Notes, Comments, and Letters to the Editor

Bargaining with endogenous information

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Abstract

This paper analyses information acquisition in ultimatum bargaining with common values. Because of an endogenous lemons problem the equilibrium payoffs of the agents are non-monotonic in the information cost. The mere possibility of information acquisition can cause no trade although the agents maintain symmetric information in equilibrium and the gain from trade is common knowledge. The agent responding to a take-it-or-leave-it offer may capture some or even the full trading surplus in a perfect Bayesian equilibrium. The implications for sequential bargaining are discussed.

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1. Introduction

A central question in the bargaining literature is why rational agents may have difficulties in reaching mutually beneficial agreements. Inefficient bargaining outcomes may take on different forms such as the failure to reach an agreement when gains from trade exist, costly delay in reaching an agreement, or settling on contractual terms that fail to fully realize all gains from trade. The bargaining literature provides asymmetric information as a dominant reason for these inefficiencies. See the survey in Ausubel et al. [3].

The present paper does not assume exogenous private information. In this model the bargainers start with symmetric information about all relevant aspects of trade but information is endogenous. In particular, it is common knowledge that there are gains from trade and the true (common) value of the asset is unknown to both agents ex ante. This paper analyses ultimatum bargaining where

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the proposer can acquire information about the value of the asset before he makes the offer, and
the responder can acquire information after seeing the offer and before he responds.

Common value uncertainty typically plays a role in real estate and financial transactions. The
intrinsic value of a piece of land is typically uncertain and an agent can acquire information
before he trades. Agents trading financial assets also face common value uncertainty because the
underlying cash flow stream of a financial asset is risky. In secondary markets the seller does
not necessarily possess better information than a potential buyer but both agents can acquire
information before they trade.¹

In contrast to stock trading in centralized markets, real estate and many other financial trans-
actions are conducted on a bilateral basis. For example, mortgage-backed securities, corporate
bonds, structured credit products, and derivatives are traded in over-the-counter-markets where
price transparency is low and bargaining is a standard feature. See Duffie et al. [13]. Also, the
block trading of stocks in upstairs markets is non-anonymous and decentralized.

This bargaining model can be interpreted as a model of over-the-counter trading with endoge-
 nous information. The main result of the paper identifies an additional source of inefficiencies
in bargaining. The mere possibility of information acquisition can already render efficient trade
unattractive although the agents do not acquire information and maintain symmetric information
in equilibrium and the gain from trade is common knowledge. This no efficient trade result is nei-
ther driven by asymmetric information about the common valuation as in Akerlof [2], Samuelson
[24], or Gresik [18] nor by asymmetric information about the private valuation as in Myer-
son and Satterthwaite [22], but by an endogenous lemons problem due to potential information
acquisition.

The intuition for this result is as follows. This paper assumes that the asset is worth \( v + \Delta \) to
the buyer and \( v - \Delta \) to the seller where \( v \) is the uncertain common value component that is either
high or low with equal probability. The total trading surplus is therefore \( 2\Delta \). By incurring the
cost \( c \), an agent can learn about the true common value. This paper assumes that the buyer makes
the offer and information acquisition is observable. The main results of the paper also hold if the
seller makes the offer or information acquisition is not observable.

Suppose the information cost is larger than the total surplus (i.e. \( c > 2\Delta \)). Then no agent
acquires information in equilibrium. However, there may also be no equilibrium with trade. To see
this, suppose that the buyer proposes the most favorable price for the seller, \( E[v] + \Delta \). If
the seller accepts the offer, he gets the expected payoff \( 2\Delta \). Alternatively, the seller can acquire
information and tries to exploit the buyer. The informed seller only accepts the offer and sells, if he
sees that the value of the asset is low. In this state he realizes the trading surplus as well as makes
some speculative profits \( \pi \). This strategy yields the expected payoff \( E[\pi] + \Delta - c \) to the seller and
dominates the first strategy if \( c < E[\pi] - \Delta \). In such a case, even if the seller is offered the full
surplus, he speculates and the buyer’s payoff is negative. Anticipating this endogenous lemons
problem, the buyer makes a low offer which the uninformed seller does not accept. Therefore, if
\( 2\Delta < c < E[\pi] - \Delta \), then no equilibrium with trade exists although the agents maintain symmetric
information and the trading gain is common knowledge.

The second main result of the paper shows that the agent responding to a take-it-or-leave-it-offer
may capture some or even the full trading surplus in a perfect Bayesian equilibrium. Whether

¹ Hedging and portfolio rebalance needs, tax-induced trades, and dividend-captured trades give rise to mutually ben-
eficial transactions. The demand for financial analysts’ coverage, rating services, Bloomberg’s and Reuters’ financial
services suggest that information acquisition is a prevalent activity in financial markets.
there is a first mover or second mover advantage in such ultimatum bargaining depends on the information cost.

The intuition is the following. Suppose the uninformed buyer wants to capture the full surplus and proposes the price $E[v] - \Delta$. If the seller speculates, then the seller does not forgo any surplus by not selling in the high state. His expected opportunity cost of speculation is zero. Speculation is profitable if $c < E[\pi]$. As argued above, if the buyer offers the seller the full surplus, the seller’s opportunity cost of speculation is $\Delta$. Consequently, if $E[\pi] - \Delta < c < E[\pi]$, there exists a critical offer which the seller accepts without information acquisition. This offer must give the seller a trading surplus what he could get by speculation. The possibility to acquire information endows the seller with a credible speculative threat. So if $2\Delta < c = E[\pi] - \Delta$, then in a perfect Bayesian equilibrium in which trade occurs, the buyer offers the seller the full trading surplus because of the endogenous lemons problem.

If the information cost is low, the buyer acquires socially wasteful information and only mixed strategy equilibria exist. Although trade only occurs with positive probability, the buyer chooses to create an actual lemons problem rather than facing an endogenous lemons problem with either no trade or giving the seller too much of the trading surplus.

This paper also discusses the implications of this endogenous lemons problem for sequential bargaining. For example, perfect Bayesian equilibria in two-period alternating offer bargaining may have the following properties: (1) The equilibrium payoff of the agent who makes the offer in the first period may increase in the discount factor of the trading surplus. (2) If the discounting of the trading surplus is lower than the discounting of the information cost, equilibrium delay arises as an optimal timing consideration and is (constraint) efficient. (3) On the other hand if the discounting of the surplus is at an intermediate level, two period bargaining may perform worse than ultimatum bargaining in terms of total expected equilibrium payoffs.

The remainder of the paper is organized as follows. The next section relates this paper to the literature. Section 3 introduces the model. Section 4 derives the equilibria. Section 5 discusses some implications of the endogenous lemons problem for sequential bargaining. Section 6 concludes. Appendix contains proofs.

2. Relation to the literature

This paper is most closely related to Shavell [25] who analyses one-sided information acquisition and the disclosure of information prior to the sale of an object through a take-it-or-leave-it-offer. He compares the equilibrium information acquisition with socially efficient information acquisition in the four constellations in which (i) information has social value versus no social value and (ii) disclosure is mandatory versus voluntary. For the case where the information cost is low and information has no social value, as it is in the present paper, Shavell [25] shows that socially wasteful information is acquired (not acquired) in equilibrium if disclosure is voluntary (mandatory).

