The Paradox of Minority Power*

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Abstract

We present a model that integrates electoral competition, legislative redistribution, and redistricting to optimize minority representation. Our analysis reveals that politically weaker minority groups benefit from concentrated districts, whereas more powerful minorities prefer dispersed voter distributions. The presence of non-minority voters supporting minority candidates has a dual effect: it helps minorities gain office but may also enhance the influence of non-minority voters and their policy benefits. Paradoxically, increasing the number of minority voters in a district can have a non-monotonic effect, sometimes leading to the election of representatives less favored by minority communities. We numerically test these propositions across various simulation environments and calculate the impact of different voting schemes on electoral success and policy benefits of minority groups. These findings shed new light on the complex trade-offs between redistricting, electoral competition, and equitable policy outcomes, offering a framework for designing more representative political institutions.

Keywords: Redistricting, Distributive Politics, Electoral Competition, Minority Representation

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1 Introduction

Two prominent themes in the political economy literature are the dynamics of voter representation and the distribution of policy benefits. The first examines electoral competition– who wins offices and which voters are represented. The second focuses on legislative institutions—how policies are made and who receives economic gains. While both strands offer valuable insights, they have traditionally stood in isolation, overlooking critical trade-offs between electoral outcomes and legislative policies.

To fill this gap, this paper analyzes the problem of redistricting in a democratic system. Redistricting lies at the intersection of electoral politics and public policy, directly shaping who gets elected (descriptive representation) and what policies are pursued (substantive representation). Given a state with a specific demographic distribution, we establish a benchmark to evaluate which districting strategies best promote minority representation. Is it better for minorities to concentrate their influence in a few districts or distribute their influence over a larger number of districts? And is electing minority representatives essential for advancing minority interests, or can non-minority legislators effectively represent those policy preferences?

In the United States, redistricting is often undertaken by partisan gerrymanders, where electoral districts are drawn to increase the chances that the majority party maintains political control or wins additional legislative seats. However, these redistricting maps are constrained by electoral laws, geographical boundaries, and protections for minority rights. Legal battles—such as the Supreme Court's decision in Shelby County v. Holder—highlight the ongoing tension between who gets elected and whose interests are represented in policymaking. The conflation of race and party further confounds redistricting. Minority voters overwhelmingly support Democrats, forming coalitions with non-minority Democrats and facing opposition from Republicans.[1] State courts increasingly grapple with whether district maps empower minority voters or conceal disenfranchisement under the guise of partisan gerrymandering. Lastly, both primary and general elections, key features of the US electoral system, significantly influence candidate success and voter choice. Our analysis incorporates these complexities, going beyond the customary focus on either partisan or racial gerrymandering to ask whether descriptive and substantive representation are complements or substitutes.

We model elections as a probabilistic process in both primaries and general elections, where

[1] We discuss recent trends of Hispanic and Latino voting later, illustrating our predictions of electoral competition and representation.

candidates can differ by party and race, and offer ideological and distributive benefits to voter groups. Voters with weaker ideological attachments or higher marginal utility from consumption tend to act as “swingy” voters, thereby giving them disproportionate power to influence political outcomes. However, these voters face a marginal trade-off: by securing greater distributive benefits, they may forgo electing candidates who share their ideological identity, reducing descriptive representation. Crossover voting underscores this tension—nonminority voters who support minority candidates increase those candidates’ chances of winning but reduce their own group’s policy gains. These trade-offs between substantive and descriptive representation are central to optimal redistricting.

By disaggregating the key drivers of substantive and descriptive representation, the theoretical findings uncover several benchmark results for evaluating how alternative districting strategies affect electoral outcomes and policy benefits. In doing so, our analysis reveals several overlooked paradoxes. Focusing first on the allocation of distributive benefits, our model shows that minorities with relatively little political power–those less motivated by distributive benefits and thus less “swingy” in elections–prefer to concentrate their voters in a few districts to elect their preferred candidate. Conversely, as minorities gain power, putting a greater weight on distributive benefits, they do best by distributing their voters more evenly across districts. Paradoxically, minorities may be better off sharing districts with members of a nonminority voter group more motivated by ideological rather than distributive benefits, independent of their party affiliation. In other words, optimal districting may require pairing minority voters with less swingy voters who could be either Democrats—appearing as partisan packing—or Republicans—appearing as partisan cracking—to increase their substantive representation.

Focusing next on ideological benefits, minority groups are better off concentrating like-minded voters into a few districts, thereby increasing the chances of electing minority candidates in a two-stage electoral competition. However, crossover by nonminority voter groups–their willingness to vote for a candidate aligned or not aligned with their partisan or identity lines–affects the election of minority candidates. For example, our results uncover that increasing the number of nonminority democrats and republicans has non-monotonic effects; in particular, replacing Republican voters, who are potentially unlikely to support a nonminority Democrat in a given district, with nonminority Democrats (appearing as Democrat packing) helps minority candidate’s chances in the general, when nonminority Democrats support along the party line, though reduces chances in the primary when nonminority Democrats potentially support their nonminority Democratic candidate. Similarly, replacing voters in one district creates externality effects for minority candidates in other districts, illustrating the nuanced trade-offs in allocating voter groups across districts with primary and general elections.

Finally, we combine the distributive and ideological analysis and identify when minority voters benefit more from a few concentrated districts or significant representation across many districts. We uncover a U-shaped relationship between minority power and the concentration of minority voters. As minority power increases, voters become more motivated by distributive benefits and prefer to be less concentrated to maximize benefits across districts. However, beyond a certain threshold, diminishing marginal returns to distributive benefits make further dispersion inefficient. At this inflection point, concentrating voters—by shifting a few into another district—can increase the likelihood of electing a minority candidate and yield greater overall benefits.

Our analysis also shows that the structure of primaries, whether open or closed, does not affect the equilibrium on voter groups' distributive benefits, though candidates' platforms may change from the primary to the general election. However, the openness of primaries plays a crucial role in determining the likelihood of candidate success and shaping the convexity of ideological benefits compared to distributive benefits, which can be concave or convex, thereby affecting redistricting.

To unpack these competing effects, we first place our analysis within the vibrant literature on redistricting and minority representation, we develop a formal model in Section 2, provide strategies for players in Section 3, and describe the distributive and ideological benefits for minority voters in Section 4, both analytically and numerically. In Section 5 we simulate various electoral maps. The concluding Section 6 explores the policy implications of our results, analyzing optimal districting schemes, including minority voter registration, crossover, and economic inequality.

1.1 Related Literature: Redistricting and Representation

This review examines the interplay between empirical findings and theoretical advancements in redistricting and representation, underscoring the inherent link between electoral politics and legislative outcomes. For the most part, the research on redistricting's effect on political and economic outcomes bifurcates into studies on partisan politics and ones addressing racial/ethnic minority groups. The former includes seminal work on seat-votes curve biases (Tufte, 1973; King, 1989; Gelman and King, 1990; Lublin et al., 2020), incumbent protection (Cox and Katz, 2002; Ansolabehere and Snyder Jr., 2004), and party power consolidation (Butler and Cain, 1991; Issacharoff, 2002; Persily, 2002).

The literature on racial redistricting, on the other hand, explores the impact on Black office-holding in the South (Davidson and Grofman, 1994), the descriptive versus substantive representation trade-off (Cameron et al., 1996; Epstein and O'Halloran, 1999; Lublin, 1997b), its role in congressional partisan shifts (Lublin and Voss, 2000), and voting polarization and segregation (Stephanopoulos, 2016). The legal revolution and the considerations of descriptive versus substantive representation are discussed in detail by Canon (2022) from a political science perspective and Ross II (2024) from a legal perspective.

Other empirical studies, such as Jeong and Shenoy (2022), document "packing-and-cracking" gerrymandering tactics against African American voters, who typically support Democratic candidates. They show that after a Republican redistricting, minority voters are more likely to be segregated into black districts. Cameron et al. (1996) and Lublin (1997a) argue that such concentrated majority-minority districts might inadvertently diminish minority policy impact. Canon (1999) contests this point, emphasizing minority legislators' behind-the-scenes influence in policy-making.[2] The key point of contention revolves around the extent of white-crossover voting. While Ansolabehere et al. (2010) argue for its prevalence, Lublin et al. (2020) claim it is declining, suggesting that minorities' power lies more in optimal voter composition than in winning white votes. Regardless, such a strategy results in highly polarized districts, where neither a nonminority candidate nor a minority candidate wins the Democratic primary, and neither a nonminority candidate nor a minority candidate wins the Republican primary. Despite its richness, the empirical evidence provides little relief in assessing optimal districting strategies as conflicting redistricting measures and minority incorporation continue to prevail. This limitation is particularly acute with the Supreme Court's reticence to review partisan districting claims, making all potential challenges about race (Tofighbakhsh, 2020).

Formal redistricting models tend to prioritize partisan over racial divisions, with studies like Musgrove (1977) and Owen and Grofman (1988) focusing on maximizing majority party seats. Coate and Knight (2007) provides a more subtle analysis of partisan redistricting, where partisans and independents calculate the districting schemes that maximize overall social utility. Partisan voters are depicted as point masses at the extremes of a one-dimensional policy space. In contrast, independent voters are uniformly distributed along some intervals in the space's interior. In this

[2] Issacharoff (2002); Grofman et al. (1992); Epstein and O'Halloran (2006) note that such back-room influence is, however, difficult to measure, suggesting that the promotion of majority-minority districts via Section 5 of the Voting Rights Act may undermine minority influence, even as it increases the number of Republicans elected to office.

context, the problem is ensuring that the average legislator has preferences near these independents. Bouton et al. (2023) analyzes strategic incentives of partisan gerrymandering when voter turnout varies across parties and tests their predictions with the U.S. redistricting cycle of 2020. They document how parties benefit from matching supporters with low turnout, opponents with high turnout, and supporters and opponents with intermediate turnout. Kolotilin and Wolitzky (2024) take a different approach, modelling a designer who faces district-level uncertainty about a party's number of votes and voter-level uncertainty about which voter will cast their ballot for a particular candidate. The designer sorts voters into segregated pairs, packing them in weaker districts and cracking them in stronger ones. These patterns are driven by voters' partisan preferences. Our analysis focuses on ideological and distributive preferences, where packing and cracking emerge from competition over both candidate selection and shares of legislative benefits—factors that may diverge from partisanship alone.

Notably, racial considerations in redistricting are less frequently addressed. Exceptions include Shotts (2001), who presents a model of racial redistricting with partisan control of the redistricting process. The partisan gerrymanderer allocates a continuum of voters with observable identities and receives a noisy signal of their preferences. The analysis finds that majority-minority district requirements do not affect liberal gerrymanders; they can, however, limit the options of conservative districters. Friedman and Holden (2008) offer a model of this tradeoff, analyzing optimal redistricting where a gerrymanderer observes a noisy signal of voter preferences across a continuum of voters. The study demonstrates that cracking districts is never optimal. While their analysis does not consider the Voting Rights Act, using their results, one could predict that spreading (minority) voters with strong partisan preferences across many districts would not be optimal. In our analysis, we explicitly incorporate partisanship and identity, illustrating alternative considerations for both packing and cracking outcomes along these dimensions.

The above studies and models draw political boundaries by allocating voters to districts to maximize social welfare, measured against the median voter's preferred electoral outcomes and level of economic distribution. Furthermore, they focus on voters' ideological and partisan preferences but do not consider the influence different voter groups have in the electoral process and how they can influence candidate selection and distributive outcomes. Our analysis fills this gap; we distinguish between descriptive representation and distributive representation, enabling us to uncover cracking-and-packing patterns along the dimensions of ideology and partisanship as well as the distribution of voter group power.

Our analysis of voters' trade-offs between ideological and distributive benefits contributes to the literature on electoral competition for legislative outcomes. Myerson (1993) develops a model of strategic political competition, where individuals and groups seek economic benefits through "rent-seeking," often at the expense of other groups' policy goals. Similarly, Lindbeck and Weibull (1987, 1993) builds a model of economic redistribution, emphasizing that voter preferences for altruism, risk aversion, and income redistribution shape distributive policy outcomes. Dixit and Londregan (1996) introduce a model of redistribution that incorporates economic inequality across voter groups, showing how differences in average income affect the allocation of economic benefits—a framework we adopt in our analysis below.

1.2 Courts in the United States: Partisan and Racial Gerrymandering

The above literature highlights the importance of institutions like redistricting on political representation and economic inequality. Therefore, it is important to disentangle these dimensions as an ill-considered restricting strategy can lead to minority segregation and/or increased inequality. The history of legal challenges to state redistricting plans emphasizes this dilemma.

For example, the *Georgia v. Ashcroft* Supreme Court ruling (539 U.S. 461) highlighted this tradeoff. The question was whether a proposed redistricting scheme violated Section 5 of the *1965 Voting Rights Act* (VRA) because it "unpacked" minority voters, spreading their influence more evenly across districts. The Court ruled that state legislators attempted to increase minorities' overall influence on policy (substantive representation), concluding the scheme was not "retrogressive" even though it might result in electing fewer minority representatives (descriptive representation). The *Cooper v. Harris* case extended this logic to North Carolina's redistricting plan that "packed" minority voters into a few districts. The Court noted that in the absence of racially polarized voting, usually owed to white crossover voters, concentrating minorities into a few districts does not necessarily increase minority power; indeed, it may dilute it.[3] By disentangling partisan from racial gerrymandering, the Court upended the long-standing practice of Republican-dominated state legislatures packing minority voters into a few districts while drawing electorally safe Republican districts elsewhere.[4] In a radical reversal, the Supreme Court asserted in the 2019 *Rucho v. Common Cause* case that federal courts could not adjudicate partisan

[3] Writing for the majority, Justice Kagan noted that unlawful racial gerrymander can occur when "legislators have placed a significant number of voters within or without a district predominantly because of their race, regardless of their ultimate objective in taking that step." 137 S. Ct. 1455, p. 11 (2017).

[4] 137 S. Ct. 1455 (2017). See Hasen (2017) for an analysis.

gerrymandering claims, ignoring the intrinsic link between race and party and placing the legal enforcement of redistricting plans squarely in the hands of the states that perpetrated them.[5]

2 General Model

We now move to a systematic analysis of redistricting and representation, specifically from the perspective of minority voters' benefits, and adapt the Dixit and Londregan (1996) model of electoral competition.[6] In this model, voters ascribe ideological attachments to different candidates, and these candidates then compete for office by promising group-specific policy benefits. This model is well-suited to our purposes: it captures that voters of one identity may prefer representatives of the same identity and candidate competition over policy outcomes.

To provide a preview of our model's various steps, we consider 1) a Democratic primary for minority- and nonminority-Democratic voters; 2) a general election with the Democratic primary winner and a Republican candidate; 3) for each election, all candidates announce redistributive platforms to each group within the district; 4) voters benefit from redistributive policies through the legislature and ideological benefits from their legislator; 5) an optimal districting map of voter groups maximizes minority group utility across all districts.[7] Hence, we provide a detailed model that provides a mapping from a districting scheme in a two-dimensional simplex, $\mathbf{D} = \mathbf{D}(S^2)$, to an electoral outcome describing a legislature of the districts' winning candidates, $\mathbf{L} = \mathbf{L}(\mathbf{D}(S^2))$, and to legislative outcomes with distributive benefits for districts and voter groups implemented by the legislature, $\mathbf{P} = \mathbf{P}(\mathbf{L}(\mathbf{D}(S^2)))$.

2.1 Districts

Assume a population of voters, V, divided into a given number of identifiable groups Θ; these may be defined according to voters' ethnicity, language, economic status, religion, political party, etc. Thus, there is a partition from the set of voters V to groups, $\nu : V \to \Theta$.

For simplicity, we assume a state population divided along ethnic and partisan lines with voter types $\Theta = i \in \{mD, nD, R\}$, for minority-Democrats, nonminority-Democrats, and Republicans, respectively. Their statewide populations are $\mathbf{N}_{mD}$, $\mathbf{N}_{nD}$, and $\mathbf{N}_R$, with $\sum_i \mathbf{N}_i = \mathbf{N}$, the total

[5] Indeed, in 2022, the Supreme Court denied review of claims of partisan over gerrymandering by Pennsylvania and Maryland. For a review of the implications of *Rucho* for redistricting, see Tofighbakhsh (2020).

[6] We can also embed the Baron and Ferejohn (1989) model of legislative bargaining.

[7] We maximize minority utility to set a benchmark to evaluate alternative voter allocations in the numerical simulations below.

state population. Since population proportions must sum to 1, we can represent the mix of voter types statewide—or in any given district—as a point in the two-dimensional simplex, S^2, as illustrated above.

