Redistricting and Representation: The Paradox of Minority Power*

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Abstract

We present a model that integrates electoral competition of majority-minority redistricting with legislative redistribution to optimize minority representation. Analyzing voter allocation's impact, we find that minorities with limited political power benefit from concentrated districts, while stronger minorities prefer dispersed voter distributions. Majority voters voting for minorities has two effects: it helps minorities gain offices, but it may increase majority voters' influence and policy benefits. Paradoxically, adding minorities to a district is non-monotonic and can result in representatives less favored by minorities. The interplay between redistricting, electoral competition, and policy distribution offers novel insights into equitable minority representation and public policy.

Keywords: Redistricting, Minority Representation, Distributive Politics, Electoral Competition

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1 Introduction

Two prominent themes in political economy are the dynamics of electoral representation and the processes underlying policymaking. The first focuses on electoral competition, influencing who gets elected and thus represented. The second revolves around legislative institutions, shaping policy selection and the distribution of economic benefits. Traditionally, these themes have stood in isolation. In contrast, our paper aims to bridge this analytical gap, examining the effects of electoral districting rules on representation and the role of legislative bargaining in converting policy preferences into legislative outcomes.

We apply our integrated approach to investigate how electoral and legislative institutions affect the representation and economic policy benefits of minority communities, yielding fresh perspectives on democratic governance and equitable distribution. The analysis raises questions about the optimal distribution of minority voters across districts, the trade-offs between descriptive (identity-based) and substantive (policy-based) representation, and the development of a generalizable methodology to estimate these effects.

This paper presents a novel formal characterization of gerrymandering, aiming to describe benefits for diverse voter groups through a combination of electoral strategies and legislative bargaining. In particular, our analysis integrates the strategic forces in electoral competition with distributive and ideological benefits (Dixit and Londregan, 1996), legislative bargaining (Baron and Ferejohn, 1989), and voter group competition in a primary and general election (Snyder and Ting, 2011), focusing on how these elements interact to optimize minority benefits in a political environment characterized by partisanship and identity.

The model begins at the onset of the election cycle when candidates announce redistributive platforms targeting specific groups within their districts. We then examine the probabilistic outcomes of primary and general elections, considering how these influence subsequent legislative bargaining and the distribution of redistributive benefits. The process culminates in a social planner optimizing district designs to maximize minority group utility across political jurisdictions.

Our integrative approach allows us to simulate the effects of voter groups shifting along partisanship and identity lines on optimal districting schemes and voter payoffs. We analyze the dynamics of voter groups trading ideological alignment for distributive benefits and assess the resulting influence on electoral and legislative outcomes. The more likely a voter group can be swayed by a party and its candidates with distributive benefits, in a sense, the "swinginess" of the group along partisanship or identity, the more competition arises for its voters and the greater the group's influence in a district.

An example illustrates our point.[1] Let us suppose that a state designs a district with 40% minority and 60% non-minority voters, of which 80% of non-minority voters are Republicans. The district's demographics then are 52% Democrats and 48% Republicans. Further, assume that no non-minority voter crosses over and votes for a minority candidate in the primary or general elections. In this case, the minority candidate wins the Democratic primary, the non-minority wins the Republican primary, and the Republicans win the general election. Now let 20% of non-minorities cross over and vote for the minority candidate in the primary and general elections. Again, the minority wins the Democratic primary, the non-minority wins the Republican primary, but now the minority candidate wins the election. While adding non-minority Republican voters to concentrated minority districts helps dilute Republicans' influence in surrounding districts, crossover voting is still necessary for minorities to win the general election.[2]

Our findings uncover counterintuitive results with significant policy implications. The results show that minorities with relatively little political power prefer to concentrate their voters in a few districts and shift the weight of the bargaining problem to the legislature. Conversely, as minorities gain power, they do best by distributing their voters more evenly across districts. Paradoxically, powerful minorities may be better off sharing districts with members of the majority least predisposed to their interests. Furthermore, declining majority racism has two competing effects on minorities. It helps them by making it easier to elect minorities to office; it can also hurt them by making majority voters more influential, increasing their relative power at minorities' expense. In addition, adding more minority voters to a given district is non-monotonic and, in some cases, can have the perverse effect of electing a candidate less favored by the minority community.

This paper contributes to the robust literature on minority representation by defining the districting problem in a spatial context, developing a formal model, and solving the social planner's districting problem. We then provide simulation results to evaluate the robustness of our analysis under various scenarios. Our concluding section discusses the policy implications of our results and analyzes optimal districting schemes under varying conditions, such as changes in minority voter registration and partisan polarization.

[1]Figure 11 in Appendix B.1 illustrates the example with our simplex methodology presented in Section 2.
[2]We show below that optimizing minority gains depends on the district's composition and the election's influence.

1.1 Related Literature: Redistricting and Representation

This review examines the interplay between empirical findings and theoretical advancements in redistricting and representation, underscoring the inherent link between electoral politics and legislative outcomes. For the most part, the research on redistricting's effect on political and economic outcomes bifurcates into studies on partisan politics and ones addressing racial/ethnic minority groups. The former includes seminal works on seats-votes curve biases (Tufte, 1973; King, 1989; Gelman and King, 1990; Lublin et al., 2020), incumbent protection (Cox and Katz, 2002; Ansolabehere and Snyder Jr., 2004), and party power consolidation (Butler and Cain, 1991; Issacharoff, 2002; Persily, 2002).

The literature on racial redistricting, on the other hand, explores the impact on Black office-holding in the South (Davidson and Grofman, 1994), the descriptive vs. substantive representation tradeoff (Cameron et al., 1996; Epstein and O'Halloran, 1999; Lublin, 1997b), its role in Congressional partisan shifts (Lublin and Voss, 2000), and voting polarization and segregation (Stephanopoulos, 2016). Another vein of research underscores the impact of districting on electoral competition and policy outcomes. For instance, districting schemes can allocate minority voters across many districts (known as "cracking"), maximizing the number of elected officials they can directly influence via the ballot box. Alternatively, districting strategies can concentrate minority voters (known as "packing") in fewer districts, shifting the weight of coalition building to the legislature.

Rigorous empirical studies, such as Jeong and Shenoy (2022), document "packing-and-cracking" gerrymandering tactics against African American voters, who typically support Democratic candidates. They show that after a Republican redistricting, minority voters are more likely to be segregated into black districts. Cameron et al. (1996) and Lublin (1997a) argue that such concentrated majority-minority districts might inadvertently diminish minority policy impact. Canon (1999) contests this point, emphasizing minority legislators' behind-the-scenes influence in policy-making.[3] The key point of contention revolves around the extent of white-crossover voting. While Ansolabehere et al. (2010) argue for its prevalence, Lublin et al. (2020) claim it is declining, suggesting that minorities' power lies more in optimal voter composition than in winning white votes. Regardless, such a strategy results in highly polarized districts, where no non-minority candidate

[3] Issacharoff (2002); Grofman et al. (1992); Epstein and O'Halloran (2006) note that such back-room influence is, however, difficult to measure, suggesting that the promotion of majority-minority districts via Section 5 of the Voting Rights Act may undermine minority influence, even as it increases the number of Republicans elected to office.

wins the Democratic primary, and no minority candidate wins in the Republican primary. Despite its richness, the empirical evidence provides little relief in assessing optimal districting strategies as conflicting redistricting measures and minority incorporation continue to prevail. This limitation is particularly acute with the Supreme Court's reticence to review partisan districting claims, making all potential challenges about race (Tofighbakhsh, 2020).

Formal redistricting models tend to prioritize partisan over racial divisions, with studies like Musgrove (1977) and Owen and Grofman (1988) focusing on maximizing majority party seats. Kolotilin and Wolitzky (2020) adopts a novel methodology that sorts voter types into districts and identifies the optimal conditions for packing or cracking districts when allocating voters. Bouton et al. (2023) analyze strategic incentives of partisan gerrymandering when voter turnout varies across parties and test their predictions with the US redistricting cycle of 2020. They document how parties benefit from mixing supporters with low turnout, opponents with high turnout, and supporters and opponents with intermediate turnout. Coate and Knight (2007) provides a more subtle analysis of partisan redistricting, where partisans and independents calculate the districting schemes that maximize overall social utility.

Notably, racial considerations in redistricting are less frequently addressed. Exceptions include Shotts (2001), who presents a model of racial redistricting with partisan control of the redistricting process. The analysis finds that majority-minority district requirements do not affect liberal gerrymanders; they can, however, limit the options of conservative districters. Friedman and Holden (2008) offers a model of this tradeoff, analyzing optimal redistricting where a gerrymanderer observes a noisy signal of voter preferences. The study demonstrates that cracking districts and spreading minorities across many districts is never optimal.

The above models draw political boundaries by allocating voters to districts to maximize social welfare, measured against the median voter's preferred electoral and policy outcomes. Nevertheless, these studies do not consider the nature of coalition formation and aggregation bias that may occur in the policymaking process at the electoral and legislative stages, leaving the mechanism of how minorities wield power incomplete. This omission is significant because political competition affects economic benefits and the identity of the officeholder. Furthermore, how the political process aggregates voter preferences can tip the gains of electoral redistricting plans from one group to another.

Myerson (1993) highlights these insights, developing a model of strategic political competition, where individuals and groups seek economic benefits through the political process at the expense of other groups' policy goals by "rent-seeking." Similarly, Lindbeck and Weibull (1987, 1993) builds a model of economic redistribution that underscores the role of voter preferences in shaping policy outcomes, asserting that voter altruism, risk aversion, and income redistribution can also influence distributive policies. And Dixit and Londregan (1996) proposes a redistributive model that incorporates economic inequality across different groups of voters and shows how variations in average income impact the allocation of economic benefits.

Our study introduces a novel theoretical framework to analyze the effects of redistricting strategies on electoral outcomes, legislative bargaining, and the equitable distribution of benefits, with a specific focus on minority representation. Our approach addresses gaps in the literature by analyzing tradeoffs between descriptive (electing minority candidates) and substantive representation (enacting policies beneficial to minorities) through the lens of politicians' strategic policy choices and the complex dynamics of voter behavior. This approach enables the simulation of districting maps optimized for enhancing minority benefits, shedding light on the strategic nuances of redistricting and the impacts of redistricting on coalition formations and voter alignments across party lines. The insights garnered from our study offer a deeper understanding of how redistricting strategies influence minority group representation and economic well-being, contributing significantly to the discourse on maintaining a robust and equitable democratic system.

After placing our analysis within the vibrant literature on redistricting and minority representation, we present the districting problem in a spatial context and offer initial results in Section 2. Building on this intuition, we develop the formal model in Section 3, provide strategies for the players in Section 4, and describe distributive and ideological benefits for minority voters in Section 5. We then solve the social planner's districting problem in Section 6 and illustrate various electoral maps. The concluding Section 7 explores the policy implications of our results, analyzing optimal districting schemes under varying circumstances, including increasing minority voter registration, increasing crossover along identity, and increased partisan polarization. In doing so, we offer valuable insights into the complexities of minority representation in a majoritarian democracy.

2 Triangles: Districting Made Simple(x)

In the following sections, we present a novel simplex approach to illustrate redistricting maps with three voter groups followed by our general model that merges models of electoral competition with a primary and general election between two candidates, legislative bargaining over distributive benefits allocated across districts and voter groups, and optimal districting. Both the simplex illustrations and the model's predictions are used to illustrate distributive, ideological, and total benefits and simulate redistricting maps that maximize minority benefits.

We first provide a novel graphical context to analyze districting alternatives. For simplicity, we take the electorate to consist of three groups: majority-Democrats (MD), minority-Democrats (mD), and Republicans (R). These three groups exist in specific proportions in the state overall, and we look at possible districting schemes that divide these voters into some equally sized districts within the state.

2.1 Districting Alternatives

A valid districting scheme is a matrix like the one illustrated in Table 1. Each row is a district, giving the number of MD, mD, and R voters (where each quantity is, naturally, non-negative). The sum of the entries in each row must equal the total population N divided by the number of districts K. Furthermore, the sums of the entries in each column are the group populations N_i. Of course, we can also express all values in percentages for each type.

$$\begin{array}{ccc}\left(\begin{array}{ccc} MD_1 & mD_1 & R_1 \\ MD_2 & mD_2 & R_2 \\ \vdots & \vdots & \vdots \\ MD_K & mD_K & R_K \end{array}\right) & \begin{array}{c} \frac{N}{K} \\ \frac{N}{K} \\ \vdots \\ \frac{N}{K} \end{array} \\ \begin{array}{ccc} N_{MD} & N_{mD} & N_R \end{array} & \end{array}$$

Table 1: Sample Districting Matrix.

To visualize the alternatives, we use an equilateral triangle, as in Figure 1, representing the two-dimensional simplex of possible percentages of each group in a given electorate. The corners thus indicate an electorate with only one type of voter: majority-Democrat in the bottom left, minority-Democrat in the bottom right, and Republican on top. The center point is an electorate with an equal division of all three types, each comprising one-third of the district population. We could also divide the triangle into four smaller triangles that highlight electorates with majorities of each group or electorates with no majority. The bottom left triangle indicates electorates with

MD majorities, the bottom right triangle one with mD majorities, and the top triangle one with R majorities. Hence, point b indicates a majority-minority electorate. The center triangle indicates no majority among the three groups, though they would be Democrat-majority due to the sum of minority- and majority-Democratic voters.

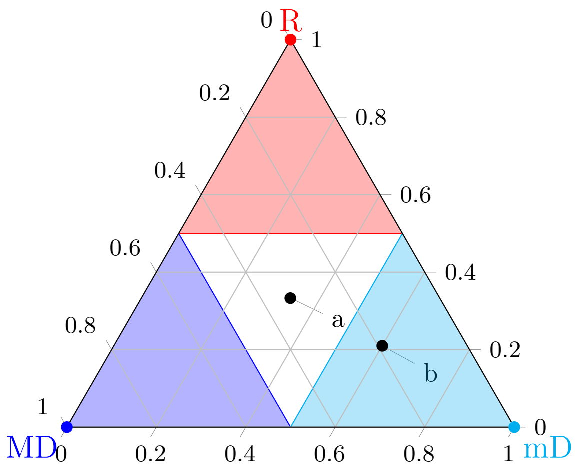


Figure 1: Possible Electorates.

State and Districts. The state has a given percent of each type of voter, so it can also be represented as a point on the triangle. Then, a valid districting scheme is a set of points that average the statewide population proportions.[4] Figure 2 illustrates a state with five districts. The statewide distribution of voters is marked by point S, while the other five points represent the districts, one of which is majority-minority.

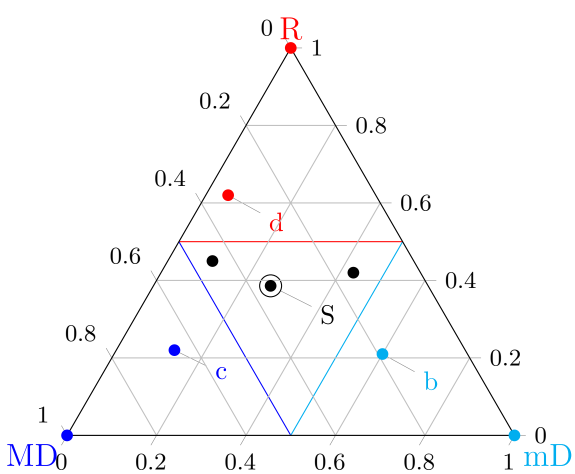


Figure 2: Sample State with Five Districts.

The figure also illustrates trade-offs in districting. Suppose the statewide distribution S favors

[4]We provide the numerical values of Figure 2 in Appendix B.2

a Democrat, most likely a majority-Democrat. Then by making all districts in a state have demographics equal to S — making them microcosms of the state as a whole — majority-Democrats will be the favorites in all district elections. But suppose the probability of success for a majority-Democrat in such a district is close to 50%. In that case, risk-averse legislators might prefer to make some districts heavily Democrat (toward the bottom of the triangle, such as point c or b), which comes at the expense of creating districts leaning heavily Republican (towards the top of the triangle, such as point d).

Similarly, the requirement that some majority-minority districts be created requires that these districts must be located near the bottom right, such as point b, pushing the other districts up and to the left, possibly increasing the likelihood that Republicans will win elsewhere– either decisively in a few districts (closer to R compared to S) or more likely in more districts (closer to S compared to R).

2.2 Voting and Elections

Given district characteristics, let us now turn to the question of which type of candidate will win. We analyze a two-stage electoral cycle, consisting of a primary pitting a majority-Democratic candidate against a minority-Democrat, with the winner facing a Republican in the general election. For now, and generalizing our assumptions later in Section 3, we make the following simplifying assumptions for our initial illustrations here.

1. In the primary election,
   (a) all mD voters cast their ballots for the mD candidate;
   (b) a fraction a of MD voters cross over to cast their ballots for the mD candidate, with $a \geq 0$.
2. In the general election,
   (a) all mD voters cast their ballots against the R candidate, so they vote for whichever type of Democrat won the primary;
   (b) all R voters cast their ballots for the R candidate;
   (c) with a mD candidate against an R candidate, a fraction b of MD voters cast their ballots for the mD candidate, with $a \leq b \leq 1$; and

(d) with a MD candidate against a R candidate, a fraction c of MD voters cast their ballots for the MD candidate, with $b \leq c \leq 1$.

These conditions are fairly natural: the mD and R voters are "extreme" and will vote only for their party's candidates. Furthermore, the mD voters are homogeneous enough that they cross over and vote for MD candidates at a lower rate than MD voters will vote for a minority candidate.[5] Majority-Democratic voters in a minority candidate vs. Republican general election will vote for a minority candidate at a higher rate than they crossed over in the primary (identity motive) when they prefer a Democratic candidate to a Republican candidate winning the general election (partisan motive). Finally, majority-Democratic voters will vote for a majority-Democratic candidate versus a Republican opponent at a higher rate than they will vote for a minority-Democratic candidate.[6] For our main analysis, we focus on the empirically relevant cases of sincere voting but also illustrate in Appendix B.3 that strategic voting in the primary has only limited effects. Given these assumptions, we can provide the first predictions of election outcomes depending on district demographics and the electoral process.

Primary Elections. In a Democratic primary, the minority candidate will win the nomination in a district with n_{mD} minority-Democratic and n_{MD} majority-Democratic voters if

$$n_{mD} + a n_{MD} \geq (1-a) n_{MD} \;\Rightarrow\; \frac{n_{mD}}{n_{MD}} \geq 1 - 2a. \tag{2.1}$$

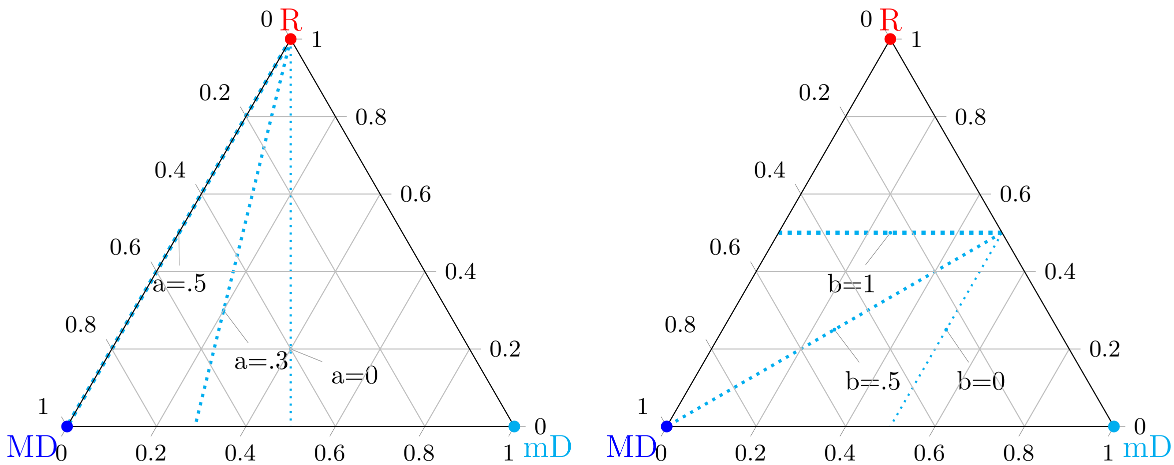
The greater the value of a, the cross-over of majority-Democratic voters, the fewer minority voters are necessary for a minority candidate to win the primary. The critical value of minority voters reaches zero when half of the majority-Democratic voters cross over. For any $a > 1/2$, a minority candidate would win any primary independent of the number of majority-Democratic and minority-Democratic voters in the district.

The set of points in the triangle satisfying (2.1) is illustrated in Figure 3(a) as a function of majority-Democratic crossover in the primary election, a. Generally, a minority candidate will win any district to the right of the illustrated lines, starting at the top apex and going to the bottom horizontal line. At $a = 0$, the minority candidate would win any primary in a district

[5] The results below do not change qualitatively if we allow for minority crossover as well; see the analysis in Section 6 below.

[6] See for example Washington (2006) who shows that White Democratic and Republican voters are less likely to support a Black candidate of their party.

in the right half of the triangle. Still, as a increases, more districts would be winnable until all primaries nominate a minority candidate as $a \geq 1/2$.



(a) Primary (b) General

Figure 3: Minority Candidate Winning.

General Elections. For the minority candidate to then win the general election against a Republican opponent, normalizing the district size to 1 and using the fact that there are only three types of voters with $n_{MD} + n_{mD} + n_R = 1$,[7] we get

$$n_{mD} + bn_{MD} \geq (1-b)n_{MD} + n_R \Rightarrow n_{mD} + bn_{MD} \geq \frac{1}{2}. \tag{2.2}$$

Similarly, as for the primary, the greater the value of b, the partisanship attachment of majority-Democratic voters, the fewer minority voters are necessary for a minority candidate to win the election.

We again illustrate the set of points satisfying (2.2) in Figure 3(b) but this time as a function of majority-Democratic crossover in the general election, b. At $b = 1$ when majority-Democratic voters' partisanship dominates, the line is horizontal, meaning that minority and majority-Democratic voters are perfect substitutes. Republicans can only win if they comprise more than half the district (top triangle). At $b = 0$ when majority-Democratic voters' identity dominates their partisanship, the line goes down to the midpoint of the bottom edge of the triangle, meaning that minority candidates can only win if they are over half the population (bottom right triangle). At $b = 1/2$ when majority-Democratic voters' partisanship equals their identity voting, minority candidates

[7]We will denote each group's vote share with π_{mD}, π_{MD}, and π_R later.

can only win if minority-Democratic voters outweigh the number of Republican voters in any district.

2.3 District Outcomes

Combining both outcomes in the primary and general elections, we can predict the winners for each district as a function of the district's demographics. Figure 4 shows a typical scenario for who wins the overall election, drawn for $a = 0.3$, $b = 0.7$, and $c = 1$. The top region represents districts in which a Republican wins; the bottom left triangle represents the districts with a majority-Democrat winner; and the bottom right represents districts with minority-Democrat winners. Note the asymmetry around the center is due to the fact that if it would be easier for a majority-Democrat to defeat a Republican opponent than it is for a minority-Democrat to win, given greater majority-Democratic support for majority-Democratic candidates as opposed to minority candidates – i.e., if $b < c$ for the general election.

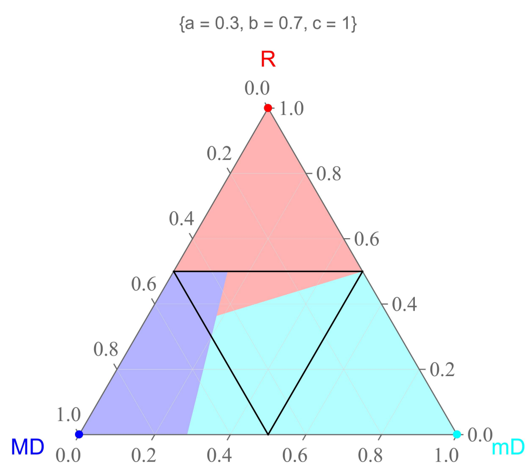


Figure 4: Electoral Winners.

