L))+\tfrac{1}{2}\sigma_{B3}(\tfrac{1}{2}\Delta-\tfrac{1}{2}(v_H-v_L))+\sigma_{B2}\tfrac{1}{2}\Delta$$

$$EU^S(1,m,h) = \tfrac{1}{2}\sigma_{B4}\tfrac{1}{2}\Delta+\tfrac{1}{2}\sigma_{B2}(\tfrac{1}{2}\Delta+\tfrac{1}{2}(v_H-v_L))-c$$

$$EU^S(1,l,h) = \tfrac{1}{2}\sigma_{B4}\tfrac{1}{2}\Delta+\tfrac{1}{2}(\sigma_{B3}+\sigma_{B4})\ \tfrac{1}{2}\Delta+\tfrac{1}{2}\sigma_{B1}\tfrac{1}{2}\Delta+\tfrac{1}{2}\sigma_{B2}(\tfrac{1}{2}\Delta+\tfrac{1}{4}(v_H-v_L))-c$$

$EU^S(1,h,h) \quad = \frac{1}{2}\,\sigma_{B4}\frac{1}{2}\,\Delta - c$

Since the pure strategy *(1,l,l)* is strictly dominated by the pure strategy *(0,l)*, the buyer chooses σ_{B5}=0. Since *(1,h,h)* is strictly dominated by *(0,h)*, the seller chooses σ_{S5}=0.

Step 2

(a) This step analyses the best responses of the buyer.

(i) Strategy (1,*l*,*m*) weakly dominates (1,*l*,*h*) if

$$\Leftrightarrow \quad \tfrac{1}{2}\,\sigma_{S2}(\tfrac{1}{2}\,\Delta+\tfrac{1}{2}\,(v_H-v_L))-c \geq \tfrac{1}{2}\,(\sigma_{S3}+\sigma_{S4})\tfrac{1}{2}\,\Delta+\tfrac{1}{2}\,\sigma_{S1}\tfrac{1}{2}\,\Delta+\tfrac{1}{2}\,\sigma_{S2}(\tfrac{1}{2}\,\Delta+\tfrac{1}{4}\,(v_H-v_L))-c$$

$$\Leftrightarrow \quad \tfrac{1}{8}\,\sigma_{S2}(v_H-v_L)) \geq \tfrac{1}{4}\,(\sigma_{S1}+\sigma_{S3}+\sigma_{S4})\,\Delta$$

$$\Leftrightarrow \quad \sigma_{S2}(v_H-v_L)) \geq 2(1-\sigma_{S2})\,\Delta$$

$$\Leftrightarrow \quad \sigma_{S2} \geq \frac{2\Delta}{(2\Delta+v_H-v_L)} \equiv I_{(l,h)}^{(l,m)}$$

(ii) Strategy (1,*l*,*m*) weakly dominates the strategy (0,*l*) if

$$\sigma_{S2} \geq \frac{4c}{(\Delta+v_H-v_L)} \equiv I_{l}^{(l,m)}$$

(iii) Strategy (1,*l*, *m*) weakly dominates the strategy (0,*m*) if

$$\sigma_{S2} \geq \frac{4c-\frac{1}{2}\sigma_{S4}(v_H-v_L)}{(v_H-v_L-\Delta)} - \sigma_{S3} \equiv I_{m}^{(l,m)}$$

(iv) Strategy (0,*m*) weakly dominates the strategy (0,*l*) if

$$\sigma_{S2} \geq \frac{(2\sigma_{S3}+\sigma_{S4})(v_H-v_L)}{4\Delta} - \frac{1}{2}\sigma_{S3} \equiv I_{l}^{m}$$

(v) Strategy (1,*l*,*h*) weakly dominates the strategy (0,*l*) if

$$\sigma_{S2} \geq \frac{2(4c-\Delta)}{v_H-v_L} \equiv I_{l}^{(l,h)}$$

(vi) Strategy (1,*l*,*h*) weakly dominates the strategy (0,*m*) if

$$\sigma_{S2} \geq \frac{8c+2\Delta(-\sigma_{S1}+\sigma_{S3}-\sigma_{S4})-(\sigma_{S3}+\sigma_{S4})(v_H-v_L)}{(v_H-v_L-2\Delta)} \equiv I_{m}^{(l,h)}$$

(b) Analogously for the seller. E.g., if $\sigma_{B2} \geq I_{(l,h)}^{(l,m)}$,then $EU^S(1,m,h) \geq EU^S(1,l,h)$.