The two main results of this paper do not occur in Shavell’s [25] model. (1) The no trade equilibrium does not arise in Shavell [25] since he assumes that the seller always wants to sell. In the terminology of the present paper where $u^B = v + \Delta$ and $u^S = v - \Delta$, $\Delta$ is assumed to be large.

Matthews [21] and Hausch and Li [19] show that bidders acquire excessive information in pure common value auctions. See also Hirshleifer [20]. Bergemann and Valimäki [4] employ a mechanism design approach and a local efficiency concept and show that any ex post efficient allocation mechanism causes an ex ante information acquisition inefficiency.
so that the speculative loss is not severe relative to the realization of the trading gain even if a lemons problem exists. Additionally, this paper shows that if $\Delta$ is large, the no trade result does not arise as an equilibrium outcome because the proposer is not concerned about the endogenous lemons problem. The responder only has an incentive to speculate, if $c < E[\pi] - \Delta$ and this does occur if $E[\pi] - \Delta < 0$.

(2) In the present paper the equilibrium payoffs of the agents are non-monotonic in the information cost. In particular, the responder may capture some or even the full surplus in a perfect Bayesian equilibrium. In Shavell [25] the responder always receives zero payoff in equilibrium. The reason for the different results is that information can only be acquired prior to the bargaining stage in [25], while the present model assumes that information can be acquired during the bargaining process. The responder can acquire information after seeing the offer. This assumption endows the responder with a credible speculative threat so that he may obtain a share of the surplus. If this model assumed that information can only be acquired prior to the bargaining stage, the responder would have no speculative threat, capture no surplus and not acquire information in equilibrium because of a hold-up problem.

A second related line of research is the work by Cremer and Khalil [7] and Cremer et al. [9] who analyze one-sided information acquisition in a principal-agent framework, where a principal contracts with an agent for the production of goods. In [7] ([9]) the agent can acquire socially wasteful information about the production cost after (before) the principal offers him a contract. In [7] the optimal contract induces no information acquisition even for low information cost and the agent captures no rent. In [9] if the information cost is low (intermediate), then the agent acquires information (randomizes information acquisition) in equilibrium. The rent the agent captures decreases monotonically in the information cost.4

This paper differs from [7,9] in four important aspects. (i) The key difference is that [7,9] are private value models where there is uncertainty about the size of the rent, while this paper is a common value model and assumes that the magnitude of the surplus is fixed but there is uncertainty about the common valuation.5 (ii) This difference gives rise to very different strategic reasons for information acquisition and different uses of information. In [7,9] if the agent acquires information, he knows the exact production rent before he signs the contract and he may be able to capture more of it. In the present model better information has a speculative use and the motive for information acquisition is to exploit the opponent or to avoid being exploited and suffering a speculative loss. The key concern of the bargainers in this model is the endogenous lemons problem and it is not present in [7,9].

(iii) These different strategic incentives imply very different equilibrium predictions. This can be seen as follows. The sequence of moves in the present model is similar to [7], i.e. the responder can acquire information after seeing the offer, but because of the different strategic motives the equilibrium outcome is very different. If the information cost is low, the responder faces an

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3 Shavell [25, p. 25] states “In the absence of such an assumption, the complicating issue would arise that the seller without information might not sell, a problem similar to the ‘lemons’ problem in Akerlof [2]”. Also, in contrast to [25], this paper assumes that information cannot be disclosed credibly, but this assumption is not crucial. In the no trade equilibrium and the equilibrium in which the seller gets some surplus, the buyer does not acquire information because it is not worthwhile to do it. There is nothing to disclose.

4 Cremer et al. [8] analyze contract design where the agent can acquire socially useful information after seeing the contract offer and they characterize how the optimal contract induces efficient information acquisition.

5 In [7,9], the surplus is $\Delta = V(q) - \beta \cdot q$ where $V$ is a function and $\beta$ is the uncertain production cost of the agent. The agent learns $\beta$ for free when he produces. Efficient production occurs if the agent chooses the quantity $q$ that maximizes $\Delta$, and full rent extraction means that the principal captures $\Delta$. 
informed proposer, he randomizes information acquisition and captures no surplus. If the cost is intermediate, the responder is offered some surplus so that he does not acquire information. None of these outcomes occurs in [7]. If a sequence of moves as in [9] is assumed, then the equilibrium outcome in this bargaining model is also completely different. Since the responder can only acquire information before seeing the offer, he has no speculative threat and captures no surplus. In addition, he does not acquire information because of a hold-up problem and trade always occurs in equilibrium.

(iv) The economic environment these papers address are also different. The models in [7,9] are relevant when the uncertainty about the contracting surplus is a major concern such as in a task delegation problem. This model assumes that common value uncertainty is the key concern in the transaction. Because of this focus on common values this bargaining paper applies more to financial transactions and to the discussion of the performance of centralized versus decentralized trading when the information of the traders is endogenous.

This paper shows that if the information cost is high, an efficient equilibrium exists in decentralized trading. Dang [11] analyses information acquisition in a simplified version of the Reny and Perry [23] type double auction environment. Reny and Perry [23] provide a strategic foundation for an efficient and fully revealing rational expectations equilibrium under exogenous private information. Dang [11] shows that an efficient equilibrium may fail to exist if information is endogenous and costly. As the number of traders increases, the equilibria are inefficient even if the information cost is high. In a large market an informed trader can make more speculative profits because there are potentially more uninformed traders to exploit. Therefore, the uninformed traders face an endogenous lemons problem, even if the information cost is high. In such a case decentralized trading may outperform centralized trading.6

3. The model

Two risk neutral agents play an ultimatum bargaining game and seek to agree on a price $p$ at which to trade an asset. It is common knowledge that the asset is worth $v + \Delta$ to the buyer and $v - \Delta$ to the seller where $\Delta$ is a constant and $v$ is the uncertain common value component that is either $v_L$ or $v_H$ with equal probability and $v_H > v_L > \Delta > 0$. If trade occurs, the surplus $2\Delta$ is realized and $U^B = (v + \Delta) - p$ and $U^S = p - (v - \Delta)$. If no agreement is reached, the payoffs of the agents are normalized to zero.

The buyer’s action is to acquire $n_B \in \{0, 1\}$ unit of information and then to choose an offer $b \geq 0$. Upon seeing the offer $b$, the seller’s action is to acquire $n_S \in \{0, 1\}$ unit of information and to choose a response $s \in \{Y, N\}$. If $s = Y$, trade occurs at the price $b$. Otherwise there is no trade. The information cost is $c > 0$ and an informed agent knows true value $v$. Information acquisition is observable and private information cannot be disclosed credibly.7 The solution concept is perfect Bayesian equilibrium.

6 Duffie et al. [13] analyze how search costs affect the equilibrium price and surplus division in dynamic decentralized trading under symmetric information. The present paper shows how information costs affect the equilibrium price and surplus division in decentralized trading with endogenous information.