Majority-Democrats' Identity vs. Partisanship. The outcomes above depend on majority-Democrats' crossover rates, which reflect the value voters place on identity and partisanship. For instance, suppose majority-Democrats vote purely based on identity and do not support a minority-Democratic candidate, $a = b = 0$. In that case, a minority candidate can only win majority-minority districts, and Republican candidates win districts where minority voters have the largest share because majority-Democrats vote for the Republican candidate. If majority-Democrats vote mostly on identity in the primary election, but partisanship dominates in the general election, for example, $a = 0$ and $b = 1$, minority candidates can win districts in which minority voters have the largest vote share, and Republicans cannot benefit from the identity conflict among Democratic voters. Finally, if majority-Democrats vote purely along partisan lines, for example, $a = 0.5$ and $b = 1$, minority candidates can win any district but majority-Republican ones. We illustrate all three scenarios in Figure 13 of the Appendix from left to right.

Crossover in Primary vs. General Election. We explore crossover rates in primary and general elections based on our prior analysis. Increasing crossover benefits minority candidates by securing more districts and helping parties optimize districts and seats. However, the magnitude of the effect hinges on the timing of crossover, causing the impact on minority representation and party dynamics to vary throughout the electoral process.

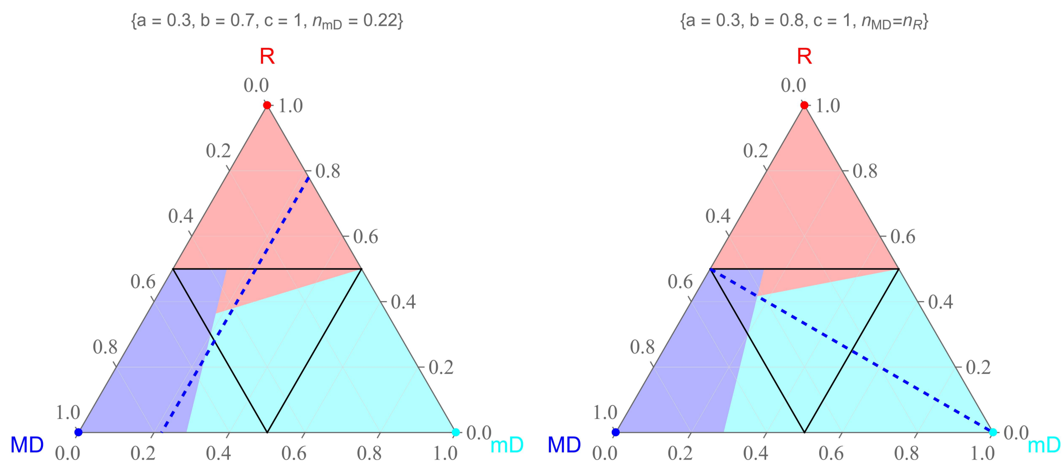
Consider the effects of majority-Democratic crossover voting in primary elections more closely. We modify parameter a and hold b and c constant, as depicted in Figure 14 in the Appendix. The analysis shows that as primary crossover voting increases (meaning a rises), minority candidates can secure more districts. However, these gains have a downside: a higher likelihood of potential Republican wins. This occurs because minority-Democratic candidates more frequently face Republican challengers in the general election and may not obtain the same level of support from majority-Democratic voters, which lowers their chances of beating a Republican opponent. Increasing primary crossover voting can improve minority representation but may also inadvertently lead to more Republican victories.

On the other hand, higher crossover rates in general elections benefit minority candidates without helping Republicans, as illustrated in Figure 15 of the Appendix. If majority-Democratic voters' support for minority candidates increases, Republicans can win fewer contested districts, eliminating inner partisan conflict about minority-candidate wins and total potential wins. In contrast to Lublin et al. (2020), which focuses on the mix of Republicans and minorities, these illustrations indicate that district crossover rates can impact elections. The analysis also shows that it is crucial to differentiate between primary and general elections when analyzing crossover effects, emphasizing the intricate balance between minority interests and party dynamics in the political arena.

2.4 Preliminary Districting Analysis: Minority Voter Share

The triangle diagrams also offer a few intriguing and cautionary tales when we consider the predicted winners of any district and the share of minority voters relative to other voter groups.

Constant Minority Vote Share. Suppose we hold the share of minority-democratic voters constant and consider the predicted winners, illustrated by the dashed diagonal line in Figure 5(a). The contour line goes from an area where a majority-Democratic candidate wins to a minority-Democratic winner, on to a Republican winner. So there is more to districting than just specifying the percentage of minority voters; the composition of the rest of the district and crossover voting also matter. Even more to the point, adding Republicans to a district can be good for minority voters; as the figure shows, this can allow a minority-Democrat to win the primary and then the general, whereas a majority-Democrat would have won before. So supposedly conservative shifts in districting can aid minority constituents.



(a) Constant n_{mD}

(b) Increasing n_{mD}

Figure 5: Share of Minority Voters and Predicted Winners.

Increasing Minority Vote Share. We can also consider the opposite by adding more minority voters to a district while keeping the ratio of majority-Democratic to Republican voters constant (dashed line in Figure 5(b)). Again, the impact is non-monotonic. We first move from a situation where a majority-Democrat faces a Republican, and then a minority-Democrat – and these patterns can be quite sensitive (Figure 16).[8] So, a pro-minority shift can have adverse consequences

[8]If we consider $a = .3$, $b = 0.75$, and $c = .95$ (left in Figure 16 of the Appendix), then we can find even that we move from a majority-Democratic winner to a Republican winner, back to majority-Democratic winner, and finally

if the entire district composition and potential crossover rates are not considered.[9]

2.5 Voter Registration and Turnout.

Our analysis of the districting maps and possible electoral outcomes assumed that the population in the district is registered (Census data vs. voter registration) and, more importantly, voters participate in primary and general elections (voter turnout). We will add this dimension of voter turnout to our formal analysis in the next part.

3 General Model

The triangle plots help us understand the districting problem and some of its subtleties. We now move to a more systematic analysis of redistricting, specifically from the perspective of minority voters' benefits. Given a state with a certain demographic distribution, which districting plans maximize substantive minority representation? Is it better for minorities to have much influence in a few districts or more modest influence over a wider area? Is the election of minority representatives a necessary part of substantive representation, or can minority constituents have their policy interests championed by non-minority representatives?

To address these questions, we generalize the triangle analysis above by adapting the Dixit and Londregan (1996) model of electoral competition.[10] In this model, voters ascribe ideological attachments to different candidates, and these candidates then compete for office by promising group-specific policy benefits. This model seems well-suited to our purposes: it captures the fact that voters of one identity may prefer representatives of the same identity but also competition by candidates over policy outcomes. As we will see, it allows us to address the impact of increasing or decreasing racism, greater registration and turnout by minority constituents, and changing partisan attachments in the electorate. Comparing our model to Coate and Knight (2007) also offers valuable insights. In their model, partisan voters can be depicted as point masses at the extremes of a one-dimensional policy space. In contrast, independent voters are uniformly distributed along some interval interior to the space. In this context, the problem is ensuring that the average legislator has preferences near these independents.

to a minority-Democratic winner. Or, if we consider a slight variation of $a = .3$, $b = 0.8$, and $c = 1$, then we predict only majority-Democratic or minority-Democratic winners.

[9] This non-monotonicity requires some degree of sincere voting; if all voters were sophisticated, then the line could not re-enter the Republican region.

[10] We also embed the Baron and Ferejohn (1989) model of legislative bargaining.

To provide a preview of the model's various steps, we consider 1) a Democratic primary for minority- and majority-Democratic voters; 2) a general election with the Democratic primary winner and a Republican candidate; 3) for each election, all candidates announce redistributive platforms to each group within the district; 4) after elections, the representatives of each district bargain over a potentially heterogenous redistributive policy for all districts and their district members; 5) voters benefit from redistributive policies through the legislature and ideological benefits from their legislator; 6) a social planner considers how to design the districting map of voter groups to maximize minority group utility across districts. In the following, we provide a detailed model that provides a mapping from a districting scheme in a two-dimensional simplex, $\mathbf{D} = \mathbf{D}(S^2)$, to an electoral outcome describing a legislature of the districts' winning candidates, $\mathbf{L} = \mathbf{L}(\mathbf{D}(S^2))$, and to legislative outcomes with distributive benefits for districts and voter groups implemented by the legislature, $\mathbf{P} = \mathbf{P}(\mathbf{L}(\mathbf{D}(S^2)))$. This formalizes our illustrations from the previous section and integrates them with our upcoming analysis.

3.1 Districts

Assume a population of voters, V, divided into a given number of identifiable groups Θ; these may be defined according to voters' ethnicity, language, economic status, religion, political party, etc. Thus, there is a partition from the set of voters V to groups, $\nu : V \to \Theta$.

For simplicity, we assume a state population divided along ethnic and partisan lines with voter types $\Theta = i \in \{mD, MD, R\}$, for minority-Democrats, majority-Democrats, and Republicans, respectively. Their statewide populations are $\mathbf{N}_{mD}$, $\mathbf{N}_{MD}$, and $\mathbf{N}_R$, with $\sum_i \mathbf{N}_i = \mathbf{N}$, the total state population. Since population proportions must sum to 1, we can represent the mix of voter types statewide—or in any given district—as a point in the two-dimensional simplex, S^2, as illustrated above.

A district is a vector $\mathbf{d} = (N_{mD}, N_{MD}, N_R)$ of voters with $N_i \geq 0$. Let $\mathcal{D}$ be the set of all possible districts, and assume that the state will be divided into K districts, K odd,[11] with N_{ik} representing the number of voters of type i in district k. We denote the number of all voters in district k with N_k. Then a districting scheme is a function $\mathbf{D} : S^2 \to \mathcal{D}^K$, yielding a list $(\mathbf{d}_1, \mathbf{d}_2, ..., \mathbf{d}_K)$ of districts. Furthermore, a *valid* districting scheme is a districting scheme such that in any given district, $\sum_i N_{ik} = N_k = \mathbf{N}/K$, and across districts $\sum_k N_{ik} = \mathbf{N}_i$ for all voter

[11] We will assume that each district elects one legislator and the odd number of legislators avoids legislative ties.

types i – i.e., all districts are equally sized, and all voters are assigned to a district.[12]

3.2 Candidates and Elections

Suppose in each of the K districts three candidates are competing for a seat in the legislature, these candidates are also of types $\theta = j \in \{mD, MD, R\}$. Candidates try to maximize their vote share with platforms that offer a proportion T_i of the district's redistributive benefits as transfers to voters of type, which captures *substantative representation*. Denote the redistributive platform of candidate j towards group i in district k as T_{ijk}; then, campaign platforms must satisfy $\sum_i T_{ijk} = 1$ for each j and k.[13]

Candidates attain office according to a two-stage electoral cycle: first, each district holds a primary election, in which the mD candidate faces a MD opponent; second, there is a general election in each district where the primary winner squares off against a Republican. Only mD and MD voters cast ballots in the primary, whereas all voters participate in the general election. We assume that candidates are committed to a single platform for the entire electoral cycle; they cannot, for instance, change platforms between the primary and general elections.

Represent a candidate by a vector $c = (\theta, T_{mD}, T_{MD}, T_R)$, where θ is the candidate's type, and let $\mathcal{C}$ be the set of all possible candidates. Let $\mathbf{c}_k$ be the list of three candidates from district k, and $\mathbf{C} = \{\mathbf{c}_1, \mathbf{c}_2, ..., \mathbf{c}_K\}$ be the entire set of (3K) candidates in all districts. Then an election is a mapping to a legislature $\mathbf{L} : \mathcal{D}^K \times \mathcal{C}^{3K} \rightarrow \mathcal{C}^K$, producing a representative for each district with a given type and committed to a given platform.

To smooth out the response functions, we assume probabilistic voting so that the probability a candidate wins a given election rises with the expected proportion of votes she receives. Given expected vote proportion v, let the probability of winning the election be $\Psi(v)$, with $\Psi' > 0$, $\Psi(0) = 0$, $\Psi(1) = 1$, and $\Psi(1 - v) = 1 - \Psi(v)$. We assume here the simplest linear function $\Psi(v) = v$, so that, for instance, a candidate expecting to receive 60% of the vote wins with a 60% probability.[14]

The winners of the K district elections then go to a legislature $\mathbf{L} \in \mathcal{C}^K$. Considering candidates'

[12]Equivalently, as in the triangle analysis above, the average of the percentages of each group in the K districts must equal their statewide population proportion $\mathbf{N_i}/\mathbf{N}$. See the numerical example in Appendix B.2 for illustrations above.

[13]We assume both parties and all candidates have equal abilities to distribute benefits as in Dixit and Londregan (1996). Without loss of generality, the model could be extended to allow each candidate to evaluate the marginal value spent for any program on the voter group's marginal utility. We discuss this possibility below.

[14]The qualitative results derived below do not depend on our assumption of probabilistic voting. See, for instance, the results in Table 4.

equilibrium strategies, then elections transform a districting scheme into a legislature; that is, $\mathbf{L} = \mathbf{L}(\mathbf{D}(S^2))$.

3.3 Legislative Policies

The legislature then passes a redistributive policy, dividing K dollars across all districts. They do so via a Baron and Ferejohn (1989)-open rule bargaining process: a legislator is selected randomly to offer a proposed budget division. The entire legislature then votes on the proposal (under a closed rule); if it is adopted, the game ends. If a majority vote rejects it, discounting occurs (all payoffs are lowered by a factor of δ, $0 < \delta \leq 1$), and the legislative subgame starts again with another member chosen randomly to make an offer. In this game, members try to maximize the benefits directed toward their district.[15]

The outcome of this legislative process will be a vector $(B_1, B_2, ..., B_K)$ of district-specific benefits, with $B_k \geq 0$ and $\sum_k B_k = K$, allocating a given budget.[16] So the legislative policy function is $\mathbf{P} : \mathcal{C}^K \to \mathbb{R}_+^K$. This follows from the results of the elections, which in turn depend on the districting scheme, so $\mathbf{P} = \mathbf{P}(\mathbf{L}(\mathbf{D}(S^2)))$.

Any funds allocated to district k in the legislative process are divided according to the platform adopted by that district's representative. So if the type j representative from district k ran on a platform promising T_{ijk} to members of a group i, then voters in this group will receive $T_{ijk} * B_k$ in total benefits, with individual benefits $b_{ijk} = (T_{ijk} * B_k)/N_{ik}$.

3.4 Voters

Voters enjoy distributive benefits from legislative outcomes and ideological benefits from elected candidates. We adopt Dixit and Londregan (1995, 1996)'s characterization of utilities where voters from group i receive utility $U_i(.)$ from consumption and an ideological attachment to winning

[15] Our analysis streamlines the fiscal budgetary process with a focus on a single legislature, discretionary spending, and a given budget amount. The analysis ignores the complexities of redistribution that arise when considering federal vs. state legislatures in a multi-level system of redistribution, legislative budgeting vs. executive budget implementation in a system with separate powers, discretionary vs. mandatory spending in an environment of multi-means redistribution, or balanced budget vs. flexible budget in a fiscal setting with tax collection, government borrowing or lending, and endogenous budget amounts. Here, we focus on who gets elected and the policy outcome enacted by the legislature. For a budget process with separation of powers, see Grossman and Helpman (2008); for redistribution with the same different abilities of parties to collect taxes and distribute benefits, see Dixit and Londregan (1996).

[16] We ignore considerations of balanced budgets and taxation and instead assume that a given budget is divided across districts as the federal government can borrow or has revenues from various other tax sources unrelated to the distributive policies of interest here. For balanced budget implications and redistributive policies, see Cox and McCubbins (1986) and Lindbeck and Weibull (1987).

candidates.[17] In particular, assume that the utility from consumption, b, is given by:

$$U_i(b) = \kappa_i \frac{b^{1-\epsilon}}{1-\epsilon} \tag{3.1}$$

with $\epsilon > 0$ and $\epsilon \neq 1$. Then the marginal utility of an additional dollar of consumption and the return to consumption are

$$U_i'(b) = \kappa_i b^{-\epsilon} > 0 \text{ and } U_i''(b) = -\epsilon\kappa_i b^{-\epsilon-1} < 0. \tag{3.2}$$

As b increases from 0 to ∞, the marginal utility falls from ∞ to 0, and this assumption avoids corner solutions. A one percent increase in b causes an ϵ percent decrease in marginal utility, so ϵ captures the degree of diminishing returns in consumption.[18] Furthermore, the parameter κ_i captures the relative weight of consumption to ideological benefits for voter group i; higher values of κ_i imply that voters of group i are more responsive to distributive than ideological benefits. Together, ϵ and κ_i allow characterizing the trade-offs between economic and ideological benefits.

Voters' ideological benefits depend on their district's winning candidate and are described by X^j for a candidate of type j. The overall utility for a voter of type i a representative of type j offering distributive benefits b_{ij} is the sum of their ideological and distributive benefits: $U_i = X_i^j + E[U_i(b_{ij})]$. Thus, for instance, a voter with ideological preference of X^{mD} for minority-Democratic candidates and X^R for Republicans gets extra utility $X^{mD} - X^R \gtreqless 0$ from seeing a minority-Democrat win office instead of a Republican. The voter with a positive ideological gain will, therefore, prefer the minority-Democrat candidate unless the Republican offers her sufficiently greater consumption value:

$$E[U_i(b_{iR})] - E[U_i(b_{imD})] > X^{mD} - X^R. \tag{3.3}$$

Define the critical value, or "cutpoint" X_i for group i in an election between two candidates labeled 1 and 2 by:

$$X_i^e \equiv U_i(b_{i1}) - U_i(b_{i2}), \tag{3.4}$$

[17] Lindbeck and Weibull (1987) also consider utility functions with additively separable benefits from consumption and ideological benefits and a positive decreasing marginal utility of consumption.

[18] In other words, individuals have a willingness to trade off consumption for ideology depending on their consumption level. Individuals with low values of ϵ remain quite sensitive to transfers even when they receive a lot of transfers. In contrast, individuals with high values of ϵ are less willing to compromise on ideology with greater consumption levels, and it would be easier to sway voters with fewer consumption opportunities.

where e indicates the type of election being contested– i.e. a primary election or a general election with either mD vs. R or MD vs. R candidates. Voters are assumed to cast their ballots sincerely for the candidate offering them higher utility. Then, group i voters with values of X_i less than X_i^e will vote for Candidate 1, while the others will vote for Candidate 2. If Candidate 1 offers an additional dollar to each member of the group i, then the critical value will shift in her favor by $U_i'(b_{i1}) = \kappa_i b_{i1}^{-\epsilon}$.[19]

Votes Let Φ_i^e be the concave cumulative distribution of voters of a group i in an election of type e, so that, given the campaign platforms, a proportion $\Phi_i^e(X_i)$ will vote for candidate 1. Given N_i voters of type i, this candidate will receive $N_i\Phi_i^e(X_i)$ votes from group i, with total votes of:

$$V_1^e = \sum_{i \in \Theta} N_i \Phi_i^e(X_i). \tag{3.5}$$

The opposing candidate will then get votes of:

$$V_2^e = \sum_{i \in \Theta} N_i[1 - \Phi_i^e(X_i)] = N - V_1^e. \tag{3.6}$$

Crossover Voting. The distribution functions $\Phi_i^e(X_i)$ play an important role in the following analysis. They indicate the ideological preference of a given voter i for one candidate over another. These preferences could arise partly from a spatial policy model, measuring the degree to which voters agree with the policy choices of their representatives. But they could also arise from group voting preferences: voters might want to support candidates of one type θ over those of another type. In the legal literature, this is what is meant by polarized voting: the willingness, or lack thereof, of voters to cross over and vote for candidates of another race and ethnicity. We assume for simplicity that if the distribution of type i voters in the entire population is $\Phi_i^e(.)$, then this is also the distribution of the type i voters in any given district.[20]

Notice that the rates at which different types of voters cast their ballots for various candidates are given by the $\Phi_i^e(0)$ functions for group i in an election of type e, where for convenience we label the primary as election $e = 1$, a general election of mD vs. R as type $e = 2$, and a general election of MD vs. R as $e = 3$. For instance, in an mD vs. MD primary, a proportion $\Phi_{mD}^1(0)$

[19] We assume throughout that voter groups may differ in their consumption and ideological preferences but neglect within group-differences across districts.

[20] We also assume that the number of voters in each district is large enough that we can calculate expected voter utility as the integral of $\Phi_i^e(.)$ for voter types.

Election	**Candidate** / **Group**	mD	MD	R
Primary, $e = 1$	mD	a^1_{mD}	$1 - a^1_{mD}$	
	MD	a^1_{MD}	$1 - a^1_{MD}$	
General mD **vs.** R, $e = 2$	mD	a^2_{mD}		$1 - a^2_{mD}$
	MD	a^2_{MD}		$1 - a^2_{MD}$
	R	a^2_R		$1 - a^2_R$
General MD **vs.** R, $e = 3$	mD		a^3_{mD}	$1 - a^3_{mD}$
	MD		a^3_{MD}	$1 - a^3_{MD}$
	R		a^3_R	$1 - a^3_R$

Table 2: Crossover Rates.

of minority voters will vote for the minority candidate, and the remaining $1 - \Phi^1_{mD}(0)$ will vote for the majority-Democrat candidate.

We redefine these quantities as *crossover rates*, following the usual standard for voting studies, letting a^e_Θ represent the rate at which voters of a group Θ vote for the more liberal candidate in election e.[21] Thus, a proportion a^1_{MD} of majority-Democrats cross over to vote for the minority candidate in the primary, while $1 - a^1_{MD}$ vote for the majority-Democratic candidate. Similarly, a proportion a^2_R of Republican voters prefer the minority-Democrat in a general election. For reference, a table of these crossover rates is given in Table 2.

Winning Probabilities. Then, for instance, the minority candidate will be expected to win the primary, ignoring the notation for district k, if:

$$a^1_{mD}N_{mD} + a^1_{MD}N_{MD} \geq (1 - a^1_{mD})N_{mD} + (1 - a^1_{MD})N_{MD} \Rightarrow \frac{N_{mD}}{N_{MD}} \geq \frac{1 - 2a^1_{MD}}{2a^1_{mD} - 1}, \tag{3.7}$$

similar to (2.1) and Figure 3(a) above. Similarly, we apply a few assumptions on the relative magnitudes of crossover rates: $a^e_{mD} > a^e_{MD} > a^e_R$, reflecting closer ideological alignment among Democrats.

Let Ψ^e_θ represent the probability a type θ candidate wins election e, and Ψ_θ be the probability that the candidate wins overall. Given that the proportion of votes a candidate receives equals

[21] Assuming for the purposes of definition that minority-Democrats are more liberal than majority-Democrats, who are more liberal than Republicans.

her probability of winning, we have the following for each election type:

$$\Psi^1_{mD} = \frac{a^1_{mD}N_{mD} + a^1_{MD}N_{MD}}{N_{mD} + N_{MD}} \quad \text{and } \Psi^1_{MD} = 1 - \Psi^1_{mD}; \tag{3.8}$$

$$\Psi^2_{mD} = \frac{a^2_{mD}N_{mD} + a^2_{MD}N_{MD} + a^2_{R}N_{R}}{N_{mD} + N_{MD} + N_R} \quad \text{and } \Psi^2_{R} = 1 - \Psi^2_{mD}; \tag{3.9}$$

$$\Psi^3_{MD} = \frac{a^3_{mD}N_{mD} + a^3_{MD}N_{MD} + a^3_{R}N_{R}}{N_{mD} + N_{MD} + N_R} \quad \text{and } \Psi^3_{R} = 1 - \Psi^3_{MD}, \tag{3.10}$$

which describes the probabilities of winning the district for each candidate type with

$$\Psi_{mD} = \Psi^1_{mD}\Psi^2_{mD},\ \Psi_{MD} = \Psi^1_{MD}\Psi^3_{MD}, \text{ and } \Psi_R = 1 - \Psi_{mD} - \Psi_{MD}. \tag{3.11}$$

These equations define a surface on S^2 similar to that illustrated in Figure 4 and 5, but with smoothly increasing election probabilities for each type rather than sharply demarcated regions illustrating deterministic electoral outcomes purely based on voter demographics and partisanship. To simplify the analysis and account for voting nuances, we assume probabilistic voting outcomes, rather than ex-ante assigned electoral winners based on demographics and party identity, which account for the effects of voter registration and turnout, which may or may not vary across partisanship and identity and which we do not model here explicitly.[22]

3.5 Order of Play

To summarize, the order of play is as follows:

1. Given state demographics of $\mathbf{N}_{mD}$, $\mathbf{N}_{MD}$, and $\mathbf{N}_R$ and K districts, a valid districting scheme $\mathbf{D}$ is enacted.