A district is a vector $\mathbf{d} = (N_{mD}, N_{nD}, N_R)$ of voters with $N_i \geq 0$. Let $\mathcal{D}$ be the set of all possible districts, and assume that the state will be divided into K districts with N_{ik} representing the number of voters of type i in district k. We denote the number of all voters in district k with N_k. Then a districting scheme is a function $\mathbf{D} : S^2 \rightarrow \mathcal{D}^K$, yielding a list $(\mathbf{d}_1, \mathbf{d}_2, ..., \mathbf{d}_K)$ of districts. Furthermore, a *valid* districting scheme is a districting scheme such that in any given district, $\sum_i N_{ik} = N_k = \mathbf{N}/K$, and across districts $\sum_k N_{ik} = \mathbf{N}_i$ for all voter types i – i.e., all districts are equally sized, and all voters are assigned to a district.[8]

We use an equilateral triangle, as in Figure 1(a), to represent the two-dimensional simplex S^2 of possible percentages of each group in a given electorate. The corners thus indicate an electorate with only one type of voter: nD in the bottom left, mD in the bottom right, and R on top. The center point (a) is an electorate with an equal division of all three types, each comprising one-third of the district population. We could also divide the triangle into four smaller triangles, highlighting electorates with majorities or no majority. The bottom left triangle indicates electorates with nD majorities, the bottom right triangle with mD majorities, and the top triangle with R majorities. Hence, point (b) indicates a majority-minority electorate. The center triangle indicates no majority among the three groups, though they would be Democrat-majority due to the sum of mD and nD voters. Figure 1(b) illustrates a state with five districts.[9] The statewide distribution of voters is marked by point (S), while the other five points represent the districts, one of which is majority-minority.

2.2 Candidates and Elections

Suppose in each of the K districts, three candidates are competing for a seat in the legislature; these candidates are also of types $\theta = j \in \{mD, nD, R\}$. Candidates try to maximize their vote share with platforms that offer a proportion T_i of the district's redistributive benefits as transfers to voters of type, which captures *substantative representation.* Denote the redistributive platform of candidate j towards group i in district k as T_{ijk}; then, campaign platforms must

[8] Equivalently, as in the triangle analysis above, the average of the percentages of each group in the K districts must equal their statewide population proportion $\mathbf{N_i}/\mathbf{N}$. See the numerical example in Appendix A.1 for illustrations above.

[9] We provide the numerical values of Figure 1(b) in Appendix A.1

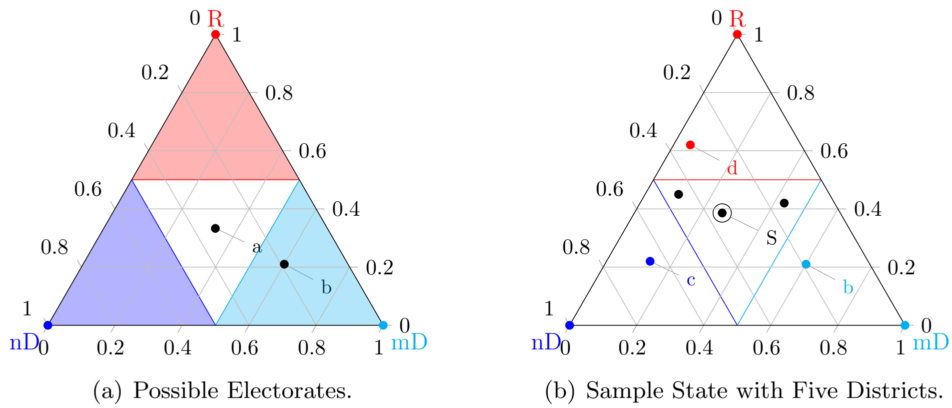


Figure 1: Possible Electorates and Districts.

satisfy $\sum_i T_{ijk} = 1$ for each j and k.[10]

Candidates attain office according to a two-stage electoral cycle: first, each district holds a primary election, in which the mD candidate faces a nD opponent; second, there is a general election in each district where the primary winner squares off against a Republican. We consider closed and open primaries and discuss how candidates' platforms may adjust from the primary to the general election stage.

Represent a candidate by a vector $c = (\theta, T_{mD}, T_{nD}, T_R)$, where θ is the candidate's type, and let $\mathcal{C}$ be the set of all possible candidates. Let $\mathbf{c}_k$ be the list of three candidates from district k, and $\mathbf{C} = \{\mathbf{c}_1, \mathbf{c}_2, ..., \mathbf{c}_K\}$ be the entire set of (3K) candidates in all districts. Then an election is a mapping to a legislature $\mathbf{L} : \mathcal{D}^K \times \mathcal{C}^{3K} \rightarrow \mathcal{C}^K$, producing a representative for each district with a given type and committed to a given platform.

To smooth out the response functions, we assume probabilistic voting so that the probability a candidate wins a given election rises with the expected proportion of votes she receives. Given expected vote proportion v, let the probability of winning the election be $\Psi(v)$, with $\Psi' > 0$, $\Psi(0) = 0$, $\Psi(1) = 1$, and $\Psi(1 - v) = 1 - \Psi(v)$. We assume here the simplest linear function $\Psi(v) = v$, so that, for instance, a candidate expecting to receive 60% of the vote wins with a 60% probability.[11]

The winners of the K district elections then go to a legislature $\mathbf{L} \in \mathcal{C}^K$. Considering candidates' equilibrium strategies, elections transform a districting scheme into a legislature; that is, $\mathbf{L} = \mathbf{L}(\mathbf{D}(S^2))$.

[10] We assume both parties and all candidates have equal abilities to distribute benefits as in Dixit and Londregan (1996). Without loss of generality, the model could be extended to allow each candidate to evaluate the marginal value spent for any program on the voter group's marginal utility. We discuss this possibility below.

[11] The qualitative results derived below do not depend on our assumption of probabilistic voting.

2.3 Legislative Policies

The legislature then passes a redistributive policy $\mathbf{P}$, dividing K dollars across all districts. Any funds allocated to district k in the legislative process are divided according to the platform adopted by that district's representative. So if the type j representative from district k ran on a platform promising T_{ijk} to members of a group i, then voters in this group will receive $T_{ijk} * B_k$ in total benefits, with individual benefits $b_{ijk} = (T_{ijk} * B_k)/N_{ik}$. Considering a Baron and Ferejohn (1989)-closed-rule bargaining process, we can quickly state that each legislator and district receives one dollar in expected terms – i.e., $B_k = 1$.[12]

2.4 Voters

Voters enjoy distributive benefits from legislative outcomes and ideological benefits from elected candidates. We adopt Dixit and Londregan (1995, 1996)'s characterization of utilities where voters from group i receive utility $U_i(.)$ from consumption and an ideological attachment to winning candidates.[13] In particular, assume that the utility from consumption, b, is given by:

$$U_i(b) = \kappa_i \frac{b^{1-\epsilon}}{1-\epsilon} \tag{2.1}$$

with $\epsilon > 0$ and $\epsilon \neq 1$. Then the marginal utility of an additional dollar of consumption and the return to consumption are

$$U_i'(b) = \kappa_i b^{-\epsilon} > 0 \text{ and } U_i''(b) = -\epsilon\kappa_i b^{-\epsilon-1} < 0. \tag{2.2}$$

As b increases from 0 to ∞, the marginal utility falls from ∞ to 0, and this assumption avoids corner solutions. A one percent increase in b causes an ϵ percent decrease in marginal utility, so ϵ captures the degree of diminishing returns in consumption.[14] Furthermore, the parameter κ_i captures the relative weight of consumption to ideological benefits for voter group i; higher values of κ_i imply that voters of group i are more responsive to distributive than ideological benefits. Together, ϵ and κ_i characterize the trade-offs between economic and ideological benefits.

[12]See the derivation of the assumed solution in Appendix A.2.

[13]Lindbeck and Weibull (1987) also consider utility functions with additively separable benefits from consumption and ideological benefits and a positive decreasing marginal utility of consumption.

[14]In other words, individuals have a willingness to trade off consumption for ideology depending on their consumption level. Individuals with low values of ϵ remain quite sensitive to transfers, even when they receive numerous transfers; in contrast, individuals with high values of ϵ are less sensitive to transfers. Hence, it is easier and less costly to sway voters with greater sensitivity.

Voters' ideological benefits depend on their district's winning candidate and are described by X^j for a candidate of type j, illustrating *descriptive representation.* The overall utility for a voter of type i a representative of type j offering distributive benefits b_{ij} is the sum of their ideological and distributive benefits: $U_i = X_i^j + E[U_i(b_{ij})]$. Thus, for instance, a voter with ideological preference of X^{mD} for minority-Democratic candidates and X^R for Republicans gets extra utility $X^{mD} - X^R \gtreqless 0$ from seeing a minority-Democrat win office instead of a Republican. The voter with a positive ideological gain will, therefore, prefer the minority-Democrat candidate unless the Republican offers her a sufficiently greater consumption value:

$$E[U_i(b_{iR})] - E[U_i(b_{imD})] > X^{mD} - X^R. \tag{2.3}$$

We define the critical value, or "cutpoint" X_i for group i in an election between two candidates labeled 1 and 2 by:

$$X_i^e \equiv U_i(b_{i1}) - U_i(b_{i2}), \tag{2.4}$$

where e indicates the type of election being contested– i.e. a primary election or a general election with either mD vs. R or nD vs. R candidates. Voters are assumed to cast their ballots sincerely for the candidate offering them higher utility. Then, group i voters with values of X_i less than X_i^e will vote for Candidate 1, while the others will vote for Candidate 2. If Candidate 1 offers an additional dollar to each member of the group i, then the critical value will shift in her favor by $U_i'(b_{i1}) = \kappa_i b_{i1}^{-\epsilon}$.[15]

Votes Let Φ_i^e be the concave cumulative distribution of voters of a group i in an election of type e, so that, given the campaign platforms, a proportion $\Phi_i^e(X_i)$ will vote for candidate 1. Given N_i voters of type i, this candidate will receive $N_i\Phi_i^e(X_i)$ votes from group i, with total votes of:

$$V_1^e = \sum_{i \in \Theta} N_i \Phi_i^e(X_i). \tag{2.5}$$

The opposing candidate will then get votes of:

$$V_2^e = \sum_{i \in \Theta} N_i[1 - \Phi_i^e(X_i)] = N - V_1^e. \tag{2.6}$$

[15]We assume throughout that voter groups may differ in their consumption and ideological preferences but neglect within group-differences across districts.

Crossover Voting. The distribution functions $\Phi_i^e(X_i)$ play an important role in the following analysis. They indicate the ideological preference of a given voter i for one candidate over another. These preferences could arise partly from a spatial policy model, measuring the degree to which voters agree with the policy choices of their representatives. But they could also arise from group voting preferences: voters might want to support candidates of one type θ over those of another type. In the legal literature, this is what is meant by polarized voting: the willingness, or lack thereof, of voters to cross over and vote for candidates of another race and ethnicity. We assume for simplicity that if the distribution of type i voters in the entire population is $\Phi_i^e(.)$, then this is also the distribution of the type i voters in any given district.[16]

Notice that the rates at which different types of voters cast their ballots for various candidates are given by the $\Phi_i^e(0)$ functions for group i in an election of type e, where for convenience we label the primary as election $e = 1$, a general election of mD vs. R as type $e = 2$, and a general election of nD vs. R as $e = 3$. For instance, in an mD vs. nD primary, a proportion $\Phi_{mD}^1(0)$ of minority voters will vote for the minority candidate, and the remaining $1 - \Phi_{mD}^1(0)$ will vote for the nonminority-Democrat candidate.

We redefine these quantities as *crossover rates*, following the usual standard for voting studies, letting a_Θ^e represent the rate at which voters of a group Θ vote for the more liberal candidate in election e.[17] Thus, a proportion a_{nD}^1 of nonminority Democrats cross over to vote for the minority candidate in the primary, while $1 - a_{nD}^1$ vote for the nonminority Democratic candidate. Similarly, a proportion a_R^2 of Republican voters prefer the minority-Democrat in a general election. For reference, a table of these crossover rates is given in Table 1.

Crossover Voting and Group Power. In the above section, we introduced voters' willingness to trade off ideological and distributive benefits, which determines their electoral power or their "swinginess" between candidates. There is a link between the swinginess and crossover rates. Initially, we keep them separate when sequentially solving for distributive and ideological benefits. Section 5, where we simulate optimal redistricting schemes, discusses possible mappings and links.

[16] We also assume that the number of voters in each district is large enough that we can calculate expected voter utility as the integral of $\Phi_i^e(.)$ for voter types.

[17] Assuming for the purposes of definition that minority-Democrats are more liberal than nonminority-Democrats, who are more liberal than Republicans.

Election	Group \ Candidate	mD	nD	R
Primary,	mD	a^1_{mD}	$1-a^1_{mD}$	
$e=1$	nD	a^1_{nD}	$1-a^1_{nD}$	
open primary	R	a^1_{R}	$1-a^1_{R}$	
General mD **vs.** R,	mD	a^2_{mD}		$1-a^2_{mD}$
$e=2$	nD	a^2_{nD}		$1-a^2_{nD}$
	R	a^2_{R}		$1-a^2_{R}$
General nD **vs.** R,	mD		a^3_{mD}	$1-a^3_{mD}$
$e=3$	nD		a^3_{nD}	$1-a^3_{nD}$
	R		a^3_{R}	$1-a^3_{R}$

Table 1: Crossover Rates.

Winning Probabilities. Then, for instance, the minority candidate will be expected to win a closed primary, ignoring the notation for district k, if:

$$a^1_{mD}N_{mD} + a^1_{nD}N_{nD} \geq (1-a^1_{mD})N_{mD} + (1-a^1_{nD})N_{nD} \;\Rightarrow\; \frac{N_{mD}}{N_{nD}} \geq \frac{1-2a^1_{nD}}{2a^1_{mD}-1}. \tag{2.7}$$

Similarly, we apply a few assumptions on the relative magnitudes of crossover rates: $a^e_{mD} > a^e_{nD} > a^e_R$, reflecting closer ideological alignment among Democrats.[18]

Let Ψ^e_θ represent the probability that a type θ candidate wins election e, and Ψ_θ be the probability that the candidate wins overall. Given that the proportion of votes a candidate receives equals her probability of winning, we have the following for each election type:

$$\text{closed primary} - \Psi^1_{mD} = \frac{a^1_{mD}N_{mD} + a^1_{nD}N_{nD}}{N_{mD}+N_{nD}} \quad \text{and } \Psi^1_{nD} = 1-\Psi^1_{mD}; \tag{2.8}$$

$$\text{open primary} - \hat{\Psi}^1_{mD} = \frac{a^1_{mD}N_{mD} + a^1_{nD}N_{nD} + a^1_R N_R}{N_{mD}+N_{nD}+N_R} \quad \text{and } \hat{\Psi}^1_{nD} = 1-\hat{\Psi}^1_{mD}; \tag{2.9}$$

$$\Psi^2_{mD} = \frac{a^2_{mD}N_{mD} + a^2_{nD}N_{nD} + a^2_R N_R}{N_{mD}+N_{nD}+N_R} \quad \text{and } \Psi^2_R = 1-\Psi^2_{mD}; \tag{2.10}$$

$$\Psi^3_{nD} = \frac{a^3_{mD}N_{mD} + a^3_{nD}N_{nD} + a^3_R N_R}{N_{mD}+N_{nD}+N_R} \quad \text{and } \Psi^3_R = 1-\Psi^3_{nD}, \tag{2.11}$$

which describes the probabilities of winning the district for each candidate type with

$$\Psi_{mD} \;=\; \Psi^1_{mD}\Psi^2_{mD},\; \Psi_{nD} = \Psi^1_{nD}\Psi^3_{nD}, \text{ and } \Psi_R = 1-\Psi_{mD}-\Psi_{nD} \text{ (closed primary)}$$

[18] These crossover rates are for the same election e. One can consider different crossover rates across types for each election. For example, Washington (2006) shows that White Democratic and Republican voters are less likely to support a Black candidate of their party – $a^2_{nD} < a^3_{nD}$.

$$\hat{\Psi}_{mD} = \hat{\Psi}^1_{mD}\Psi^2_{mD},\ \hat{\Psi}_{nD} = \hat{\Psi}^1_{nD}\Psi^3_{nD},\ \text{and } \hat{\Psi}_R = 1 - \hat{\Psi}_{mD} - \hat{\Psi}_{nD} \text{ (open primary)}. \quad (2.12)$$

These equations define a surface on S^2 with smoothly increasing election probabilities for each type. This set-up simplifies the analysis and accounts for voting nuances such as the effects of voter registration and turnout, which may or may not vary across partisanship and identity and which we do not model here explicitly.[19]

2.5 Order of Play

To summarize, the order of play is as follows:

1. Given state demographics of $\mathbf{N}_{mD}$, $\mathbf{N}_{nD}$, and $\mathbf{N}_R$ and K districts, a valid districting scheme $\mathbf{D}$ is enacted.

2. Candidates of type j in each district k announce their platforms offering distribute benefits T_{ijk} for $i, j \in \{mD, nD, R\}$.

3. Voters elect candidates in primary and general elections, yielding legislature $\mathbf{L}$ and distributing $\mathbf{P}$.