Step 3

Claim: There exists no mixed strategy equilibrium in which the informed buyer and informed seller choose the “honest” strategy (1,*l*,*h*), with positive probability.

Proof: For the buyer, (1,*l*,*h*) and (1,*l*,*m*) are the two potential strategies with information acquisition for being a candidate in a mixed strategy equilibrium. The buyer does not choose (1,*l*,*h*) with positive probability if it is strictly dominated by (1,*l*,*m*). Suppose that the strategy (1,*l*,*h*) weakly dominates (1,*l*,*m*), i.e. $\sigma_{S2} \le I^{(l,m)}_{(l,h)}$. It is easy to see that $I^{(l,m)}_{(l,h)} < I^{(l,h)}_{l}$. Consequently, if the strategy (1,*l*,*h*) weakly dominates (1,*l*,*m*), then (1,*l*,*h*) is strictly dominated by the strategy (0,*l*) because in this case $\sigma_{S2} < I^{(l,h)}_{l}$.[30] Therefore, if the seller randomizes such that the buyer is indifferent between (1,*l*,*h*) and (1,*l*,*m*); or (1,*l*,*h*) dominates (1,*l*,*m*), then the buyer chooses σ_{B1}=1, i.e. he does not acquire information. Analogously for the seller, if the strategy (1,*l*,*h*) weakly dominates (1,*m*,*h*), then (1,*l*,*h*) is strictly dominated by the strategy (0,*h*). Consequently, in a mixed strategy equilibrium (where information must be acquired with positive probability), the strategy (1,*l*,*h*) must be a strictly dominated strategy and one must have $\sigma_{B4}=\sigma_{S4}=0$.[31]

Step 4

Claim: In a mixed strategy equilibrium the traders get zero expected payoff.

Proof: For $\sigma_{B4}=\sigma_{S4}=0$, $EU^B(0,l)=EU^S(0,h)=0$ since the buyer does not bid more and the seller does not demand less than the price $m= E[v]+\frac{1}{2}\Delta$, respectively. In other words, if the buyer is indifferent between (1,*l*,*m*) and (0,*l*) or indifferent between (0,*m*) and (0,*l*), then his expected payoff is zero. In order to find a mixed strategy equilibrium in which the traders get positive expected payoffs, the following is required: For the buyer, he should be indifferent between (1,*l*,*m*) and (0,*m*); and (0,*m*) should strictly dominate (0,*l*), i.e. the buyer chooses σ_{B1}=0.[32] The buyer is indifferent between (1,*l*,*m*) and (0,*m*) if $\sigma_{S2} = I^{(l,m)}_{m}$. For σ_{S4}=0,

$$\sigma_{S2} = \frac{4c - \frac{1}{2}\sigma_{S4}(v_H - v_L)}{(v_H - v_L - \Delta)} - \sigma_{S3}$$

$$\Leftrightarrow \qquad \sigma_{S2} = \frac{4c}{(v_H - v_L - \Delta)} - \sigma_{S3}.$$

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[30] If the buyer is indifferent between (1,*l*,*m*) and (1,*l*,*h*), then (0,*l*) strictly dominates both (1,*l*,*m*) and (1,*l*,*h*), since $\sigma_{S2} = I^{(l,m)}_{(l,h)}$ and c>0.5Δ imply $\sigma_{S2} < I^{(l,m)}_{l}$.

[31] Suppose the buyer only randomizes over (0,*l*) and (0,*m*). If the seller also does not acquire information, then he chooses $s=m$ with probability 1. Given the seller's response (0,*m*), the buyer's best response is (1,*l*,*m*). So there exists no mixed strategy equilibrium in which information is acquired with zero probability.

[32] The seller should be indifferent between (1,*m*,*h*) and (0,*m*); and (0,*m*) should strictly dominate (0,*h*), i.e. he chooses σ_{S1}=0.

Since $c<\frac{1}{4}(v_H-v_L-\Delta)$, this implies that $\frac{c}{v_H-v_L-\Delta}<1$. Therefore $\sigma_{S2}+\sigma_{S3}<1$, which means that there is some probability "left", i.e. σ_{S1} must be larger than zero. In order to make the buyer indifferent between (1,*l*,*m*) and (0,*m*), the seller must choose (0,*h*) with positive probability.