2. Candidates of type j in each district k announce their platforms offering distribute benefits T_{ijk} for $i, j \in \{mD, MD, R\}$.

3. Voters elect candidates in primary and general elections, yielding legislature $\mathbf{L}$.

4. The legislature enacts a distributive policy $\mathbf{P}$.

5. All voters receive their utilities, and the game ends.

[22]Without loss of generality, we could consider uneven levels of turnout and registration, which would change the results by the respective proportions. For a recent analysis of partisan gerrymandering and voter turnout see Bouton et al. (2023).

All preferences and institutional rules are common knowledge, and actions are observable. Hence, there is perfect information in the described game. Beginning from the last stage forward, we will solve the game for a subgame-perfect Nash equilibrium that fulfills specific distributional characteristics we will describe now.

3.6 Evaluation of Districting Plans

We will evaluate districting plans according to their impact on minority voters, assuming that a social planner wishes to maximize minority voters' overall welfare. Let L_k be the legislator elected from district k, and let $\theta(L_k)$ be her type. Then given the utility functions above, the social planner selects

$$\mathbf{D}^* \in \arg\max_{\mathbf{D}\in\mathcal{D}^K} \sum_{i=1}^{\mathbf{N}_{mD}} X_i^{\theta(L_k)} + E[U_i(b_i)|\mathbf{P}(\mathbf{L}(\mathbf{D}))]. \tag{3.12}$$

The social planner, then, must allocate the different types of voters across districts, taking into account the impact of the chosen districting scheme on minority voters' distributive and ideological benefits. For instance, concentrating minority voters into a few districts will increase the probability of electing minority representatives at the potential cost of electing more Republicans elsewhere. This strategy also promises large distributive benefits in the concentrated-minority districts but makes it less likely that these representatives will be included in winning legislative coalitions. Spreading voters out means that minorities can influence outcomes in more districts. Yet it also raises the possibility that they will be marginalized everywhere, electing no minorities to office and gaining only paltry distributive benefits. The question is how voters weigh these considerations under changing ideological distributions, different groups' population proportions, and group power variations.

4 Platforms and Policy Benefits

We first solve the legislative game describing policy benefits enacted by the legislature and then characterize candidates' electoral platforms offering voter groups shares of those policy benefits.

4.1 Legislative Outcomes

The legislative game is elementary; in equilibrium, the legislator chosen to make the first offer constructs a random minimum-winning coalition of $(K-1)/2$ other legislators and keeps the remainder for herself.[23] Let l be the legislator who makes the offer, C be the legislators selected to be in the coalition, and O be the remaining legislators. Then equilibrium offers to share the K being distributed are:

$$B_k = \begin{cases} \frac{(2-\delta)K+\delta}{2} & \text{if } k = l; \\ \delta & \text{if } k \in C; \\ 0 & \text{if } k \in O. \end{cases} \tag{4.1}$$

Since the game is symmetric, each legislator has an expected return of 1 from the legislative bargaining session. If a candidate promises a group T_{ijk} in transfers during the election, this is also their expected total legislative payout if that candidate wins office.

Since the game is symmetric, each legislator has an expected return of 1 from the legislative bargaining session. This means that if a group is promised T_{ijk} in transfers from a given candidate's platform, this is also their expected total legislative payout if that candidate wins office.

In our setup, the chosen legislator constructs a minimum-winning coalition at the lowest possible costs for herself, which describes its Shapley value. Hence, the cost of swaying other legislators may or may not be related to partisan control of the legislators, but it is less if legislators are driven by securing transfers for their districts and competition for joining the minimum-winning coalition dominates.

4.2 Candidate Platforms and Group Powers

Candidates adopt platforms to maximize their votes, balancing their offers to various groups. In equilibrium, the candidates adopt identical redistributive platforms: $b_{i1k} = b_{i2k}$ and $T_{i1k} = T_{i2k}$ for each group i in a given district k.[24] Consequently, voters cast their ballots for the candidate with whom they have the higher ideological affinity.

Furthermore, the individual benefits and share of the benefits offered to group Θ by candidate j for district k in equilibrium are

$$b_{ijk} = \frac{\pi_i}{\sum_i \pi_i N_{ik}} B_k \text{ and } T_{ijk} = \frac{\pi_i N_{ik}}{\sum_i \pi_i N_{ik}}, \tag{4.2}$$

[23]See Baron and Ferejohn (1989)'s Proposition 3 for a stationary subgame-perfect equilibrium with infinite sessions, majority and closed rule, and n-(odd)-legislators as well as $n = K$ districts/legislators and a total distributive benefit of K instead of 1.

[24]See Dixit and Londregan (1996)'s description and existence of an equilibrium in pure strategies of platforms. The existence conditions of Glickberg's Theorem are fulfilled, and the constrained maximization problem is derived using (3.5) and (3.6) for each candidate's objective function and $\sum_i N_{ik} b_{ijk} = B_k$ as a constraint. The Nash equilibrium follows from a simultaneous solution for all first-order conditions and Lagrange parameters. See Appendix A.1 for the formal characterization.

where

$$\pi_i = [\kappa_i \phi_i(0)]^{1/\epsilon} \tag{4.3}$$

and $\phi(.) = \Phi'(.)$.[25] The π_i parameters represent each group's political power. The value of π_i increases for groups with larger values of κ_i; so groups get a bigger share of the legislative pie the more they care about distributive rather than ideological issues.

A group's power also grows with $\phi_i(0)$, which is the density of their distribution function when voters are indifferent between the two candidates running for office. This term captures a group's "swinginess:" the greater the percentage of members indifferent between the candidates or close to it, the more benefits the group and each member receive. The intuition behind this result is straightforward. First, in equilibrium, the candidates offer the same platform to voters, so this will make no difference in voters' decisions. Since the candidates' promises cancel out, those voters who are indifferent between the parties in equilibrium are those for whom $X_i^e = 0$ in the first place. When deciding whether to transfer funds from one group to another, then, it is these marginal voters who will gain or lose; hence, the candidates pay off the groups in ratios proportional to their $\phi_i(0)$ values, and the group's members enjoy greater distributive benefits. In addition, the parameter ϵ represents the degree of diminishing returns in consumption; as the additional consumption matters less, the group's power declines, and distributive benefits are more even across groups. For example, as $\epsilon \to \infty$, we get $\pi_i \to 1$ and benefits and shares become simple averages: $b_{ijk} \to B_k/N_k$ and $T_{ijk} \to N_{ik}/N_k$.

An individual's benefits and a group's share of district benefits depend on their group's power and on either the district representative's ability to deliver distributive benefits, B_k, or the district's demographics, N_{ik}. For example, the larger the district-specific transfers, the greater the individual's gains in consumption from the group's size, which is independent of voters' benefits. On the other hand, the legislator allocates a larger share of district benefits to larger groups, $\partial T_{ijk}/\partial N_{ik} > 0$; though the share of the pie is independent of the pie's size.

5 Distributive and Ideological Benefits

To understand the properties of the equilibrium, we break the analysis into three stages. First, we examine the implications of per-voter distributive benefits b_{ijk}. Ignoring the ideological benefits of

[25] The solution follows from applying the utility function of (3.1) and its marginal utility (3.2) to the Nash equilibrium's first-order conditions. More details are in Appendix A.1.

electing different types of representatives for the moment, we ask how one would allocate minority voters across districts to maximize their total (or average) distributive returns. We next analyze the ideological utility arising from the different types of elected representatives. We then combine both types of utilities and characterize the optimal districts maximizing minority voters' returns.

5.1 Distributive Benefits and Minority Power

Characterizing districting schemes that provide the most benefits to minorities depends on the behavior of (4.2) on the two-dimensional simplex S^2. We are particularly interested in its behavior on the surface $N_{mDk} + N_{MDk} + N_{Rk} = N_k = \mathbf{N}/K$. We thus rewrite (4.2), minorities' distributive benefits as a group's share in a given district by any candidate,[26] as

$$T_{mDk} = f(N_{mDk}, N_{MDk}) = \frac{\pi_{mD} N_{mDk}}{\pi_{mD} N_{mDk} + \pi_{MD} N_{MDk} + \pi_R N_{Rk}} \quad (5.1)$$

$$= \frac{\pi_{mD} N_{mDk}}{(\pi_{mD} - \pi_R) N_{mDk} + (\pi_{MD} - \pi_R) N_{MDk} + \pi_R N_k} \geq 0. \quad (5.2)$$

Note that the denominator is positive throughout. Thus, we can define $\Pi_k \equiv (\pi_{mD} - \pi_R)N_{mDk} + (\pi_{MD} - \pi_R)N_{MDk} + \pi_R N_k$ – i.e., the aggregate group power of district k. We then write the derivatives of the minorities' benefits with respect to the groups' relative powers as

$$\frac{\partial f}{\partial \pi_{mD}} = \frac{N_{mDk}\left(\pi_R(N_k - N_{mDk} - N_{MDk}) + \pi_{MD} N_{MDk}\right)}{\Pi_k^2} > 0; \quad (5.3)$$

$$\frac{\partial f}{\partial \pi_{MD}} = -\frac{\pi_{mD} N_{mDk} N_{MDk}}{\Pi_k^2} < 0; \quad (5.4)$$

$$\frac{\partial f}{\partial \pi_R} = -\frac{\pi_{mD} N_{mDk}(N_k - N_{mDk} - N_{MDk})}{\Pi_k^2} < 0. \quad (5.5)$$

Assigning signs to the derivatives with $\pi_i > 0$ and $N_{ik} > 0$ shows that increases in the minority group's power is beneficial while increasing the power of either other group decreases the minority's utility.

5.1.1 Districting Scheme

We now turn to the districting question: how to maximize minority voters' utility by changing the numbers of different types of voters across districts. That is, we seek a valid districting scheme

[26] We can ignore the candidate subscript as candidates in the same district promise the same benefits.

$\tilde{\mathbf{D}}^*$ such that

$$\tilde{\mathbf{D}}^* \in \underset{\mathbf{D}\in\mathcal{D}^K}{\arg\max} \sum_{i=1}^{N_{mD}} E\left[U_i(b_i)|\mathbf{P}(\mathbf{L}(\mathbf{D}))\right]. \tag{5.6}$$

The solution to the utility-maximizing districting scheme may not be unique. Hence, let the set of all possible schemes be $\tilde{\mathcal{D}}^*$ and $\tilde{\mathbf{D}}^*$ a representative element. To determine the characteristics of an optimal districting scheme, we first evaluate the derivatives of minority voters' benefits of (5.2) with respect to the populations of voters:

$$\frac{\partial f}{\partial N_{mDk}} = \frac{\pi_{mD}\left(\pi_{MD}N_{mDk} + \pi_R(N_k - N_{mDk})\right)}{\Pi_k^2} > 0, \tag{5.7}$$

$$\frac{\partial f}{\partial N_{MDk}} = \frac{\pi_{mD}N_{mDk}(\pi_R - \pi_{MD})}{\Pi_k^2} \gtreqless 0. \tag{5.8}$$

As one would expect, the first derivative is always positive; adding more minority voters to a district increases their share of distributive benefits. However, the sign of the first derivative of (5.8) is ambiguous and depends on the other groups' relative power. Minority voters benefit if voters from the more powerful non-minority group are replaced with voters from the less powerful group. Suppose Republicans are politically more powerful than majority-Democrats, $\pi_R > \pi_{MD}$. In that case, minority voters' benefits increase as the number of majority-Democratic voters reduces the number of Republicans in a district, and vice versa. In the districting process, however, changes in voters must be balanced across districts. Hence, minority gains in one district, where less powerful voters increase, accompany another district's minority voters' loss as more powerful voters join. We can state

Proposition 1. *If majority voter groups' power differs, $\pi_{MD} \neq \pi_R$, and for any two districts with different minority voter concentration and aggregate group power, $N_{mDk} \neq N_{mDl}$ and $\Pi_k \neq \Pi_l$ with $k \neq l$, then any districting scheme that maximizes minority distributive benefits concentrates less powerful majority voters into minority-populated, less powerful districts and more powerful majority voters into majority-populated, more powerful districts.*

All proofs are in Appendix A. Figure 6 and (5.8) illustrate the intuition of the proof. Take any two districts with $N_{mD1} > N_{mD2}$ and $\Pi_1 < \Pi_2$. Focus attention on the interior of the simplex; the goal is to shift voters of the more powerful majority group from the minority-populated district to the other district. As illustrated in Figure 6, to accomplish this goal, Republicans will be moved from k_1 to k_2 and majority-Democrats from k_2 to k_1 where they have greater influence. If District 1 is less powerful than District 2 ($\Pi_1 \leq \Pi_2$), then minority benefits in the district with more minority voters will increase (k_1) by more than the minority benefits fall in the district with fewer minority voters (k_2). Accordingly, average payoffs for minority voters across districts increase, but the district's power across groups decreases. This process continues until one district collides with the border of the simplex. The process can be re-iterated by using the remaining interior district with any other district in the interior that fulfills $N_{mDk} \neq N_{mDl}$ and $\frac{N^2_{mDk}}{\Pi^2_k} > \frac{N^2_{mDl}}{\Pi^2_l}$ (districts k_2 and k_3 in Figure 6). In equilibrium, at least one district will lie at the simplex border. The results imply that optimal districting schemes have a high concentration of minority voters combined with sharing their districts with the less powerful majority group. In contrast, more powerful majority voters concentrate in districts with few minority voters. Table 3 provides simulations of optimal districts with varying group power, further corroborating our results.

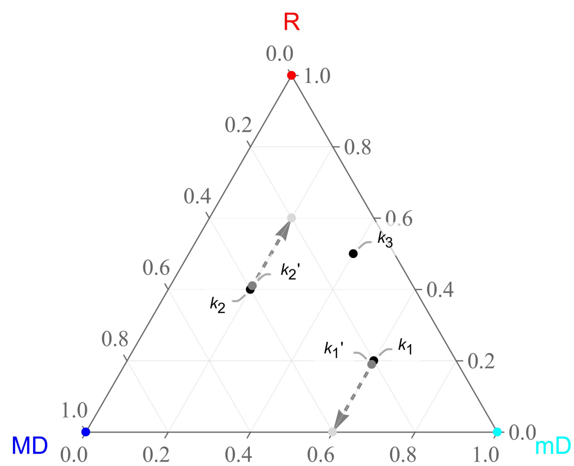


Figure 6: Optimal Districting Process for $\pi_R > \pi_{MD}$ and $\Pi_1 \leq \Pi_2$ with $K = 3$.

5.1.2 Voter Distribution and Minority Distributive Benefits

All that remains to characterize $\tilde{\mathcal{D}}^*$ completely is to determine the optimal distribution of minority voters across districts. The surfeit of boundary conditions makes the usual maximization solution via Lagrange multipliers opaque. Still, we can gain insight into the solution by examining the concavity/convexity of the payoff function with respect to the number of minority voters in the district. We thus calculate the determinants of the principal minors of the Hessian matrix:

$$H = \begin{bmatrix} \frac{\partial^2 f}{\partial N^2_{mDk}} & \frac{\partial^2 f}{\partial N_{mDk} N_{MDk}} \\ \frac{\partial^2 f}{\partial N_{MDk} N_{mDk}} & \frac{\partial^2 f}{\partial N^2_{MDk}} \end{bmatrix}. \tag{5.9}$$

with

$$\frac{\partial^2 f}{\partial N_{mDk}^2} = \frac{2\pi_{mD}(\pi_R - \pi_{mD})\left(\pi_R N_k + (\pi_{MD} - \pi_R)N_{MDk}\right)}{\Pi_k^3}, \tag{5.10}$$

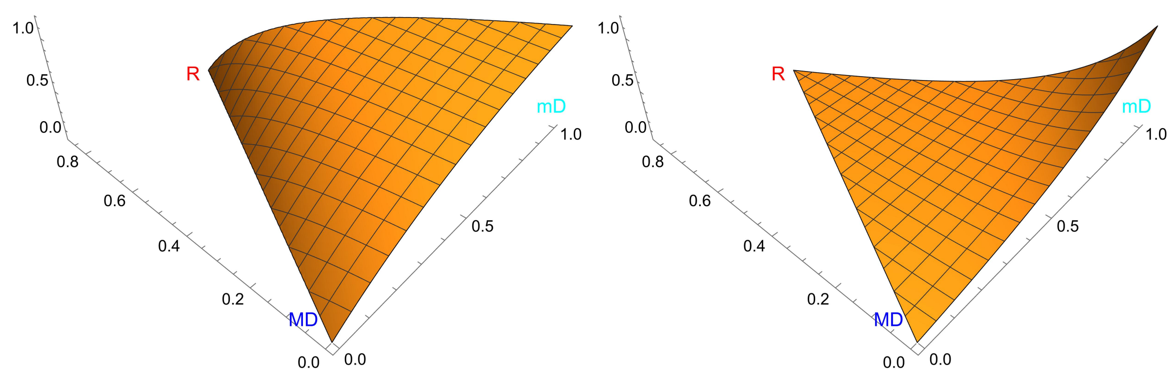
$$\frac{\partial^2 f}{\partial N_{mDk} N_{MDk}} = \frac{\pi_{mD}(\pi_{MD} - \pi_R)\left((\pi_{mD} - \pi_R)N_{mDk} + (\pi_R - \pi_{MD})N_{MDk} - \pi_R N_k\right)}{\Pi_k^3}, \tag{5.11}$$

$$\frac{\partial^2 f}{\partial N_{MDk}^2} = \frac{2\pi_{mD}(\pi_{MD} - \pi_R)^2 N_{mDk}}{\Pi_k^3}, \tag{5.12}$$

and

$$det(H) = \frac{\pi_{mD}^2(\pi_R - \pi_{MD})^2}{\Pi_k^4}. \tag{5.13}$$

The determinant of the entire H matrix is positive for $\pi_{MD} \neq \pi_R$, but the value of $\frac{\partial^2 f}{\partial N_{mDk}^2}$ is indeterminate, indicating that the H matrix can be positive definite, negative definite, or neither, depending on the parameter values. For optimization, the surface could be either concave or convex. Figure 7(a) illustrates a concave function for minority distributive benefits when the minority group's power is larger than others. On the other hand, when minority power is lower than the majority groups' power, the function is convex, as illustrated in Figure 7(b).[27]



(a) Concave: $\pi_{mD} = 10$, $\pi_{MD} = 5$, $\pi_R = 2$

(b) Convex: $\pi_{mD} = 2$, $\pi_{MD} = 5$, $\pi_R = 10$

Figure 7: Concave and Convex Minority Distributive Benefits.

The importance of this difference is clear. If we wish to maximize the overall return to minorities, then in the concave case, a social planner would divide minority voters more evenly across districts than they would with a convex payoff function. Note that the difference between the curvatures of the two surfaces lies in the relative power of minorities compared with the other

[27] We provide additional examples of nonconcave and nonconvex payoffs in Figure 17 of Appendix B.8 that arise when minority group power lies between both majority groups' power.

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{MD}	π_R	n_{mD1}	n_{MD1}	n_{R1}	n_{mD2}	n_{MD2}	n_{R2}	n_{mD3}	n_{MD3}	n_{R3}	Total	Aver.	$V(\mathbf{D})$
1	3	1	75%	0%	25%	0%	95%	5%	0%	25%	75%	0.750	0.25	75%
2	3	1	31%	20%	49%	0%	100%	0%	44%	0%	56%	0.974	0.32	44%
3	3	1	39%	0%	61%	0%	100%	0%	36%	20%	44%	1.167	0.39	39%
4	3	1	37%	20%	43%	0%	100%	0%	38%	0%	62%	1.300	0.43	38%
5	3	1	30%	35%	35%	15%	85%	0%	30%	0%	70%	1.426	0.48	15%
1	3	3	0%	97%	3%	0%	21%	79%	75%	2%	23%	0.500	0.17	75%
2	3	3	0%	4%	96%	0%	100%	0%	75%	16%	9%	0.667	0.22	75%
3	3	3	58%	23%	19%	12%	57%	30%	5%	40%	55%	0.750	0.25	53%
4	3	3	25%	55%	20%	25%	49%	26%	25%	16%	59%	0.923	0.31	0%
5	3	3	25%	57%	18%	25%	46%	29%	25%	17%	58%	1.071	0.36	0%
1	3	5	0%	9%	91%	0%	86%	14%	75%	25%	0%	0.500	0.17	75%
2	3	5	0%	47%	53%	0%	48%	52%	75%	25%	0%	0.667	0.22	75%
3	3	5	0%	95%	5%	0%	0%	100%	75%	25%	0%	0.750	0.25	75%
4	3	5	35%	60%	5%	0%	0%	100%	40%	60%	0%	0.876	0.29	40%
5	3	5	38%	62%	0%	37%	58%	5%	0%	0%	100%	0.987	0.33	38%

Table 3: Districting Plans Maximizing Minority Distributive Benefits: $N_{mD} = 25\%$, $N_{MD} = 40\%$, and $N_R = 35\%$.

groups: concave for more powerful minorities and convex for less powerful. This forms the basis for the following proposition:

Proposition 2. *If $\pi_{mD} = \max_{i\in\Theta}\{\pi_i\}$, then T_{mDk} is concave on S^2; if $\pi_{mD} = \min_{i\in\Theta}\{\pi_i\}$, then T_{mDk} is convex.*

Since optimal values of N_{mD} on a concave surface will be less dispersed than on a convex surface, we have the result that as minority voters gain power, all else being equal, optimal gerrymanders for distributive benefits divide these voters more equally across districts.[28] Formally, let

$$V(\mathbf{D}) = \max_{d_k, d_l \in \mathbf{D}^*} N_{mDk} - N_{mDl}, \tag{5.14}$$

be the maximum difference between the minority population of any two districts in an optimal districting scheme. Then $\frac{\partial V(\mathbf{D})}{\partial \pi_{mD}} \leq 0$, so that minority voters are (weakly) spread out less as their power increases. Combining these results with Proposition 1, we can say that optimal districting schemes will concentrate minority voters in a few districts when their power is low, spread them out when their power is high, and combine them as much as possible with the less powerful of the other two groups.

These results are illustrated in Table 3, which details optimal districts for varying levels of groups' power, done for a state with three districts in which the population proportions of minority-Democratic, majority-Democratic, and Republican voters are 25%, 40%, and 35%, respectively.

[28] In fact, optimal districts when T_{mDk} is convex concentrate all minority voters into as few districts as possible. Conversely, when T_{mDk} is concave, $N_{mD} > 0$ for all districts.

The power of majority-Democratic voters in the simulations is fixed at $\pi_{MD} = 3$, while the other two groups' power varies between 1 and 5. Note that, as predicted, the variance $V(\mathbf{D})$ declines and minorities' utility rises within each set of observations as π_{mD} increases. Where possible, minority voters are designated into districts with more voters from the less powerful of the other groups. Furthermore, when minority voters are the least powerful group, then they are highly concentrated (rows highlighted in blue). In contrast, when they are the most powerful group, and the other groups are uniformly less powerful, minority voters are equally represented in all districts (rows highlighted in orange).