4. All voters receive their utilities, and the game ends.

All preferences and institutional rules are common knowledge, and actions are observable. Hence, there is perfect information in the described game. Beginning from the last stage forward, we will solve the game for a subgame-perfect Nash equilibrium that fulfills specific distributional characteristics.

2.6 Evaluation of Districting Plans

We evaluate districting plans based on their impact on the overall welfare of minority voters. Let L_k be the legislator elected from district k, and let $\theta(L_k)$ be her type. Then, the plan that maximizes minority groups' utility maximizes the function:

$$\mathbf{D}^* \in \arg\max_{\mathbf{D}\in\mathcal{D}^K} \sum_{i=1}^{\mathbf{N}_{mD}} X_i^{\theta(L_k)} + E[U_i(b_i)|\mathbf{P}(\mathbf{L}(\mathbf{D}))]. \quad (2.13)$$

[19] Without loss of generality, we could consider uneven levels of turnout and registration, which would change the results by the respective proportions. For a recent analysis of partisan gerrymandering and voter turnout, see Bouton et al. (2023).

The optimal districting plan allocates different types of voters across districts, taking into account the impact of the districting scheme on minority voters' distributive and ideological benefits. For instance, concentrating minority voters into a few districts will increase the probability of electing minority representatives at the potential cost of electing more Republicans elsewhere. This strategy also promises large distributive benefits in the concentrated-minority districts but makes it less likely that these representatives will be included in winning legislative coalitions. Spreading voters out means that minorities can influence outcomes in more districts. Yet it also raises the possibility that they will be marginalized everywhere, electing no minorities to office and gaining only paltry distributive benefits. The question is how voters weigh these considerations under changing ideological distributions, different population proportions of groups, and variations in group power.

3 Platforms and Policy Benefits

Candidates adopt platforms to maximize their votes in primary and general elections, balancing their offers to various groups. In equilibrium, the candidates adopt identical redistributive platforms: $b_{i1k} = b_{i2k}$ and $T_{i1k} = T_{i2k}$ for each group i in a given district k.[20] Consequently, voters cast their ballots for the candidate with whom they have the higher ideological affinity.

Note that, independent of whether primaries are open or closed, the distributive platforms are determined in the general election, when all voter groups participate. For example, if primaries are closed, both Democrat candidates offer identical distributive platforms to minority and non-minority Democrat voters. When the winner faces a Republican, competing for Republican voters, too, the Democrat candidate adjusts promises strategically from the primary to the general and offers the same as the Republican's distributive platform in equilibrium.[21] However, if primaries are closed, then Democrats offer, ignoring turnout differences, identical platforms across primary and general elections as they compete for Republican votes at each stage. In summary, the structure of primaries and a change in platforms across elections do not affect distributive benefits, but will affect later ideological benefits.

[20]See Dixit and Londregan (1996)'s description and existence of an equilibrium in pure strategies of platforms. The existence conditions of Glickberg's Theorem are fulfilled, and the constrained maximization problem is derived using (2.5) and (2.6) for each candidate's objective function and $\sum_i N_{ik} b_{ijk} = B_k$ as a constraint. The Nash equilibrium follows from a simultaneous solution for all first-order conditions and Lagrange parameters. See Appendix A.3 for the formal characterization.

[21]We ignore commitment or credibility problems across election stages and illustrate empirically observed strategic adjustments by candidates across election rounds (di Tella et al., 2023).

Furthermore, the individual benefits and share of the distributive benefits offered to group Θ by candidate j for district k in equilibrium are

$$b_{ijk} = \frac{\pi_i}{\sum_i \pi_i N_{ik}} B_k \text{ and } T_{ijk} = \frac{\pi_i N_{ik}}{\sum_i \pi_i N_{ik}}, \tag{3.1}$$

where

$$\pi_i = [\kappa_i \phi_i(0)]^{1/\epsilon} \tag{3.2}$$

and $\phi(.) = \Phi'(.)$.[22] The distributive benefits and the groups' distributive shares depend on i) the group's influence to swing election outcomes of π_i, ii) the district's distributive benefits of B_k allocated by the legislature's policy $\mathbf{P}$, and iii) the distribution of voters N_{ik} derived from the districting scheme $\mathbf{D}$.

Voter Group's Power As a group's political power increases, the greater the value of π_i, its share of the legislative pie increases. The group's ability to affect electoral outcomes increases as

1. Weight on distributive benefits, κ_i, increases;

   Groups with larger values of κ_i care more about distributive rather than ideological issues, and these groups get a bigger share of the legislative pie.

2. Group's candidate indifference, $\phi_i(0)$, increases;

   A group's power also grows with $\phi_i(0)$, which is the density of their distribution function when voters are indifferent between the two candidates running for office. This term captures a group's "swinginess:" the greater the percentage of members indifferent between the candidates or close to it, the more benefits the group and each member receive. The intuition behind this result is straightforward. First, in equilibrium, the candidates offer the same platform to voters, so this will make no difference in voters' decisions. Since the candidates' promises cancel out, those voters who are indifferent between the parties in equilibrium are those for whom $X_i^e = 0$ in the first place. When deciding whether to transfer funds from one group to another, then, it is these marginal voters who will gain or lose; hence, the candidates pay off the groups in ratios proportional to their $\phi_i(0)$ values, and the group's members enjoy greater distributive benefits.

[22]The solution follows from applying the utility function of (2.1) and its marginal utility (2.2) to the Nash equilibrium's first-order conditions. More details are in Appendix A.3.

3. Returns in consumption, $1/\epsilon$, are greater;

   The parameter ϵ represents the degree of diminishing returns in consumption; as the additional consumption matters less, the group's power declines, and distributive benefits are more even across groups. For example, as $\epsilon \to \infty$, we get $\pi_i \to 1$ and benefits and shares become simple averages: $b_{ijk} \to B_k/N_k$ and $T_{ijk} \to N_{ik}/N_k$.

Clearly, one group's power adversely affects other groups' distributive benefits as there is a fixed legislative pie and the distribution of benefits can be seen as a contest function of (3.1) with a group's total power relative to all groups' powers determining outcomes.

Legislative Policies and Districting An individual's distributive benefits and a group's share of district benefits also depend on the district representative's ability to deliver distributive benefits, B_k, and the district's demographics, N_{ik}. For example, the larger the district-specific transfers, B_k, the greater the individual's gains in consumption from the group's size, which is independent of voters' benefits. On the other hand, the legislator allocates a larger share of district benefits to larger groups, $\partial T_{ijk}/\partial N_{ik} > 0$; though the pie's share, T_{ijk}, is independent of the pie's size, B_k.

4 Distributive and Ideological Benefits

To understand the properties of the equilibrium, we break the analysis into three stages. First, we examine the implications of per-voter distributive benefits b_{ijk}. Ignoring the ideological benefits of electing different types of representatives for the moment, we ask how one would allocate minority voters across districts to maximize their total (or average) distributive returns. We next analyze the ideological utility arising from the different types of elected representatives. We then combine both types of utilities and characterize the optimal districts maximizing minority voters' returns.

4.1 Distributive Benefits and Minority Power

Characterizing districting schemes that provide the most benefits to minorities depends on the behavior of (3.1) on the two-dimensional simplex S^2. We are particularly interested in its behavior on the surface $N_{mDk} + N_{nDk} + N_{Rk} = N_k = \mathbf{N}/K$. We thus rewrite (3.1), minorities' distributive benefits as a group's share in a given district by any candidate,[23] as

$$T_{mDk} = f(N_{mDk}, N_{nDk}) = \frac{\pi_{mD} N_{mDk}}{\pi_{mD} N_{mDk} + \pi_{nD} N_{nDk} + \pi_R N_{Rk}} \tag{4.1}$$

$$= \frac{\pi_{mD} N_{mDk}}{(\pi_{mD} - \pi_R) N_{mDk} + (\pi_{nD} - \pi_R) N_{nDk} + \pi_R N_k} \geq 0. \tag{4.2}$$

Note that the denominator is positive throughout. Thus, we can define $\Pi_k \equiv (\pi_{mD} - \pi_R) N_{mDk} + (\pi_{nD} - \pi_R) N_{nDk} + \pi_R N_k$ – i.e., the aggregate group power of district k. We then write the derivatives of the minorities' benefits with respect to the groups' relative powers as

$$\frac{\partial f}{\partial \pi_{mD}} = \frac{N_{mDk} \left(\pi_R (N_k - N_{mDk} - N_{nDk}) + \pi_{nD} N_{nDk} \right)}{\Pi_k^2} \geq 0; \tag{4.3}$$

$$\frac{\partial f}{\partial \pi_{nD}} = -\frac{\pi_{mD} N_{mDk} N_{nDk}}{\Pi_k^2} \leq 0; \tag{4.4}$$

$$\frac{\partial f}{\partial \pi_R} = -\frac{\pi_{mD} N_{mDk} (N_k - N_{mDk} - N_{nDk})}{\Pi_k^2} \leq 0.^{24} \tag{4.5}$$

Assigning signs to the derivatives with $\pi_i > 0$ and $N_{ik} > 0$ shows that increases in the minority group's power is beneficial while increasing the power of either other group decreases the minority's utility.

4.1.1 Districting Scheme

We now turn to the districting question: how to maximize minority voters' utility by changing the numbers of different types of voters across districts. That is, we seek a valid districting scheme $\tilde{\mathbf{D}}^*$ such that

$$\tilde{\mathbf{D}}^* \in \arg\max_{\mathbf{D} \in \mathcal{D}^K} \sum_{i=1}^{N_{mD}} E \left[U_i(b_i) | \mathbf{P}(\mathbf{L}(\mathbf{D})) \right]. \tag{4.6}$$

The solution to the utility-maximizing districting scheme may not be unique. Hence, let the set of all possible schemes be $\tilde{\mathcal{D}}^*$ and $\tilde{\mathbf{D}}^*$ a representative element. To determine the characteristics of an optimal districting scheme, we first evaluate the derivatives of minority voters' benefits of (4.2) with respect to the populations of voters:

$$\frac{\partial f}{\partial N_{mDk}} = \frac{\pi_{mD} \left(\pi_{nD} N_{mDk} + \pi_R (N_k - N_{mDk}) \right)}{\Pi_k^2} > 0, \tag{4.7}$$

$$\frac{\partial f}{\partial N_{nDk}} = \frac{\pi_{mD} N_{mDk} (\pi_R - \pi_{nD})}{\Pi_k^2} \gtrless 0. \tag{4.8}$$

[23] We can ignore the candidate subscript as candidates in the same district promise the same benefits.

[24] For $N_k = N_{mDk}$, we have $\frac{\partial f}{\partial \pi_{mD}} = \frac{\partial f}{\partial \pi_{nD}} = \frac{\partial f}{\partial \pi_R} = 0$.

As one would expect, the first derivative is always positive; adding more minority voters to a district increases their share of distributive benefits. However, the sign of the first derivative of (4.8) is ambiguous and depends on the other groups' relative power. Minority voters benefit if voters from the more powerful non-minority group are replaced with voters from the less powerful group. Suppose Republicans are politically more powerful than nonminority Democrats, $\pi_R > \pi_{nD}$. In that case, the benefits for minority voters increase as the number of nonminority Democratic voters decreases the number of Republicans in a district, and vice versa. In the districting process, however, changes in voters must be balanced across districts. Hence, minority gains in one district, where less powerful voters increase in number, accompany another district's minority voters' loss as more powerful voters join. We can state

Proposition 1. *If nonminority voter groups' power differs, $\pi_{nD} \neq \pi_R$, and for any two districts with different minority voter concentration and aggregate group power, $N_{mDk} \neq N_{mDl}$ and $\Pi_k \neq \Pi_l$ with $k \neq l$, then any districting scheme that maximizes minority distributive benefits concentrates less powerful nonminority voters into minority-populated, less powerful districts and more powerful nonminority voters into nonminority-populated, more powerful districts.*

All proofs are in Appendix A. Figure 2 and (4.8) illustrate the intuition of the proof. Take any two districts with $N_{mD1} > N_{mD2}$ and $\Pi_1 < \Pi_2$. Focus attention on the interior of the simplex; the goal is to shift voters of the more powerful nonminority group from the minority-populated district to the other district. As illustrated in Figure 2, to accomplish this goal, Republicans will be moved from k_1 to k_2 and nonminority Democrats from k_2 to k_1, where they have greater influence. If District 1 is less powerful than District 2 ($\Pi_1 \leq \Pi_2$), then minority benefits in the district with more minority voters will increase (k_1) by more than the minority benefits fall in the district with fewer minority voters (k_2). Accordingly, average payoffs for minority voters across districts increase, but the district's power across groups decreases. This process continues until one district collides with the simplex's border and can be re-iterated by using the remaining interior district with any other district in the interior that fulfills $N_{mDk} \neq N_{mDl}$ and $\frac{N_{mDk}^2}{\Pi_k^2} > \frac{N_{mDl}^2}{\Pi_l^2}$ (districts k_2 and k_3 in Figure 2). In equilibrium, at least one district will lie at the simplex border. The results imply that optimal districting schemes have a high concentration of minority voters combined with sharing their districts with the less powerful nonminority group. In contrast, more powerful nonminority voters concentrate in districts with few minority voters. Table 2 provides simulations of optimal districts with varying group power, corroborating our results.

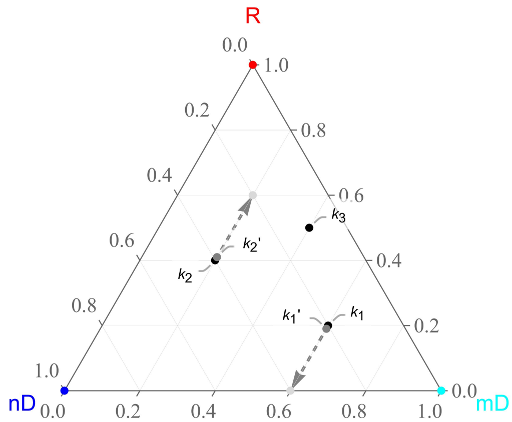


Figure 2: Optimal Districting Process for $\pi_R > \pi_{nD}$ and $\Pi_1 \leq \Pi_2$ with $K = 3$.

4.1.2 Voter Distribution and Minority Distributive Benefits

All that remains to characterize $\tilde{\mathcal{D}}^*$ completely is to determine the optimal distribution of minority voters across districts. The surfeit of boundary conditions makes the usual maximization solution via Lagrange multipliers opaque. Still, we can gain insight into the solution by examining the concavity/convexity of the payoff function with respect to the number of minority voters in the district. We thus calculate the determinants of the principal minors of the Hessian matrix:

$$H = \begin{bmatrix} \frac{\partial^2 f}{\partial N_{mDk}^2} & \frac{\partial^2 f}{\partial N_{mDk} N_{nDk}} \\ \frac{\partial^2 f}{\partial N_{nDk} N_{mDk}} & \frac{\partial^2 f}{\partial N_{nDk}^2} \end{bmatrix}. \tag{4.9}$$

with

$$\frac{\partial^2 f}{\partial N_{mDk}^2} = \frac{2\pi_{mD}(\pi_R - \pi_{mD})\left(\pi_R N_k + (\pi_{nD} - \pi_R)N_{nDk}\right)}{\Pi_k^3}, \tag{4.10}$$

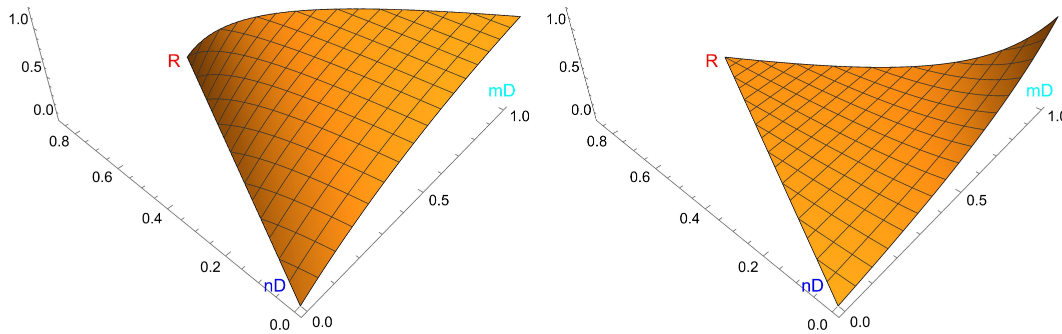
$$\frac{\partial^2 f}{\partial N_{mDk} N_{nDk}} = \frac{\pi_{mD}(\pi_{nD} - \pi_R)\left((\pi_{mD} - \pi_R)N_{mDk} + (\pi_R - \pi_{nD})N_{nDk} - \pi_R N_k\right)}{\Pi_k^3}, \tag{4.11}$$

$$\frac{\partial^2 f}{\partial N_{nDk}^2} = \frac{2\pi_{mD}(\pi_{nD} - \pi_R)^2 N_{mDk}}{\Pi_k^3}, \tag{4.12}$$

and

$$det(H) = \frac{\pi_{mD}^2(\pi_R - \pi_{nD})^2}{\Pi_k^4}. \tag{4.13}$$

The determinant of the entire H matrix is positive for $\pi_{nD} \neq \pi_R$, but the value of $\frac{\partial^2 f}{\partial N_{mDk}^2}$ is indeterminate, indicating that the H matrix can be positive definite, negative definite, or neither, depending on the parameter values. For optimization, the surface could be either concave or



(a) Concave: $\pi_{mD} = 10$, $\pi_{nD} = 5$, $\pi_R = 2$ (b) Convex: $\pi_{mD} = 2$, $\pi_{nD} = 5$, $\pi_R = 10$

Figure 3: Concave and Convex Minority Distributive Benefits.

convex. Figure 3(a) illustrates a concave function for minority distributive benefits when the minority group's power is larger than others. On the other hand, when minority power is lower than the nonminority groups' power, the function is convex, as illustrated in Figure 3(b).[25]

The importance of this difference is clear. If we wish to maximize the overall return to minorities, then in the concave case, we would divide minority voters more evenly across districts than with a convex payoff function. Note that the difference between the curvatures of the two surfaces lies in the relative power of minorities compared to other groups: concave for more powerful minorities and convex for less powerful ones. This forms the basis for the following proposition:

Proposition 2. *If $\pi_{mD} = \max_{i \in \Theta}\{\pi_i\}$, then T_{mDk} is concave on S^2; if $\pi_{mD} = \min_{i \in \Theta}\{\pi_i\}$, then T_{mDk} is convex.*

Since optimal values of N_{mD} on a concave surface will be less dispersed than on a convex surface, we have the result that as minority voters gain power, all else being equal, optimal gerrymanders for distributive benefits divide these voters more equally across districts.[26] Formally, let

$$R(\mathbf{D}) = \max_{d_k, d_l \in \mathbf{D}^*} \left(N_{mDk} - N_{mDl}\right), \tag{4.14}$$

be the range, the maximum difference between the minority population, of any two districts in an optimal districting scheme. Then $\frac{\partial R(\mathbf{D})}{\partial \pi_{mD}} \leq 0$, so that minority voters are (weakly) spread out less

[25] We provide additional examples of nonconcave and nonconvex payoffs in Figure 7 of Appendix A.5 that arise when minority group power lies between both nonminority groups' power.