On the other hand, if the seller chooses (0,*h*) with positive probability he must be indifferent between (0,*m*) and (0,*h*). Since $EU^S(0,h)=0$, $EU^S(0,m)$ must be zero, too. Otherwise, the seller is not indifferent. Consequently, the expected payoff of the seller must be zero in a mixed strategy equilibrium.[33]

Step 5

Claim: In the (non-degenerated) mixed strategy equilibrium the buyer randomizes over (0,*l*), (0,*m*) and (1,*l*,*m*) according to σ_B and the seller randomizes over (0,*h*), (0,*m*) and (1,*m*,*h*) according to σ_S where

$$\sigma_B=\sigma_S=\left(1-\frac{4c}{v_H-v_L-\Delta},\ \frac{4c}{v_H-v_L+\Delta},\ \frac{8c\Delta}{(v_H-v_L)^2-\Delta^2}\right).$$

Proof: Note, $\Delta<c<\frac{1}{4}(v_H-v_L-\Delta)$. The buyer is indifferent between (1,*l*,*m*) and (0,*l*) if the seller chooses σ_{S2} such that $\sigma_{S2}=I_l^{(l,m)}$; and the buyer is indifferent between (1,*l*,*m*) and (0,*m*), if the seller chooses σ_{S2} and σ_{S3} such that $\sigma_{S2}=I_m^{(l,m)}$.[34] So $I_l^{(l,m)}=I_m^{(l,m)}$ implies

$$\frac{4c}{v_H-v_L+\Delta}=\frac{4c}{v_H-v_L-\Delta}-\sigma_{S3}$$

$$\Leftrightarrow\qquad \sigma_{S3}=\frac{8c\Delta}{(v_H-v_L+\Delta)(v_H-v_L-\Delta)}=\frac{8c\Delta}{(v_H-v_L)^2-\Delta^2}.$$

In addition, the seller chooses

$$\sigma_{S1}=1-\sigma_{S2}-\sigma_{S3}=1-\frac{4c}{v_H-v_L-\Delta}.$$

Step 6

Claim: The outcome in a mixed strategy BNE has the following properties. (i) The probability of trade is $\frac{16c^2}{(v_H-v_L)^2-4\Delta^2}$. (ii) The only trading price is $p=E[v]+\frac{1}{2}\Delta$ and not fully revealing.

Proof: The buyer randomizes over (0,*l*), (0,*m*) and (1,*l*,*m*). The seller randomizes over (0,*h*), (0,*m*) and (1, *m*,*h*).

[33] Analogously, the buyer must choose (0,*l*) with positive probability in order to make the seller indifferent between (1,*m*,*h*) and (0,*m*), i.e. his expected payoff is zero in a mixed strategy equilibrium.

[34] Alternatively, the buyer should be indifferent between (0,*m*) and (0,*l*) and this yields the same condition.

(i) Trade occurs in the following events: (a) both the buyer and the seller choose $(0,m)$; (b) the buyer chooses $(0,m)$ and the seller chooses $(1,m,h)$ and the true state is v_L; and (c) the buyer chooses $(1,l,m)$, the seller chooses $(0,m)$ and the true state is v_H. The probability of trade is given as follows:

$$prob(trade) = \sigma_{B2}\sigma_{S2} + \tfrac{1}{2}\sigma_{B2}\sigma_{S3} + \tfrac{1}{2}\sigma_{B3}\sigma_{S2} = \sigma_{B2}\sigma_{S2} + \sigma_{B2}\sigma_{S3}$$

$$\Leftrightarrow \qquad prob(trade) = \frac{16c^2}{(v_H - v_L + 2\Delta)^2} + \frac{64c^2\Delta}{(v_H - v_L + 2\Delta)^2(v_H - v_L - 2\Delta)}$$

$$\Leftrightarrow \qquad prob(trade) = \frac{16c^2}{(v_H - v_L)^2 - 4\Delta^2}.$$

(ii) The buyer bids at most $E[v]+\frac{1}{2}\Delta$ and the seller demands at least $E[v]+\frac{1}{2}\Delta$. Therefore, no trade occurs at the prices v_L and v_H. Trade only occurs if at least one trader is uninformed. If the uninformed trader observes trade, he cannot distinguish whether he makes a fair deal and realizes $\frac{1}{2}\Delta$, or suffers a speculative loss. Although the uninformed trader updates his belief, he does not know the true state when observing $p=E[v]+\frac{1}{2}\Delta$.