7 Both assumptions are not crucial for the two main results. See the discussion at the end of Section 4. Smith et al. [26] provide empirical evidence that suggests that the market makers in upstairs markets are able to identify whether the counter party is informed (and has acquired information) or not. Information motivated trades are sent downstairs. Germaine and Moskowitz [17] document that in commercial real estate transactions, agents respond to information disparities by not purchasing assets about which they are uninformed, focusing on assets that are easier to evaluate (like nearby properties and properties with long income histories), and avoiding trades with identifiably informed. Limited participation, selective offering, and market segmentation suggest that (i) agents with private information can be identified and (ii) the credible disclosure of private information and the writing of state contingent contracts are difficult in these transactions.
4. The analysis

Since there are gains from trade and information has no social value, the efficient outcome is trade without costly information acquisition. This section shows that the equilibrium payoffs of the agents are non-monotonic in the information cost and the set of perfect Bayesian equilibria (PBE) has the following properties: (i) If the information cost $c$ is low, the buyer acquires information and only he captures some surplus in a PBE in mixed strategies. If $c$ is in an intermediate range, then no agent acquires information in equilibrium and three cases can arise. (ii) No PBE with trade exists. (iii) In the unique PBE both agents capture some surplus. (iv) A PBE exists in which the seller captures the full surplus. (v) If $c$ is high, then in the unique PBE no agent acquires information and the buyer captures the full surplus.

The reason for the no efficient trade result under symmetric information is the following. Suppose the buyer does not acquire information and proposes the most favorable price for the seller, $b = E[v] + \Delta$. The seller has two potentially profitable responses. (i) He accepts this offer and gets $EU^S = \frac{1}{2}[(v_L + \Delta) - p] = \frac{1}{2}(v_H - v_L) + \Delta - c$ and dominates the first response if $c < \frac{1}{4}(v_H - v_L) - \Delta$. This condition has a simple economic interpretation. While $\frac{1}{4}(v_H - v_L)$ is the expected speculative profit the informed seller makes, $\Delta$ can be interpreted as the expected opportunity cost of speculation. If the seller speculates, he does not sell in the high state and ex ante he forgoes the surplus $\frac{1}{2}(v_H - v_L)$ with probability 0.5.

Suppose the buyer acquires information. Then a signaling game arises. The maximum surplus the buyer can capture is $2k\Delta$ where $k$ denotes the probability of trade in a mixed strategy equilibrium and is a function of the parameters $v_L$, $v_H$, and $\Delta$. For a formal statement of $k$ see Step 3d in Appendix which shows that $k > 0.5$. 8 If the surplus $2k\Delta$ is smaller than the information cost, then no PBE with trade exists although the buyer and the seller maintain symmetric information in equilibrium and the gain from trade is common knowledge.

Proposition 1. If $2k\Delta < c < \frac{1}{4}(v_H - v_L) - \Delta$, then the set of PBE is given as follows. The buyer chooses $n_B = 0$ and $b < v_L - \Delta + 2c$, and the seller chooses $n_S = 0$ and $s = N$. No PBE with trade exists.

Proposition 1 identifies an additional source of inefficiencies in bargaining. It is not only actual asymmetric information but the mere possibility of information acquisition can already render efficient trade unattractive. 9 The next proposition describes the second main result of the paper and shows that due to the endogenous lemons problem which the proposer faces, the agent responding to a take-it-or-leave-offer captures some surplus in a PBE.

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8 The Appendix also shows that this alternative yields a higher payoff to the buyer than the following strategy. The buyer induces the seller to acquire information and the uniformed buyer accounts for the lemons problem. In this case trade only occurs in the low state and $k = 0.5$.

9 Dang [11] shows that this no-trade result also holds in simultaneous offer bargaining.
Proposition 2. If \( \max\left\{ \frac{1}{3}(v_H - v_L) - \Delta, \frac{2}{3}(k - 1) + \frac{1}{6}(v_H - v_L) \right\} < c < \frac{1}{3}(v_H - v_L) \), then in the unique PBE the buyer chooses \( n_B = 0 \) and \( b = v_H - \Delta - 2c \), and the seller chooses \( n_S = 0 \) and \( s = Y \). The surplus is shared as follows: \( EU^B = 2\Delta + 2c - \frac{1}{2}(v_H - v_L) \) and \( EU^S = \frac{1}{2}(v_H - v_L) - 2c \).

The intuition for Proposition 2 is the following. The arguments above show that if the buyer proposes \( E[v] + \Delta \) and \( c > \frac{1}{4}(v_H - v_L) - \Delta \), then the seller accepts the offer and gets \( EU^S = 2\Delta \). If the buyer proposes \( E[v] - \Delta \) and \( c < \frac{1}{4}(v_H - v_L) \), the seller acquires information and speculates instead of just getting \( EU^S = 0 \). Therefore, if \( \frac{1}{4}(v_H - v_L) - \Delta < c < \frac{1}{3}(v_H - v_L) \), there exists a critical offer which the seller accepts without information acquisition. This offer must give the seller a surplus what he could get by speculation. The buyer’s payoff is \( EU^B = 2\Delta + 2c - \frac{1}{2}(v_H - v_L) \). The second term in the maximum bracket in Proposition 2 is the condition for this payoff to be larger than \( 2k\Delta - c \), i.e. the payoff the buyer gets when he acquires information.

In such a case the possibility to acquire information endows the seller with a credible speculative threat. If he does not get enough trading surplus, he acquires information and exploits the buyer. If \( c = \frac{1}{2}(v_H - v_L) - \Delta \), then in order to prevent the seller from speculation, the uninformed buyer must offer the seller the full surplus.

Proposition 3. If \( \frac{2}{3}(k - 1) + \frac{1}{6}(v_H - v_L) \leq c = \frac{1}{3}(v_H - v_L) - \Delta \), then the set of PBE has the following properties. In any PBE the buyer gets \( EU^B = 0 \). There exists a PBE in which the buyer chooses \( n_B = 0 \) and \( b = \frac{1}{2}(v_H + v_L) + \Delta \), and the seller chooses \( n_S = 0 \) and \( s = Y \), and obtains \( EU^S = 2\Delta \).

If the information cost is higher than the speculative profit, then the buyer is not concerned about the endogenous lemons problem. As in the standard take-it-or-leave-it-offer setting, the buyer captures the full surplus in equilibrium.

Proposition 4. If \( c \geq \frac{1}{4}(v_H - v_L) \), then in the unique PBE the buyer chooses \( n_B = 0 \) and \( b = \frac{1}{2}(v_H + v_L) - \Delta \), and the seller chooses \( n_S = 0 \) and \( s = Y \). The payoffs are \( EU^B = 2\Delta \) and \( EU^S = 0 \).

The next proposition completes the analysis. If the information cost is low, the buyer acquires information and only he gets some surplus in a mixed strategy PBE. The buyer chooses to create an actual lemons problem rather than facing an endogenous lemons problem with either no trade or giving the seller too much of the trading surplus. For a formal statement of the offer \( b_L \) and the equilibrium randomizations in Proposition 5 see Step 3 in Appendix.