[26] In fact, optimal districts when T_{mDk} is convex concentrate all minority voters into as few districts as possible. Conversely, when T_{mDk} is concave, $N_{mD} > 0$ for all districts.

Group Power			*District 1*			*District 2*			*District 3*			**Benefits**		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	75%	0%	25%	0%	64%	36%	0%	56%	44%	0.750	0.250	75%
2	3	1	44%	0%	56%	0%	100%	0%	31%	20%	49%	0.974	0.325	44%
3	3	1	39%	0%	61%	36%	20%	44%	0%	100%	0%	1.167	0.389	39%
4	3	1	0%	100%	0%	37%	20%	43%	38%	0%	62%	1.300	0.433	38%
5	3	1	30%	35%	35%	30%	0%	70%	15%	85%	0%	1.426	0.475	15%
1	3	3	0%	54%	46%	0%	54%	46%	75%	13%	12%	0.500	0.167	75%
2	3	3	0%	53%	47%	0%	51%	49%	75%	15%	10%	0.667	0.222	75%
3	3	3	25%	40%	35%	25%	40%	35%	25%	40%	35%	0.750	0.250	0%
4	3	3	25%	49%	26%	25%	33%	42%	25%	38%	37%	0.923	0.308	0%
5	3	3	25%	39%	36%	25%	40%	35%	25%	41%	34%	1.071	0.357	0%
1	3	5	0%	93%	7%	0%	2%	98%	75%	25%	0%	0.500	0.167	75%
2	3	5	75%	25%	0%	0%	3%	97%	0%	92%	8%	0.667	0.222	75%
3	3	5	0%	93%	7%	0%	2%	98%	75%	25%	0%	0.750	0.250	75%
4	3	5	40%	60%	0%	35%	60%	5%	0%	0%	100%	0.876	0.292	40%
5	3	5	37%	58%	5%	38%	62%	0%	0%	0%	100%	0.987	0.329	38%

Table 2: Districting Plans Maximizing Minority Distributive Benefits: $N_{mD} = 25\%$, $N_{nD} = 40\%$, and $N_R = 35\%$.

as their power increases. Combining these results with Proposition 1, we can say that optimal districting schemes will concentrate minority voters in a few districts when their power is low, spread them out when their power is high, and combine them as much as possible with the less powerful of the other two groups.

These results are illustrated in Table 2, which details optimal districts for varying levels of groups' power, done for a state with three districts in which the population proportions of minority-Democrat, nonminority-Democrat, and Republican voters are 25%, 40%, and 35%, respectively. The power of nonminority Democrat voters in the simulations is fixed at $\pi_{nD} = 3$, while the other two groups' power varies between 1 and 5. Note that, as predicted, the range $R(\mathbf{D})$ declines and minorities' utility rises within each set of observations as π_{mD} increases. Where possible, minority voters are designated into districts with more voters from the less powerful of the other groups, concentrating other powerful voter groups (see, for example, the last row). Furthermore, when minority voters are the least powerful group, they are highly concentrated (rows highlighted in blue). In contrast, when they are the most powerful group, and the other groups are uniformly less powerful, minority voters are equally represented in all districts (rows highlighted in orange).

These four patterns concerning benefits, range, power of nonminority voters within minority concentrated districts, and equal spread of minority voters if they are more powerful than equally powerful nonminority voters are consistent for i) variations in group powers, as illustrated in Table 9 in Appendix B.1, ii) variations in state demographics, as illustrated in Table 10 in Appendix B.2, and iii) variations in the number of districts, as illustrated in Table 11 in Appendix B.3.

4.2 Ideological Benefits

We now turn to the ideological benefit that minority voters gain from their representatives. We first examine the likelihood that minority candidates are elected and then discuss the expected ideological utilities for minority voters.

4.2.1 Likelihood of Successful Minority Candidates

In the first step, we evaluate the likelihood that minority candidates will be elected, analyzing the first and second derivatives of Ψ_{mD} for closed primaries and $\hat{\Psi}_{mD}$ for open primaries from (2.12). We can state

Proposition 3. *The probability of electing a minority candidate:*

1. *increases with the number of minority-Democratic voters;*
2. *is ambiguous in the number of nonminority-Democrat voters in a district (flexible N_k);*
3. *is ambiguous in the number of nonminority-Democrat voters replacing Republican voters in a district with a closed primary (fixed N_k); and*
4. *increases with the number of nonminority-Democrat voters replacing Republican voters in a district with an open primary if $a^1_{nD} > a^1_R$ (fixed N_k).*

Finally, $\hat{\Psi}_{mD}$ is convex on S^2 and Ψ_{mD} is convex on S^2 if $a^2_R < (a^2_{mD} - a^2_{nD})N_{nDk}/N_k$.

The results of adding minority voters to a district are not surprising; they can only increase the probability that a minority candidate wins both the primary and general elections, independent of whether a primary election is closed or open. Neither are the results of adding nonminority-Democrat voters mysterious; these voters may support minority candidates in the general election but favor nonminority-Democrat candidates in the primary, and it is only when the former effect dominates the latter that the overall chances of electing a minority candidate to office rise. When we consider a given district and replace Republican voters with nonminority-Democrat voters, adding nonminority-Democrat voters has adverse effects for minority candidates in a closed primary but positive effects in the general elections, as they are more likely than Republican voters to vote for the minority-Democrat candidate. However, in open primaries, the effects are strictly positive as the number of Republican voters decreases in both the primary and general elections. Straightforward as this assertion may be, its logical counterpart (really just a restatement under different terms) may still surprise some observers: one may be able to increase the probability of electing a minority-Democrat from a given district by increasing the number of Republican voters, especially in states with closed primaries or growing populations.

The fact that $\hat{\Psi}_{mD}(.)$ for open primaries is convex or $\Psi_{mD}(.)$ for closed primaries is convex at low levels of Republican crossover (a_R^2) implies that under these conditions, districting schemes that maximize the number of minority-Democrats elected will concentrate minority voters in as few districts as possible. This accords with empirical findings on the subject (see, for instance, Cameron et al. (1996)), although it has never been shown in a general theoretical context before. Two interesting points emerge from the analysis here: first, the relation between electing minority-Democrats and concentrating minority voters depends on low crossover rates for closed primaries; when a_R^2 is higher, optimal schemes for descriptive representation spread minority voters more evenly across districts. Second, the convexity of $\Psi_{mD}(.)$ derives from the two-step primary-general election process. Adding minority voters to a district increases the chances a minority candidate wins both the primary and general elections. And since $\hat{\Psi}_{mD}(.)$ or $\Psi_{mD}(.)$ is the product of these two probabilities, adding minority voters at the margin has a quadratic impact on the overall chances of electing minority candidates to office.[27]

4.2.2 Expected Minority Ideological Benefits

With these expected electoral outcomes, we can examine the ideological benefits that minority voters anticipate from candidates joining the legislature as their representatives. We can define the average utility per voter of a given type i for a j type representative:

$$\bar{X}_i^j = \int_{-\infty}^{\infty} X_i^j d[\Phi(X_i)]. \tag{4.15}$$

Then the total utility to voters electing a type j representative is $N_{ij}\bar{X}_i^j$. For convenience, recalibrate utilities so that $\bar{X}_{mD}^{mD} = 1$ and $\bar{X}_{mD}^{R} = 0$, and define $\beta \equiv \bar{X}_{mD}^{nD}$, with $0 \leq \beta \leq 1$. Overall expected utility for minority voters includes both the type elected and their average attachment to representatives of that type:

$$\begin{aligned} E[X] &= \Psi_{mD}\bar{X}_{mD}^{mD} + \Psi_{nD}\bar{X}_{mD}^{nD} + \Psi_R\bar{X}_{mD}^{R} \\ &= \Psi_{mD} + \Psi_{nD}\beta - \text{closed primaries} \end{aligned}$$

[27]In fact, looking at the primary and general elections independently, we see that the election function is concave in N_{mDk} for the primary and linear in N_{mDk} for the general, making the overall convexity all the more interesting.

$$= \hat{\Psi}_{mD} + \hat{\Psi}_{nD}\beta - \text{open primaries}. \tag{4.16}$$

It is natural to ask whether the districting schemes that maximize minority voters' overall expected ideological utility are the same as those that elect minority representatives.

Proposition 4. *There exists a $\tilde{\beta} > 0$ such that for $\beta < \tilde{\beta}$, $E(X)$ is convex on S^2.*

When the extra utility of electing a minority-Democrat is high enough (β is close to 0), the $E(X)$ function is convex, independent of whether primaries are closed or open. Districting schemes that maximize overall utility coincide with those that elect as many minority-Democrats as possible to office. Conversely, when it is more important to avoid electing Republicans (β is close to 1), the function becomes concave, and optimal schemes spread minority voters more across districts. As partisan concerns rise, then, minority voters prefer to work more through electoral coalitions, joining with nonminority-Democratic voters to minimize the number of Republicans elected to office.[28,29]

5 Optimal Districts

We identify districting schemes that maximize minority voters' utility by combining the distributive and ideological benefits derived above. On the one hand, these benefits are additive; seemingly, the task is to add up the above-mentioned effects. On the other hand, this rosy scenario is complicated by the two effects being inextricably linked: groups receive greater distributive benefits with increasing "swinginess," their density at $\phi_i(0)$ rises, but this quantity also indicates the amount of crossover voting by that group.

This observation cuts two ways. First, as nonminority voters are increasingly willing to cross over and vote for minority candidates, the chances of electing minorities to office rise, which raises the average ideological utility of minority voters. However, this greater willingness to crossover means that nonminority voters are now more swingy and decisive, so they will receive larger shares of distributive benefits B_k in equilibrium. From minorities' point of view, then, the price for greater electoral support from other groups is a loss of distributive benefits.

[28] As a special case for elections with closed primaries, we note that the $E(X)$ function is also convex when minority voters are more likely to support a minority candidate in the general election ($a^2_{mD} > a^2_{nD}$ by assumption), Republican voters more likely support a nonminority-Democrat than a minority candidate ($a^3_R\beta > a^2_R$), with Democrats similarly voting against a Republican candidate ($a^3_{nD} \geq a^3_{mD}$).

[29] As a special case for elections with open primaries, we note that the $E(X)$ function is also convex when minority voters are more likely to support a minority candidate in the general election ($a^2_{mD} > a^2_R$ by assumption) and Republican voters are more likely to support a nonminority-Democrat candidate than minority voters ($a^3_{mD} \leq a^3_R$).

Second, the more politically cohesive the minority-Democrats are, the more they vote only for minority-Democrats running for office, the less influential they are compared to other groups, and thus, the less distributive benefits they receive. In this sense, the model captures the notion that the most loyal democratic supporters are also the most easily "taken for granted" by their elected representatives. Thus, decreased racial or ethnic polarization in voting patterns is a mixed blessing for minorities, involving as it does a tradeoff between ideological and distributive benefits.

How do these considerations affect the nature of optimal districting schemes as minorities gain power? We know that the distributive payoff function T_{ijk} becomes concave as π_{mD} rises; how does this interact with ideological utility, given that $E(X)$ is convex under certain circumstances? We can state

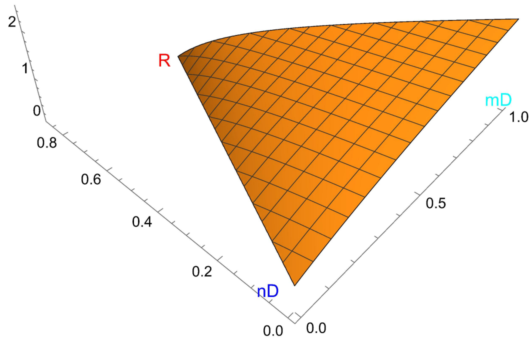
Proposition 5. *Districting schemes that maximize minorities' utility concentrate minority voters less as their power increases.*

If minority voters are motivated more by distributional than ideological benefits (κ_i is increasing π_i) and their voting rates are decreasing in each election round, making them more influential ($\phi_i(0)$ is increasing π_i), the concavity of minority distributive benefits will eventually outweigh any convexity in minority ideological benefits. Hence, more powerful minorities are sufficiently motivated by distributive benefits, resulting in a greater spread of minority voters across districts and a greater realization of distributive benefits. Less powerful minority voters are more concentrated and gain ideological benefits in those districts, but on average, they receive fewer distributive benefits. Overall, then, if minority voters prioritize tangible benefits over political beliefs, the spread of benefits among them will outweigh the concentration of benefits. Powerful minorities, therefore, benefit more from a spread of minority voters, while less powerful ones gain more from concentration. Minority voters become more influential as their voting rates decrease.

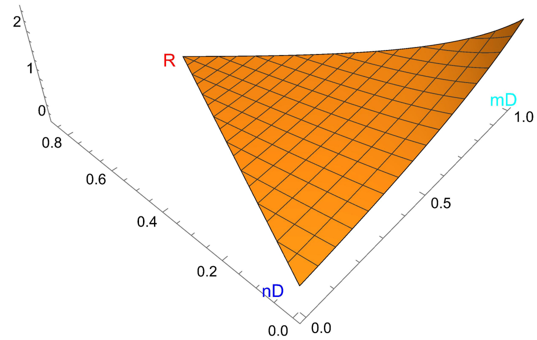
Figure 4 is the overall utility for minority voters, combining the concave (convex) distributive benefits from (4.2) and the expected (convex) minority ideological gains from (4.16). Thus, we have Figures 4 with concave (convex) total benefits that illustrate the expected benefits from

$$\begin{aligned} EU_{mD} &= \frac{\pi_{mD} N_{mDk}}{\sum_i \pi_i N_{ik}} + \Psi_{mD} + \Psi_{nD}\beta - \text{closed primary}, \\ &= \frac{\pi_{mD} N_{mDk}}{\sum_i \pi_i N_{ik}} + \hat{\Psi}_{mD} + \hat{\Psi}_{nD}\beta - \text{open primary}. \end{aligned} \tag{5.1}$$

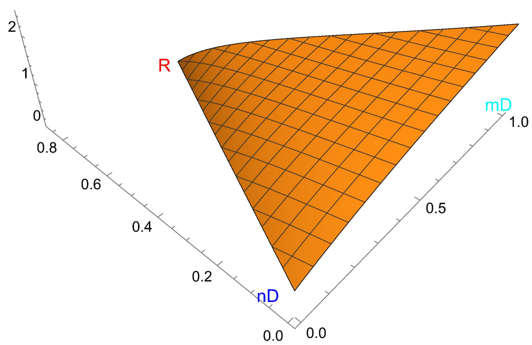
Our illustrations follow similar patterns as in Figure 3 but are bounded between 0 and 2,



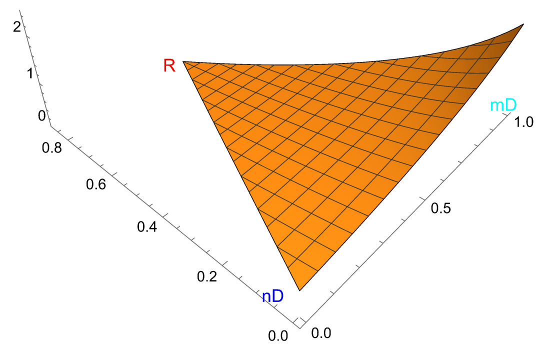
(a) $\pi_{mD} = 10$, $\pi_{nD} = 5$, $\pi_R = 2$, closed primary



(b) $\pi_{mD} = 2$, $\pi_{nD} = 5$, $\pi_R = 10$, closed primary



(c) $\pi_{mD} = 10$, $\pi_{nD} = 5$, $\pi_R = 2$, open primary



(d) $\pi_{mD} = 2$, $\pi_{nD} = 5$, $\pi_R = 10$, open primary

Figure 4: Minority Total Benefits – $a^1_{nD} = .3$, $a^2_{nD} = .7$, $\beta = .5$.

compared to 0 and 1 for distributive shares/benefits. The graphs illustrate smooth payoff functions as both distributive and ideological benefits are determined by probabilistic voting and resulting contest functions for voter groups. Though the structure of primaries matters for the numerical values of ideological benefits, we see little visual difference comparing the figures vertically for closed and open primaries.