These four patterns concerning benefits, variance, majority voters within minority concentrated districts, and equal spread of minority voters if they are more powerful than equally powerful majority voters are consistent for i) variations in group powers, as illustrated in Table 5 in Appendix B.9, ii) variations in state demographics, as illustrated in Table 6 in Appendix B.10, and iii) variations in the number of districts, as illustrated in Table 7 in Appendix B.11.

5.2 Ideological Benefits

We now turn to the ideological benefit minority voters gain from their representatives. We first examine the likelihood that minority candidates are elected and then examine the expected ideological utilities for minority voters.

5.2.1 Likelihood of Successful Minority Candidates

In the first step, we evaluate the likelihood that minority candidates will be elected, analyzing the first and second derivatives of Ψ_{mD} from (3.11). We can state

Proposition 3. *The probability of electing a minority candidate:*

1. *increases with the number of minority-Democratic voters;*
2. *is ambiguous in the absolute number of majority-Democrat in a district (flexible N_k); and*
3. *is ambiguous in the number of majority-Democrat voters replacing Republican voters in a district (fixed N_k).*

Finally, Ψ_{mD} is convex on S^2 if $a_R^2 < (a_{mD}^2 - a_{MD}^2)N_{MDk}/N_k$.

The results of adding minority voters to a district are not surprising; they can only increase the probability that a minority candidate wins both the primary and general elections. Neither are the results of adding majority-Democratic voters mysterious; these voters support minority candidates in the general election but favor majority-Democratic candidates in the primary, and it is only when the former effect dominates the latter that the overall chances of electing a minority candidate to office rise. Straightforward as this assertion may be, its logical counterpart (really just a restatement under different terms) may still surprise some observers: one may be able to increase the probability of electing a minority-Democrat from a given district by increasing the number of Republican voters.

The fact that $\Psi_{mD}(.)$ is convex at low levels of Republican crossover (a_R^2) implies that under these conditions, districting schemes that maximize the number of minority-Democrats elected will concentrate minority voters in as few districts as possible. This accords with empirical findings on the subject (see, for instance, Cameron et al. (1996)), although it has never been shown in a general theoretical context before. Two interesting points emerge from the analysis here: first, the relation between electing minority-Democrats and concentrating minority voters depends on low crossover rates; when a_R^2 is higher, optimal schemes for descriptive representation spread minority voters more evenly across districts. Second, the convexity of $\Psi_{mD}(.)$ derives from the two-step primary-general election process. Adding minority voters to a district, that is, increases the chances a minority candidate wins both the primary and general elections, and since $\Psi_{mD}(.)$ is the product of these two probabilities, adding minority voters at the margin has a quadratic impact on the overall chances of electing minority candidates to office.[29]

5.2.2 Expected Minority Ideological Benefits

With these expected election outcomes, we can examine minority voters' expected ideological benefits from candidates joining the legislature as their representatives. We can define the average utility per voter of a given type i for a j type representative:

$$\bar{X}_i^j = \int_{-\infty}^{\infty} X_i^j d[\Phi(X_i)]. \tag{5.15}$$

Then the total utility to voters electing a type j representative is $N_{ij}\bar{X}_i^j$. For convenience, recalibrate utilities so that $\bar{X}_{mD}^{mD} = 1$ and $\bar{X}_{mD}^{R} = 0$, and define $\beta \equiv \bar{X}_{mD}^{MD}$, with $0 \leq \beta \leq 1$. Overall expected utility for minority voters includes both the type elected and their average attachment

[29]In fact, looking at the primary and general elections independently, we see that the election function is concave in N_{mDk} for the primary and linear in N_{mDk} for the general, making the overall convexity all the more interesting.

to representatives of that type:

$$E[X] = \Psi_{mD}\bar{X}^{mD}_{mD} + \Psi_{MD}\bar{X}^{MD}_{mD} + \Psi_R\bar{X}^{R}_{mD}$$
$$= \Psi_{mD} + \Psi_{MD}\beta. \tag{5.16}$$

It is natural to ask whether the districting schemes that maximize minority voters' overall expected ideological utility are the same as those that elect minority representatives.

Proposition 4. *There exists a $\tilde{\beta} > 0$ such that for $\beta < \tilde{\beta}$, $E(X)$ is convex on S^2.*

When the extra utility of electing a minority-Democrat is high enough (β is close to 0), the $E(X)$ function is convex. Districting schemes that maximize overall utility coincide with those that elect as many minority-Democrats as possible to office. Conversely, when it is more important to avoid electing Republicans (β is close to 1), the function becomes concave, and optimal schemes spread minority voters more across districts. As partisan concerns rise, then, minority voters prefer to work more through electoral coalitions, joining with majority-Democratic voters to minimize the number of Republicans elected to office.[30]

6 Optimal Districts

We identify districting schemes that maximize minority voters' utility by combining the distributive and ideological benefits derived above. On the one hand, these benefits are additive; seemingly, the task is to add up the above-mentioned effects. On the other hand, this rosy scenario is complicated by the two effects being inextricably linked: groups receive greater distributive benefits with increasing "swinginess," their density at $\phi_i(0)$ rises, but this quantity also indicates the amount of crossover voting by that group.

This observation cuts two ways. First, as majority voters are increasingly willing to cross over and vote for minority candidates, the chances of electing minorities to office rise, which raises the average ideological utility of minority voters. However, this greater willingness to crossover means that majority voters are now more swingy and decisive, so they will receive larger shares of distributive benefits B_k in equilibrium. From minorities' point of view, then, the price for greater electoral support from other groups is a loss of distributive benefits.

[30] As a special case we note that the $E(X)$ function is also convex when minority voters are more likely to support a minority candidate in the general election ($a^2_{mD} > a^2_{MD}$ by assumption), Republican voters more likely support a majority-Democrat than a minority candidate ($a^3_R\beta > a^2_R$), with Democrats similarly voting against a Republican candidate ($a^3_{MD} \geq a^3_{mD}$).

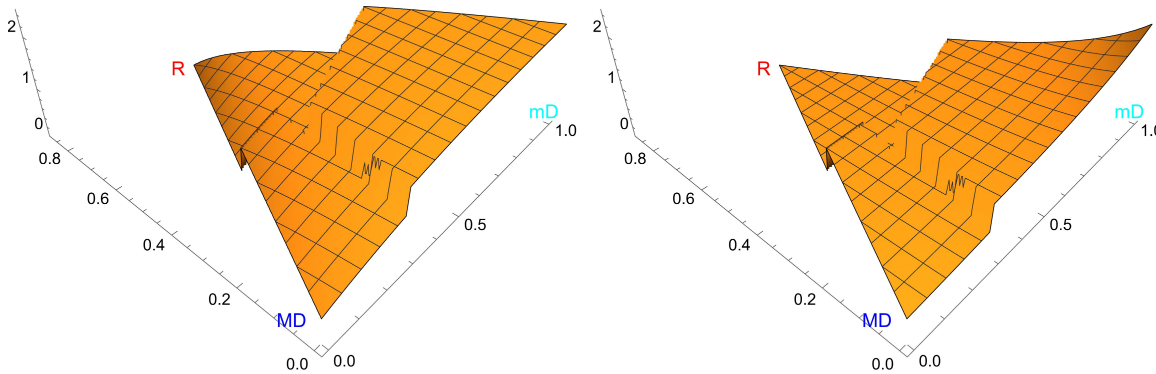
Second, the more politically cohesive minority-Democrats are—the more they vote only for minority-Democrats running for office—the less influential they are compared to other groups, and thus the less distributive benefits they receive. In this sense, the model captures the notion that the most loyal democratic supporters are also the most easily "taken for granted" by their elected representatives. Thus decreased racial or ethnic polarization in voting patterns is a mixed blessing for minorities, involving as it does a tradeoff between ideological and distributive benefits.

How do these considerations affect the nature of optimal districting schemes as minorities gain power? We know that the distributive payoff function T_{ijk} becomes concave as π_{mD} rises; how does this interact with ideological utility, given that $E(X)$ is convex under certain circumstances? We can state

Proposition 5. *Districting schemes that maximize minorities' utility concentrate minority voters less as their power increases.*

If minority voters are motivated more by distributional than ideological benefits (κ_i is increasing π_i) and their voting rates are decreasing in each election round, making them more influential ($\phi_i(0)$ is increasing π_i), the concavity of minority distributive benefits will eventually outweigh any convexity in minority ideological benefits. Hence, more powerful minorities are sufficiently motivated by distributive benefits that will result in a greater spread of minority voters across districts, realizing more distributive benefits. Whereas less powerful minority voters are more concentrated and gain ideological benefits in those districts but overall less distributive benefits. Overall, then, if minority voters prioritize tangible benefits over political beliefs, the spread of benefits among them will outweigh the concentration of benefits. Powerful minorities, therefore, benefit more from a spread of voters, while less powerful ones gain more from concentration. Minority voters become more influential as their voting rates decrease.

Figure 8 is the overall utility for minority voters combining the concave (convex) distributive benefits from Figure 7 and the different types of representatives winning elections from Figure 4. Thus, we have Figure 7 with "cliffs" following the electoral outcomes from Figure 4 showing minority voters' extra utility from electing a minority-Democrat ($\bar{X}^{mD}_{mD} = 1$) or majority-Democrat ($\bar{X}^{MD}_{mD} = \beta = .5$), using Republicans as the baseline ($\bar{X}^{R}_{mD} = 0$). This extra dimension (literally and figuratively) to the analysis adds an incentive to create concentrated minority districts where minority candidates can get elected or at least have a majority-Democrat elected rather than a Republican. These districts are of the type in which minority candidates can eke out a win; that is,



(a) $\pi_{mD} = 10$, $\pi_{MD} = 5$, $\pi_R = 2$

(b) $\pi_{mD} = 2$, $\pi_{MD} = 5$, $\pi_R = 10$

Figure 8: Minority Total Benefits – $a = .3$, $b = .7$, $\beta = .5$.

they sit near the region's border in which minority candidates win and do not waste extra votes that could be more fruitfully used elsewhere. These specific regions have been noted as "sweet spots" where minority voters are sufficiently decisive across many districts electing minority candidates (Lublin et al., 2020). However, these districts may not deliver maximum minority benefits, as illustrated in Figure 8. We can see that districts with greater minority voter concentrations may realize greater utility from ideological and distributive benefits (top right corner in each graph).

The magnitude of the cliffs is decreasing in i) minority voters' extra utility from electing a minority-Democrat (larger β) as illustrated in Figure 18 in Appendix B.12, ii) in majority-Democratic voters' crossover voting in the primary as illustrated in Figure 19 in Appendix B.13, and iii) in majority-Democratic voters' crossover voting in the general election as illustrated in Figure 20 in Appendix B.14.

The cliffs in Figure 8 show a discontinuity in minority voters' utility or payoffs based on electoral outcome changes. Counterintuitively, as minority voters' preference for a minority-Democrat increases, the difference between the expected utility of electing a minority-Democrat versus a non-minority-Democrat decreases. In this scenario, partisan effects outweigh the identity ones. As more majority-Democrats engage in crossover voting (voting for a candidate from the other party in the primary), the difference between the expected utility of electing a Democratic and a Republican candidate decreases. In this case, identity effects outweigh partisan ones. Furthermore, majority-Democrats' increased crossover in the general election reduces the difference between the expected utility of electing a Democratic and a Republican candidate because the

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{MD}	π_R	n_{mD1}	n_{MD1}	n_{R1}	n_{mD2}	n_{MD2}	n_{R2}	n_{mD3}	n_{MD3}	n_{R3}	Total	Aver.	$V(\mathbf{D})$
1	3	1	0%	54%	46%	0%	66%	34%	75%	0%	25%	2.75	0.92	75%
2	3	1	18%	45%	36%	18%	58%	24%	39%	16%	45%	3.28	1.09	20%
3	3	1	20%	50%	31%	20%	49%	31%	35%	21%	44%	4.00	1.33	16%
4	3	1	21%	54%	25%	21%	41%	38%	32%	26%	42%	4.18	1.39	11%
5	3	1	24%	60%	16%	27%	0%	73%	24%	60%	16%	3.41	1.14	3%
6	3	1	22%	53%	25%	21%	41%	38%	32%	26%	42%	4.47	1.49	11%
7	3	1	26%	64%	10%	25%	0%	75%	25%	56%	20%	3.65	1.22	1%
8	3	1	24%	51%	26%	20%	42%	37%	31%	27%	42%	4.68	1.56	11%
9	3	1	24%	50%	26%	20%	43%	37%	31%	27%	42%	4.77	1.59	11%
10	3	1	24%	50%	26%	20%	43%	37%	31%	28%	42%	4.84	1.61	11%
1	3	3	0%	51%	49%	0%	55%	45%	75%	14%	11%	2.50	0.83	75%
2	3	3	0%	64%	36%	0%	52%	48%	75%	5%	20%	2.67	0.89	75%
3	3	3	24%	39%	37%	10%	44%	46%	41%	37%	22%	3.25	1.08	31%
4	3	3	25%	48%	26%	20%	43%	37%	30%	28%	42%	3.92	1.31	11%
5	3	3	25%	49%	26%	20%	43%	37%	30%	28%	42%	4.07	1.36	11%
1	3	5	0%	45%	55%	0%	50%	50%	75%	25%	0%	2.00	0.67	75%
2	3	5	5%	45%	50%	0%	50%	50%	70%	25%	5%	2.61	0.87	70%
3	3	5	5%	45%	50%	0%	50%	50%	70%	25%	5%	2.71	0.90	70%
4	3	5	14%	36%	50%	32%	45%	24%	29%	40%	31%	3.27	1.09	18%
5	3	5	32%	43%	24%	14%	36%	50%	29%	41%	31%	3.40	1.13	18%

Table 4: Districting Plans Maximizing Minority Total Benefits: $N_{mD} = 25\%$, $N_{MD} = 40\%$, $N_R = 35\%$, $a = .3$, $b = .7$, and $\beta = .5$.

probability of electing a minority increases.

To illustrate these tradeoffs identified in Proposition 5, we calculate optimal districting schemes for the same values of π_{mD}, π_{MD}, and π_R power as in Table 3, and using the same overall population proportions. The extra utility of electing a majority-Democrat is assumed to be 0.5 and 1 for electing a minority candidate relative to a baseline of 0 for a Republican. The majority-Democrat primary crossover rate is 30%, while the general election crossover rates are 70%, as illustrated in Figure 4. The variances are generally higher in Table 4 compared to Table 3 resulting from the increased desire to concentrate minorities up to the point where a minority candidate can be elected in some districts. Note also that the rule stating that $V(D)$ weakly decreases within each subgroup of five simulations still holds. It is also still the case that when minorities are the least powerful group, at least one district has no minority voters, and when minorities are the most powerful, $N_{mDk} > 0$ for all districts k.

Our simulations of districting plans also underscore the paradox of minority voting power. The two highlighted rows in Table 4 indicate that a lower concentration of minority voters, when they are more powerful, may come at the expense of lower total payoffs compared to a slightly higher concentration but with majority-Democratic voters in all districts. This tension arises when majority-Democratic voters are more powerful than Republican ones ($\pi_{MD} > \pi_R$). Proposition 1 predicts concentrating more powerful minorities with less powerful majority voters maximizes minority voters' distributive benefits; however, when we also consider minority voters' ideological benefits ($\beta = .5$), a majority-Democratic candidate is preferred to a Republican one.

The four patterns concerning benefits, variance, majority voters within minority concentrated districts, and equal spread of minority voters if they are more powerful than equally powerful majority voters are generally consistent for i) variations in group powers, as illustrated in Table 8 in Appendix B.15, ii) variations in state demographics, as illustrated in Table 9 in Appendix B.16, iii) variations in the number of districts, as illustrated in Table 10 in Appendix B.17, iv) variations in minority ideological benefits, as illustrated in Tables 11 and 12 in Appendix B.18, and v) variations in primary and general crossover rates, as illustrated in Tables 13 and 14 in Appendix B.19. Still, there are few cases where the variance is not decreasing monotonically in minority power. Again, the results highlight potential cliffs and the interactions of how minority voters may be powerful due to their swinginess or focus on legislative outcomes, other groups' voting power, state demographics, and crossover voting patterns in primary and general elections, that affect the optimal redistricting when we consider both descriptive and substantive representation and partisan and identity effects within and across parties.

7 Discussion and Conclusion

This paper offers a comprehensive approach to redistricting and representation. We examine how voters' ideological and policy preferences align with a candidate's identity and partisan affiliation to determine minorities' political power (swinginess), and the policy benefits they receive (distributive gains) in a majoritarian political process. We observe optimal redistricting as a nuanced tradeoff between ideological and distributive benefits. First, grouping less influential minority voters with less influential majority voters is more effective in promoting minority interests than grouping influential minority voters with influential majority voters, regardless of their political party. Second, concentrated minority districts work well when minority voters are less swingy and prioritize ideological benefits, emphasizing coalition formation at the electoral stage. In contrast, spreading minority voters across multiple districts is more effective when they are swingy and focused on distributive benefits, thereby shifting coalition building to the legislature. Third, optimal districting depends on the likelihood of electing a Democrat or a Republican candidate in the primary and general elections.

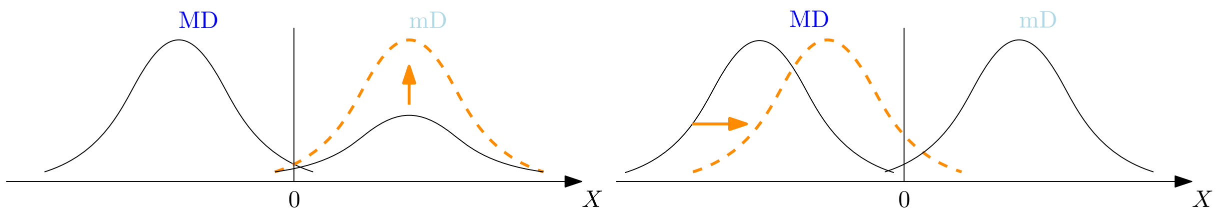
The analysis highlights that redistricting is a multi-dimensional problem. In contrast, other studies search for the "sweet spot" between Republicans and minority voters. When minority voters emphasize ideological benefits to the detriment of distributive policies, the situation resembles the one-dimensional scenario analyzed by Lublin et al. (2020). However, when minority voters value ideological and distributive benefits with varying levels of importance, the goal becomes not just about electing a specific candidate but also about achieving influence within the legislature. In this context, minority groups gain by allocating as many voters to as many districts as possible. When minority voters become influential in determining district elections, the emphasis shifts to finding the optimal combination of voters to enhance minority legislative power. Thus, the effectiveness of redistricting in maximizing minority representation depends on the power of minority voters and the weight they assign to distributive benefits.

To conclude our analysis, we apply the framework developed in the previous sections to examine the impact of various changes in the political landscape, increasing Black voter registration, the defection of white Democrats to the Republican party, and decreasing racism on minority electoral success, policy benefits, and optimal redistricting plans.

Increasing Minority Registration and Voting Turnout. Before the passage of the 1965 Voting Rights Act (VRA), many Southern states enacted laws to de facto disenfranchise Blacks. Such devices as the grandfather clause, poll taxes, and white-only primaries, not to mention direct intimidation, minimized Blacks' participation in politics. When one form of discrimination was outlawed, the states would switch to another. This macabre game of wack-a-mole continued until the VRA swept away all such "tests and devices," and its Section 5 preclearance provisions required covered states, those with historical patterns of discrimination, to obtain the permission of the federal government before adopting any new law that might impact minorities' ability to vote. The most direct result of passing the VRA was thus to greatly increase Blacks' participation to the point where now, in most areas of the South, minorities register and vote at rates at or above those of white voters.[31]

From the model above, the impact of an increase in statewide nonminority-Democrats is (usually) unambiguous: it acts just like an increase in power π_{mD}, and so both increase the flow of benefits to minority constituents and make it easier to elect minorities to office, thereby increasing their ideological benefits. These electoral benefits are illustrated in Figure 5(b): increasing the

[31]By 2020, Black registration and electoral turnout in Southern states only differed by 5 percentage points.



(a) Minority Voter Registration. (b) Majority-Democratic Crossover.

Figure 9: Relative Power of Majority and minority-Democratic Voters.

percentage of minority voters while keeping the ratio of MD and R voters constant nearly always makes it easier to elect minority candidates. We say "nearly always" because, as pointed out in the earlier discussion of the figure, there are some exceptional regions where, under sincere voting, Republicans get elected rather than Democrats (Figure 16). Other than this, though, the overall effect should be to increase descriptive representation. The shift in legislative benefits is illustrated in Figure 9(a), where the horizontal axis shows the ideological distributions of the MD and mD voters, with the 0, or indifference, point in the middle. The increase in the size of the minority electorate increases their power π_{mD} by raising $\phi_{mD}(0)$ while also increasing the number of districts that could elect an mD candidate. Cascio and Washington (2014) document how the reforms following the VRA increased voter turnout and public spending for counties with a higher Black population.

Furthermore, according to the model, the first response of state district drawers as the number of Blacks registered and voting increases from low numbers should be to create concentrated minority districts. Indeed, this happened in the 1970s and 1980s, with one rule of thumb stating that districts had to be at least 65% Black to be "effectively" majority-minority. As minority participation continues to increase, the response should be to concentrate minority voters less, spreading them out more evenly across districts. Some worry that reducing the majorities in these districts will dilute their influence over policy and reduce the number of minorities in office, thereby giving back some of the hard-won gains of the civil rights movement. Others see it as a natural progression of minorities into mainstream politics and a way to spread their influence over greater areas.

Increasing Crossover. Finally, we come to the increased willingness of non-minority voters of all stripes to vote for minority candidates due to steadily decreasing racism. Figures 3(a) and 3(b) above show the impact of such changes in the values of a and b, expanding the region where an mD candidate can win the general election. Thus decreasing racism does help minorities win office, and indeed, the number of elected minorities in the South has skyrocketed since adopting the VRA.[32]

Nevertheless, as mentioned above and illustrated in Figure 9(b), the impact on distributive benefits is complex. For the majority to be less racist, they must be less ideologically averse to minorities' holding office, represented by a righthand shift in the distribution of X-values as in Figure 9(b). This shift increases MD voters' density at $X = 0$; as minorities become more influential in elections, they will enjoy more legislative benefits. These gains continue until the central hump of the distribution passes the 0 threshold, past which decreased racism (increased crossover) also leads to less of a share of the legislative pie.

Since less than 50% of majority voters reliably support minority candidates, though, we may assume that we still reside on the upward slope of the distribution function. Hence majority voters may be gaining increasing benefits from candidates' platforms at minority voters' expense. Of course, the tradeoff regarding increased descriptive representation may be worthwhile. However, it is still interesting that decreased racism is not an unalloyed good for minority voters. There have long been rumblings that Democrats in office, white and Black alike, take their minority constituents for granted and give them less than their fair share of insiders' benefits. While this may be true, and if so, the model given here provides a plausible rationale for why it would happen.

Increased Partisan Polarization. Another notable development concerns the breakup of the formerly "Solid South" Democratic party and the associated increase in partisan divisions. Since Reconstruction, Southerners had identified the Republicans as the party of Lincoln and the North and thus voted nearly unanimously for Democratic candidates. However, Democratic support of the VRA and other civil rights measures in the 1960s led inexorably to the defection of many Southerners to the Republican party, which had a solid conservative message that appealed to many voters. As we move from a situation where MD voters dominate the political landscape to one where Republican (R) voters are also in evidence, it becomes easier to elect minority-Democrat (mD) candidates to office.[33] Thus, the electoral and, hence, ideological impact on Black voters is

[32]See the essays Davidson and Grofman (1994) for detailed state-by-state analyses attributing the rise in Black office holding directly to the VRA. Indeed, 2020 voter data shows that whites register at a 75% rate, Blacks 68%, Asians 64%, and Hispanics 59%—similarly, Whites turnout at a 69% rate, Blacks 61%, Asians 59%, and Hispanics 52%. See kff.org.