Proposition 5. If \( c < \min\{2k\Delta, \frac{2}{3}(k - 1) + \frac{1}{6}(v_H - v_L)\} \), then a PBE in mixed strategies has the following properties. The buyer chooses \( n_B = 1 \). If \( v = v_L \), the buyer chooses \( b_L \). If \( v = v_H \), the buyer randomizes over \( b_L \) and \( b_H = v_H - \Delta \). The seller chooses the following response: If he sees \( b_H \), he chooses \( s = Y \). If he sees \( b_L \), he randomizes over \( n_S = 0 \) and \( n_S = 1 \). An informed seller chooses \( s = Y \) if \( v = v_L \), and \( s = N \) if \( v = v_H \). An uninformed seller randomizes over \( s = Y \) and \( s = N \). Trade occurs with probability \( k > 0.5 \) and \( EU^B = 2k\Delta - c \) and \( EU^S = 0 \).

Two numerical examples are illustrated in Fig. 1 which plots the equilibrium payoffs of the agents as a function of the information cost. In Fig. 1(a), Propositions 1–5 arise consecutively.
and $k \approx 0.52$. In Fig. 1(b) Propositions 5, 2 and 4 arise consecutively and $k \approx 0.56$. There is a discrete jump in the buyer’s payoff at $c = 0$ from $2\Delta$ to $2k\Delta$, since in a mixed strategy equilibrium the probability $k$ of trade is strictly bounded away from one.\(^{10}\)

This section closes with a discussion of the assumptions: (1) If the informed buyer can credibly disclose his private information, then trade occurs with probability one, his payoff is $2\Delta - c$, and $k = 1$ in all propositions. (2) If the signal the agents acquire is not perfect, this only changes the potential speculative profit and the critical values of the information cost for the different types of equilibria to arise but not the qualitative implications.

(3) Suppose information acquisition is not observable. The qualitative results hold but Proposition 3. If the uninformed seller is to accept the price $E[v] + \Delta$, then the buyer speculates, i.e. he acquires information and proposes this offer only if he sees $v_H$. Formally, if $c < \frac{1}{4}(v_H - v_L) - \frac{1}{2}\Delta$, then no pure strategy PBE with trade exists. In a mixed strategy PBE the buyer randomizes information acquisition and the seller faces a maybe informed buyer.\(^{11}\)

(4) Suppose the seller can only acquire information prior to the bargaining stage. Then an informed seller faces a hold-up problem. If the buyer sees that the seller has acquired information, an uninformed buyer offers $v_L - \Delta$ to account for the lemons problem, while an informed buyer offers $v_L - \Delta$ at $v_L$ and $v_H - \Delta$ at $v_H$. The seller may accept the offer since the information cost is sunk and the seller’s payoff is $-c$ in both cases. In the unique PBE the buyer chooses $n_B = 0$, $b = E[v] - \Delta$, the seller chooses $n_S = 0$, $s = Y$, and $EU^B = 2\Delta$ and $EU^S = 0$.

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\(^{10}\) If $c = 0$, then two PBE exist. The buyer chooses $n_B = 1$, $b_L = v_L - \Delta$ and $b_H = v_H - \Delta$. If the seller sees $b_H$, he chooses $n_S = 0$ or $n_S = 1$ and $s = Y$. If the seller sees $b_L$, he chooses $n_S = 1$ and $s_L = Y$ and $s_H = N$. In both PBE $U^B = 2\Delta$ and $U^S = 0$.

\(^{11}\) Proposition 2 holds if the condition $\frac{1}{4}(v_H - v_L) - \Delta$ in the maximum bracket is replaced by $\frac{1}{4}(v_H - v_L) - \frac{1}{2}\Delta$. For example, if $c = \frac{1}{4}(v_H - v_L) - \frac{1}{2}\Delta > \Delta$, then in the unique PBE, the buyer chooses $n_B = 0$, $b = E[v]$, and the seller chooses $n_S = 0$, $s = Y$, and $EU^B = EU^S = \Delta$. 
5. Implications for sequential bargaining

This section discusses the implications of the endogenous lemons problem for two period alternating offer bargaining and infinite horizon bargaining where information can be acquired prior to making an offer and a response in each bargaining period $t$. The trading surplus and the information cost are discounted as follows: $\Delta_t = \delta^t \Delta$ and $c_t = \beta^t c$ where $\delta, \beta \in [0, 1]$. To focus on the interesting case, it is assumed that $c < \frac{1}{4} (v_{H} - v_{L}) - \Delta$, i.e. there is an endogenous lemons problem in all periods. If trade occurs in equilibrium, at least one agent acquires information.

5.1. Two period alternating offer bargaining

The buyer makes the offer in the first period and the seller makes the offer in the second period. For a formal statement of the following results as well as other results see Dang [10].

1. The equilibrium payoff of the buyer (first period proposer) can increase in the discount factor $\delta$ of the trading surplus. The intuition is the following. Suppose $2\delta \Delta < \beta c$ and the agents reach the second period without information acquisition. Then no trade occurs in the second period, too. In such a case the continuation payoff of the seller is zero. In the first period the buyer compares the following two alternatives.

   (a) If the buyer acquires information, he increases the continuation payoff of the seller from zero to $\delta \Delta$. If the seller rejects any offer in the first period and proposes $v_{H} + \delta \Delta$ in the second period, the informed buyer accepts this offer at $v = v_{H}$. Information acquisition exerts a positive externality. The informed buyer can obtain at most $EU^B = k(2\Lambda - \delta \Delta) - c$.

   (b) If the buyer does not acquire information but induces the seller to acquire information in the first period by just compensating him for the information cost $c$, the buyer can keep the continuation payoff of the seller at zero. The uninformed buyer accounts for the lemons problem. In the first period trade only occurs in the low state. If there is no trade, then in the second period the surplus $2\delta \Delta$ realizes with probability $k_1$ since the seller is informed. With an appropriate offer, the buyer can extract this surplus in the first period. His payoff is $EU^B = 0.5(2\Lambda + k_1 2\delta \Delta) - c = \Lambda + k_1 \delta \Delta - c$. For some parameter values, there exists a range for $\delta$, such that the buyer chooses this alternative and his equilibrium payoff increases in $\delta$.

2. If the discounting of the trading surplus is lower than the discounting of the information cost (i.e. $\delta > \beta$), then the equilibrium delay of information acquisition and trade is (constraint) efficient. To highlight the intuition, consider the extreme case where $\delta = 1$ and $\beta = 0$, i.e. information is free in the second period because there is a public announcement of $v$. The continuation payoff of the seller is $2\Lambda$. In the first period the buyer captures no surplus and does not acquire information. Due to the endogenous lemons problem no trade occurs in the first period. In the second period trade occurs and $EU^B = 0$ and $EU^S = 2\Delta$.\footnote{In this case, the delay of trade is not caused by signaling since there is symmetric information in the period of no trade but by an optimal timing argument subject to an endogenous lemons constraint. The bargaining literature provides as a dominant reason for delay a signaling or screening story due to asymmetric information. Admati and Perry [1] and Cramton [6] show that asymmetric information about the private valuation can cause delay. Evans [14] and Vincent [27] show that asymmetric information about the common valuation can lead to delay, too. See also Cho [5], Feinberg and Skrzypacz [15], Fernandez and Glaser [16] and Watson [28].}

3. If $\delta$ is in an intermediate range, then two period alternating offer bargaining may perform worse than ultimatum bargaining. If the agents are only allowed to bargain for one period, then the equilibrium payoffs are $EU^B = 2k\Lambda - c$ and $EU^S = 0$ (as in Proposition 5). If the bargaining
lasts for two period, then no trade may occur at all. The intuition is the following. Extending the length of bargaining may shift some bargaining power from the buyer to the seller. If both agents have not enough bargaining power so as to capture enough surplus to cover the information cost, then no agent acquires information and no trade occurs at all.