To illustrate these tradeoffs identified in Proposition 5, we calculate optimal districting schemes for the same values of π_{mD}, π_{nD}, and π_R power as in Table 2, using the same overall population proportions. The extra utility of electing a nonminority Democrat is assumed to be $\beta = 0.5$ and 1 for electing a minority candidate relative to a baseline of 0 for a Republican. The nonminority-Democrat primary crossover rate is 30%, while the general election crossover rates are 70%.[30] The ranges are generally higher in Tables 3 and 4 compared to Table 2, resulting from the increased desire to concentrate minorities to increase the likelihood a minority candidate can be elected in

[30]Formally, we assume $a^e_{mD} = 1$, $a^e_R = 0$, and $a^e_{nD} = \{0.3, 0.7, 1\}$.

Group Power			*District 1*			*District 2*			*District 3*			**Benefits**		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	75%	0%	25%	0%	70%	30%	0%	50%	50%	2.172	0.724	75%
2	3	1	46%	0%	54%	0%	100%	0%	29%	20%	51%	2.370	0.790	46%
3	3	1	35%	20%	45%	0%	100%	0%	40%	0%	60%	2.562	0.854	40%
4	3	1	37%	20%	43%	0%	100%	0%	38%	0%	62%	2.695	0.898	38%
5	3	1	35%	0%	65%	6%	94%	0%	34%	26%	40%	2.795	0.932	28%
6	3	1	29%	36%	35%	16%	84%	0%	30%	0%	70%	2.905	0.968	14%
7	3	1	28%	39%	33%	19%	81%	0%	28%	0%	72%	3.007	1.002	8%
8	3	1	27%	41%	32%	21%	79%	0%	27%	0%	73%	3.097	1.032	6%
9	3	1	27%	43%	31%	23%	77%	0%	26%	0%	74%	3.177	1.059	4%
10	3	1	26%	44%	30%	24%	76%	0%	25%	0%	75%	3.248	1.083	3%
1	3	3	0%	100%	0%	0%	20%	80%	75%	0%	25%	1.922	0.641	75%
2	3	3	0%	98%	2%	0%	22%	78%	75%	0%	25%	2.089	0.696	75%
3	3	3	0%	100%	0%	0%	20%	80%	75%	0%	25%	2.172	0.724	75%
4	3	3	40%	0%	60%	0%	100%	0%	35%	20%	45%	2.284	0.761	40%
5	3	3	19%	81%	0%	25%	39%	36%	31%	0%	69%	2.424	0.808	11%
1	3	5	0%	61%	39%	0%	34%	66%	75%	25%	0%	1.883	0.628	75%
2	3	5	75%	25%	0%	0%	32%	68%	0%	63%	37%	2.049	0.683	75%
3	3	5	0%	90%	10%	0%	5%	95%	75%	25%	0%	2.133	0.711	75%
4	3	5	0%	0%	100%	32%	63%	5%	43%	57%	0%	2.202	0.734	43%
5	3	5	38%	62%	0%	37%	58%	5%	0%	0%	100%	2.312	0.771	38%

Table 3: Districting Plans Maximizing Minority Total Benefits: $N_{mD} = 25\%$, $N_{nD} = 40\%$, $N_R = 35\%$, $a^1_{nD} = .3$, $a^2_{nD} = .7$, $\beta = .5$, closed primaries.

some districts. Note also that the rule stating that $R(D)$ weakly decreases within each subgroup of five simulations still holds. It is also still the case that when minorities are the least powerful group, at least one district has no minority voters, and when minorities are the most powerful, $N_{mDk} > 0$ for all districts k.

Comparing the simulation results for a state with either closed or open primaries, we see similar patterns of increasing benefits, decreasing concentration, and sorting according to differences in group power. Yet there are numerical differences for the concentration of minority voters, $R(D)$. Tables 3 and 4 address whether a state with closed or open primaries would yield a greater concentration of minority voters. The results illustrate inconsistent level effects between the two types of primary structures. We will explore this discrepancy further in the next part when we focus on the light-gray highlighted rows in Table 4.

The four patterns concerning benefits, range, power of nonminority voters within minority concentrated districts, and equal spread of minority voters if they are more powerful than equally powerful nonminority voters are generally consistent for i) variations in group powers, as illustrated in Table 12 and 13 in Appendix B.4, ii) variations in state demographics, as illustrated in Table 14 and 15 in Appendix B.5, iii) variations in minority ideological benefits, as illustrated in Tables 16 and 17 with closed primaries and Tables 18 and 19 with open primaries in Appendix B.6, and iv)

Group Power			*District 1*			*District 2*			*District 3*			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	0%	100%	0%	0%	20%	80%	75%	0%	25%	2.069	0.69	75%
2	3	1	59%	0%	41%	0%	100%	0%	16%	20%	64%	2.183	0.73	59%
3	3	1	38%	0%	62%	0%	100%	0%	37%	20%	43%	2.370	0.79	38%
4	3	1	35%	0%	65%	0%	100%	0%	40%	20%	40%	2.505	0.83	40%
5	3	1	30%	38%	32%	18%	82%	0%	27%	0%	73%	2.637	0.88	11%
6	3	1	28%	43%	29%	23%	77%	0%	24%	0%	76%	2.764	0.92	5%
7	3	1	27%	45%	28%	25%	75%	0%	23%	0%	77%	2.874	0.96	5%
8	3	1	27%	46%	27%	26%	74%	0%	22%	0%	78%	2.969	0.99	6%
9	3	1	27%	47%	26%	27%	73%	0%	21%	0%	79%	3.052	1.02	6%
10	3	1	27%	48%	25%	28%	72%	0%	20%	0%	80%	3.125	1.04	7%
1	3	3	0%	95%	5%	0%	0%	100%	75%	25%	0%	1.880	0.63	75%
2	3	3	0%	95%	5%	0%	0%	100%	75%	25%	0%	2.046	0.68	75%
3	3	3	0%	95%	5%	0%	0%	100%	75%	25%	0%	2.130	0.71	75%
4	3	3	35%	60%	5%	0%	0%	100%	40%	60%	0%	2.205	0.74	40%
5	3	3	38%	62%	0%	36%	58%	7%	2%	0%	98%	2.317	0.77	36%
1	3	5	0%	0%	100%	0%	95%	5%	75%	25%	0%	1.880	0.63	75%
2	3	5	0%	95%	5%	0%	0%	100%	75%	25%	0%	2.046	0.68	75%
3	3	5	0%	95%	5%	0%	0%	100%	75%	25%	0%	2.130	0.71	75%
4	3	5	27%	68%	5%	0%	0%	100%	48%	52%	0%	2.194	0.73	48%
5	3	5	40%	60%	0%	0%	0%	100%	35%	60%	5%	2.304	0.77	40%

Table 4: Districting Plans Maximizing Minority Total Benefits: $N_{mD} = 25\%$, $N_{nD} = 40\%$, $N_R = 35\%$, $a^1_{nD} = .3$, $a^2_{nD} = .7$, $\beta = .5$, open primaries.

variations in primary and general crossover rates, as illustrated in Tables 20 and 21 with closed primaries Tables 22 and 23 with open primares in Appendix B.7.

5.1 Minority Power and U-Shaped Concentration

Our simulation results in Table 4 indicate another potential paradox. As minority power increases, the concentration of minority voters decreases as predicted by Proposition 5. Still, we can see a non-monotonic relationship between group power π_{mD} and $R(D)$, as highlighted by the light gray rows, where concentration increases for greater values. When we explore this further, increasing the values for minority power while holding other groups' power constant, we can illustrate the u-shaped relationship for states with closed and open primaries in Table 7. Mathematically, the result arises from the concavity of distributive benefits, the convexity of ideological benefits, and the bounded payoff between 0 and 2. The u-shaped relationship is also apparent in the case of open primaries due to the stronger convexity of ideological benefits (Propositions 3 and 4). Graphically, it depends on the shape of the surface S^2 as illustrated in Figures 8 and 9, where most of the surface, illustrating possible districts, is flat for greater values of π_{mD} and the concavity is mostly along the nD-R diagonal with $N_{mDk}/N_k \to 0$.

Intuitively, and focusing on the interaction of $\pi_{mD} * N_{mDk}$ in (5.1), when minority power is

Group Power – **Minority Power**					
π_{mD}	π_{nD}	π_R	Total	Average	$R(\mathbf{D})$
1	3	1	2.172	0.72	75%
2	3	1	2.370	0.79	46%
3	3	1	2.562	0.85	40%
4	3	1	2.695	0.90	38%
5	3	1	2.795	0.93	28%
6	3	1	2.905	0.97	14%
7	3	1	3.007	1.00	8%
8	3	1	3.097	1.03	6%
9	3	1	3.177	1.06	4%
10	3	1	3.248	1.08	3%
15	3	1	3.503	1.17	3%
20	3	1	3.662	1.22	4%
50	3	1	4.025	1.34	5%
100	3	1	4.176	1.39	3%
150	3	1	4.231	1.41	0%
200	3	1	4.259	1.42	3%
250	3	1	4.277	1.43	7%
300	3	1	4.290	1.43	11%
400	3	1	4.307	1.44	18%
500	3	1	4.318	1.44	23%

Table 5: Closed Primaries.

Group Power – **Minority Power**					
π_{mD}	π_{nD}	π_R	Total	Average	$R(\mathbf{D})$
1	3	1	2.069	0.69	75%
2	3	1	2.183	0.73	59%
3	3	1	2.370	0.79	38%
4	3	1	2.505	0.83	40%
5	3	1	2.637	0.88	11%
6	3	1	2.764	0.92	5%
7	3	1	2.874	0.96	5%
8	3	1	2.969	0.99	6%
9	3	1	3.052	1.02	6%
10	3	1	3.125	1.04	7%
15	3	1	3.388	1.13	10%
20	3	1	3.550	1.18	12%
50	3	1	3.921	1.31	17%
100	3	1	4.078	1.36	22%
150	3	1	4.137	1.38	26%
200	3	1	4.169	1.39	30%
250	3	1	4.189	1.40	34%
300	3	1	4.204	1.40	37%
400	3	1	4.225	1.41	43%
500	3	1	4.240	1.41	47%

Table 6: Open Primaries.

Table 7: Minority Voter Concentration and Benefits – $N_{mD} = 25\%$, $N_{nD} = 40\%$, and $N_R = 35\%$.

low, a map designer concentrates minority voters in a few districts as they do not receive much distributive benefits in electoral competition but can influence the election of the candidate's type. Hence, for low minority power, the optimization focuses on descriptive representation. As minority power increases, and candidates compete for their votes, distributive benefits rise. It is then optimal to spread minority voters across districts to maximize total benefits across districts. However, at some point, further increases in π_{mD} have a small marginal impact on T_{mDk} for each district due to the arising concavity from minority power. These diminishing marginal returns allow for the reallocation of N_{mDk} across districts – keeping distributive benefits across k districts relatively equal but increasing ideological benefits in at least one district.

Group Power and Crossover Note that our simulations above increased π_{mD} while assuming $a^e_{mD} = 1$, $a^e_R = 0$, and $a^e_{nD} = \{0.3, 0.7, 1\}$. Obviously, electoral crossover, a^e_i, and group power, π^e_i, are not independent as a voter group's swinginess is defined by its crossover along partisanship and identity. However, the functional form mapping these two variables is an empirical question. To illustrate the robustness of the observation above, we impose a negative relationship between π_i and a_i. As we increase the value of minority group power, we let the crossover value converge to one-half –reflecting the order of π_{nD}, π_R, a^e_{nD}, and a^e_R but varying the mapping between π_{mD} and a^e_{mD}. Our simulations in Tables 8 show again that as minority power increases– and crossover

Group Power			Crossover							Closed Primaries		Open Primaries	
π_{mD}	π_{nD}	π_R	a^1_{mD}	a^2_{mD}	a^3_{mD}	a^1_{nD}	a^2_{nD}	a^3_{nD}	a^e_R	Total	$R(\mathbf{D})$	Total	$R(\mathbf{D})$
1	3	1	1	1	1	0.3	0.7	0.7	0	2.046	75%	1.980	75%
2	3	1	1	0.8	0.8	0.3	0.7	0.7	0	2.115	45%	2.006	46%
3	3	1	1	0.7	0.7	0.3	0.7	0.7	0	2.238	39%	2.141	38%
4	3	1	1	0.6	0.6	0.3	0.7	0.7	0	2.301	34%	2.223	22%
5	3	1	1	0.5	0.5	0.3	0.7	0.7	0	2.368	12%	2.312	8%
1	3	1	1	1	1	0.3	0.7	0.7	0	2.046	75%	1.980	75%
2	3	1	1	0.8	0.8	0.3	0.7	0.7	0	2.115	45%	2.006	46%
3	3	1	1	0.7	0.7	0.3	0.7	0.7	0	2.238	39%	2.141	38%
10	3	1	1	0.6	0.6	0.3	0.7	0.7	0	2.908	3%	2.854	6%
20	3	1	1	0.5	0.5	0.3	0.7	0.7	0	3.264	7%	3.222	10%
1	3	1	1	1	1	0.3	0.7	0.7	0	2.046	75%	1.980	75%
2	3	1	1	0.8	0.8	0.3	0.7	0.7	0	2.115	45%	2.006	46%
3	3	1	1	0.7	0.7	0.3	0.7	0.7	0	2.238	39%	2.141	38%
25	3	1	1	0.6	0.6	0.3	0.7	0.7	0	3.435	8%	3.387	11%
50	3	1	1	0.5	0.5	0.3	0.7	0.7	0	3.629	10%	3.589	14%

Table 8: Districting Plans Maximizing Minority Total Benefits: Minority Power and Crossover.

converges to 0.5– the concentration of minority voters is u-shaped.

We recognize that this result may be primarily theoretical, as the values of π_{mD} relative to the other groups' values need to be tremendous. Nevertheless, they illustrate the trade-offs in redistricting when we consider the structure of the electoral process, voter groups' influence on elections and platforms, and the resulting representation of voter groups.

6 Discussion and Conclusion

This paper offers a comprehensive approach to redistricting and representation. We examine how voters' ideological and policy preferences align with a candidate's identity and partisan affiliation to determine minorities' political power (swinginess), and the policy benefits they receive (distributive gains) in a majoritarian political process. We observe optimal redistricting as a nuanced tradeoff between ideological and distributive benefits. First, grouping less influential minority voters with less influential nonminority voters is more effective in promoting minority interests than grouping influential minority voters with influential nonminority voters, regardless of their political party. Second, concentrated minority districts are effective when minority voters are less swingy and prioritize ideological benefits, thereby emphasizing coalition formation at the electoral stage. In contrast, spreading minority voters across multiple districts is more effective when they are swingy and focused on distributive benefits, thereby shifting coalition building to the legislature. Third, optimal districting depends on the likelihood of electing a Democrat or a Republican candidate in the primary and general elections.

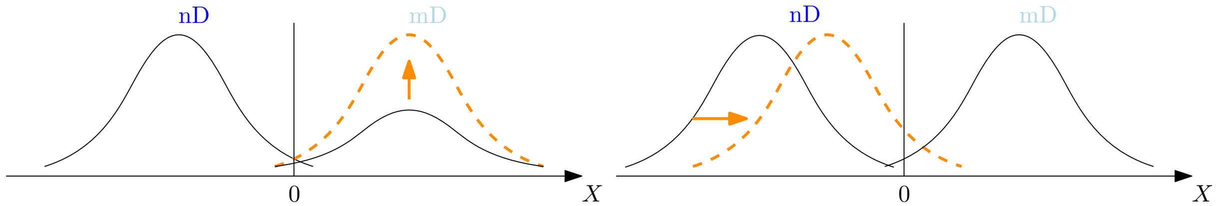
To conclude our analysis, we apply the framework developed in the previous sections to examine the impact of various changes in the political landscape, including increased Black voter registration, the defection of white Democrats to the Republican Party, and decreased racism on minority electoral success, policy benefits, and optimal redistricting plans.