[33]The electoral impact of this shift has been investigated in Figure 5(a).

positive.

In addition, the impact of majority-Democractic voters on minority benefits from (5.8) shows that minorities gain policy benefits if the weaker group replaces the politically stronger group of non-minority voters. Recall that we define power in terms of $\phi_i(0)$. High-power groups are decisive and sufficiently motivated by distributive benefits or vice versa. Low-power groups are neither influential nor motivated by distributive gains. Given that MD voters are more centrist than Republicans, the change is in the desired direction, allowing minorities to compete more equally for their legislative pie. Thus the switch from majority-Democrats to Republicans is also in the minority group's favor.[34]

Hispanic and Latino Vote. Recent trends in the Hispanic/Latino vote in the United States are complex and multifaceted. Historically, Hispanics have aligned with the Democratic party and tend to be concentrated in minority districts largely overlooked by the Democratic party. However, as Hispanics increasingly become swing voters, shifting between the Republican and Democratic parties, their ability to gain distributive gains will inevitably increase.

One factor that appears to be driving this shift is working-class labor trends, with some Hispanic/Latino voters moving towards the Republican Party in response to economic concerns and a desire for greater job security. Evidence suggests that conservative social issues, such as abortion and same-sex marriage, also play a role in this shift. Another factor is the diversity within the Hispanic/Latino community, which includes individuals with a wide range of cultural, linguistic, and socioeconomic backgrounds. Consequently, there is significant variation in voting patterns based on individual and community-level preferences.

Yet the political stakes have never been higher; partisan polarization increases the costs of not winning an election. The backsliding of abortion, the introduction of work requirements for food stamps, and the book purge all have had significant impacts. What is increasingly clear is that the Hispanic population is less readily aligned with most Democrats, who are increasingly affluent, educated, and white. As a result, it is difficult to generalize about the political preferences of this group as a whole, as there is significant variation in voting patterns across different subgroups of Hispanic/Latino voters. These factors will likely continue to play a role in shaping Hispanic political preferences.

[34] The logic provided by Dixit and Londregan (1996) prevails here. The group at the center of the voter distribution, the influential voters, benefit most from redistributive politics. Groups that are numerous but concentrated at large positive (or negative values) of X will not partake in these benefits; they will be written off by one party and taken for granted by the other.

Future Research We end by pointing to several possible extensions to our model. First, our current legislative model is simple to focus on the logic of changing preferences and group powers without building an incentive to form party-based coalitions in the legislature. Hence there are no real legislative parties or permanent coalitions. Nevertheless, if one wanted to investigate the impact of changing legislative rules, committee powers, or party leadership on districting, these elements could be incorporated into the legislative model.

Voter Demographics and Partisanship. Our analysis centers around the districting, electoral, and legislative characteristics and demographics in the United States. We consider two parties and divide the population into a minority and a majority. Both demographics and partisanship describe the groups and their political motivations. However, our analysis can be generalized to majoritarian systems in which parties compete in single-representative districts, holding some form of primaries to choose a candidate from within the party who faces other parties' candidates, a legislature awarding distributive benefits across districts and voting groups, and voting groups identified by partisanship and identity. The two voters' identities can follow ethnic, racial, religious, economic, or gender attributes that may define a group and describe the group's voting behavior. For more than two voter identities, groups may still be similar regarding their willingness to cross partisan and identity lines. For example, many minority voters, such as Black, Afro-American, Hispanic, Asian-American, or LGBTQIA+, tend to support in various degrees the Democratic party in the United States, and their willingness or reluctance to vote for Republican candidates may unify them. The key tensions in our analysis arise from i) a group's willingness to trade off distributive benefits from a legislature for ideological benefits from a candidate's identity, ii) a group's willingness to cross partisan and identity lines in elections (e.g., minority voters' attachment to a party, majority voters' tradeoff between minority candidate from the same party or majority candidate from the other party), and iii) the institutional principles of districting.

Comparative Studies of Electoral Rules. One could also examine the impact of other electoral rules. Here, minorities must win a primary and then a general election via plurality votes to gain office. One could use single transferable votes, multi-member districts, approval voting, or any other popular election method and see how that changes the results, both in getting minorities elected and in the policy favors paid by candidates of one type to their supporters of another type. Such a comparative analytical study has yet to be undertaken. Yet in doing so, it is worthwhile to recall Key (1949). Southern politics after Reconstruction was divided not simply because Blacks had more liberal policy preferences than whites but because the underlying bedrock of politics at all levels was a desire to deny Blacks any representation. Formal models of political institutions in divided societies should incorporate the notion that in such circumstances, race can be more than a symbol, more than a summary of policy positions across various issues; often, race is *the* issue.

Electoral Platforms and Fiscal Policies. Our results follow Dixit and Londregan (1996)'s characterization of electoral platforms in which candidates make identical promises regarding redistribution of consumption benefits and taxes for each voter group in a district, all under the assumption that both parties have identical abilities to distribute benefits and raise taxes. Our analysis streamlines the budget process and neglects the collection of taxes, assuming that there is sufficient fiscal capacity and discretion in the budget. Instead, we focus on distributive benefits only, fiscal transfers from the legislature's budget to the district(s), that come as subsidies, tax credits, tax deductions, or welfare payments. If candidates were to run on in-kind transfers or public goods, where voter groups may have different preferences across programs, each candidate would need to evaluate the marginal value of a dollar spent for any program on the voter group's marginal utility from the program affecting the change in the voter group's vote. We leave this interesting angle for future research.

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A Online Appendix: Derivations and Proofs

A.1 Derivation of Solution to Candidate Platforms

The derivation and solution of the subgame follow Dixit and Londregan (1996) and incorporate the districting plans and characterization of minority benefits from our model.

Candidate Platforms First, we characterize the candidates' platforms. Candidates adopt platforms to maximize their votes subject to an allocation of district benefits across voter groups. In each election, two candidates announce simultaneously. Candidate 1's problem is

$$\max V_1^e = \sum_{i\in\Theta} N_i \Phi_i^e(X_i) \text{ s.t. } \sum_{i\in\Theta} T_{i1k} B_k = \sum_{i\in\Theta} N_{ik} b_{i1k} \leq B_k \tag{A.1}$$

and candidate 2's is equivalently

$$\max V_2^e = \sum_{i\in\Theta} N_i [1 - \Phi_i^e(X_i)] \text{ s.t. } \sum_{i\in\Theta} T_{i2k} B_k = \sum_{i\in\Theta} N_{ik} b_{i2k} \leq B_k. \tag{A.2}$$

For the existence of a Nash equilibrium, Glickberg's Theorem requires that each candidate's payoffs are a quasi-concave function of their strategy and a continuous function of other players' strategies. First, the distributive benefits for voters, b_{ij}, are an increasing linear function of the candidates' platforms, T_{ijk}. Second, the voters' cutoff for differences in candidates' promised benefits, $X_i^e = U_i(b_{i1}) - U_i(b_{i2})$, is increasing and concave in b_{ij}. Finally, the expected candidate's number of vote, $V_1^e = \sum_{i\in\Theta} N_i \Phi_i^e(X_i)$ and $V_2^e = \sum_{i\in\Theta} N_i(1 - \Phi_i^e(X_i))$, is increasing in the cutoff X_i and concave due to the concavity of $\Phi_i(.)$. Hence, existence holds.

For the solution of the Nash equilibrium, we use Lagrange parameters λ_1 and λ_2 for each respective candidate and solve all first-order conditions simultaneously. Consider candidate 1 first:

$$L = \sum_{i\in\Theta} N_i \Phi_i^e(X_i) + \lambda_1 \left(B_k - \sum_{i\in\Theta} N_{ik} b_{i1k} \right) \tag{A.3}$$

with

$$\frac{\partial L}{\partial T_{i1k}} = N_{ik} \phi_i^e(X_i) U_i'(b_{i1k}) \frac{\partial b_{i1k}}{\partial T_{i1k}} - \lambda_1 N_{ik} \frac{\partial b_{i1k}}{\partial T_{i1k}} = 0, \tag{A.4}$$

which can be written as

$$\lambda_1 = \phi_i^e(X_i) U_i'(b_{i1k}) \;\Leftrightarrow\; b_{i1k} = H_i\left(\frac{\lambda_1}{\phi_i^e(X_i)} \right), \tag{A.5}$$

where $H_i(.)$ is the inverse of the marginal utility function. Since $U_i(.)$ is a decreasing function, $H_i(.)$ is decreasing as well, and there is a unique solution for λ_1.

Candidate 2's problem is symmetric, and we have

$$L = \sum_{i \in \Theta} N_i(1 - \Phi_i^e(X_i)) + \lambda_2 \left(B_k - \sum_{i \in \Theta} N_{ik} b_{i2k} \right) \tag{A.6}$$

with

$$\frac{\partial L}{\partial T_{i2k}} = -N_{ik}\phi_i^e(X_i)(-U_i'(b_{i2k}))\frac{\partial b_{i2k}}{\partial T_{i2k}} - \lambda_2 N_{ik}\frac{\partial b_{i2k}}{\partial T_{i2k}} = 0, \tag{A.7}$$

which can be written as

$$\lambda_2 = \phi_i^e(X_i)U_i'(b_{i2k}) \;\Leftrightarrow\; b_{i2k} = H_i\left(\frac{\lambda_2}{\phi_i^e(X_i)}\right), \tag{A.8}$$

which provides a unique solution for λ_2.

The Lagrange parameters are independent of candidates' characteristics, which implies both candidates face the same shadow value in equilibrium, $\lambda_1 = \lambda_2$. As a result, a voter's marginal utility in distributive benefits is equal across both candidates:

$$\lambda_1 = \lambda_2 \;\Leftrightarrow\; U_i'(b_{i1k}) = U_i'(b_{i2k}). \tag{A.9}$$

Due to $U_i(.)$ being a continuous, increasing function, we have that the distributive benefits are identical across both candidates, $b_{i1k} = b_{i2k}$, which implies that both candidates choose identical platforms, $T_{i1k} = T_{i2k}$, and distributive promises cancel each other out such that voters choose based on ideological alignments.

Group and Member Benefits Now, we describe the distribution of district benefits across groups and their members. We use the first-order conditions above with the voters' utility function described by (3.1) and (3.2):

$$\lambda_j = \phi_i^e(0)U_i'(b_{ijk}) \;\Rightarrow\; b_{ijk} = \left(\frac{\phi_i(0)\kappa_i}{\lambda_j}\right)^{\frac{1}{\epsilon}} = \frac{\pi_i}{\lambda_j^{1/\epsilon}} \;\Leftrightarrow\; \lambda_j^{1/\epsilon} = \frac{\pi_i}{b_{ijk}}. \tag{A.10}$$

Applying $\lambda_1 = \lambda_2$ and $b_{i1k} = b_{i2k}$, we get for group i compared to group $h \neq i$ that

$$b_{ijk} = \frac{\pi_i b_{hjk}}{\pi_h}. \tag{A.11}$$

Using the budget constraint of $\sum_i N_{ik} b_{ijk} = B_k$ with (A.11), we get

$$b_{hjk} = \frac{\pi_h}{\sum_i N_{ik}\pi_i} B_k, \tag{A.12}$$

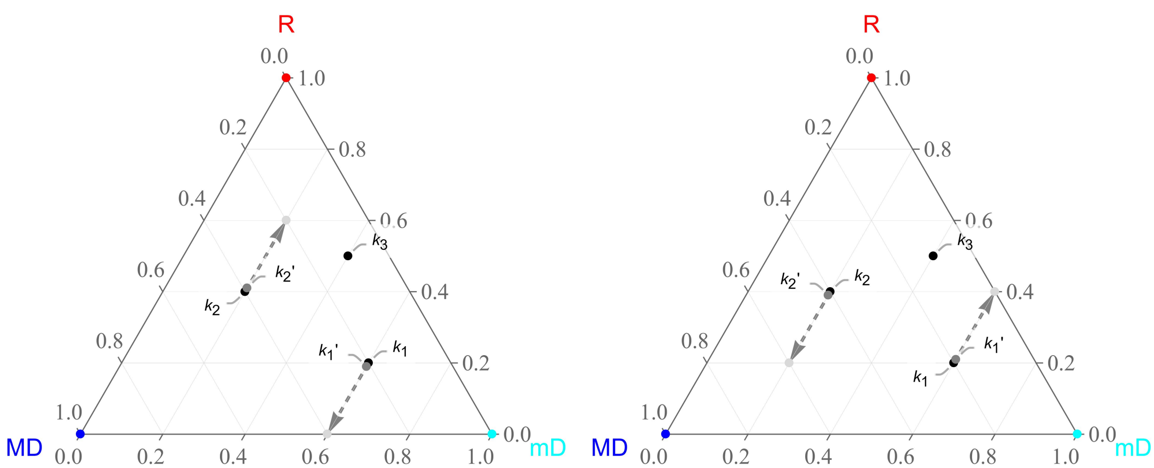
which provides the individual benefits for a member of group Θ in (4.2). The group shares follow from rearranging $b_{ijk} = (T_{ijk} * B_k)/N_{ik}$ and applying (A.12):

$$T_{hjk} = \frac{b_{hjk} N_{ik}}{B_k} = \frac{\pi_h N_{hk}}{\sum_i N_{ik}\pi_i}, \tag{A.13}$$

which completes (4.2).

A.2 Proof of Proposition 1

Consider a valid districting scheme **D**, and assume to the contrary that $N_{ik} > 0$ for all $i \in \Theta$ and k. First, assume that Republicans are more powerful than majority-Democrats, $\pi_{MD} < \pi_R$. Consider two districts k_1 and k_2, with N_{mD1} minority voters in k_1 and N_{mD2} in k_2, and assume that $N_{mD1} > N_{mD2}$. This is illustrated in Figure 10(a).



(a) $\pi_{MD} < \pi_R$ and $\Pi_1 \leq \Pi_2$.

(b) $\pi_{MD} > \pi_R$ and $\Pi_1 \leq \Pi_2$.

Figure 10: Optimal Districting Process with $K = 3$.

We now move one Republican voter from k_1 to k_2 and one majority-Democratic voter from k_2 and k_1, while holding minority voters constant in each district. Such a change preserves the validity of the districting scheme, and the arrows in Figure 10(a) illustrate the direction of changes. For optimality, it has to increase minority voters' distributive benefits. Hence, we compare any gains and losses across the two districts.

Considering the changes in minority benefits from (5.8), minority voters' distributive benefits in k_1 are increasing when a Republican is replaced with a less powerful majority-Democratic voter, by

$$N_{mD1}\frac{\partial f}{\partial N_{MD1}} = N_{mD1}^2\frac{\pi_{mD}(\pi_R - \pi_{MD})}{\Pi_1^2}; \tag{A.14}$$

while minority voters' distributive benefits in k_2 are decreasing when a more powerful Republican replaces the majority-Democratic voter, by

$$N_{mD2}\frac{\partial f}{\partial N_{MD2}} = N_{mD2}^2\frac{\pi_{mD}(\pi_R - \pi_{MD})}{\Pi_2^2}. \tag{A.15}$$

Comparing A.14 with A.15, we get

$$\frac{N_{mD1}^2}{\Pi_1^2} \gtreqless \frac{N_{mD2}^2}{\Pi_2^2}. \tag{A.16}$$

Given the assumption of $N_{mD1} > N_{mD2}$, the minority gains in k_1 outweigh minority losses in k_2 if $\Pi_1^2 \leq \Pi_2^2$, and such redistricting would be optimal as it increases net gains for minority voters.

Second, assume that majority-Democratic voters are more powerful than Republican voters, $\pi_{MD} > \pi_R$, and the number of minorities differs, $N_{mD1} > N_{mD2}$. Then it would be beneficial to move a majority-Democrat from k_1 to k_2, and a Republican vice versa, if the minority gains in k_1 are greater than the minority losses in k_2. This comparison follows again

$$\frac{N_{mD1}^2}{\Pi_1^2} \gtreqless \frac{N_{mD2}^2}{\Pi_2^2} \text{ with } \pi_{MD} > \pi_R. \tag{A.17}$$

If the minority-concentrated district is less powerful, then this would be beneficial. Figure 10(b) illustrates this process.

Hence, the proposed districting scheme **D** cannot be optimal. Through re-iteration of the process – the district's power decreases when less powerful majority voters replace more powerful majority voters, $\frac{\partial \Pi_k}{\partial N_{MD}} = \pi_{MD} - \pi_R$ – minority populated districts with low district power or majority populated districts with high district power will not lie in the interior of S^2.

A.3 Proof of Proposition 2

Consider a valid districting scheme **D** and a district $\mathbf{d}^* = (N_{mD}, N_{MD}, N_R)$. Define $t = N_{mD}/N_k$ and $\alpha = N_{MD}/(N_{MD} + N_R)$, and let $l = \{\mathbf{d} \in \mathcal{D} | \alpha = N_{MD}/(N_{MD} + N_R)\}$. Thus l is a line running through $\mathbf{d}^*$, connecting it to $(1,0,0)$, which is the corner of S^2 where the district composes of minority voters only ("mD" in triangles) while keeping the ratio of majority voters constant throughout. Applying (5.1) divided by N_k with $t = N_{mDk}/N_k$, $(1-t)*\alpha = \frac{N_{MDk}+N_{Rk}}{N_k} * \frac{N_{MDk}}{N_{MDk}+N_R} = N_{MDk}/N_k$, and $(1-t)*(1-\alpha) = \frac{N_{MDk}+N_{Rk}}{N_k} * \frac{N_{Rk}}{N_{MDk}+N_R} = N_{Rk}/N_k$, the parameterized path for the minority payoff function is

$$g(t) = \frac{\pi_{mD} t}{\pi_{mD} t + \pi_{MD}(1-t)\alpha + \pi_R(1-t)(1-\alpha)} \tag{A.18}$$

Note that the denominator is positive, and we can evaluate the curvature of the path with its second derivative with respect to t:

$$g''(t) = -\frac{2\pi_{mD}\overbrace{(\alpha\pi_{MD} + (1-\alpha)\pi_R)}^{(+)}\overbrace{(\pi_{mD} - \alpha\pi_{MD} - (1-\alpha)\pi_R)}^{(?)}}{\underbrace{(\pi_{mD} t + \pi_{MD}(1-t)\alpha + \pi_R(1-t)(1-\alpha))^3}_{(+)}}. \tag{A.19}$$

The second derivative is negative if $\pi_{mD} > \alpha\pi_{MD} - (1-\alpha)\pi_R$ – i.e., when minorities' power is greater than the weighted average of the other groups' powers, based on district population. Hence, $\pi_{mD} = \max_{i\in\Theta}\{\pi_\Theta\}$ implies that $g''(t) < 0$ for all t, indicating that the entire surface is concave. Conversely, the second derivative is positive if $\pi_{mD} < \alpha\pi_{MD} - (1-\alpha)\pi_R$, which implies that for $\pi_{mD} = \min_{i\in\Theta}\{\pi_\Theta\}$ we get $g''(t) > 0$ for all t, and the surface is convex.

A.4 Proof of Proposition 3

For the first part, we substitute the primary and general election probabilities of a minority candidate winning at each stage, (3.8) and (3.9), take the respective derivative from (3.11) with respect to the number of minority voters, and get:

$$\frac{\partial\Psi_{mDk}}{\partial N_{mDk}} = \overbrace{\frac{\partial\Psi^1_{mDk}}{\partial N_{mDk}}\Psi^2_{mDk}}^{primary} + \overbrace{\Psi^1_{mDk}\frac{\partial\Psi^2_{mDk}}{\partial N_{mDk}}}^{general} \tag{A.20}$$

$$= \underbrace{\frac{(a^1_{mD} - a^1_{MD})}{(N_{mDk} + N_{MDk})^2}\underbrace{\Psi^2_{mDk}}_{(\geq 0)}}_{primary:(+)} + \underbrace{\underbrace{\Psi^1_{mDk}}_{(\geq 0)}\frac{(a^2_{mDk} - a^2_{MD})N_{MDk} + (a^2_{mD} - a^2_R)N_{Rk}}{(N_{mDk} + N_{MDk} + N_{Rk})^2}}_{general:(+)} \geq 0. \tag{A.21}$$

Given our assumption that minority voters are more likely to vote for a minority candidate than majority voters, $a^e_{mD} > a^e_{mD} > a^e_R$, we see immediately that the first term is positive,

$a^1_{mD} > a^1_{MD}$, and the two subsequent terms are positive or zero. The last term is positive or zero as well as for $a^2_{mD} \geq a^2_{MD}$ and $a^2_{mD} \geq a^2_R$. Note that our statement is independent of whether $a^2_{MD} \gtreqless a^2_R$. The same holds for an evaluation on S^2 with $N_{Rk} = N_k - N_{mDk} - N_{MDk}$.

For the second part, we repeat the substitution but take the respective derivative from (3.11) with respect to the number of majority-Democratic voters and get:

$$\frac{\partial \Psi_{mDk}}{\partial N_{MDk}} = \overbrace{\frac{\partial \Psi^1_{mDk}}{\partial N_{MDk}} \Psi^2_{mDk}}^{primary} + \overbrace{\Psi^1_{mDk} \frac{\partial \Psi^2_{mDk}}{\partial N_{MDk}}}^{general} \tag{A.22}$$

$$= \underbrace{\frac{(a^1_{MD} - a^1_{mD})}{(N_{mDk} + N_{MDk})^2} \underbrace{\Psi^2_{mDk}}_{(\geq 0)}}_{primary:(-)} + \underbrace{\underbrace{\Psi^1_{mDk}}_{(\geq 0)} \frac{(a^2_{MD} - a^2_{mD}) N_{MDk} + (a^2_{MD} - a^2_R) N_{Rk}}{(N_{mDk} + N_{MDk} + N_{Rk})^2}}_{general:(+/-)} \gtreqless 0. \tag{A.23}$$

The first term is negative due to $a^1_{mD} > a^1_{MD}$, the two subsequent terms are positive or zero, and the last term is ambiguous. We have that $a^2_{mD} > a^2_{MD}$, illustrating the negative effect in the primary election for minority candidates. The derivative may be negative overall, but that may be offset by $a^2_{mD} > a^2_{MD}$ and $a^2_{MD} > a^2_R$, which illustrates the positive effect in the general election for minority candidates – majority-Democratic voters being more likely to support a minority-Democratic candidate than Republican voters ($a^2_{MD} > a^2_R$). Here the result depends on the relationship between differences in crossover voting of majority voters.

Regarding majority-Democrats replacing Republican voters, we rewrite the probability of a minority candidate winning the election (3.9) as

$$\tilde{\Psi}_{mDk} = \left(\frac{a^1_{mD} N_{mDk} + a^1_{MD} N_{MDk}}{N_{mDk} + N_{mDk}} \right) \left(\frac{a^2_{mD} N_{mDk} + a^2_{MD} N_{MDk} + a^2_R (N_k - N_{mDk} - N_{MDk})}{N_k} \right), \tag{A.24}$$

where we employ the district's population for substitution, $N_{Rk} = N_k - N_{mD} - N_{MD}$. To illustrate the replacement effect – increasing majority-Democratic voters and decreasing Republican voters in a district – we take the derivative with respect to the number of majority-Democratic voters and get:

$$\frac{\partial \tilde{\Psi}_{mDk}}{\partial N_{MDk}} = \overbrace{\frac{\partial \Psi^1_{mDk}}{\partial N_{MDk}} \tilde{\Psi}^2_{mDk}}^{primary} + \overbrace{\Psi^1_{mDk} \frac{\partial \tilde{\Psi}^2_{mDk}}{\partial N_{MDk}}}^{general} \tag{A.25}$$

$$= \underbrace{\frac{(a^1_{MD} - a^1_{mD})}{(N_{mDk} + N_{MDk})^2} \underbrace{\Psi^2_{mDk}}_{(\geq 0)}}_{primary:(-)} + \underbrace{\underbrace{\Psi^1_{mDk}}_{(\geq 0)} \frac{a^2_{MD} - a^2_R}{N_k}}_{general:(+)} \gtreqless 0, \tag{A.26}$$

where the minority candidate's chances decrease in the primary but increase in the general election.