This observation is similar in flavor to Deneckere and Liang [12] who show that infinite horizon bargaining with common values where the seller is informed and the uninformed buyer makes all offers may perform worse than ultimatum bargaining. However, the reason is different. The intuition there is that for some parameter values of the model the uninformed buyer offers a long sequence of relatively low offers in equilibrium so that there may be long delay and the expected total payoff is lower than the one in ultimatum bargaining.

5.2. Infinite horizon bargaining

A potential difficulty in analyzing infinite horizon bargaining with endogenous information is that the continuation payoffs of the agents depend on the information acquisition as well as the discounting processes \( \{\Delta_t\} \) and \( \{c_t\} \) in a complex fashion. A reason is that information acquisition exerts a positive externality by changing the probability of trade and therefore the continuation payoff of the counter party. Three cases are discussed briefly.

(i) If \( 2\Delta < c \) and \( \delta \leq \beta \), then the agents never reach an agreement. (ii) If \( 2\Delta > c \) and \( \delta < \beta \), then the analysis of information acquisition reduces to a finite consideration. Since \( \delta < \beta \), there exists a \( t^* \) such that \( 2\Delta_t < c_t \) for \( t > t^* \). If no agent acquires information in any period \( t \leq t^* \), no trade will occur at all. So one must start with a finite backward induction argument in period \( t^* \) and go through all paths to determine information acquisition. However, once information is acquired, the game switches back to the infinite horizon version.

(iii) Suppose \( 2\Delta > c \), \( \delta = \beta \), and one agent makes all offers. Even this case may represent a non-trivial extension of Deneckere and Liang [12]. Since the responder faces a potential hold-up problem, the proposer will eventually acquire information first. Therefore, exogenously imposing the assumption that the uninformed agent makes all offers to circumvent an equilibrium selection problem may be inconsistent with equilibrium behaviors if information is endogenous.

6. Conclusion

This paper analyses ultimatum bargaining with endogenous information and common values and shows that information acquisition can cause an endogenous lemons problem and implies that the bargaining positions of the agents are endogenous. Depending on the information cost, perfect Bayesian equilibria may have the following properties: (1) No trade occurs although the agents maintain symmetric information in equilibrium and the gain from trade is common knowledge. (2) The agent responding to a take-it-or-leave-it-offer captures some or even the full trading surplus. (3) The proposer acquires information and trade only occurs with positive probability. The implications for two period alternating offer bargaining are discussed. Infinite horizon bargaining with endogenous information remains to be explored.

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This Appendix proves Propositions 1–5 together. The proof proceeds as follows. Step 1 analyses
the best response correspondence of the seller to \((n_B, b) = (0, b)\). Step 2 analyzes the buyer’s payoff
expectations at \(n_B = 0\) and different \(b\), ensuring best responses of the seller. Step 3 analyses the
best response correspondences for the case where \(n_B = 1\). Step 4 characterizes the decision of
the buyer. Step 5 summarizes the PBE paths.

Step 1: This step analyzes the best response correspondence of the seller to \((n_B, b) = (0, b)\).
If the seller does not acquire information, his strategy is denoted with \((n_S, s) = (0, s)\). If the
seller acquires information, his strategy is denoted with \((n_S, s_L, s_H) = (1, s_L, s_H)\) where \(s_L\) and
\(s_H\) describe his responses when seeing \(v_L\) and \(v_H\), respectively.

Step 1a: Case 1: If \(b < v_L - \Delta\), then the seller never wants to sell, so he has nothing to gain from
buying information. The best response to \((0, b)\) with \(b < v_L - \Delta\) is given by \((0, s)\) where \(s = N\).

Case 2: Suppose \(v_L - \Delta < b < \frac{1}{2}(v_H + v_L) - \Delta\). (a) If the seller acquires no information,
he can only loose from trading, so \((0, s)\) with \(s = Y\) is a dominated choice. His maximal payoff
without information acquisition is therefore \(EU^S = 0\). (b) If the seller buys information, then
he chooses \(s_H = N\) and \(s_L = Y\). His maximal payoff with information acquisition is \(EU^S = \frac{1}{2}\left[b - (v_L - \Delta)\right] - c\). Consequently, if \(\frac{1}{2}(b - v_L + \Delta) - c < 0\), then the seller chooses \((0, N)\).

If \(\frac{1}{2}(b - v_L + \Delta) - c > 0\), then he chooses \((1, Y, N)\). If \(\frac{1}{2}(b - v_L + \Delta) - c = 0\), the set of best responses
of the seller is given by \((0, N)\) and \((1, Y, N)\).

Case 3: Suppose \(b = \frac{1}{2}(v_H + v_L) - \Delta\). A similar argument as above shows that if \(\frac{1}{2}(b - v_L + \Delta) - c < 0\), the set of best response of the seller is given by \((0, N)\) and \((0, Y)\). In both cases
\(EU^S = 0\). For \(\frac{1}{2}(b - v_L + \Delta) - c > 0\), the best responses of the seller are given as in Case 2.

Case 4: Suppose \(\frac{1}{2}(v_H + v_L) - \Delta < b < v_H - \Delta\). (a) If the seller acquires no information,
his payoff is \(EU^S = b - \frac{1}{2}(v_H + v_L) + \Delta\). (b) If the seller buys information, then he chooses \(s_L = Y\) and \(s_H = N\). His maximal payoff with information acquisition is \(EU^S = \frac{1}{2}\left[b - (v_L - \Delta)\right] - c\), as before. It follows that if \(\frac{1}{2}(b - v_L + \Delta) - c < \frac{1}{2}(v_H + v_L) + \Delta\), the best response of the seller is given by \((0, Y)\). If \(\frac{1}{2}(b - v_L + \Delta) - c > \frac{1}{2}(v_H + v_L) + \Delta\), then he chooses \((1, Y, N)\). Otherwise the seller is indifferent between the two
responses.

Case 5: Suppose \(b = v_H - \Delta\). (a) If the seller buys information, he chooses \(s_L = Y\) and he is
willing to choose \(s_H = Y\), allowing a trade to occur albeit without any net gain to himself. His
payoff is \(EU^S = \frac{1}{2}(b - v_L + \Delta) + \frac{1}{2}(b - v_H + \Delta) - c = b - \frac{1}{2}(v_L + v_H) + \Delta - c\). (b) If the seller
does not acquire information, he chooses \(s = Y\) and \(EU^S = b - \frac{1}{2}(v_L + v_H) + \Delta\). Consequently,
buying information is a dominated action. His best response is to choose \((0, Y)\).

Case 6: Suppose \(b > v_H - \Delta\). The same argument as in Case 5 shows that the seller’s best
response to \((0, b)\) with \(b > v_H - \Delta\) is to choose \((0, Y)\).