Increasing Minority Registration and Voting Turnout. Before the passage of the 1965 Voting Rights Act (VRA), many Southern states enacted laws to de facto disenfranchise Blacks. Such devices as the grandfather clause, poll taxes, and white-only primaries, not to mention direct intimidation, minimized Blacks' participation in politics. When one form of discrimination was outlawed, the states would switch to another. This macabre game of wack-a-mole continued until the VRA swept away all such "tests and devices," and its Section 5 preclearance provisions required covered states, those with historical patterns of discrimination, to obtain the permission of the federal government before adopting any new law that might impact minorities' ability to vote. The most direct result of passing the VRA was thus to greatly increase Blacks' participation to the point where now, in most areas of the South, minorities register and vote at rates at or above those of white voters.[31]

From the model above, the impact of an increase in statewide minority-Democrats is (usually) unambiguous: it acts just like an increase in their share of legislative benefits, $T_{mDk} = \pi_{mD} N_{mDk} / \Pi_k$, and so both increase the flow of benefits to minority constituents and make it easier to elect minorities to office, thereby increasing their ideological benefits. The shift in legislative benefits is also illustrated in Figure 5(a), where the horizontal axis shows the ideological distributions of the minority and nonminority Democrat voters, with the 0, or indifference, point in the middle. The increase in the size of the minority electorate increases their power π_{mD} by raising $\phi_{mD}(0)$ while also increasing the number of districts that could elect a minority candidate. Cascio and Washington (2014) document how the reforms following the VRA increased voter turnout and public spending for counties with a higher Black population.

Furthermore, according to the model, as the number of Black registered voters increases from low numbers, the first response of state district drawers should be to create concentrated minority districts. Indeed, this happened in the 1970s and 1980s, with one rule of thumb stating that districts had to be at least 65% Black to be "effectively" majority-minority. As minority participation continues to increase, the response should be to concentrate minority voters less, spreading them

[31] By 2020, Black registration and electoral turnout in Southern states only differed by 5 percentage points.



(a) Minority Voter Registration. (b) nonminority-Democratic Crossover.

Figure 5: Relative Power of nonminority and minority-Democratic Voters.

out more evenly across districts. Some worry that reducing the majority in these districts will dilute their influence over policy and reduce the number of minorities in office, thereby giving back some of the hard-won gains of the civil rights movement. Others view it as a natural progression of minorities into mainstream politics and a means to expand their political influence.

Economic Inequality. There is robust empirical evidence that minority and nonminority voter groups differ in income and wealth levels in the United States, with some minority groups exhibiting lower economic means (Black, Hispanic, and Native American households) and other minority groups with greater economic means (Asian households). As pointed out by Dixit and Londregan (1996), voters' degree of diminishing returns to consumption affects their sensitivity in trading off distributive for ideological benefits. If one would focus on diminishing returns to consumption, all else being equal, less affluent voters are easier to sway by candidates offering distributive benefits, making their support less costly but also more competitive. In our analysis, this would imply that one would spread minority voters with less economic means across more districts to benefit them, while concentrating those minority voters who are more affluent and less responsive to distributive benefits. However, one must also consider the degree of diminishing returns, as captured by the parameter ϵ. Even minority groups with low economic means may be less sensitive to distributive benefits in their vote choice, large ϵ, and would prefer to be concentrated in few districts; while minority voter groups, even affluent ones, with high sensitivity, low ϵ, would prefer to a representation across many districts. These theoretical nuances of our model highlight the need for more empirical investigations and inform the legal challenges around descriptive and substantive representation.

Hispanic and Latino Vote. Recent trends in the Hispanic/Latino vote in the United States are complex and multifaceted. Historically, Hispanics have aligned with the Democratic party and tend to be concentrated in minority districts largely overlooked by the Democratic party.

However, as Hispanics increasingly become swing voters, shifting between the Republican and Democratic parties, their ability to gain distributive gains will inevitably increase.

One factor that appears to be driving this shift is the changing nature of working-class labor trends, with some Hispanic/Latino voters moving towards the Republican Party in response to economic concerns and a desire for greater job security. Evidence suggests that conservative social issues, such as abortion and same-sex marriage, also play a role in this shift. Another factor is the diversity within the Hispanic/Latino community, which includes individuals with a wide range of cultural, linguistic, and socioeconomic backgrounds. Consequently, there is significant variation in voting patterns based on individual and community-level preferences.

Increasing Crossover. Finally, we come to the increased willingness of nonminority voters of all stripes to vote for minority candidates due to steadily decreasing racism. Decreasing racism does help minorities win office, and indeed, the number of elected minorities in the South has skyrocketed since adopting the VRA.[32] Nevertheless, as mentioned above and illustrated in Figure 5(b), the impact on distributive benefits is complex. For nonminority voters to be less racist, they must be less ideologically averse to minorities' holding office, represented by a right-hand shift in the distribution of X-values as in Figure 5(b). This shift increases the density of nonminority Democrat voters at $X = 0$; as minorities become more influential in elections, they will enjoy greater legislative benefits. These gains continue until the central hump of the distribution passes the 0 threshold, beyond which decreased racism (increased crossover) also leads to a smaller share of the legislative pie.

Since less than 50% of nonminority voters reliably support minority candidates, though, we may assume that we still reside on the upward slope of the distribution function. Hence, nonminority voters may be gaining greater benefits from candidates' platforms at the expense of minority voters. Of course, the tradeoff regarding increased descriptive representation may be worthwhile. However, it is still interesting that decreased racism is not an unalloyed good for minorities. There have long been rumblings that Democrats in office, white and Black alike, take their minority constituents for granted and give them less than their fair share of benefits. While this may be true, and if so, the model here provides a plausible rationale for why it would happen.

[32]See the essays Davidson and Grofman (1994) for detailed state-by-state analyses attributing the rise in Black office holding directly to the VRA. Indeed, 2020 voter data shows that whites register at a 75% rate, Blacks 68%, Asians 64%, and Hispanics 59%—-similarly, Whites turnout at a 69% rate, Blacks 61%, Asians 59%, and Hispanics 52%.

Future Research We end by pointing to several possible extensions to our model. First, our current legislative model is simple to focus on the logic of changing preferences and group powers without building an incentive to form party-based coalitions in the legislature. Hence, there are no real legislative parties or permanent coalitions. Nevertheless, if one wanted to investigate the impact of changing legislative rules, committee powers, or party leadership on districting, these elements could be incorporated into the legislative model.

Voter Demographics and Partisanship. Our analysis centers around the districting, electoral, and legislative characteristics and demographics in the United States. We consider two parties and divide the population into a minority and a nonminority. Both demographics and partisanship describe the groups and their political motivations. However, our analysis can be generalized to majoritarian systems in which parties compete in single-representative districts, holding some form of primaries to choose a candidate from within the party who faces other parties' candidates, a legislature awarding distributive benefits across districts and voting groups, and voting groups identified by partisanship and identity. The two voters' identities can follow ethnic, racial, religious, economic, or gender attributes that may define a group and describe the group's voting behavior. For more than two voter identities, groups may still be similar regarding their willingness to cross partisan and identity lines. For example, many minority voters, such as Black, Afro-American, Hispanic, Asian-American, or LGBTQIA+, tend to support in various degrees the Democratic party in the United States, and their willingness or reluctance to vote for Republican candidates may unify them. The key tensions in our analysis arise from i) a group's willingness to trade off distributive benefits from a legislature for ideological benefits from a candidate's identity, ii) a group's willingness to cross partisan and identity lines in elections (e.g., minority voters' attachment to a party, nonminority voters' tradeoff between minority candidate from the same party or nonminority candidate from the other party), and iii) the institutional principles of districting.

Electoral Platforms and Fiscal Policies. Our results follow Dixit and Londregan (1996)'s characterization of electoral platforms in which candidates make identical promises regarding redistribution of consumption benefits and taxes for each voter group in a district, all under the assumption that both parties have identical abilities to distribute benefits and raise taxes. Our analysis streamlines the budget process and neglects the collection of taxes, assuming that there is sufficient fiscal capacity and discretion in the budget. Instead, we focus on distributive benefits only, fiscal transfers from the legislature's budget to the district(s) that come as subsidies, tax credits, tax deductions, or welfare payments. If candidates were to run on in-kind transfers or public goods, where voter groups may have different preferences across programs, each candidate would need to evaluate the marginal value of a dollar spent for any program on the voter group's marginal utility from the program, affecting the change in the voter group's vote.

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A Appendix: Derivations and Proofs

A.1 Sample State and Five Districts

The example of Figure 1(b) is created with $S = (.36, 0.26, .38)$ and valid districting matrix for five districts with

$$\begin{pmatrix} 0.19 & 0.6 & 0.21 \\ 0.33 & 0.05 & 0.62 \\ 0.45 & 0.1 & 0.45 \\ 0.14 & 0.43 & 0.42 \\ 0.65 & 0.13 & 0.22 \end{pmatrix}.$$

A.2 A Solution to Legislative Policies and Bargaining

The derivation and solution of a possible subgame of legislative bargaining follow Baron and Ferejohn (1989).

Close Rule Legislative Bargaining Suppose the legislature passes a redistributive policy, dividing K dollars across all districts. They do so via a closed rule bargaining process: a legislator is selected randomly to offer a proposed budget division. The entire legislature then votes on the proposal (under a closed rule); if it is adopted, the game ends. If a majority vote rejects it, discounting occurs (all payoffs are lowered by a factor of δ, $0 < \delta \leq 1$), and the legislative subgame starts again with another member chosen randomly to make an offer. In this game, members try to maximize the benefits directed toward their district.[33]

The outcome of this legislative process will be a vector $(B_1, B_2, ..., B_K)$ of district-specific benefits, with $B_k \geq 0$ and $\sum_k B_k = K$, allocating a given budget,[34] across K odd districts.[35].So

[33]Our analysis streamlines the fiscal budgetary process with a focus on a single legislature, discretionary spending, and a given budget amount. The analysis ignores the complexities of redistribution that arise when considering federal vs. state legislatures in a multi-level system of redistribution, legislative budgeting vs. executive budget implementation in a system with separate powers, discretionary vs. mandatory spending in an environment of multi-means redistribution, or balanced budget vs. flexible budget in a fiscal setting with tax collection, government borrowing or lending, and endogenous budget amounts. Here, we focus on who gets elected and the policy outcome enacted by the legislature. For a budget process with separation of powers, see Grossman and Helpman (2008); for redistribution with the same different abilities of parties to collect taxes and distribute benefits, see Dixit and Londregan (1996).

[34]We ignore considerations of balanced budgets and taxation and instead assume that a given budget is divided across districts as the federal government can borrow or has revenues from various other tax sources unrelated to the distributive policies of interest here. For balanced budget implications and redistributive policies, see Cox and McCubbins (1986) and Lindbeck and Weibull (1987).

[35]We will assume that each district elects one legislator and the odd number of legislators avoids legislative ties.

the legislative policy function is $\mathbf{P} : \mathcal{C}^K \rightarrow \mathbb{R}^K_+$. This follows from the results of the elections, which in turn depend on the districting scheme, so $\mathbf{P} = \mathbf{P}(\mathbf{L}(\mathbf{D}(S^2)))$.

Any funds allocated to district k in the legislative process are divided according to the platform adopted by that district's representative. So if the type j representative from district k ran on a platform promising T_{ijk} to members of a group i, then voters in this group will receive $T_{ijk} * B_k$ in total benefits, with individual benefits $b_{ijk} = (T_{ijk} * B_k)/N_{ik}$.

Legislative Outcomes The legislative game is elementary; in equilibrium, the legislator who makes the first offer constructs a random minimum-winning coalition of $(K-1)/2$ other legislators and keeps the remainder for herself.[36] Let l be the legislator who makes the offer, C be the legislators selected to be in the coalition, and O be the remaining legislators. Then, equilibrium offers to share the K being distributed are:

$$B_k = \begin{cases} \frac{(2-\delta)K+\delta}{2} & \text{if } k = l; \\ \delta & \text{if } k \in C; \\ 0 & \text{if } k \in O. \end{cases} \tag{A.1}$$

Since the game is symmetric, each legislator has an expected return of 1 from the legislative bargaining session. If a candidate promises a group T_{ijk} in transfers during the election, this is also their expected total legislative payout if that candidate wins office.

In this setup, the chosen legislator constructs a minimum-winning coalition at the lowest possible costs for herself, which describes its Shapley value. Hence, the cost of swaying other legislators may or may not be related to partisan control of the legislators, but it is less if legislators are driven by securing transfers for their districts and competition for joining the minimum-winning coalition dominates.

A.3 Derivation of Solution to Candidate Platforms

The derivation and solution of the subgame follow Dixit and Londregan (1996) and incorporate the districting plans and characterization of minority benefits from our model.

[36]See Baron and Ferejohn (1989)'s Proposition 3 for a stationary subgame-perfect equilibrium with infinite sessions, majority and closed rule, and n-(odd)-legislators as well as $n = K$ districts/legislators and a total distributive benefit of K instead of 1.

Elections and Voter Groups First, we characterize the candidates' platforms. Candidates adopt platforms to maximize their votes, subject to an allocation of district benefits across voter groups. The voter groups in a closed primary are $\Theta \in \{mD, nD\}$, in an open primary $\Theta \in \{mD, nD, R\}$, and in a general election $\Theta \in \{mD, nD, R\}$.

Candidate Platforms Two candidates announce simultaneously in each election. Candidate 1's problem is

$$\max V_1^e = \sum_{i \in \Theta} N_i \Phi_i^e(X_i) \text{ s.t. } \sum_{i \in \Theta} T_{i1k} B_k = \sum_{i \in \Theta} N_{ik} b_{i1k} \leq B_k \tag{A.2}$$

and candidate 2's is equivalently

$$\max V_2^e = \sum_{i \in \Theta} N_i [1 - \Phi_i^e(X_i)] \text{ s.t. } \sum_{i \in \Theta} T_{i2k} B_k = \sum_{i \in \Theta} N_{ik} b_{i2k} \leq B_k. \tag{A.3}$$

For the existence of a Nash equilibrium, Glickberg's Theorem requires that each candidate's payoffs are a quasi-concave function of their strategy and a continuous function of other players' strategies. First, the distributive benefits for voters, b_{ij}, are an increasing linear function of the candidates' platforms, T_{ijk}. Second, the voters' cutoff for differences in candidates' promised benefits, $X_i^e = U_i(b_{i1}) - U_i(b_{i2})$, is increasing and concave in b_{ij}. Finally, the expected candidate's number of vote, $V_1^e = \sum_{i \in \Theta} N_i \Phi_i^e(X_i)$ and $V_2^e = \sum_{i \in \Theta} N_i (1 - \Phi_i^e(X_i))$, is increasing in the cutoff X_i and concave due to the concavity of $\Phi_i(.)$. Hence, existence holds.

For the solution of the Nash equilibrium, we use Lagrange parameters λ_1 and λ_2 for each respective candidate and solve all first-order conditions simultaneously. Consider candidate 1 first:

$$L = \sum_{i \in \Theta} N_i \Phi_i^e(X_i) + \lambda_1 \left(B_k - \sum_{i \in \Theta} N_{ik} b_{i1k} \right) \tag{A.4}$$

with

$$\frac{\partial L}{\partial T_{i1k}} = N_{ik} \phi_i^e(X_i) U_i'(b_{i1k}) \frac{\partial b_{i1k}}{\partial T_{i1k}} - \lambda_1 N_{ik} \frac{\partial b_{i1k}}{\partial T_{i1k}} = 0, \tag{A.5}$$

which can be written as

$$\lambda_1 = \phi_i^e(X_i) U_i'(b_{i1k}) \;\Leftrightarrow\; b_{i1k} = H_i \left(\frac{\lambda_1}{\phi_i^e(X_i)} \right), \tag{A.6}$$

where $H_i(.)$ is the inverse of the marginal utility function. Since $U_i(.)$ is a decreasing function, $H_i(.)$ is decreasing as well, and there is a unique solution for λ_1.