For the last part, we employ again (A.24) and take the second derivative with respect to the number of minority voters:

$$\frac{\partial^2 \tilde{\Psi}_{mDk}}{\partial N^2_{mDk}} = \frac{2(a^1_{mD} - a^1_{MD})N_{MDk}\left((a^2_{mDk} - a^2_{MDk})N_{MDk} - a^2_R N_k\right)}{N_k(N_{mDk} + N_{MDk})^3}, \tag{A.27}$$

which is positive when

$$(a^2_{mDk} - a^2_{MDk})N_{MDk} - a^2_R N_k > 0. \tag{A.28}$$

Hence, for $a^2_R < (a^2_{mD} - a^2_{MD})N_{MDk}/N_k$, Ψ_{mD} is convex on S^2.

A.5 Proof of Proposition 4

The expected utility for minority voters from (5.16) can be rewritten with (3.8) to (3.11) and $N_R = N_k - N_{mD} - N_{MD}$ as

$$\begin{aligned} E(X) &= \Psi^1_{mDk}\Psi^2_{mDk} + \Psi^1_{MDk}\Psi^3_{MDk}\beta = \Psi^1_{mDk}\Psi^2_{mDk} + (1 - \Psi^1_{mD})\Psi^3_{MD}\beta \\ &= \left(\frac{a^1_{mD}N_{mDk} + a^1_{MD}N_{MDk}}{N_{mDk} + N_{MDk}}\right)\left(\frac{a^2_{mD}N_{mDk} + a^2_{MD}N_{MD} + a^2_R(N_k - N_{mDk} - N_{MD})}{N_k}\right) \\ &+ \left(1 - \frac{a^1_{mD}N_{mDk} + a^1_{MD}N_{MDk}}{N_{mDk} + N_{MDk}}\right)\left(\frac{a^3_{mD}N_{mDk} + a^3_{MD}N_{MDk} + a^3_R(N_k - N_{mDk} - N_{MDk})}{N_k}\right)\beta. \end{aligned} \tag{A.29}$$

The second derivative with respect to the number of minority voters in a district is

$$\frac{\partial^2 E(X)}{\partial N^2_{mDk}} = \frac{2(a^1_{mD} - a^1_{MD})N_{MDk}\left((a^2_{mD} - a^2_{MD})N_{MDk} - (a^2_R - a^3_R\beta)N_k - (a^3_{mD} - a^3_{MD})\beta N_{MDk}\right)}{N_k(N_{mDk} + N_{MDk})^3}, \tag{A.30}$$

which is positive if

$$\gamma \equiv (a^2_{mD} - a^2_{MD})N_{MDk} - (a^2_R - a^3_R\beta)N_k - (a^3_{mD} - a^3_{MD})\beta N_{MDk} > 0. \tag{A.31}$$

As $a_R^2 - a_R^3\beta \to 0$ or $a_R^2 \to 0$ and $a_R^3 \to 0$, we can state

$$\beta < \frac{a_{mD}^2 - a_{MD}^2}{a_{mD}^3 - a_{MD}^3}. \tag{A.32}$$

Note that (A.30) is also positive if $a_{mD}^2 > a_{MD}^2$ (by assumption), $a_R^3\beta > a_R^2$ (Republican voters much more likely to support a majority-Democrat than a minority candidate), and $a_{MD}^3 \geq a_{mD}^3$ (similar support among Democrats against a Republican candidate).

A.6 Proof of Proposition 5

We know from Proposition 2 that T_{ijk} becomes concave as π_{mD} rises; we wish to determine the conditions under which overall utility $\mathcal{U}_{mD} = U_{mD}(T_{ijk}) + E(X)$ is concave on S^2 with respect to π_{mD}. Recall from (4.3) that

$$\pi_i = [\kappa_i \phi_i(0)]^{1/\epsilon} \tag{A.33}$$

so that π_{mD} can increase either through a rise in κ_{mD} or $\phi_{mD}(0)$.

Taking the former, a rise in κ_{mD} indicates that minority voters prefer more distributive to ideological benefits at the margin. Since voters' overall utility is given by

$$X + \kappa_{mD}\frac{b^{1-\epsilon}}{1-\epsilon}, \tag{A.34}$$

an increase in κ_{mD} indicates that the weight placed on distributive returns increases relative to ideology. This means that the concavity of T_{ijk} will eventually dominate the sum, even if $E(X)$ is convex, making $\mathcal{U}_{mD}$ concave in π_{mD}.

Taking the latter, an increase in $\phi_{mD}(0)$ indicates that minority voters are becoming more decisive; meaning that their voting rates a_{mD}^e decline for each election type e. Taking the total derivative of (A.30) with respect to a_{mD}^e yields

$$\frac{\partial\left(\frac{\partial^2 E(X)}{\partial N_{mD}^2}\right)}{\partial a_{mD}^e} = \frac{2N_{mD}\gamma + \overbrace{2(1-\beta)(a_{mD}^1 - a_{MD}^1)}^{(+)}}{N_k(N_{mD}+N_{MD})^3}, \tag{A.35}$$

where γ follows from (A.31). If $\gamma > 0$, then $\frac{\partial^2 E(X)}{\partial N_{mD}^2}$ is positive, $E(X)$ is convex, and (A.35) is positive. So lower values of a_{mD}^e will make the surface of $E(X)$ more concave, again implying that $\mathcal{U}_{mD}$ becomes concave on S^2. If $\gamma < 0$, then $\frac{\partial^2 E(X)}{\partial N_{mD}^2}$ is negative, $E(X)$ is concave, and (A.35) is ambiguous. However, low values of a^e_{mD} will not alter much the concavity of $E(X)$, and the concavity of T_{ijk} will dominate.

B Online Appendix: Examples and Simulations

B.1 Example with Democrat and Republican Crossover

The example of the introduction has a district with $(n_{mD}, n_{MD}, n_R) = (.4, 0.12, .52)$ voters and crossover for majority-Democrats in the primary and general election as in Section 2 with $a = 0.2$ and $b = 0.2$, but also Republican crossover in the general election of $d = 0.2$, 20% of Republican voters supporting a minority-candidate in the general election. We set $c = 1$ though it is not relevant for the current example.

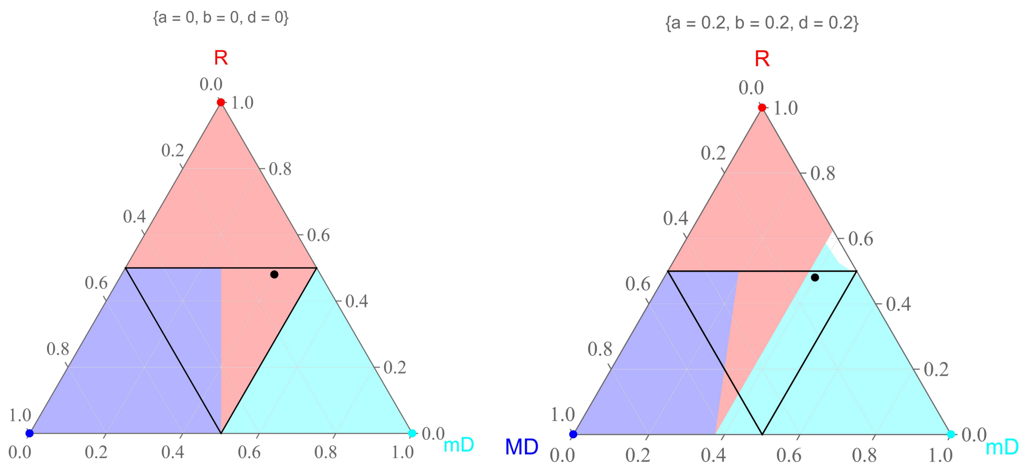


Figure 11: Crossover Voting.

B.2 Sample State and Five Districts

The example of Figure 2 is created with $S = (.36, 0.26, .38)$ and valid districting matrix for five districts with

$$\begin{pmatrix} 0.19 & 0.6 & 0.21 \\ 0.33 & 0.05 & 0.62 \\ 0.45 & 0.1 & 0.45 \\ 0.14 & 0.43 & 0.42 \\ 0.65 & 0.13 & 0.22 \end{pmatrix}.$$

B.3 Strategic Voting in Primary

One issue concerns strategic voting, or lack thereof, in the primary elections. The assumptions above imply that no one votes strategically; the net crossover is $a \leq b$ in the primary election, regardless of what happens in the general election. This makes no difference for most points in the triangle, but there is a region where the lack of strategic voting alters the results, as indicated in Figure 12. For points in the green region in the center of the triangle, naive voting would have a minority-Democratic candidate winning the primary and then losing to a Republican in the general election. Were the primary voters sophisticated, they would elect a majority-Democratic candidate instead, who would go on to beat the Republican in the general election. This type of sophisticated voting may benefit the Democrats' number of seats but is not much in evidence, so we will assume sincere primary voting hereafter and indicate which of our results depend on this lack of sophistication.

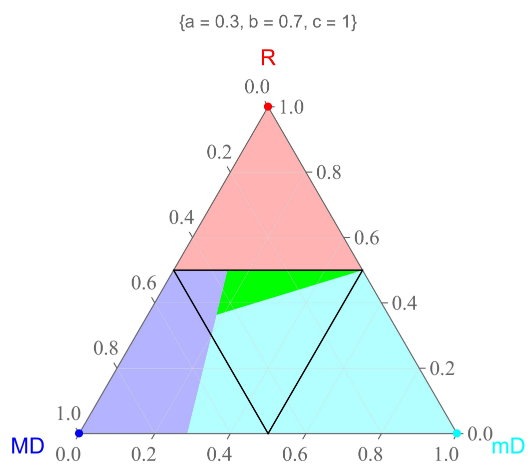


Figure 12: Electoral Winners – Strategic Voting.

B.4 District Outcomes: Identity vs. Partisanship

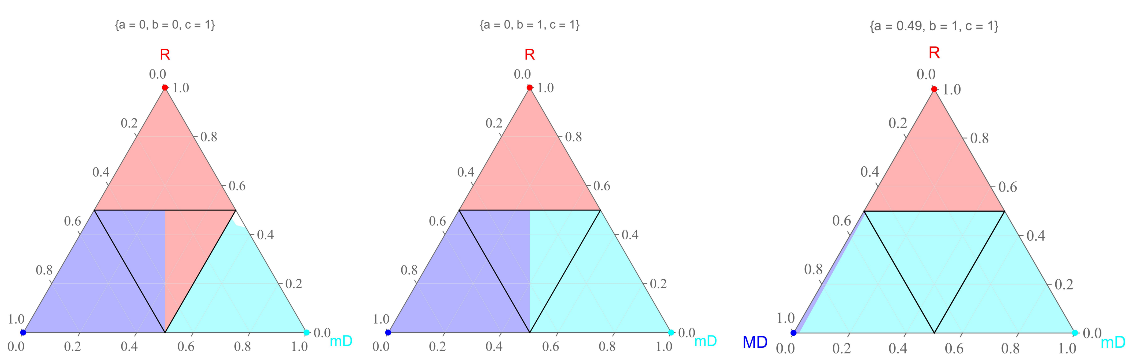


Figure 13: Majority-Democratic Voters – Identity and Partisan Voting.

B.5 District Outcomes: Role of Crossover in Primary Election

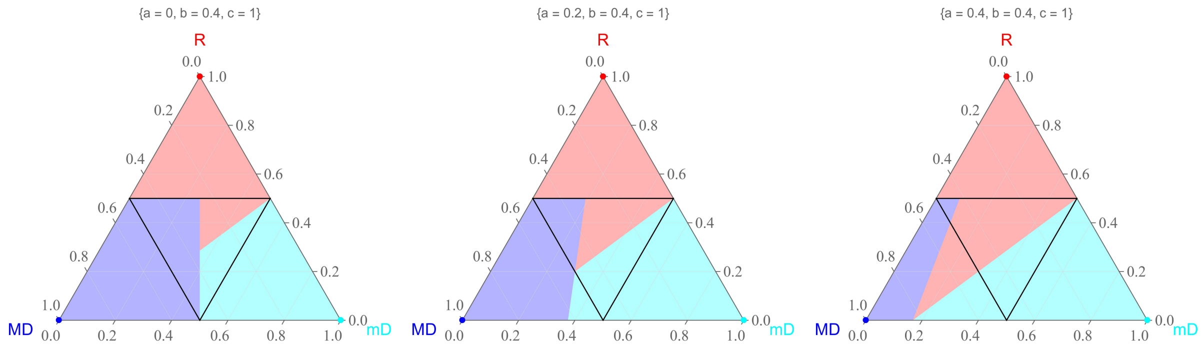


Figure 14: Majority-Democratic Voters – Role of Crossover in Primary Election.

B.6 District Outcomes: Role of Crossover in General Election

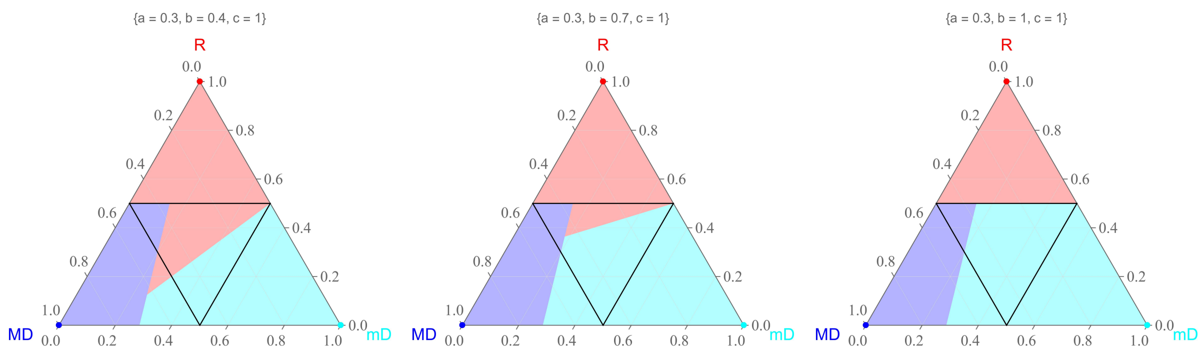


Figure 15: Majority-Democratic Voters – Role of Crossover in General Election.

B.7 District Outcomes: Increasing Minority Voter Share

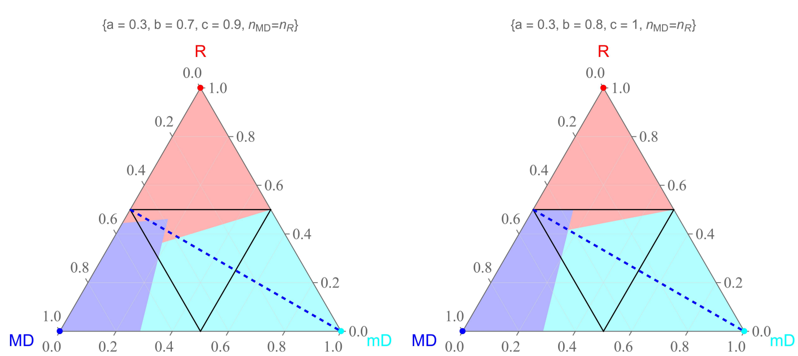
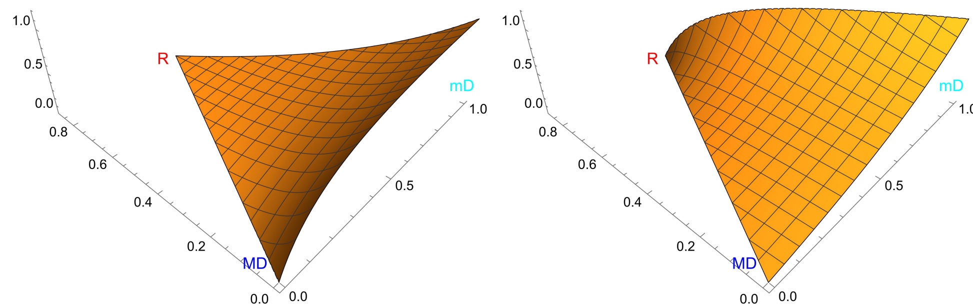


Figure 16: Increasing Minority Voter Share.

B.8 Nonconcave and Nonconvex Minority Distributive Benefits



(a) $\pi_{mD} = 7$, $\pi_{MD} = 1$, $\pi_R = 15$

(b) $\pi_{mD} = 7$, $\pi_{MD} = 15$, $\pi_R = 1$

Figure 17: Nonconcave and Nonconvex Minority Distributive Benefits.

B.9 Minority Distributive Payoffs and Voter Distribution – Group Power

Group Power – **Minority Power**					
π_{mD}	π_{MD}	π_R	Total	Average	$V(\mathbf{D})$
1	3	1	0.75	0.25	75%
3	3	1	1.17	0.39	39%
5	3	1	1.43	0.48	15%
7	3	1	1.65	0.55	5%
10	3	1	1.90	0.63	3%
1	3	2	0.60	0.20	75%
3	3	2	0.91	0.30	42%
5	3	2	1.21	0.40	11%
7	3	2	1.45	0.48	4%
10	3	2	1.71	0.57	1%
1	3	3	0.50	0.17	75%
3	3	3	0.75	0.25	33%
5	3	3	1.07	0.36	0%
7	3	3	1.31	0.44	0%
10	3	3	1.58	0.53	0%
1	3	4	0.50	0.17	75%
3	3	4	0.75	0.25	75%
5	3	4	1.00	0.33	28%
7	3	4	1.22	0.41	9%
10	3	4	1.48	0.49	3%
1	3	5	0.50	0.17	75%
3	3	5	0.75	0.25	75%
5	3	5	0.99	0.33	38%
7	3	5	1.16	0.39	26%
10	3	5	1.40	0.47	9%

Group Power – **Large Differences**					
π_{mD}	π_{MD}	π_R	Total	Average	$V(\mathbf{D})$
1	5	1	0.75	0.25	75%
3	5	1	1.09	0.36	42%
5	5	1	1.32	0.44	38%
7	5	1	1.46	0.49	35%
10	5	1	1.67	0.56	8%
1	5	3	0.50	0.17	75%
3	5	3	0.75	0.25	75%
5	5	3	0.95	0.32	41%
7	5	3	1.13	0.38	23%
10	5	3	1.37	0.46	8%
1	5	5	0.38	0.13	75%
3	5	5	0.64	0.21	75%
5	5	5	0.75	0.25	24%
7	5	5	0.95	0.32	0%
10	5	7	1.10	0.37	18%
1	5	7	0.37	0.12	75%
3	5	7	0.64	0.21	75%
5	5	7	0.75	0.25	75%
7	5	7	0.91	0.30	39%
10	5	7	1.10	0.37	18%
1	5	10	0.37	0.12	75%
3	5	10	0.64	0.21	75%
5	5	10	0.75	0.25	75%
7	5	10	0.89	0.30	40%
10	5	10	1.07	0.36	38%

Group Power – **Homogeneous Majority Power**					
π_{mD}	π_{MD}	π_R	Total	Average	$V(\mathbf{D})$
1	1	1	0.75	0.25	50%
2	1	1	1.20	0.40	0%
3	1	1	1.50	0.50	0%
4	1	1	1.71	0.57	0%
5	1	1	1.88	0.63	0%
1	3	3	0.50	0.17	75%
2	3	3	0.67	0.22	75%
3	3	3	0.75	0.25	33%
4	3	3	0.92	0.31	0%
5	3	3	1.07	0.36	0%
1	5	5	0.38	0.13	75%
2	5	5	0.55	0.18	75%
3	5	5	0.64	0.21	75%
4	5	5	0.71	0.24	75%
5	5	5	0.75	0.25	24%
1	7	7	0.30	0.10	75%
2	7	7	0.46	0.15	75%
3	7	7	0.56	0.19	75%
4	7	7	0.63	0.21	75%
5	7	7	0.68	0.23	75%
1	10	10	0.23	0.08	75%
2	10	10	0.38	0.13	75%
3	10	10	0.47	0.16	75%
4	10	10	0.55	0.18	75%
5	10	10	0.60	0.20	75%

Group Power – **Heterogeneous Majority Power**					
π_{mD}	π_{MD}	π_R	Total	Average	$V(\mathbf{D})$
1	5	1	0.75	0.25	75%
2	5	1	0.91	0.30	52%
3	5	1	1.09	0.36	42%
4	5	1	1.22	0.41	39%
5	5	1	1.32	0.44	38%
1	5	3	0.50	0.17	75%
2	5	3	0.67	0.22	75%
3	5	3	0.75	0.25	75%
4	5	3	0.85	0.28	46%
5	5	3	0.95	0.32	41%
1	5	5	0.38	0.13	75%
2	5	5	0.55	0.18	75%
3	5	5	0.64	0.21	75%
4	5	5	0.71	0.24	75%
5	5	7	0.75	0.25	75%
1	5	7	0.37	0.12	75%
2	5	7	0.55	0.18	75%
3	5	7	0.64	0.21	75%
4	5	7	0.71	0.24	75%
5	5	7	0.75	0.25	75%
1	5	10	0.37	0.12	75%
2	5	10	0.55	0.18	75%
3	5	10	0.64	0.21	75%
4	5	10	0.71	0.24	75%
5	5	10	0.75	0.25	75%

Table 5: Districting Plans Maximizing Minority Distributive Benefits – Group Power, $N_{mD} = 25\%$, $N_{MD} = 40\%$, and $N_R = 35\%$.

B.10 Minority Distributive Payoffs and Voter Distribution – Demographics

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{MD}	π_R	n_{mD1}	n_{MD1}	n_{R1}	n_{mD2}	n_{MD2}	n_{R2}	n_{mD3}	n_{MD3}	n_{R3}	Total	Aver.	$V(\mathbf{D})$
1	3	1	90%	0%	10%	0%	75%	25%	0%	0%	100%	0.900	0.30	90%
2	3	1	0%	75%	25%	45%	0%	55%	45%	0%	55%	1.241	0.41	45%
3	3	1	38%	0%	62%	14%	75%	11%	38%	0%	62%	1.447	0.48	24%
4	3	1	34%	0%	66%	23%	75%	2%	34%	0%	66%	1.625	0.54	11%
5	3	1	25%	75%	0%	32%	0%	68%	33%	0%	67%	1.770	0.59	8%
1	3	2	0%	37%	63%	0%	38%	62%	90%	0%	10%	0.818	0.27	90%
2	3	2	0%	0%	100%	0%	75%	25%	90%	0%	10%	0.900	0.30	90%
3	3	2	41%	0%	59%	8%	75%	17%	41%	0%	59%	1.106	0.37	33%
4	3	2	21%	75%	4%	35%	0%	65%	35%	0%	65%	1.291	0.43	14%
5	3	2	25%	75%	0%	33%	0%	67%	33%	0%	67%	1.450	0.48	8%
1	3	3	0%	60%	40%	0%	14%	86%	90%	1%	9%	0.750	0.25	90%
2	3	3	0%	68%	32%	0%	7%	93%	90%	0%	10%	0.857	0.29	90%
3	3	3	65%	26%	9%	14%	22%	64%	12%	27%	61%	0.900	0.30	53%
4	3	3	30%	38%	32%	30%	21%	49%	30%	16%	54%	1.091	0.36	0%
5	3	3	30%	43%	27%	30%	6%	64%	30%	26%	44%	1.250	0.42	0%
1	3	4	0%	32%	68%	0%	33%	67%	90%	10%	0%	0.750	0.25	90%
2	3	4	0%	57%	43%	0%	8%	92%	90%	10%	0%	0.857	0.29	90%
3	3	4	90%	10%	0%	0%	0%	100%	0%	65%	35%	0.900	0.30	90%
4	3	4	53%	47%	0%	0%	0%	100%	37%	28%	35%	0.998	0.33	53%
5	3	4	40%	60%	0%	22%	0%	78%	29%	15%	57%	1.126	0.38	18%
1	3	5	0%	33%	67%	0%	32%	68%	90%	10%	0%	0.750	0.25	90%
2	3	5	90%	10%	0%	0%	33%	67%	0%	32%	68%	0.857	0.29	90%
3	3	5	90%	10%	0%	0%	0%	100%	0%	65%	35%	0.900	0.30	90%
4	3	5	60%	40%	0%	0%	0%	100%	30%	35%	35%	0.967	0.32	60%
5	3	5	51%	49%	0%	0%	0%	100%	39%	26%	35%	1.070	0.36	51%

Intermediate Minority Population: $N_{mD} = 0.3,\ N_{MD} = 0.25,\ N_R = 0.45$.