Step 1b: Step 1a shows that in Cases 1, 5, and 6 the information acquisition best response of
the seller is not to acquire information. Only if \(v_L - \Delta < b < v_H - \Delta\), it is potentially worthwhile
to acquire information. In Cases 2 and 3, the information acquisition decision turns on whether

\[
\frac{1}{2}(b - v_L + \Delta) - c = 0, \tag{1}
\]
in Case 4 on whether
\[ \frac{1}{2}(b - v_L + \Delta) - c = b - \frac{1}{2}(v_H + v_L) + \Delta. \]  
(2)

Given that the left-hand side of (1) is increasing in \( b \) and the difference between the left-hand side and the right-hand side of (2) is decreasing in \( b \), information acquisition is not attractive at any price \( b \) if it is not attractive at \( b = \frac{1}{2}(v_H + v_L) - \Delta \), the upper bound of the interval defining Cases 2 and 3 and the lower bound of the interval defining Case 4. Substituting \( b = \frac{1}{2}(v_H + v_L) - \Delta \) into the left-hand side of (2) yields \( \frac{1}{4}(v_H - v_L) - c \). There are three possibilities.

**Alternative I:** \( c > \frac{1}{4}(v_H - v_L) \). In this case, at \( b = \frac{1}{2}(v_H + v_L) - \Delta \), information acquisition is not worthwhile, i.e.
\[ \frac{1}{2}\left[ \frac{1}{2}(v_H + v_L) - \Delta - v_L + \Delta \right] - c = \frac{1}{4}(v_H - v_L) - c < 0 \]
and
\[ \frac{1}{4}(v_H + v_L) - c < \frac{1}{2}(v_H + v_L) - \Delta - \frac{1}{2}(v_H + v_L) + \Delta. \]

So if \( c > \frac{1}{4}(v_H - v_L) \), then information acquisition is not worthwhile to the seller regardless of what price he expects the uninformed buyer to set. The seller’s best response to \((0, s)\) is to choose \((0, s)\) where (i) \( s = N \) if \( b < \frac{1}{2}(v_H + v_L) - \Delta \), (ii) \( s = Y \) or \( s = N \) if \( b = \frac{1}{2}(v_H + v_L) - \Delta \), and (iii) \( s = Y \) if \( b > \frac{1}{2}(v_H + v_L) - \Delta \).

**Alternative II:** \( c < \frac{1}{4}(v_H - v_L) \). In this case, at \( b = \frac{1}{2}(v_H + v_L) - \Delta \), information acquisition is worthwhile, i.e.
\[ \frac{1}{2}\left[ \frac{1}{2}(v_H + v_L) - \Delta - v_L + \Delta \right] - c = \frac{1}{4}(v_H - v_L) - c > 0 \]
and
\[ \frac{1}{4}(v_H + v_L) - c > \frac{1}{2}(v_H + v_L) - \Delta - \frac{1}{2}(v_H + v_L) + \Delta. \]

Denote \( \underline{b} \) as the price where the left-hand side of (1) is just zero and \( \overline{b} \) where the left-hand side equals the right-hand side of (2). There exist critical prices
\[ b = v_L - \Delta + 2c < \frac{1}{2}(v_H + v_L) - \Delta \]
and
\[ \overline{b} = v_H - \Delta - 2c > 2(v_H + v_L) - \Delta. \]

such that information acquisition is not worthwhile to the seller if the buyer sets \( b < \underline{b} \) or \( b > \overline{b} \). If the buyer sets \( b \in (\underline{b}, \overline{b}) \), then it is worthwhile to the seller to acquire information. (At \( \underline{b} \) and \( \overline{b} \), the seller is indifferent.)

(i) The seller’s best response to \((0, b)\) with \( b < \underline{b} \) or \( b > \overline{b} \) is to choose \((0, s)\) where \( s = N \) if \( b < \underline{b} \) and \( s = Y \) if \( b > \overline{b} \). (ii) The seller’s best response to \((0, b)\) with \( b \in (\underline{b}, \overline{b}) \), is to choose \((1, Y, N)\). (iii) For \( b = \underline{b} \), the seller is indifferent between \((0, N)\) and \((1, Y, N)\). (iv) For \( b = \overline{b} \), the seller is indifferent between \((0, Y)\) and \((1, Y, N)\).

**Alternative III:** \( c = \frac{1}{4}(v_H - v_L) \). This is the boundary between Alternatives I and II. For \( b = \frac{1}{2}(v_H + v_L) - \Delta \), the seller is indifferent between \((0, Y)\), \((0, N)\), and \((1, Y, N)\). For \( b \neq \frac{1}{2}(v_H + v_L) - \Delta \), the best response of the seller is to choose \((0, s)\) where \( s = N \) if \( b < \frac{1}{2}(v_H + v_L) - \Delta \), and \( s = Y \) if \( b > \frac{1}{2}(v_H + v_L) - \Delta \).
Step 2: This step analyses the buyer’s payoff expectations at $n_B = 0$ and $b$, ensuring best responses of the seller. As it is customary, the subsequent steps assume that if the responder is indifferent, he chooses a response from his set of best responses which the proposer prefers most.

Alternative I: $c > \frac{1}{4}(v_H - v_L)$. Given the best responses of the seller, the buyer’s payoff is zero if $b < \frac{1}{2}(v_H + v_L) - \Delta$ or if $b = \frac{1}{2}(v_H + v_L) - \Delta$ and $s = N$. The buyer’s payoff is $EU^B = \frac{1}{2}(v_L + v_H) + \Delta - b$ if $b = \frac{1}{2}(v_H + v_L) - \Delta$ and $s = Y$ or $b > \frac{1}{2}(v_H + v_L) - \Delta$.

Thus, by setting $b = \frac{1}{2}(v_L + v_H)$, the buyer can ensure himself the payoff $\Delta$. All $(0, b)$ with $b < \frac{1}{2}(v_H + v_L) - \Delta$ provides the buyer with a lower payoff than $(0, b)$ with $b = \frac{1}{2}(v_H + v_L)$. Similarly, all $(0, b)$ with $b > \frac{1}{2}(v_H + v_L) - \Delta$ provides the buyer with a worse payoff than $(0, b')$ where $\frac{1}{2}(v_H + v_L) - \Delta < b' < b$.

The only strategy without information acquisition of the buyer which is a candidate for being best response to a subform perfect strategy of the seller is thus given by $(0, b)$ with $b = \frac{1}{2}(v_H + v_L) - \Delta$. However, if this is to be best response of the buyer, it must be the case, that the seller’s response to this choice is to set $(0, Y)$, i.e. the seller must resolve his indifference by opting for trade. In this case $EU^B = 2\Delta$.

Alternative II: $c < \frac{1}{4}(v_H - v_L)$. Cases 1 and 2a: (i) If the buyer chooses $(0, b)$ with $b < b = v_L - \Delta + 2c$, the seller chooses $(0, N)$. (ii) If the buyer chooses $(0, b)$ with $b = b$, the seller is indifferent between $(0, N)$ and $(1, Y, N)$. Depending on which alternative the seller chooses, the buyer’s payoff is $EU^B = 0$ or $EU^B = \frac{1}{2}(v_L + \Delta - b) = \Delta - c$.