Candidate 2's problem is symmetric, and we have

$$L = \sum_{i \in \Theta} N_i (1 - \Phi_i^e(X_i)) + \lambda_2 \left(B_k - \sum_{i \in \Theta} N_{ik} b_{i2k} \right) \tag{A.7}$$

with

$$\frac{\partial L}{\partial T_{i2k}} = -N_{ik} \phi_i^e(X_i)(-U_i'(b_{i2k})) \frac{\partial b_{i2k}}{\partial T_{i2k}} - \lambda_2 N_{ik} \frac{\partial b_{i2k}}{\partial T_{i2k}} = 0, \tag{A.8}$$

which can be written as

$$\lambda_2 = \phi_i^e(X_i) U_i'(b_{i2k}) \;\Leftrightarrow\; b_{i2k} = H_i \left(\frac{\lambda_2}{\phi_i^e(X_i)} \right), \tag{A.9}$$

which provides a unique solution for λ_2.

The Lagrange parameters are independent of candidates' characteristics, which implies both candidates face the same shadow value in equilibrium, $\lambda_1 = \lambda_2$. As a result, a voter's marginal utility in distributive benefits is equal across both candidates:

$$\lambda_1 = \lambda_2 \;\Leftrightarrow\; U_i'(b_{i1k}) = U_i'(b_{i2k}). \tag{A.10}$$

Due to $U_i(.)$ being a continuous, increasing function, we have that the distributive benefits are identical across both candidates, $b_{i1k} = b_{i2k}$, which implies that both candidates choose identical platforms, $T_{i1k} = T_{i2k}$, and distributive promises cancel each other out such that voters choose based on ideological alignments.

Group and Member Benefits Now, we describe the distribution of district benefits across groups and their members. We use the first-order conditions above with the voters' utility function described by (2.1) and (2.2):

$$\lambda_j = \phi_i^e(0) U_i'(b_{ijk}) \;\Rightarrow\; b_{ijk} = \left(\frac{\phi_i(0)\kappa_i}{\lambda_j} \right)^{\frac{1}{\epsilon}} = \frac{\pi_i}{\lambda_j^{1/\epsilon}} \;\Leftrightarrow\; \lambda_j^{1/\epsilon} = \frac{\pi_i}{b_{ijk}}. \tag{A.11}$$

Applying $\lambda_1 = \lambda_2$ and $b_{i1k} = b_{i2k}$, we get for group i compared to group $h \neq i$ that

$$b_{ijk} = \frac{\pi_i b_{hjk}}{\pi_h}. \tag{A.12}$$

Using the budget constraint of $\sum_i N_{ik} b_{ijk} = B_k$ with (A.12), we get

$$b_{hjk} = \frac{\pi_h}{\sum_i N_{ik}\pi_i} B_k, \tag{A.13}$$

which provides the individual benefits for a member of group Θ in (3.1). The group shares follow from rearranging $b_{ijk} = (T_{ijk} * B_k)/N_{ik}$ and applying (A.13):

$$T_{hjk} = \frac{b_{hjk} N_{ik}}{B_k} = \frac{\pi_h N_{hk}}{\sum_i N_{ik}\pi_i}, \tag{A.14}$$

which completes (3.1).

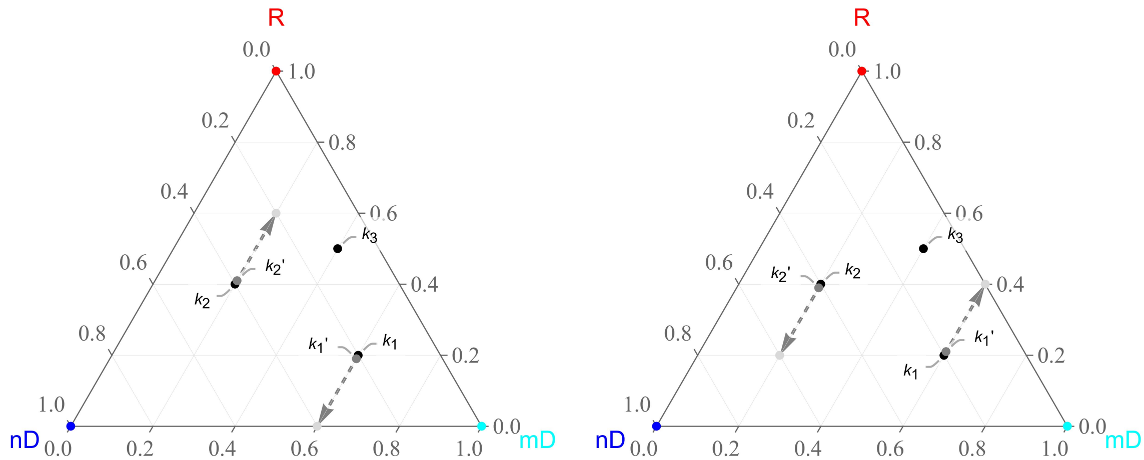
Primaries and General Election Note that candidates will offer identical platforms, but the platforms themselves depend on the voter groups and their characteristics. Hence, both Democrats offer identical platforms in a closed primary to minority and non-minority Democrat voters. Still, their platforms differ (equally) in an open primary when they also make promises to potential Republican voters, reducing promises to Democrat voters. In the general election, ignoring any credibility cost or commitments, the primary winner focuses on winning and follows a flexible platform strategy. The platform would change if the primary election were closed and the Democratic candidate competed for Republican voters, or if the primary were open but Republican turnout across the election differed.

A.4 Proof of Proposition 1

Consider a valid districting scheme $\mathbf{D}$, and assume to the contrary that $N_{ik} > 0$ for all $i \in \Theta$ and k. First, assume that Republicans are more powerful than nonminority Democrats, $\pi_{nD} < \pi_R$. Consider two districts k_1 and k_2, with N_{mD1} minority voters in k_1 and N_{mD2} in k_2, and assume that $N_{mD1} > N_{mD2}$. This is illustrated in Figure 6(a).

We now move one Republican voter from k_1 to k_2 and one nonminority-Democrat voter from k_2 and k_1, while holding minority voters constant in each district. Such a change preserves the validity of the districting scheme, and the arrows in Figure 6(a) illustrate the direction of changes. For optimality, it has to increase minority voters' distributive benefits. Hence, we compare any gains and losses across the two districts.

Considering the changes in minority benefits from (4.8), minority voters' distributive benefits in k_1 are increasing when a Republican is replaced with a less powerful nonminority-Democratic



(a) $\pi_{nD} < \pi_R$ and $\Pi_1 \leq \Pi_2$.

(b) $\pi_{nD} > \pi_R$ and $\Pi_1 \leq \Pi_2$.

Figure 6: Optimal Districting Process with $K = 3$.

voter, by

$$N_{mD1} \frac{\partial f}{\partial N_{nD1}} = N_{mD1}^2 \frac{\pi_{mD}(\pi_R - \pi_{nD})}{\Pi_1^2}; \tag{A.15}$$

while minority voters' distributive benefits in k_2 are decreasing when a more powerful Republican replaces the nonminority-Democrat voter, by

$$N_{mD2} \frac{\partial f}{\partial N_{nD2}} = N_{mD2}^2 \frac{\pi_{mD}(\pi_R - \pi_{nD})}{\Pi_2^2}. \tag{A.16}$$

Comparing A.15 with A.16, we get

$$\frac{N_{mD1}^2}{\Pi_1^2} \gtreqless \frac{N_{mD2}^2}{\Pi_2^2}. \tag{A.17}$$

Given the assumption of $N_{mD1} > N_{mD2}$, the minority gains in k_1 outweigh the minority losses in k_2 if $\Pi_1^2 \leq \Pi_2^2$, and such redistricting would be optimal as it increases net gains for minority voters.

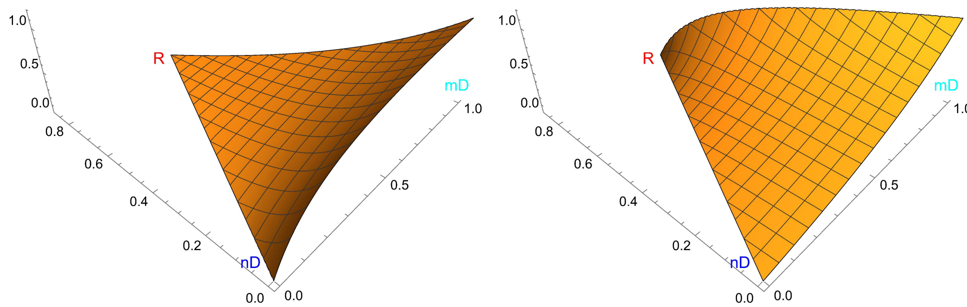
Second, assume that nonminority-Democrat voters are more powerful than Republican voters, $\pi_{nD} > \pi_R$, and the number of minorities differs, $N_{mD1} > N_{mD2}$. Then it would be beneficial to move a nonminority Democrat from k_1 to k_2, and a Republican vice versa, if the minority gains in k_1 are greater than the minority losses in k_2. This comparison follows again

$$\frac{N_{mD1}^2}{\Pi_1^2} \gtreqless \frac{N_{mD2}^2}{\Pi_2^2} \text{ with } \pi_{nD} > \pi_R. \tag{A.18}$$

If the minority-concentrated district is less powerful, then this would be beneficial. Figure 6(b) illustrates this process.

Hence, the proposed districting scheme $\mathbf{D}$ cannot be optimal. Through re-iteration of the process – the district's power decreases when less powerful nonminority voters replace more powerful nonminority voters, $\frac{\partial \Pi_k}{\partial N_{nD}} = \pi_{nD} - \pi_R$ – minority populated districts with low district power or nonminority populated districts with high district power will not lie in the interior of S^2.

A.5 Nonconcave and Nonconvex Minority Distributive Benefits



(a) $\pi_{mD} = 7$, $\pi_{nD} = 1$, $\pi_R = 15$ (b) $\pi_{mD} = 7$, $\pi_{nD} = 15$, $\pi_R = 1$

Figure 7: Nonconcave and Nonconvex Minority Distributive Benefits.

A.6 Proof of Proposition 2

Consider a valid districting scheme $\mathbf{D}$ and a district $\mathbf{d}^* = (N_{mD}, N_{nD}, N_R)$. Define $t = N_{mD}/N_k$ and $\alpha = N_{nD}/(N_{nD} + N_R)$, and let $l = \{\mathbf{d} \in \mathcal{D} | \alpha = N_{nD}/(N_{nD} + N_R)\}$. Thus l is a line running through $\mathbf{d}^*$, connecting it to $(1, 0, 0)$, which is the corner of S^2 where the district composes of minority voters only ("mD" in triangles) while keeping the ratio of nonminority voters constant throughout. Applying (4.1) divided by N_k with $t = N_{mDk}/N_k$, $(1-t) * \alpha = \frac{N_{nDk}+N_{Rk}}{N_k} * \frac{N_{nDk}}{N_{nDk}+N_R} = N_{nDk}/N_k$, and $(1-t) * (1-\alpha) = \frac{N_{nDk}+N_{Rk}}{N_k} * \frac{N_{Rk}}{N_{nDk}+N_R} = N_{Rk}/N_k$, the parameterized path for the minority payoff function is

$$g(t) = \frac{\pi_{mD} t}{\pi_{mD} t + \pi_{nD}(1-t)\alpha + \pi_R(1-t)(1-\alpha)} \tag{A.19}$$

Note that the denominator is positive, and we can evaluate the curvature of the path with its second derivative with respect to t:

$$g''(t) = -\frac{2\pi_{mD}\overbrace{(\alpha\pi_{nD} + (1-\alpha)\pi_R)}^{(+)}\overbrace{(\pi_{mD} - \alpha\pi_{nD} - (1-\alpha)\pi_R)}^{(?)}}{\underbrace{(\pi_{mD}t + \pi_{nD}(1-t)\alpha + \pi_R(1-t)(1-\alpha))^3}_{(+)}}. \tag{A.20}$$

The second derivative is negative if $\pi_{mD} > \alpha\pi_{nD} - (1-\alpha)\pi_R$ – i.e., when minorities' power is greater than the weighted average of the other groups' powers, based on district population. Hence, $\pi_{mD} = \max_{i\in\Theta}\{\pi_\Theta\}$ implies that $g''(t) < 0$ for all t, indicating that the entire surface is concave. Conversely, the second derivative is positive if $\pi_{mD} < \alpha\pi_{nD} - (1-\alpha)\pi_R$, which implies that for $\pi_{mD} = \min_{i\in\Theta}\{\pi_\Theta\}$ we get $g''(t) > 0$ for all t, and the surface is convex.

A.7 Proof of Proposition 3

We separate the proof into an analysis for a state with closed primaries and then repeat the steps for a state with open primaries.

Closed Primaries For the first part, we substitute the closed primary and general election probabilities of a minority candidate winning at each stage, (2.8) and (2.10), take the respective derivative from (2.12) with respect to the number of minority voters, and get:

$$\frac{\partial\Psi_{mDk}}{\partial N_{mDk}} = \overbrace{\frac{\partial\Psi^1_{mDk}}{\partial N_{mDk}}\Psi^2_{mDk}}^{primary} + \overbrace{\Psi^1_{mDk}\frac{\partial\Psi^2_{mDk}}{\partial N_{mDk}}}^{general} \tag{A.21}$$

$$= \underbrace{\frac{(a^1_{mD} - a^1_{nD})N_{nDk}}{(N_{mDk} + N_{nDk})^2}\underbrace{\Psi^2_{mDk}}_{(\geq 0)}}_{primary:(+)} + \underbrace{\underbrace{\Psi^1_{mDk}}_{(\geq 0)}\frac{(a^2_{mDk} - a^2_{nD})N_{nDk} + (a^2_{mD} - a^2_R)N_{Rk}}{(N_{mDk} + N_{nDk} + N_{Rk})^2}}_{general:(+)} \geq 0. \tag{A.22}$$

Given our assumption that minority voters are more likely to vote for a minority candidate than nonminority voters, $a^e_{mD} > a^e_{mD} > a^e_R$, we see immediately that the first term is positive, $a^1_{mD} > a^1_{nD}$, and the two subsequent terms are positive or zero. The last term is positive or zero as well as for $a^2_{mD} \geq a^2_{nD}$ and $a^2_{mD} \geq a^2_R$. Note that our statement is independent of whether $a^2_{nD} \gtreqless a^2_R$. The same holds for an evaluation on S^2 with $N_{Rk} = N_k - N_{mDk} - N_{nDk}$.

For the second part, we repeat the substitution but take the respective derivative from (2.12) with respect to the number of nonminority-Democrat voters and get:

$$\frac{\partial \Psi_{mDk}}{\partial N_{nDk}} = \overbrace{\frac{\partial \Psi^1_{mDk}}{\partial N_{nDk}} \Psi^2_{mDk}}^{primary} + \overbrace{\Psi^1_{mDk} \frac{\partial \Psi^2_{mDk}}{\partial N_{nDk}}}^{general} \tag{A.23}$$

$$= \underbrace{\frac{(a^1_{nD} - a^1_{mD}) N_{mDk}}{(N_{mDk} + N_{nDk})^2} \underbrace{\Psi^2_{mDk}}_{(\geq 0)}}_{primary:(-)} + \underbrace{\underbrace{\Psi^1_{mDk}}_{(\geq 0)} \frac{(a^2_{nD} - a^2_{mD}) N_{nDk} + (a^2_{nD} - a^2_{R}) N_{Rk}}{(N_{mDk} + N_{nDk} + N_{Rk})^2}}_{general:(+/-)} \gtreqless 0. \tag{A.24}$$

The first term is negative due to $a^1_{mD} > a^1_{nD}$, the two subsequent terms are positive or zero, and the last term is ambiguous. We have that $a^2_{mD} > a^2_{nD}$, illustrating the negative effect in the primary election for minority candidates. The derivative may be negative overall, but that may be offset by $a^2_{mD} > a^2_{nD}$ and $a^2_{nD} > a^2_{R}$, which illustrates the positive effect in the general election for minority candidates – nonminority-Democratic voters being more likely to support a minority-Democratic candidate than Republican voters ($a^2_{nD} > a^2_{R}$). Here, the result depends on the relationship between differences in crossover voting of nonminority voters.

For the third part, regarding replacing Republican voters with nonminority-Democrats, we rewrite the probability of a minority candidate winning the election (2.12) as

$$\tilde{\Psi}_{mDk} = \left(\frac{a^1_{mD} N_{mDk} + a^1_{nD} N_{nDk}}{N_{mDk} + N_{mDk}} \right) \left(\frac{a^2_{mD} N_{mDk} + a^2_{nD} N_{nDk} + a^2_{R} (N_k - N_{mDk} - N_{nDk})}{N_k} \right), \tag{A.25}$$

where we employ the district's population for substitution, $N_{Rk} = N_k - N_{mD} - N_{nD}$. To illustrate the replacement effect – increasing nonminority-Democratic voters and decreasing Republican voters in a district – we take the derivative with respect to the number of nonminority-Democratic voters and get:

$$\frac{\partial \tilde{\Psi}_{mDk}}{\partial N_{nDk}} = \overbrace{\frac{\partial \Psi^1_{mDk}}{\partial N_{nDk}} \tilde{\Psi}^2_{mDk}}^{primary} + \overbrace{\Psi^1_{mDk} \frac{\partial \tilde{\Psi}^2_{mDk}}{\partial N_{nDk}}}^{general} \tag{A.26}$$

$$= \underbrace{\frac{(a^1_{nD} - a^1_{mD}) N_{mDk}}{(N_{mDk} + N_{nDk})^2} \underbrace{\tilde{\Psi}^2_{mDk}}_{(\geq 0)}}_{primary:(-)} + \underbrace{\underbrace{\Psi^1_{mDk}}_{(\geq 0)} \frac{a^2_{nD} - a^2_{R}}{N_k}}_{general:(+)} \gtreqless 0, \tag{A.27}$$

where the minority candidate's chances decrease in the primary but increase in the general election.