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{MD}	π_R	n_{mD1}	n_{MD1}	n_{R1}	n_{mD2}	n_{MD2}	n_{R2}	n_{mD3}	n_{MD3}	n_{R3}	Total	Aver.	$V(\mathbf{D})$
1	3	1	0%	75%	25%	100%	0%	0%	20%	0%	80%	1.200	0.4	100%
2	3	1	2%	75%	23%	59%	0%	41%	59%	0%	41%	1.500	0.50	58%
3	3	1	48%	0%	52%	25%	75%	0%	48%	0%	52%	1.712	0.57	23%
4	3	1	48%	0%	52%	25%	75%	0%	47%	0%	53%	1.875	0.62	23%
5	3	1	46%	4%	50%	29%	71%	0%	45%	0%	55%	1.996	0.67	17%
1	3	2	0%	75%	25%	20%	0%	80%	100%	0%	0%	1.111	0.37	100%
2	3	2	20%	0%	80%	0%	75%	25%	100%	0%	0%	1.200	0.40	100%
3	3	2	51%	0%	49%	19%	75%	6%	51%	0%	49%	1.403	0.47	32%
4	3	2	47%	0%	53%	25%	75%	0%	47%	0%	52%	1.596	0.53	22%
5	3	2	34%	66%	0%	43%	9%	48%	43%	0%	57%	1.752	0.58	9%
1	3	3	0%	56%	44%	20%	19%	61%	100%	0%	0%	1.077	0.36	100%
2	3	3	0%	0%	100%	20%	75%	5%	100%	0%	0%	1.143	0.38	100%
3	3	3	15%	9%	76%	17%	55%	28%	88%	11%	2%	1.200	0.40	73%
4	3	3	40%	43%	17%	40%	20%	40%	40%	13%	47%	1.412	0.47	0%
5	3	3	40%	38%	22%	40%	23%	37%	40%	14%	46%	1.579	0.53	0%
1	3	4	20%	75%	5%	0%	0%	100%	100%	0%	0%	1.075	0.36	100%
2	3	4	20%	75%	5%	0%	0%	100%	100%	0%	0%	1.140	0.38	100%
3	3	4	0%	0%	100%	100%	0%	0%	20%	75%	5%	1.197	0.40	100%
4	3	4	0%	0%	100%	61%	39%	0%	59%	36%	5%	1.324	0.44	61%
5	3	4	48%	52%	0%	41%	23%	36%	31%	0%	69%	1.456	0.49	17%
1	3	5	0%	0%	100%	20%	75%	5%	100%	0%	0%	1.074	0.36	100%
2	3	5	0%	0%	100%	20%	75%	5%	100%	0%	0%	1.138	0.38	100%
3	3	5	0%	0%	100%	100%	0%	0%	20%	75%	5%	1.194	0.40	100%
4	3	5	0%	0%	100%	62%	38%	0%	58%	37%	5%	1.316	0.44	62%
5	3	5	0%	0%	100%	61%	39%	0%	59%	36%	5%	1.412	0.47	61%

Large Minority Population: $N_{mD} = 0.4,\ N_{MD} = 0.25,\ N_R = 0.35$.

Table 6: Districting Plans Maximizing Minority Distributive Benefits – State Demographics.

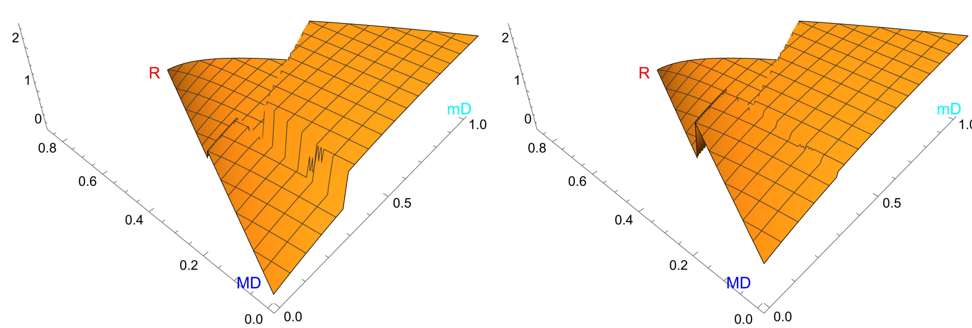
B.11 Minority Distributive Payoffs and Voter Distribution – Number of Districts

5 Districts – $N_{mD} = 0.25$, $N_{MD} = .4$, $N_R = 0.35$					
π_{mD}	π_{MD}	π_R	Total	Average	$V(\mathbf{D})$
1	3	1	1.250	0.25	87%
2	3	1	1.765	0.35	42%
3	3	1	2.045	0.41	42%
4	3	1	2.222	0.44	42%
5	3	1	2.404	0.48	15%
1	3	2	1.143	0.23	100%
2	3	2	1.250	0.25	100%
3	3	2	1.552	0.31	42%
4	3	2	1.778	0.36	25%
5	3	2	2.020	0.40	11%
1	3	3	1.100	0.22	100%
2	3	3	1.182	0.24	100%
3	3	3	1.250	0.25	50%
4	3	3	1.538	0.31	0%
5	3	3	1.786	0.36	0%
1	3	4	1.100	0.22	100%
2	3	4	1.182	0.24	100%
3	3	4	1.250	0.25	80%
4	3	4	1.464	0.29	40%
5	3	4	1.667	0.33	21%
1	3	5	1.100	0.22	100%
2	3	5	1.182	0.24	100%
3	3	5	1.250	0.25	100%
4	3	5	1.463	0.29	42%
5	3	5	1.634	0.33	39%

12 Districts – $N_{mD} = 0.25$, $N_{MD} = .4$, $N_R = 0.35$					
π_{mD}	π_{MD}	π_R	Total	Average	$V(\mathbf{D})$
1	3	1	3.000	0.25	91%
2	3	1	4.200	0.35	43%
3	3	1	4.868	0.41	41%
4	3	1	5.354	0.45	36%
5	3	1	5.856	0.49	10%
1	3	2	3.000	0.25	100%
2	3	2	3.000	0.25	100%
3	3	2	3.708	0.31	42%
4	3	2	4.281	0.36	21%
5	3	2	4.868	0.41	10%
1	3	3	3.000	0.25	100%
2	3	3	3.000	0.25	100%
3	3	3	3.000	0.25	52%
4	3	3	3.692	0.31	0%
5	3	3	4.286	0.36	0%
1	3	4	3.000	0.25	100%
2	3	4	3.000	0.25	100%
3	3	4	3.000	0.25	90%
4	3	4	3.533	0.29	39%
5	3	4	3.995	0.33	20%
1	3	5	3.000	0.25	100%
2	3	5	3.000	0.25	100%
3	3	5	3.000	0.25	95%
4	3	5	3.516	0.29	40%
5	3	5	3.955	0.33	38%

Table 7: Districting Plans Maximizing Minority Distributive Benefits – Number of Districts.

B.12 Effect of majority-Democratic Extra Utility

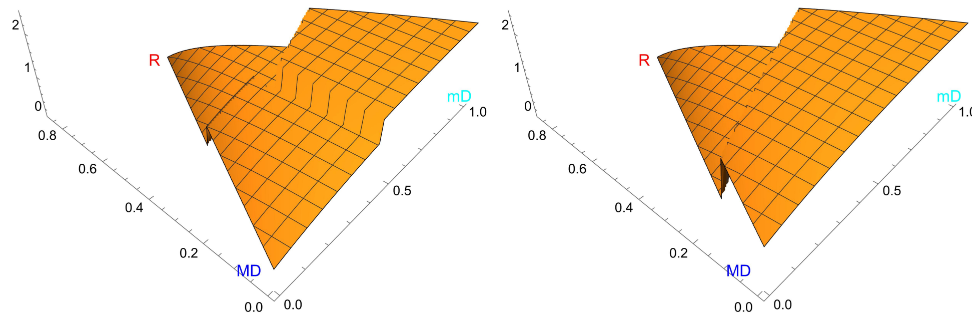


(a) $\beta = .2$.

(b) $\beta = .9$.

Figure 18: Effect of Additional Utility from majority-Democrat for Minorities – $\pi_{mD} = 10$, $\pi_{MD} = 5$, $\pi_R = 2$.

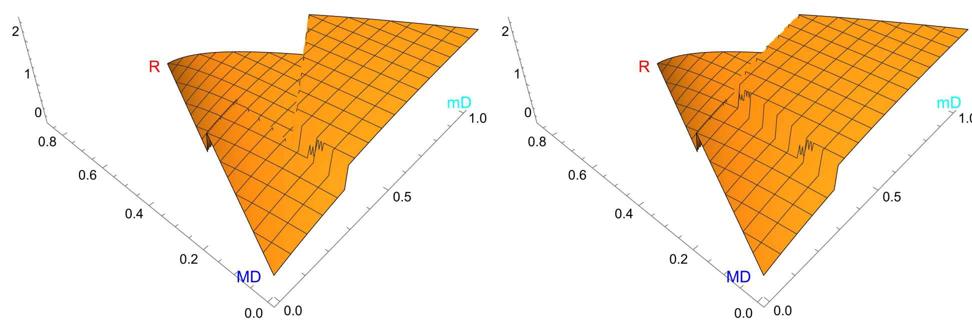
B.13 Effect of Primary Crossover Voting



(a) $a = .1$.　　(b) $a = .5$.

Figure 19: Effect of Primary Crossover Voting by majority-Democrats – $\pi_{mD} = 10$, $\pi_{MD} = 5$, $\pi_R = 2$.

B.14 Effect of General Crossover Voting



(a) $b = .5$.　　(b) $b = .9$.

Figure 20: Effect of General Crossover Voting by majority-Democrats – $\pi_{mD} = 10$, $\pi_{MD} = 5$, $\pi_R = 2$.

B.15 Minority Total Payoffs and Voter Distribution – Group Power

Group Power – **Minority Power**					
π_{mD}	π_{MD}	π_R	Total	Average	$V(\mathbf{D})$
1	3	1	2.75	0.92	75%
3	3	1	4.00	1.33	16%
5	3	1	3.41	1.14	3%
7	3	1	3.65	1.22	1%
10	3	1	4.84	1.61	11%
1	3	2	2.60	0.87	75%
3	3	2	3.86	1.29	16%
5	3	2	3.20	1.07	3%
7	3	2	4.43	1.48	11%
10	3	2	4.69	1.56	11%
1	3	3	2.50	0.83	75%
3	3	3	3.25	1.08	31%
5	3	3	4.07	1.36	11%
7	3	3	4.31	1.44	11%
10	3	3	4.57	1.52	11%
1	3	4	2.47	0.82	75%
3	3	4	2.73	0.91	70%
5	3	4	3.97	1.32	10%
7	3	4	4.20	1.40	11%
10	3	4	4.46	1.49	11%
1	3	5	2.00	0.67	75%
3	3	5	2.71	0.90	70%
5	3	5	3.40	1.13	18%
7	3	5	4.11	1.37	11%
10	3	5	4.37	1.46	11%

Group Power – **Large Differences**					
π_{mD}	π_{MD}	π_R	Total	Average	$V(\mathbf{D})$
1	5	1	2.75	0.92	75%
3	5	1	3.30	1.10	20%
5	5	1	4.07	1.36	16%
7	5	1	4.29	1.43	13%
10	5	1	4.55	1.52	11%
1	5	3	2.50	0.83	75%
3	5	3	2.75	0.92	75%
5	5	3	3.88	1.29	16%
7	5	3	4.09	1.36	11%
10	5	3	4.35	1.45	11%
1	5	5	2.38	0.79	75%
3	5	5	2.64	0.88	75%
5	5	5	3.25	1.08	31%
7	5	5	3.95	1.32	11%
10	5	7	4.07	1.36	11%
1	5	7	1.87	0.62	75%
3	5	7	2.59	0.86	70%
5	5	7	2.73	0.91	70%
7	5	7	3.35	1.12	18%
10	5	7	4.07	1.36	11%
1	5	10	1.87	0.62	75%
3	5	10	2.14	0.71	75%
5	5	10	2.70	0.90	69%
7	5	10	3.25	1.08	35%
10	5	10	3.44	1.15	17%

Group Power – **Homogeneous Majority Power**					
π_{mD}	π_{MD}	π_R	Total	Average	$V(\mathbf{D})$
1	1	1	3.25	1.08	31%
2	1	1	4.19	1.40	11%
3	1	1	4.49	1.50	11%
4	1	1	4.70	1.57	11%
5	1	1	4.86	1.62	11%
1	3	3	2.50	0.83	75%
2	3	3	2.67	0.89	75%
3	3	3	3.25	1.08	31%
4	3	3	3.92	1.31	11%
5	3	3	4.07	1.36	11%
1	5	5	2.38	0.79	75%
2	5	5	2.55	0.85	75%
3	5	5	2.64	0.88	75%
4	5	5	2.71	0.90	75%
5	5	5	3.25	1.08	31%
1	7	7	2.30	0.77	75%
2	7	7	2.41	0.80	70%
3	7	7	2.56	0.85	75%
4	7	7	2.63	0.88	75%
5	7	7	2.66	0.89	70%
1	10	10	2.23	0.74	75%
2	10	10	2.38	0.79	75%
3	10	10	2.42	0.81	70%
4	10	10	2.55	0.85	75%
5	10	10	2.57	0.86	70%

Group Power – **Heterogeneous Majority Power**					
π_{mD}	π_{MD}	π_R	Total	Average	$V(\mathbf{D})$
1	5	1	2.75	0.92	75%
2	5	1	2.41	0.80	52%
3	5	1	3.30	1.10	20%
4	5	1	3.45	1.15	21%
5	5	1	4.07	1.36	16%
1	5	3	2.50	0.83	75%
2	5	3	2.67	0.89	75%
3	5	3	2.75	0.92	75%
4	5	3	3.26	1.09	20%
5	5	3	3.88	1.29	16%
1	5	5	2.38	0.79	75%
2	5	5	2.55	0.85	75%
3	5	5	2.64	0.88	75%
4	5	5	2.71	0.90	75%
5	5	7	2.73	0.91	70%
1	5	7	1.87	0.62	75%
2	5	7	2.51	0.84	73%
3	5	7	2.59	0.86	70%
4	5	7	2.67	0.89	70%
5	5	7	2.73	0.91	70%
1	5	10	1.87	0.62	75%
2	5	10	2.05	0.68	75%
3	5	10	2.14	0.71	75%
4	5	10	2.64	0.88	71%
5	5	10	2.70	0.90	69%

Table 8: Districting Plans Maximizing Minority Total Benefits – Group Power, $N_{mD} = 25\%$, $N_{MD} = 40\%$, and $N_R = 35\%$.

B.16 Minority Total Payoffs and Voter Distribution – State Demographics

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{MD}	π_R	n_{mD1}	n_{MD1}	n_{R1}	n_{mD2}	n_{MD2}	n_{R2}	n_{mD3}	n_{MD3}	n_{R3}	Total	Aver.	$V(\mathbf{D})$
1	3	1	22%	0%	78%	0%	75%	25%	68%	0%	32%	2.40	0.80	68%
2	3	1	40%	14%	46%	0%	61%	39%	50%	0%	50%	3.64	1.21	50%
3	3	1	23%	58%	19%	29%	0%	71%	38%	17%	45%	3.36	1.12	15%
4	3	1	23%	59%	18%	28%	0%	72%	39%	16%	45%	3.55	1.18	15%
5	3	1	24%	59%	17%	27%	0%	73%	39%	16%	45%	3.71	1.24	15%
1	3	2	23%	39%	38%	14%	36%	50%	53%	0%	47%	3.03	1.01	40%
2	3	2	18%	45%	36%	0%	30%	70%	72%	0%	28%	2.87	0.96	72%
3	3	2	24%	59%	17%	28%	0%	72%	39%	16%	45%	3.07	1.02	15%
4	3	2	35%	22%	43%	15%	39%	46%	40%	14%	46%	3.75	1.25	24%
5	3	2	27%	48%	25%	32%	0%	68%	31%	27%	42%	3.44	1.15	5%
1	3	3	9%	30%	61%	18%	45%	36%	63%	0%	37%	2.46	0.82	54%
2	3	3	0%	27%	73%	18%	45%	36%	72%	3%	25%	2.76	0.92	72%
3	3	3	34%	26%	40%	12%	39%	49%	44%	10%	46%	3.40	1.13	32%
4	3	3	30%	41%	29%	30%	6%	64%	30%	29%	41%	3.09	1.03	0%
5	3	3	30%	46%	24%	30%	0%	70%	30%	29%	41%	3.25	1.08	0%
1	3	4	0%	2%	98%	19%	45%	37%	71%	29%	0%	2.52	0.84	71%
2	3	4	18%	45%	36%	0%	1%	99%	72%	28%	0%	2.74	0.91	72%
3	3	4	18%	46%	35%	0%	0%	100%	71%	29%	0%	2.88	0.96	71%
4	3	4	53%	47%	0%	0%	0%	100%	37%	28%	35%	3.00	1.00	53%
5	3	4	37%	18%	45%	10%	0%	90%	43%	57%	0%	3.12	1.04	33%
1	3	5	19%	46%	35%	0%	0%	100%	71%	29%	0%	2.51	0.84	71%
2	3	5	19%	45%	37%	0%	2%	98%	71%	29%	0%	2.73	0.91	71%
3	3	5	18%	45%	36%	14%	2%	85%	58%	28%	14%	2.77	0.92	44%
4	3	5	60%	40%	0%	0%	0%	100%	30%	35%	35%	2.97	0.99	60%
5	3	5	51%	49%	0%	0%	0%	100%	39%	26%	35%	3.07	1.02	51%

Intermediate Minority Population – D Majority: $N_{mD} = 0.3,\ N_{MD} = 0.25,\ N_R = 0.45.$

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{MD}	π_R	n_{mD1}	n_{MD1}	n_{R1}	n_{mD2}	n_{MD2}	n_{R2}	n_{mD3}	n_{MD3}	n_{R3}	Total	Aver.	$V(\mathbf{D})$
1	3	1	1%	14%	85%	56%	1%	44%	18%	45%	36%	2.66	0.89	55%
2	3	1	13%	6%	81%	44%	9%	47%	18%	45%	36%	2.93	0.98	31%
3	3	1	13%	5%	81%	44%	9%	47%	18%	46%	36%	3.17	1.06	30%
4	3	1	12%	6%	81%	44%	8%	48%	18%	46%	36%	3.34	1.11	32%
5	3	1	12%	6%	81%	45%	8%	48%	18%	46%	36%	3.48	1.16	32%
1	3	2	0%	15%	85%	57%	0%	43%	18%	45%	36%	2.48	0.83	57%
2	3	2	0%	14%	86%	57%	0%	43%	18%	45%	36%	2.72	0.91	57%
3	3	2	17%	0%	83%	40%	15%	46%	18%	45%	36%	2.91	0.97	23%
4	3	2	13%	5%	81%	43%	9%	47%	18%	45%	36%	3.08	1.03	30%
5	3	2	9%	11%	80%	34%	23%	43%	32%	26%	42%	3.21	1.07	24%
1	3	3	0%	15%	85%	57%	0%	43%	18%	45%	36%	2.37	0.79	57%
2	3	3	0%	15%	85%	57%	0%	43%	18%	45%	36%	2.60	0.87	57%
3	3	3	5%	1%	94%	43%	12%	45%	26%	47%	26%	2.75	0.92	38%
4	3	3	16%	0%	84%	29%	32%	39%	30%	28%	41%	2.92	0.97	14%
5	3	3	10%	10%	80%	33%	24%	43%	32%	26%	42%	3.05	1.02	23%
1	3	4	0%	0%	100%	57%	15%	29%	18%	45%	36%	2.32	0.77	57%
2	3	4	0%	0%	100%	57%	15%	29%	18%	45%	36%	2.53	0.84	57%
3	3	4	0%	0%	100%	51%	23%	26%	24%	37%	39%	2.68	0.89	51%
4	3	4	0%	0%	100%	34%	23%	43%	41%	37%	22%	2.81	0.94	41%
5	3	4	9%	12%	79%	37%	18%	44%	29%	30%	41%	2.92	0.97	28%
1	3	5	0%	0%	100%	57%	15%	29%	18%	45%	36%	2.29	0.76	57%
2	3	5	0%	0%	100%	18%	45%	36%	57%	15%	29%	2.48	0.83	57%
3	3	5	0%	0%	100%	50%	23%	26%	24%	37%	39%	2.62	0.87	50%
4	3	5	0%	0%	100%	32%	25%	42%	43%	35%	23%	2.75	0.92	43%
5	3	5	0%	0%	100%	35%	22%	43%	40%	38%	22%	2.86	0.95	40%

Intermediate Minority Population – R Majority: $N_{mD} = 0.25,\ N_{MD} = 0.2,\ N_R = 0.55.$

Table 9: Districting Plans Maximizing Minority Total Benefits – State Demographics.

B.17 Minority Total Payoffs and Voter Distribution – Number of Districts

5 Districts – $N_{mD} = 0.25$, $N_{MD} = .4$, $N_R = 0.35$					
π_{mD}	π_{MD}	π_R	Total	Average	$V(\mathbf{D})$
1	3	1	5.46	1.09	51%
2	3	1	5.87	1.17	50%
3	3	1	6.20	1.24	37%
4	3	1	6.50	1.30	28%
5	3	1	7.24	1.45	3%
1	3	2	4.76	0.95	57%
2	3	2	4.68	0.94	69%
3	3	2	5.96	1.19	41%
4	3	2	6.73	1.35	8%
5	3	2	6.48	1.30	13%
1	3	3	4.61	0.92	65%
2	3	3	4.98	1.00	63%
3	3	3	5.75	1.15	20%
4	3	3	6.04	1.21	10%
5	3	3	6.79	1.36	3%
1	3	4	4.57	0.91	64%
2	3	4	4.92	0.98	62%
3	3	4	5.19	1.04	66%
4	3	4	5.89	1.18	25%
5	3	4	6.12	1.22	17%
1	3	5	4.53	0.91	62%
2	3	5	4.88	0.98	58%
3	3	5	5.16	1.03	62%
4	3	5	5.34	1.07	54%
5	3	5	6.00	1.20	25%

Table 10: Districting Plans Maximizing Minority Total Benefits – Number of Districts.