Cases 2b, 3, 4a: (i) If the buyer chooses $(0, b)$ with $b = v_L - \Delta + 2c < b < v_H - \Delta - 2c$, the seller chooses $(1, Y, N)$ and $EU^B = \frac{1}{2}(v_L + \Delta - b)$. (ii) If the buyer chooses $(0, b)$ with $b = b$, the seller is indifferent between choosing $(0, Y)$ and $(1, Y, N)$. If the seller chooses the first response, then $EU^B = \frac{1}{2}(v_L + v_H) + \Delta - b = 2\Delta + 2c - \frac{1}{2}(v_H - v_L)$. If the seller chooses the second response, then $EU^B = \frac{1}{2}(v_L + \Delta - b) = \Delta + c - \frac{1}{2}(v_H - v_L)$. So the buyer has a strict preference to have the seller resolve his indifference by not acquiring information.

Cases 4b, 5, 6: If the buyer chooses $(0, b)$ with $b > b = v_H - \Delta - 2c$, the seller chooses $(0, Y)$ and $EU^B = \frac{1}{2}(v_L + v_H) + \Delta - b < 2\Delta + 2c - \frac{1}{2}(v_H - v_L)$.

Given these observations, any choice $(0, b)$ with $b > b$ is obviously worse for the buyer than the choice $(0, b)$ with $b = \frac{1}{2}(b + b)$. Similarly, any choice $(0, b)$ with $b < b < b$ is worse for the buyer than $(0, b)$ with $b = \frac{1}{2}(b + b)$; as is the choice $(0, b)$ with $b = b$ followed by information acquisition of the seller, i.e. $(1, s_L, s_H)$ with $s_L = Y$ and $s_H = N$.

The only strategies without information acquisition of the buyer which remain as possible candidates for being best responses to a subform perfect strategy of the seller are the following: (i) $(0, b)$ with $b = b$, assuming that this is followed by the seller choosing $(0, Y)$, (ii) $(0, b)$ with $b = b$, followed by $(1, Y, N)$, (iii) $(0, b)$ with $b < b$, followed by $(0, N)$. Path (i) implies $EU^B = 2\Delta + 2c - \frac{1}{2}(v_H - v_L)$, path (ii) implies $EU^B = \Delta - c$, and path (iii) implies $EU^B = 0$.

Alternative III: $c = \frac{1}{4}(v_H - v_L)$. As above, the only strategy without information acquisition of the buyer which is a candidate for being best response to a subform perfect strategy of the seller is given by $(0, b)$ with $b = \frac{1}{2}(v_H + v_L) - \Delta$, assuming that the seller chooses $(0, Y)$. Then $EU^B = 2\Delta$ and $EUS^B = 0$.

Step 3: This step analyses best responses for the case where the buyer chooses $n_B = 1$:

(a) The following arguments show that no best responses in pure strategies exist once the buyer acquires information. Suppose the informed buyer is honest and chooses $b = v_L - \Delta$ at $v_L$ and $b = v_H - \Delta$ at $v_H$. In this case the seller is willing to choose $s = Y$. However, if the seller always chooses $s = Y$, the buyer has an incentive always to choose $b = v_L - \Delta$. (If the seller always
chooses \( s = N \) when seeing \( b < v_H - \Delta \) then the buyer always chooses \( b = v_H - \Delta \) if \( v = v_H \). In this case seeing \( b = v_L - \Delta \) is fully revealing of \( v_L \) and the seller may choose \( s = Y \).

(b) It is easy to see that choosing \( s = Y \) when seeing \( b = v_L - \Delta \) is a weakly dominated strategy. The seller never gets some surplus but may suffer a lemons problem.

**Step 3a (Mixed strategies):** Define \( b_L \equiv v_L - \Delta + z \) for \( 0 < z \leq 2\Delta \) and \( b_H \equiv v_H - \Delta \). (Note, the informed buyer does not choose \( b > v_L + \Delta \) if \( v = v_L \). So any \( b > v_L + \Delta \) reveals that \( v \neq v_L \).

1. Suppose the buyer considers the following strategy. If the buyer sees \( v = v_H \), then he chooses \( b = b_H \) with probability \( 1 - \gamma_1 \), and \( b = b_L \) with probability \( \gamma_1 \) (where \( b_L \) and \( b_H \) are as defined above). If he sees \( v = v_L \), then he chooses \( b = b_L \).

2. Suppose the seller considers the following strategies. If the seller sees \( b_H \), he chooses \( s = Y \.

If he sees \( b_L \), two cases arises. (a) If \( z \geq 2c \), he may choose \( n_S = 1 \) with probability \( 1 - \beta \), and \( n_S = 0 \) with probability \( \beta \). An informed seller chooses \( s = Y \) if \( v = v_L \); and \( s = N \) if \( v = v_H \). An uninformed seller chooses \( s = Y \) with probability \( \gamma_1 \) and \( s = N \) with probability \( 1 - \gamma_1 \). (b) If \( z < 2c \), he chooses \( n_S = 0 \) and \( s = Y \) with probability \( \gamma_0 \) and \( s = N \) with probability \( 1 - \gamma_0 \).

**Step 3b (Making the buyer indifferent at \( v = v_H \)):** (1) Suppose the seller chooses \( n_S = 0 \) and randomizes his decision as described above. At \( v = v_H \), if the buyer chooses \( b = b_H \), then his payoff is \( U^B = 2\Delta \). If the buyer chooses \( b = b_L \), then \( EU^B = \gamma_0 \cdot (v_H + \Delta - (v_L - \Delta + z)) \). The buyer is indifferent between choosing \( b = b_L \) and \( b = b_H \) at \( v = v_H \) if \( \gamma_0 \cdot (v_H + \Delta - (v_L - \Delta + z)) \equiv 2\Delta \). (Note, \( c \) is sunk at this stage.) In order to make the buyer indifferent the seller chooses \( \gamma_0 \equiv 2\Delta/(v_H - v_L + 2\Delta - z) \).

2. Suppose the seller chooses \( n_S = 1 \) with probability \( 1 - \beta \); and \( n_S = 0 \) with probability \( \beta \) and randomizes his decision as described above. In this case, at \( v_H \) if the buyer chooses \( b = b_L \) then \( EU^B = (1 - \beta) \cdot 0 + \beta \cdot \gamma_1 \cdot (v_H + \Delta - (v_L - \Delta + z)) \). The buyer is indifferent between choosing \( b = b_L \) and \( b = b_H \) at \( v = v_H \) if \( \beta \cdot \gamma_1 (v_H - v_L + 2\Delta - z) = 2\Delta \). In order to make the buyer indifferent the seller chooses \( \beta \cdot \gamma_1 = 2\Delta/(v_H - v_L + 2\Delta - z) \).

**Step 3c (Making the seller indifferent when seeing \( b = b_L \)):** Case 1: \( z < 2c \). The seller never chooses \( n_S = 1 \); see Case 2 below. If the uninformed seller sees \( b_L \) and chooses \( s = Y \), then \( EU^S = \frac{1}{2} [v_L - \Delta + z - (v_L - \Delta)] + \frac{1}{2} z_0 \cdot (v_H - v_L + 2\Delta - z) \). If the seller chooses \( s = N \), then \( U^S = 0 \). In order to make the seller indifferent the buyer chooses \( z_0 = z/(v_H - v_L - z) \).