Remarks

(1) σ_{i3} is the equilibrium probability of information acquisition. It decreases in v_H-v_L and increases in the information cost c which seems unintuitive. In order to make the other trader indifferent between his pure strategies randomization requires it.

(2) For $c<\frac{1}{4}(v_H-v_L-\Delta)$, as $(v_H-v_L)\rightarrow\infty$, then the probability that the buyer chooses $(0,v_L)$ and the seller chooses $(0,v_H)$ converges to one.

Characterization of mixed strategy BNE

(1) Suppose the agents can choose any real numbers as bid and ask prices. Define $l=v_L+\frac{1}{2}\Delta$, $m=E[v]+\frac{1}{2}\Delta$, and $h=v_H+\frac{1}{2}\Delta$. A set of mixed strategy BNE from a continuous distribution is the following. (a) The buyer chooses (i) a density $f_0(b)$ over the set of pure strategies $(0,b)$ with continuous bid prices b on the interval $[0, l]$ where $\int f_0(b)db = \sigma_{B1}$; (ii) a probability σ_{B2} on the pure strategy $(0,m)$; and (iii) a density $f_1(b)$ over the set of pure strategies $(1,b,m)$ with continuous bid prices b on the interval $[0, l]$ where $\int f_1(b)db = \sigma_{B3}$. (b) The seller chooses (i) a density $g_0(s)$ over the set of pure strategies $(0,s)$ with continuous ask prices s on the interval $[h, \infty)$ where $\int g_0(s)ds = \sigma_{S1}$; (ii) a probability σ_{S2} on the pure strategy $(0,m)$; and (iii) a

density $g_1(s)$ over the set of pure strategies $(1,m,s)$ with continuous ask prices s on the interval $[h, \infty)$ where $\int f_1(s)ds = \sigma_{S3}$.

(2) In any mixed strategy BNE, the set of pure strategies $(0,b)$ with bid prices b on the interval $[0, l]$ is played with “probability” $\int f_0(b)db = \sigma_{B1} > 0$ so as to make the other trader indifferent between information acquisition and no information acquisition. Since this no trade strategy yields zero net payoff to the buyer and he is indifferent between this and other pure strategies, all other pure strategies yield zero expected payoff as well. **QED**

Proof Proposition 3

Define Q^* such that $\frac{1}{4}(2Q^*-1)(v_H-v_L)>c$. If $M,N>Q^*$, Lemma 5 shows that in a BNE with positive volume of trade, some traders acquire information. To save on notations, suppose $M \geq N$.

Case (i): $\frac{1}{2}\Delta<c<\frac{1}{2}N\Delta$. No pair of informed B- and S-trader can jointly cover their information cost by trading one unit. Suppose one S-trader acquires information. Because of the lemons problem, all uninformed B-traders behaves defensively, trade may occur at $p=v_L+\Delta$ and $EU_S=\frac{1}{2}N\Delta-c>0$. Since there is a profit, as a best response another S-trader acquires information and sells for $p=v_L+\Delta-\varepsilon$ and gets all demand. If the price is such that $EU_S=\frac{1}{2}N\Delta-c=0$, then only one S-trader acquires information. But if only one S-trader acquires information, he chooses $v_L+\Delta$ at v_L and $EU_S=\frac{1}{2}N\Delta-c>0$. A standard Bertrand type of arguments with sunk information costs shows that there is no pure strategy equilibrium with trade.

Case (ii): $c>\frac{1}{2}N\Delta$. If only one side of the market acquires information, the other side of the market behaves defensively and the informed trader has negative payoff. (See Case (i).) So there is no pure strategy BNE with trade.

In any mixed strategy equilibrium uninformed traders are indifferent between trade and no trade (i.e. submitting a defensive order with U=0). Since in a mixed strategy equilibrium, traders are indifferent between becoming informed and staying uninformed all traders have EU=0.

Case (iii): $c=\frac{1}{2}N\Delta$. The following is a pure strategy BNE with positive volume of trade. One S-agent acquires information and all other traders do not. All uninformed S-agents demand s=v_H for selling one unit, all B-agent bid b=$v_L+\Delta$ for buying one unit. The informed S-agent chooses to sell N units at the ask price s=$v_L+\Delta$ in state v_L and s=$v_H+\Delta$ in state v_H. Trade occurs in state v_L. All traders have zero expected payoff.**QED**

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