B.18 Minority Total Payoffs and Voter Distribution – Ideological Benefits

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{MD}	π_R	n_{mD1}	n_{MD1}	n_{R1}	n_{mD2}	n_{MD2}	n_{R2}	n_{mD3}	n_{MD3}	n_{R3}	Total	Aver.	$V(\mathbf{D})$
1	3	1	0%	21%	79%	0%	99%	1%	75%	0%	25%	1.75	0.58	75%
2	3	1	25%	23%	52%	0%	97%	3%	50%	0%	50%	1.96	0.65	50%
3	3	1	50%	0%	50%	3%	79%	17%	22%	41%	38%	3.08	1.03	47%
4	3	1	20%	50%	30%	28%	70%	2%	27%	0%	73%	3.25	1.08	8%
5	3	1	29%	31%	41%	13%	65%	21%	33%	24%	43%	3.34	1.11	20%
1	3	2	0%	60%	40%	0%	60%	40%	75%	0%	25%	1.60	0.53	75%
2	3	2	75%	0%	25%	0%	100%	0%	0%	20%	80%	1.75	0.58	75%
3	3	2	49%	1%	50%	0%	81%	19%	26%	38%	36%	2.88	0.96	49%
4	3	2	29%	30%	41%	13%	65%	21%	33%	24%	43%	3.04	1.01	20%
5	3	2	21%	49%	30%	27%	67%	6%	27%	4%	69%	3.20	1.07	7%
1	3	3	0%	0%	100%	0%	100%	0%	75%	20%	5%	1.50	0.50	75%
2	3	3	0%	97%	3%	0%	23%	77%	75%	0%	25%	1.67	0.56	75%
3	3	3	29%	63%	8%	19%	5%	77%	28%	52%	20%	2.75	0.92	10%
4	3	3	25%	54%	21%	25%	44%	31%	25%	23%	52%	2.92	0.97	0%
5	3	3	25%	54%	21%	25%	44%	31%	25%	23%	52%	3.07	1.02	0%
1	3	4	75%	25%	0%	0%	40%	60%	0%	55%	45%	1.50	0.50	75%
2	3	4	0%	83%	17%	0%	12%	88%	75%	25%	0%	1.67	0.56	75%
3	3	4	0%	0%	100%	0%	95%	5%	75%	25%	0%	1.75	0.58	75%
4	3	4	36%	59%	5%	0%	0%	100%	39%	61%	0%	2.88	0.96	39%
5	3	4	33%	55%	12%	35%	65%	0%	7%	0%	93%	3.00	1.00	28%
1	3	5	0%	6%	94%	0%	89%	11%	75%	25%	0%	1.50	0.50	75%
2	3	5	0%	48%	52%	0%	47%	53%	75%	25%	0%	1.67	0.56	75%
3	3	5	0%	95%	5%	0%	0%	100%	75%	25%	0%	1.75	0.58	75%
4	3	5	40%	60%	0%	0%	0%	100%	35%	60%	5%	2.88	0.96	40%
5	3	5	38%	62%	0%	0%	0%	100%	37%	58%	5%	2.99	1.00	38%

Low Minority Benefit from Majority Democract Candidate: $\beta = 0$.

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{MD}	π_R	n_{mD1}	n_{MD1}	n_{R1}	n_{mD2}	n_{MD2}	n_{R2}	n_{mD3}	n_{MD3}	n_{R3}	Total	Aver.	$V(\mathbf{D})$
1	3	1	7%	43%	50%	0%	77%	23%	68%	0%	32%	2.22	0.74	68%
2	3	1	54%	0%	46%	0%	78%	22%	21%	42%	38%	3.16	1.05	54%
3	3	1	20%	50%	31%	20%	50%	31%	35%	21%	44%	4.00	1.33	16%
4	3	1	21%	54%	25%	21%	41%	38%	32%	26%	42%	4.18	1.39	11%
5	3	1	24%	60%	16%	27%	0%	73%	24%	60%	16%	3.41	1.14	3%
1	3	2	0%	60%	40%	0%	60%	40%	75%	0%	25%	2.10	0.70	75%
2	3	2	0%	52%	48%	0%	68%	32%	75%	0%	25%	2.25	0.75	75%
3	3	2	20%	50%	31%	20%	49%	31%	35%	21%	44%	3.86	1.29	16%
4	3	2	23%	45%	32%	18%	51%	30%	34%	24%	43%	3.28	1.09	15%
5	3	2	24%	60%	16%	27%	0%	73%	24%	60%	16%	3.20	1.07	3%
1	3	3	0%	70%	30%	18%	45%	36%	57%	5%	38%	2.62	0.87	57%
2	3	3	4%	46%	50%	0%	50%	50%	71%	24%	5%	2.15	0.72	71%
3	3	3	24%	39%	37%	10%	44%	46%	41%	37%	22%	3.00	1.00	31%
4	3	3	25%	48%	26%	20%	43%	37%	30%	28%	42%	3.92	1.31	11%
5	3	3	25%	49%	26%	20%	43%	37%	30%	28%	42%	4.07	1.36	11%
1	3	4	0%	95%	5%	0%	0%	100%	75%	25%	0%	1.75	0.58	75%
2	3	4	5%	45%	50%	0%	50%	50%	70%	25%	5%	2.12	0.71	70%
3	3	4	5%	45%	50%	0%	50%	50%	70%	25%	5%	2.23	0.74	70%
4	3	4	32%	43%	24%	14%	36%	50%	28%	41%	31%	3.09	1.03	18%
5	3	4	26%	47%	27%	19%	44%	37%	30%	29%	41%	3.97	1.32	10%
1	3	5	0%	44%	56%	0%	51%	49%	75%	25%	0%	1.75	0.58	75%
2	3	5	0%	50%	50%	0%	45%	55%	75%	25%	0%	1.92	0.64	75%
3	3	5	5%	45%	50%	0%	50%	50%	70%	25%	5%	2.21	0.74	70%
4	3	5	14%	36%	50%	35%	50%	15%	26%	35%	40%	3.02	1.01	21%
5	3	5	32%	43%	24%	14%	36%	50%	29%	41%	31%	3.15	1.05	18%

Low Minority Benefit from Majority Democract Candidate: $\beta = 0.25$.

Table 11: Districting Plans Maximizing Minority Total Benefits – Low Maj. Democrat Benefit.

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{MD}	π_R	n_{mD1}	n_{MD1}	n_{R1}	n_{mD2}	n_{MD2}	n_{R2}	n_{mD3}	n_{MD3}	n_{R3}	Total	Aver.	$V(\mathbf{D})$
1	3	1	0%	55%	45%	0%	65%	35%	75%	0%	25%	3.25	1.08	75%
2	3	1	18%	45%	36%	18%	58%	24%	39%	16%	45%	3.53	1.18	20%
3	3	1	20%	50%	31%	20%	49%	31%	35%	21%	44%	4.00	1.33	16%
4	3	1	21%	54%	25%	21%	41%	38%	32%	26%	42%	4.18	1.39	11%
5	3	1	18%	58%	24%	23%	39%	38%	34%	23%	43%	4.09	1.36	16%
1	3	2	0%	60%	40%	0%	60%	40%	75%	0%	25%	3.10	1.03	75%
2	3	2	0%	70%	30%	0%	50%	50%	75%	0%	25%	3.25	1.08	75%
3	3	2	20%	50%	31%	20%	50%	31%	35%	21%	44%	3.86	1.29	16%
4	3	2	21%	54%	25%	21%	41%	38%	32%	26%	42%	4.03	1.34	11%
5	3	2	23%	52%	25%	21%	42%	37%	31%	26%	42%	4.19	1.40	11%
1	3	3	0%	51%	49%	0%	55%	45%	75%	14%	11%	3.00	1.00	75%
2	3	3	5%	45%	50%	0%	50%	50%	70%	25%	5%	3.14	1.05	70%
4	3	3	25%	46%	29%	25%	37%	38%	25%	36%	39%	3.92	1.31	0%
5	3	3	25%	46%	29%	25%	38%	37%	25%	36%	39%	4.07	1.36	0%
1	3	4	0%	50%	50%	0%	55%	45%	75%	15%	10%	2.97	0.99	75%
2	3	4	5%	45%	50%	0%	50%	50%	70%	25%	5%	3.12	1.04	70%
3	3	4	5%	45%	50%	0%	50%	50%	70%	25%	5%	3.23	1.08	70%
4	3	4	13%	37%	50%	13%	38%	49%	49%	45%	6%	3.34	1.11	36%
5	3	4	26%	47%	27%	24%	37%	39%	24%	37%	39%	3.97	1.32	2%
1	3	5	0%	45%	55%	0%	50%	50%	75%	25%	0%	2.25	0.75	75%
2	3	5	2%	48%	50%	18%	45%	36%	54%	27%	19%	3.25	1.08	52%
3	3	5	5%	45%	50%	0%	50%	50%	70%	25%	5%	3.21	1.07	70%
4	3	5	8%	42%	50%	27%	33%	40%	40%	45%	15%	3.53	1.18	32%
5	3	5	14%	36%	50%	8%	42%	50%	53%	42%	5%	3.40	1.13	45%
5	3	5	32%	43%	24%	14%	36%	50%	29%	41%	31%	3.15	1.05	18%

High Minority Benefit from Majority Democrat Candidate: $\beta = 0.75$.

We dropped $\pi_i = 3$ for computational reasons.

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{MD}	π_R	n_{mD1}	n_{MD1}	n_{R1}	n_{mD2}	n_{MD2}	n_{R2}	n_{mD3}	n_{MD3}	n_{R3}	Total	Aver.	$V(\mathbf{D})$
1	3	1	0%	50%	50%	0%	70%	30%	75%	0%	25%	3.55	1.18	75%
2	3	1	0%	84%	16%	25%	36%	39%	50%	0%	50%	3.82	1.27	50%
3	3	1	20%	50%	31%	20%	49%	31%	35%	21%	44%	4.00	1.33	16%
4	3	1	21%	54%	25%	21%	41%	38%	32%	26%	42%	4.18	1.39	11%
5	3	1	18%	58%	24%	23%	39%	38%	34%	23%	43%	4.24	1.41	16%
1	3	2	0%	60%	40%	0%	60%	40%	75%	0%	25%	3.40	1.13	75%
2	3	2	0%	53%	47%	0%	67%	33%	75%	0%	25%	3.55	1.18	75%
3	3	2	20%	50%	31%	20%	50%	31%	35%	21%	44%	3.86	1.29	16%
4	3	2	21%	54%	25%	21%	41%	38%	32%	26%	42%	4.03	1.34	11%
5	3	2	23%	52%	25%	21%	42%	37%	31%	26%	42%	4.19	1.40	11%
1	3	3	0%	51%	49%	0%	55%	45%	75%	14%	11%	3.30	1.10	75%
2	3	3	5%	45%	50%	0%	50%	50%	70%	25%	5%	3.44	1.15	70%
4	3	3	25%	46%	29%	25%	37%	38%	25%	36%	39%	3.92	1.31	0%
5	3	3	25%	46%	29%	25%	38%	37%	25%	36%	39%	4.07	1.36	0%
1	3	4	0%	50%	50%	0%	53%	47%	75%	17%	8%	3.27	1.09	75%
2	3	4	5%	45%	50%	0%	50%	50%	70%	25%	5%	3.42	1.14	70%
3	3	4	5%	45%	50%	0%	50%	50%	70%	25%	5%	3.53	1.18	70%
4	3	4	13%	37%	50%	13%	38%	49%	49%	45%	6%	3.64	1.21	36%
5	3	4	26%	47%	27%	24%	37%	39%	24%	37%	39%	3.97	1.32	2%
1	3	5	12%	38%	50%	0%	50%	50%	63%	32%	5%	3.17	1.06	63%
2	3	5	2%	48%	50%	18%	45%	36%	54%	27%	19%	3.40	1.13	52%
3	3	5	5%	45%	50%	19%	45%	37%	52%	30%	18%	3.55	1.18	47%
4	3	5	8%	42%	50%	27%	33%	40%	40%	45%	15%	3.68	1.23	32%
5	3	5	14%	36%	50%	8%	42%	50%	53%	42%	5%	3.70	1.23	45%
5	3	5	32%	43%	24%	14%	36%	50%	29%	41%	31%	3.15	1.05	18%

Table 12: Districting Plans Maximizing Minority Total Benefits – High Maj. Democrat Benefit.

B.19 Minority Total Payoffs and Voter Distribution – Crossover Rates

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{MD}	π_R	n_{mD1}	n_{MD1}	n_{R1}	n_{mD2}	n_{MD2}	n_{R2}	n_{mD3}	n_{MD3}	n_{R3}	Total	Aver.	$V(\mathbf{D})$
1	3	1	7%	43%	50%	0%	77%	23%	68%	0%	32%	2.72	0.91	68%
2	3	1	0%	92%	8%	22%	28%	50%	53%	0%	47%	2.94	0.98	53%
3	3	1	9%	41%	50%	16%	79%	5%	50%	0%	50%	3.05	1.02	41%
4	3	1	40%	14%	46%	8%	73%	19%	27%	33%	40%	3.70	1.23	33%
5	3	1	28%	31%	41%	13%	65%	21%	33%	24%	43%	3.84	1.28	20%
1	3	2	0%	60%	40%	0%	60%	40%	75%	0%	25%	2.60	0.87	75%
2	3	2	0%	56%	44%	0%	64%	36%	75%	0%	25%	2.75	0.92	75%
3	3	2	22%	28%	50%	12%	79%	9%	41%	13%	46%	2.87	0.96	29%
4	3	2	29%	30%	41%	13%	65%	21%	33%	24%	43%	3.54	1.18	20%
5	3	2	28%	35%	37%	15%	59%	26%	32%	26%	42%	3.68	1.23	17%
1	3	3	0%	54%	46%	0%	54%	46%	75%	12%	13%	2.50	0.83	75%
2	3	3	0%	54%	46%	0%	53%	47%	75%	13%	12%	2.67	0.89	75%
3	3	3	16%	37%	47%	22%	58%	20%	37%	25%	38%	2.75	0.92	21%
4	3	3	31%	39%	29%	17%	48%	36%	27%	33%	40%	3.42	1.14	15%
5	3	3	30%	38%	32%	16%	51%	33%	28%	31%	41%	3.56	1.19	14%
1	3	4	0%	50%	50%	0%	54%	46%	75%	16%	9%	2.47	0.82	75%
2	3	4	3%	47%	50%	0%	50%	50%	72%	23%	5%	2.63	0.88	72%
3	3	4	4%	46%	50%	2%	48%	50%	68%	27%	5%	2.73	0.91	66%
4	3	4	22%	28%	50%	8%	42%	50%	45%	50%	5%	2.84	0.95	38%
5	3	4	28%	57%	15%	20%	30%	50%	27%	33%	40%	2.98	0.99	7%
1	3	5	0%	60%	40%	0%	35%	65%	75%	25%	0%	2.00	0.67	75%
2	3	5	5%	45%	50%	0%	50%	50%	70%	25%	5%	2.61	0.87	70%
3	3	5	4%	46%	50%	0%	50%	50%	71%	24%	5%	2.72	0.91	71%
4	3	5	11%	39%	50%	12%	38%	50%	52%	43%	5%	2.80	0.93	41%
5	3	5	21%	29%	50%	12%	38%	50%	42%	53%	5%	2.91	0.97	30%

Low Majority Democratic Primary Crossover: $a = 0.1$.

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{MD}	π_R	n_{mD1}	n_{MD1}	n_{R1}	n_{mD2}	n_{MD2}	n_{R2}	n_{mD3}	n_{MD3}	n_{R3}	Total	Aver.	$V(\mathbf{D})$
1	3	1	39%	0%	61%	0%	100%	0%	36%	20%	44%	2.65	0.88	39%
2	3	1	39%	0%	61%	0%	100%	0%	36%	20%	44%	2.97	0.99	39%
3	3	1	38%	1%	61%	0%	100%	0%	37%	19%	44%	3.16	1.05	38%
4	3	1	37%	20%	43%	0%	100%	0%	38%	0%	62%	3.30	1.10	38%
5	3	1	30%	35%	35%	15%	85%	0%	30%	0%	70%	3.43	1.14	15%
1	3	2	2%	49%	49%	0%	71%	29%	73%	0%	27%	2.58	0.86	73%
2	3	2	0%	29%	71%	0%	91%	9%	75%	0%	25%	2.75	0.92	75%
3	3	2	39%	0%	61%	0%	100%	0%	36%	20%	44%	2.91	0.97	39%
4	3	2	11%	89%	0%	29%	31%	40%	35%	0%	65%	3.06	1.02	24%
5	3	2	33%	32%	35%	6%	69%	25%	37%	19%	44%	4.15	1.38	31%
1	3	3	0%	39%	61%	0%	71%	29%	75%	10%	15%	2.50	0.83	75%
2	3	3	0%	39%	61%	0%	71%	29%	75%	10%	15%	2.66	0.89	75%
3	3	3	4%	39%	57%	11%	81%	8%	60%	0%	40%	2.75	0.92	56%
4	3	3	25%	39%	36%	25%	44%	31%	25%	38%	37%	3.92	1.31	0%
5	3	3	25%	52%	23%	25%	48%	27%	25%	21%	54%	3.07	1.02	0%
1	3	4	0%	35%	65%	0%	71%	29%	75%	14%	11%	2.47	0.82	75%
2	3	4	0%	26%	74%	7%	62%	31%	68%	32%	0%	2.63	0.88	68%
3	3	4	75%	25%	0%	0%	95%	5%	0%	0%	100%	2.75	0.92	75%
4	3	4	39%	61%	0%	36%	59%	5%	0%	0%	100%	2.88	0.96	39%
5	3	4	35%	65%	0%	33%	55%	12%	7%	0%	93%	3.00	1.00	28%
1	3	5	0%	22%	78%	0%	73%	27%	75%	25%	0%	2.50	0.83	75%
2	3	5	0%	22%	78%	0%	73%	27%	75%	25%	0%	2.67	0.89	75%
3	3	5	75%	25%	0%	0%	95%	5%	0%	0%	100%	2.75	0.92	75%
4	3	5	40%	60%	0%	35%	60%	5%	0%	0%	100%	2.88	0.96	40%
5	3	5	37%	58%	5%	38%	62%	0%	0%	0%	100%	2.99	1.00	38%

High Majority Democratic Primary Crossover: $a = 0.5$.

Table 13: Districting Plans Maximizing Minority Total Benefits – Primary Crossover.

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{MD}	π_R	n_{mD1}	n_{MD1}	n_{R1}	n_{mD2}	n_{MD2}	n_{R2}	n_{mD3}	n_{MD3}	n_{R3}	Total	Aver.	$V(\mathbf{D})$
1	3	1	0%	56%	44%	0%	64%	36%	75%	0%	25%	2.75	0.92	75%
2	3	1	13%	37%	50%	12%	83%	5%	50%	0%	50%	2.89	0.96	38%
3	3	1	25%	61%	14%	27%	0%	73%	24%	59%	17%	3.06	1.02	3%
4	3	1	24%	60%	16%	27%	0%	73%	24%	60%	15%	3.25	1.08	3%
5	3	1	25%	61%	14%	27%	0%	73%	23%	59%	18%	3.41	1.14	4%
1	3	2	0%	60%	40%	0%	60%	40%	75%	0%	25%	2.60	0.87	75%
2	3	2	0%	55%	45%	0%	65%	35%	75%	0%	25%	2.75	0.92	75%
3	3	2	27%	69%	4%	23%	0%	77%	24%	51%	24%	2.86	0.95	4%
4	3	2	27%	68%	5%	23%	1%	75%	25%	51%	25%	3.04	1.01	4%
5	3	2	24%	60%	16%	27%	0%	73%	24%	60%	16%	3.20	1.07	3%
1	3	3	0%	51%	49%	0%	55%	45%	75%	14%	11%	2.50	0.83	75%
2	3	3	0%	64%	36%	0%	52%	48%	75%	5%	20%	2.67	0.89	75%
3	3	3	27%	47%	26%	15%	32%	52%	33%	40%	27%	2.75	0.92	18%
4	3	3	27%	67%	6%	23%	3%	74%	25%	50%	25%	2.92	0.97	4%
5	3	3	27%	67%	6%	23%	3%	73%	25%	50%	25%	3.07	1.02	4%
1	3	4	0%	50%	50%	0%	54%	46%	75%	16%	9%	2.47	0.82	75%
2	3	4	3%	47%	50%	0%	50%	50%	72%	23%	5%	2.63	0.88	72%
3	3	4	5%	45%	50%	0%	50%	50%	70%	25%	5%	2.73	0.91	70%
4	3	4	11%	39%	50%	32%	45%	23%	32%	36%	32%	3.33	1.11	21%
5	3	4	33%	55%	12%	7%	0%	93%	35%	65%	0%	3.00	1.00	28%
1	3	5	0%	45%	55%	0%	50%	50%	75%	25%	0%	2.00	0.67	75%
2	3	5	5%	45%	50%	0%	50%	50%	70%	25%	5%	2.61	0.87	70%
3	3	5	5%	45%	50%	0%	50%	50%	70%	25%	5%	2.71	0.90	70%
4	3	5	35%	60%	5%	0%	0%	100%	40%	60%	0%	2.88	0.96	40%
5	3	5	0%	0%	100%	38%	62%	0%	37%	58%	5%	2.99	1.00	38%

Low Majority Democratic General Crossover: $b = 0.1$.

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{MD}	π_R	n_{mD1}	n_{MD1}	n_{R1}	n_{mD2}	n_{MD2}	n_{R2}	n_{mD3}	n_{MD3}	n_{R3}	Total	Aver.	$V(\mathbf{D})$
1	3	1	0%	58%	42%	0%	62%	38%	75%	0%	25%	2.75	0.92	75%
2	3	1	0%	92%	8%	26%	27%	47%	49%	1%	50%	3.44	1.15	49%
3	3	1	21%	49%	30%	21%	52%	27%	33%	19%	48%	4.00	1.33	12%
4	3	1	24%	59%	17%	25%	34%	40%	26%	27%	47%	4.19	1.40	2%
5	3	1	26%	64%	11%	28%	25%	48%	22%	31%	47%	4.35	1.45	6%
1	3	2	0%	60%	40%	0%	60%	40%	75%	0%	25%	2.60	0.87	75%
2	3	2	5%	82%	12%	16%	38%	46%	54%	0%	46%	3.21	1.07	48%
3	3	2	20%	49%	31%	21%	53%	26%	34%	18%	48%	3.86	1.29	14%
4	3	2	23%	57%	20%	24%	37%	38%	28%	25%	47%	4.04	1.35	5%
5	3	2	24%	60%	16%	26%	32%	42%	26%	28%	46%	4.19	1.40	2%
1	3	3	0%	52%	48%	0%	54%	46%	75%	14%	11%	2.50	0.83	75%
2	3	3	4%	46%	50%	0%	50%	50%	71%	24%	5%	2.65	0.88	70%
3	3	3	26%	50%	24%	19%	37%	44%	30%	33%	37%	3.75	1.25	10%
4	3	3	25%	58%	17%	25%	34%	41%	25%	28%	47%	3.92	1.31	0%
5	3	3	25%	58%	17%	25%	34%	41%	25%	28%	47%	4.07	1.36	0%
1	3	4	0%	50%	50%	0%	54%	46%	75%	16%	9%	2.47	0.82	75%
2	3	4	3%	47%	50%	0%	50%	50%	72%	23%	5%	2.63	0.88	72%
3	3	4	0%	50%	50%	22%	31%	47%	53%	39%	8%	3.21	1.07	53%
4	3	4	34%	55%	12%	25%	28%	47%	16%	37%	46%	3.84	1.28	17%
5	3	4	29%	58%	13%	25%	28%	47%	21%	34%	44%	3.98	1.33	7%
1	3	5	0%	45%	55%	0%	50%	50%	75%	25%	0%	2.00	0.67	75%
2	3	5	5%	45%	50%	0%	50%	50%	70%	25%	5%	2.61	0.87	70%
3	3	5	2%	48%	50%	16%	37%	46%	57%	34%	9%	3.18	1.06	56%
4	3	5	34%	54%	12%	25%	28%	47%	16%	38%	46%	3.78	1.26	18%
5	3	5	33%	56%	11%	25%	27%	47%	17%	37%	46%	3.90	1.30	16%

High Majority Democratic General Crossover: $b = 0.9$.

Table 14: Districting Plans Maximizing Minority Total Benefits – General Crossover.