Case 2: \( z \geq 2c \). The seller may choose \( n_S = 1 \). If the seller chooses \( n_S = 1 \) and sees \( v_L \), then he chooses \( s = Y \). Otherwise he chooses \( s = N \). \( EU^S = \frac{1}{2} [v_L - \Delta + z - (v_L - \Delta)] - c = \frac{1}{2} z - c \). If the seller chooses \( n_S = 0 \) then \( EU^S = \frac{1}{2} z + \frac{1}{2} z_1 (v_L - v_H + z) \). The seller is indifferent between \( n_S = 0 \) and \( n_S = 1 \) if \( z - c = \frac{1}{2} z + \frac{1}{2} z_1 (v_L - v_H + z) \). In order to make the seller indifferent the buyer chooses \( z_1 = 2c/(v_H - v_L - z) \).

**Step 3d (Choosing the optimal \( z \)):** Case 1: \( z < 2c \). The expected payoff of the buyer is

\[
EU^B = \frac{1}{2} \gamma_0 [v_L + \Delta - (v_L - \Delta + z)] + \frac{1}{2} [(1 - z_0)(v_H + \Delta - (v_H - \Delta)) + z_0 \gamma_0 (v_H + \Delta - (v_L - \Delta + z))] - c = \Delta + (2\Delta - z)/(v_H - v_L + 2\Delta - z) - c.
\]

Case 2: \( z \geq 2c \). The expected payoff of the buyer is

\[
EU^B = \frac{1}{2} \beta \gamma_1 [v_L + \Delta - (v_L - \Delta + z)] + \frac{1}{2} [(1 - z_1)(v_H + \Delta - (v_H - \Delta)) + z_1 \beta \gamma_1 (v_H + \Delta - (v_L - \Delta + z))] - c = \Delta + (2\Delta - z)/(v_H - v_L + 2\Delta - z) + \Delta - c.
\]
Both responses of the seller yield the same payoff to the buyer. The buyer chooses \( z \in (0, 2\Delta) \) to maximize his payoff and therefore,

\[
z = \frac{1}{2} (-v_H + v_L - \Delta) + \sqrt{\frac{1}{4} (-v_H + v_L - \Delta)^2 + \Delta (v_H - v_L) + \Delta^2} > 0.
\]

Define \( k \) such that \( 2k\Delta = \Delta + \Delta(2\Delta - z)/(v_H - v_L + 2\Delta - z) \) then

\[
k = \frac{1}{2} + \frac{1}{2}(2\Delta - z)/(v_H - v_L + 2\Delta - z).
\]

So the payoff of the buyer is \( EU^B = 2k\Delta - c \). \(^{13}\)

**Step 4 (The buyer’s decision): Alternative I, II: \( c \geq \frac{1}{4}(v_H - v_L) \).** The best response of the buyer is to choose \( n_B = 0 \) and \( b = \frac{1}{2}(v_H + v_L) - \Delta \), assuming that this is followed by \( n_S = 0 \) and \( s = Y \). Trade occurs with probability 1 and \( EU^B = 2\Delta \) and \( EU^S = 0 \).

**Alternative III: \( c < \frac{1}{4}(v_H - v_L) \).** The set of candidates without information acquisition for being best responses is the following: (a) \( n_B = 0 \) and \( b = v_H - \Delta - 2c \), assuming it is followed by \( n_S = 0 \) and \( s = Y \). (b) \( n_B = 0 \) and \( b = v_L - \Delta + 2c \), assuming it is followed by \( n_S = 1 \) and \( s_L = Y \) and \( s_H = N \). (c) \( n_B = 0 \) and \( b = v_L - \Delta + 2c \), followed by \( n_S = 0 \) and \( s = N \).

(d) A candidate with information acquisition for being best responses is described in Step 3. The buyer’s payoff is given as follows: (a) \( 2\Delta + 2c - \frac{1}{4}(v_H - v_L) \), (b) \( \Delta - c \), (c) 0, and (d) \( 2k\Delta - c \). (Since \( k > 0.5 \), strategy (d) dominates strategy (b)).

Case 1: \( c > \frac{2}{3}\Delta(k-1) + \frac{1}{6}(v_H - v_L) \). Strategy (a) dominates strategy (d). So the buyer compares strategy (a) with (c). If \( c < \frac{1}{4}(v_H - v_L) - \Delta \), the buyer chooses strategy (c). If \( c > \frac{1}{4}(v_H - v_L) - \Delta \), the buyer chooses strategy (a). If \( c = \frac{1}{4}(v_H - v_L) - \Delta \), the buyer is indifferent between the two choices.

Case 2: \( c < \frac{2}{3}\Delta(k-1) + \frac{1}{6}(v_H - v_L) \). Strategy (d) dominates (a). The buyer compares strategy (d) with (c). If \( c > 2k\Delta \), the buyer chooses strategy (c). If \( c < 2k\Delta \), the buyer chooses (d). If \( c = 2k\Delta \) the buyer is indifferent between the two strategies.

Case 3: \( c = \frac{2}{3}\Delta(k-1) + \frac{1}{6}(v_H - v_L) \).

The buyer is indifferent between strategy (a) and (d). So the buyer compares (a,d) with (c). If \( c > 2k\Delta \) the buyer chooses (c). If \( c < 2k\Delta \), the buyer is indifferent between the alternatives (a) and (d). If \( c = 2k\Delta \) the buyer is indifferent between the three strategies.

**Step 5 (Equilibrium paths):** Summarizing Step 4 yields the Propositions 1–5 which describe all potential PBE paths. In Proposition 2 the buyer chooses the prescribed strategy if \( \frac{2}{3}\Delta(k-1) + \frac{1}{6}(v_H - v_L) < c < \min(2k\Delta, \frac{1}{4}(v_H - v_L) - \Delta) \) or \( \max(2k\Delta, \frac{1}{4}(v_H - v_L) - \Delta) < c < \frac{1}{4}(v_H - v_L) \). Combining the two conditions yields the condition in Proposition 2. The seller’s payoff is \( EU^S = b - (E[v] - \Delta) = v_H - \Delta - 2c - (\frac{1}{2}(v_L + v_H) - \Delta) = \frac{1}{2}(v_H - v_L) - 2c \). In Proposition 3 two types of PBE exist. (i) The buyer chooses \( n_B = 0 \) and \( b < v_L - \Delta + 2c \) and the seller chooses \( n_S = 0 \) and \( s = N \). No trade occurs. (ii) The buyer chooses \( n_B = 0 \) and \( b = v_H - \Delta - 2c = \frac{1}{2}(v_H + v_L) + \Delta \) and the seller chooses \( n_S = 0 \) and \( s = Y \). Trade occurs and \( EU^B = 0 \) and \( EU^S = \frac{1}{2}(v_H - v_L) - 2c = 2\Delta \).

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\(^{13}\) Note, \( k \) is also the probability of trade \( \frac{1}{2} \gamma_0 + \frac{1}{2}(1 - \gamma_0) \gamma_0 \gamma_0 = \frac{1}{2} + \frac{1}{2} \gamma_0 - \gamma_0 \gamma_0 \). If \( v_H - v_L \) is large, then the optimal \( z > 2\Delta \). In this case the buyer wants to choose \( z \) as close as possible to 2\( \Delta \). So no best response exists.
References