For the last part, we employ again (A.25) and take the second derivative with respect to the number of minority voters:

$$\frac{\partial^2 \tilde{\Psi}_{mDk}}{\partial N_{mDk}^2} = \frac{2(a_{mD}^1 - a_{nD}^1)N_{nDk}\left((a_{mD}^2 - a_{nD}^2)N_{nDk} - a_R^2 N_k\right)}{N_k(N_{mDk} + N_{nDk})^3}, \tag{A.28}$$

which is positive when

$$(a_{mD}^2 - a_{nD}^2)N_{nDk} - a_R^2 N_k > 0. \tag{A.29}$$

Hence, for $a_R^2 < (a_{mD}^2 - a_{nD}^2)N_{nDk}/N_k$, Ψ_{mD} is convex on S^2.

Open Primaries For the first part, we substitute the open primary and general election probabilities of a minority candidate winning at each stage, (2.9) and (2.10), take the respective derivative from (2.12) with respect to the number of minority voters, and get:

$$\begin{aligned} \frac{\partial \hat{\Psi}_{mDk}}{\partial N_{mDk}} &= \overbrace{\frac{\partial \hat{\Psi}_{mDk}^1}{\partial N_{mDk}}\Psi_{mDk}^2}^{primary} + \overbrace{\hat{\Psi}_{mDk}^1 \frac{\partial \Psi_{mDk}^2}{\partial N_{mDk}}}^{general} && \text{(A.30)} \\ &= \underbrace{\frac{(a_{mD}^1 - a_{nD}^1)N_{nDk} + (a_{mD}^1 - a_R^1)N_{Rk}}{(N_{mDk} + N_{nDk} + N_{Rk})^2}\underbrace{\Psi_{mDk}^2}_{(\geq 0)}}_{primary:(+)} \\ &\quad + \underbrace{\underbrace{\hat{\Psi}_{mDk}^1}_{(\geq 0)}\frac{(a_{mDk}^2 - a_{nD}^2)N_{nDk} + (a_{mD}^2 - a_R^2)N_{Rk}}{(N_{mDk} + N_{nDk} + N_{Rk})^2}}_{general:(+)} \geq 0. && \text{(A.31)} \end{aligned}$$

Given our assumption that minority voters are more likely to vote for a minority candidate than nonminority voters, $a_{mD}^e > a_{mD}^e > a_R^e$, we see immediately that the first term is positive, $a_{mD}^1 > a_{nD}^1$, and the two subsequent terms are positive or zero. The last term is positive or zero as well as for $a_{mD}^2 \geq a_{nD}^2$ and $a_{mD}^2 \geq a_R^2$. Note that our statement is independent of whether $a_{nD}^2 \gtreqless a_R^2$. The same holds for an evaluation on S^2 with $N_{Rk} = N_k - N_{mDk} - N_{nDk}$.

For the second part, we take the respective derivative from (2.12) with respect to the number of nonminority-Democrat voters and get:

$$\begin{aligned} \frac{\partial \hat{\Psi}_{mDk}}{\partial N_{nDk}} &= \overbrace{\frac{\partial \hat{\Psi}_{mDk}^1}{\partial N_{nDk}}\Psi_{mDk}^2}^{primary} + \overbrace{\hat{\Psi}_{mDk}^1 \frac{\partial \Psi_{mDk}^2}{\partial N_{nDk}}}^{general} && \text{(A.32)} \\ &= \underbrace{\frac{(a_{nD}^1 - a_{mD}^1)N_{mDk} + (a_{nD}^1 - a_R^1)N_{Rk}}{(N_{mDk} + N_{nDk} + N_{Rk})^2}\underbrace{\Psi_{mDk}^2}_{(\geq 0)}}_{primary:(+/-)} \end{aligned}$$

$$+\underbrace{\underbrace{\hat{\Psi}^1_{mDk}}_{(\geq 0)}\frac{(a^2_{nD}-a^2_{mD})N_{nDk}+(a^2_{nD}-a^2_{R})N_{Rk}}{(N_{mDk}+N_{nDk}+N_{Rk})^2}}_{general:(+/-)} \gtreqless 0. \tag{A.33}$$

The first term is this time ambiguous due to $a^1_{nD} < a^1_{nD}$ and $a^1_{nD} > a^1_R$, the two subsequent terms are positive or zero, and the last term is ambiguous, too. With an open primary, the negative effect in the closed primary election for minority candidates no longer holds. Here again, the result depends on the relationship between differences in crossover voting of nonminority voters, both in the primary and general election, where it was only dependent on the general election with closed primaries.

For the third part, regarding nonminority-Democrats replacing Republican voters, we rewrite the probability of a minority candidate winning the election (2.12) as

$$\begin{aligned}\hat{\tilde{\Psi}}_{mDk} &= \left(\frac{a^1_{mD}N_{mDk}+a^1_{nD}N_{nDk}+a^1_R(N_k-N_{mDk}-N_{nDk})}{N_k}\right) \\ &\quad\times\left(\frac{a^2_{mD}N_{mDk}+a^2_{nD}N_{nDk}+a^2_R(N_k-N_{mDk}-N_{nDk})}{N_k}\right),\end{aligned} \tag{A.34}$$

where we employ the district's population for substitution, $N_{Rk} = N_k - N_{mD} - N_{nD}$. To illustrate the replacement effect – increasing nonminority-Democratic voters and decreasing Republican voters in a district – we take the derivative with respect to the number of nonminority-Democratic voters and get:

$$\frac{\partial\hat{\tilde{\Psi}}_{mDk}}{\partial N_{nDk}} = \overbrace{\frac{\partial\hat{\tilde{\Psi}}^1_{mDk}}{\partial N_{nDk}}\tilde{\Psi}^2_{mDk}}^{primary}+\overbrace{\hat{\tilde{\Psi}}^1_{mDk}\frac{\partial\tilde{\Psi}^2_{mDk}}{\partial N_{nDk}}}^{general} \tag{A.35}$$

$$= \underbrace{\frac{a^1_{nD}-a^1_R}{N_k}\underbrace{\tilde{\Psi}^2_{mDk}}_{(\geq 0)}}_{primary:(+)}+\underbrace{\underbrace{\hat{\tilde{\Psi}}^1_{mDk}}_{(\geq 0)}\frac{a^2_{nD}-a^2_R}{N_k}}_{general:(+)} \geq 0, \tag{A.36}$$

where the minority candidate's chances increase in the primary and general elections when nD voters are crossing more over than R voters.

For the last part, we employ again (A.34) and take the second derivative with respect to the number of minority voters:

$$\frac{\partial^2\hat{\tilde{\Psi}}_{mDk}}{\partial N^2_{mDk}} = \frac{2(a^1_{mD}-a^1_R)(a^2_{mD}-a^2_R)}{N^2_k} > 0, \tag{A.37}$$

which is positive and hence, $\hat{\hat{\Psi}}_{mDk}$ is convex on S^2.

A.8 Proof of Proposition 4

We separate the proof into an analysis for a state with closed primaries and then repeat the steps for a state with open primaries.

Closed Primaries The expected utility for minority voters from (4.16) can be rewritten with (2.8) to (2.12) and $N_R = N_k - N_{mD} - N_{nD}$ as

$$\begin{aligned} E(X) &= \Psi^1_{mDk}\Psi^2_{mDk} + \Psi^1_{nDk}\Psi^3_{nDk}\beta = \Psi^1_{mDk}\Psi^2_{mDk} + (1-\Psi^1_{mDk})\Psi^3_{nDk}\beta \\ &= \left(\frac{a^1_{mD}N_{mDk} + a^1_{nD}N_{nDk}}{N_{mDk}+N_{nDk}}\right)\left(\frac{a^2_{mD}N_{mDk} + a^2_{nD}N_{nDk} + a^2_R(N_k - N_{mDk} - N_{nDk})}{N_k}\right) \\ &\quad + \left(1 - \frac{a^1_{mD}N_{mDk} + a^1_{nD}N_{nDk}}{N_{mDk}+N_{nDk}}\right)\left(\frac{a^3_{mD}N_{mDk} + a^3_{nD}N_{nDk} + a^3_R(N_k - N_{mDk} - N_{nDk})}{N_k}\right) \end{aligned} \tag{A.38}$$

The second derivative with respect to the number of minority voters in a district is

$$\frac{\partial^2 E(X)}{\partial N^2_{mDk}} = \frac{2(a^1_{mD} - a^1_{nD})N_{nDk}\left((a^2_{mD} - a^2_{nD})N_{nDk} - (a^2_R - a^3_R\beta)N_k - (a^3_{mD} - a^3_{nD})\beta N_{nDk}\right)}{N_k(N_{mDk}+N_{nDk})^3}, \tag{A.39}$$

which is positive if

$$\gamma \equiv (a^2_{mD} - a^2_{nD})N_{nDk} - (a^2_R - a^3_R\beta)N_k - (a^3_{mD} - a^3_{nD})\beta N_{nDk} > 0. \tag{A.40}$$

As $a^2_R - a^3_R\beta \to 0$ or $a^2_R \to 0$ and $a^3_R \to 0$, we can state

$$\beta < \frac{a^2_{mD} - a^2_{nD}}{a^3_{mD} - a^3_{nD}}. \tag{A.41}$$

Note that (A.39) is also positive if $a^2_{mD} > a^2_{nD}$ (by assumption), $a^3_R\beta > a^2_R$ (Republican voters much more likely to support a nonminority-Democrat than a minority candidate), and $a^3_{nD} \geq a^3_{mD}$ (similar support among Democrats against a Republican candidate).

Open Primaries The expected utility for minority voters from (4.16) can be rewritten with (2.8) to (2.12) and $N_R = N_k - N_{mD} - N_{nD}$ as

$$E(X) = \hat{\Psi}^1_{mDk}\Psi^2_{mDk} + \hat{\Psi}^1_{nDk}\Psi^3_{nDk}\beta = \hat{\Psi}^1_{mDk}\Psi^2_{mDk} + (1-\hat{\Psi}^1_{mDk})\Psi^3_{nDk}\beta$$

$$= \left(\frac{a^1_{mD}N_{mDk} + a^1_{nD}N_{nDk} + a^1_R N_{Rk}}{N_k}\right)\left(\frac{a^2_{mD}N_{mDk} + a^2_{nD}N_{nDk} + a^2_R(N_k - N_{mDk} - N_{nDk})}{N_k}\right)$$
$$+ \left(1 - \frac{a^1_{mD}N_{mDk} + a^1_{nD}N_{nDk} + a^1_R N_{Rk}}{N_k}\right)\left(\frac{a^3_{mD}N_{mDk} + a^3_{nD}N_{nDk} + a^3_R(N_k - N_{mDk} - N_{nDk})}{N_k}\right) \quad \text{(A.42)}$$

The second derivative with respect to the number of minority voters in a district is

$$\frac{\partial^2 E(X)}{\partial N^2_{mDk}} = \frac{2(a^1_{mD} - a^1_R)\left(a^2_{mD} - a^2_R - \beta(a^3_{mD} - a^3_R)\right)}{N^2_k}, \quad \text{(A.43)}$$

which is positive if

$$\delta \equiv a^2_{mD} - a^2_R - \beta(a^3_{mD} - a^3_R) > 0. \quad \text{(A.44)}$$

We can state

$$\beta < \frac{a^2_{mD} - a^2_R}{a^3_{mD} - a^3_R}. \quad \text{(A.45)}$$

A.9 Proof of Proposition 5

We know from Proposition 2 that T_{ijk} becomes concave as π_{mD} rises; we wish to determine the conditions under which overall utility $\mathcal{U}_{mD} = U_{mD}(T_{ijk}) + E(X)$ is concave on S^2 with respect to π_{mD}. Recall from (3.2) that

$$\pi_i = [\kappa_i \phi_i(0)]^{1/\epsilon} \quad \text{(A.46)}$$

so that π_{mD} can increase either through a rise in κ_{mD} or $\phi_{mD}(0)$.

Taking the former, a rise in κ_{mD} indicates that minority voters prefer more distributive to ideological benefits at the margin. Since voters' overall utility is given by

$$X + \kappa_{mD}\frac{b^{1-\epsilon}}{1-\epsilon}, \quad \text{(A.47)}$$

an increase in κ_{mD} indicates that the weight placed on distributive returns increases relative to ideology. This means that the concavity of T_{ijk} will eventually dominate the sum, even if $E(X)$ is convex, making $\mathcal{U}_{mD}$ concave in π_{mD}.

Closed Primaries Taking the latter, an increase in $\phi_{mD}(0)$ indicates that minority voters are becoming more decisive; meaning that their voting rates a^e_{mD} decline for each election type e.

Taking the total derivative of (A.39) with respect to a^e_{mD} yields

$$\frac{\partial\left(\frac{\partial^2 E(X)}{\partial N^2_{mDk}}\right)}{\partial a^e_{mD}} = \frac{2N_{mD}\gamma + \overbrace{2(1-\beta)(a^1_{mD} - a^1_{nD})}^{(+)}}{N_k(N_{mDk} + N_{nDk})^3}, \tag{A.48}$$

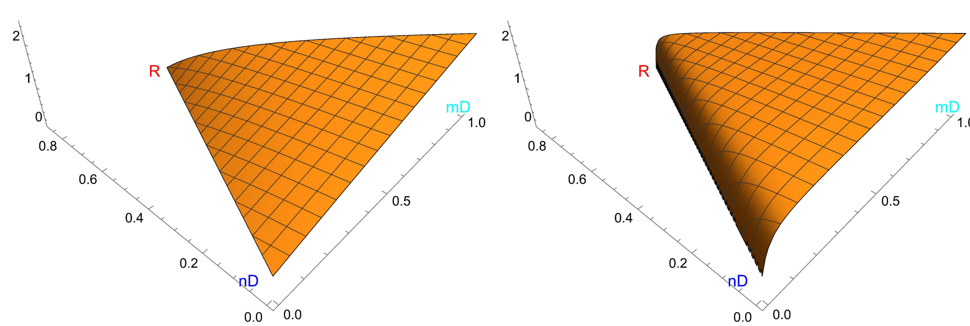
where γ follows from (A.40). If $\gamma > 0$, then $\frac{\partial^2 E(X)}{\partial N^2_{mDk}}$ is positive due to $a^1_{mD} > a^1_{nD}$, $E(X)$ is convex, and (A.48) is positive. So lower values of a^e_{mD} will make the surface of $E(X)$ more concave, again implying that $\mathcal{U}_{mD}$ becomes concave on S^2. If $\gamma < 0$, then $\frac{\partial^2 E(X)}{\partial N^2_{mDk}}$ is negative, $E(X)$ is concave, and (A.48) is ambiguous. However, low values of a^e_{mD} will not alter much the concavity of $E(X)$, and the concavity of T_{ijk} will dominate.

Open Primaries Taking the total derivative of (A.43) with respect to a^e_{mD} yields

$$\frac{\partial\left(\frac{\partial^2 E(X)}{\partial N^2_{mDk}}\right)}{\partial a^e_{mD}} = \frac{2\left(\delta + \overbrace{(a^1_{mD} - a^1_{R})(1-\beta)}^{(+)}\right)}{N^2_k}, \tag{A.49}$$

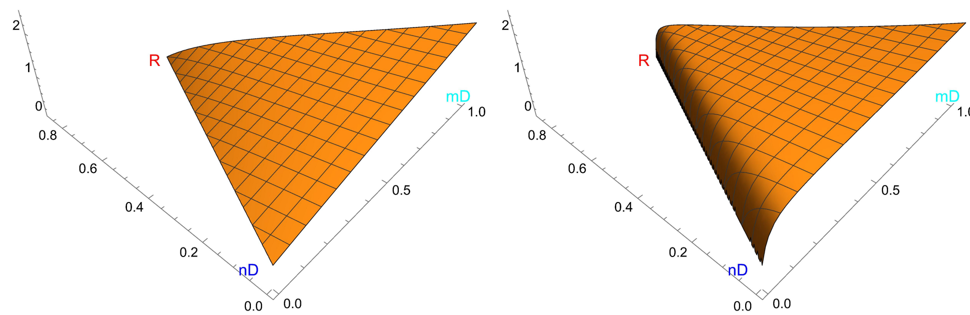
where δ follows from (A.44). If $\delta > 0$, then $\frac{\partial^2 E(X)}{\partial N^2_{mDk}}$ is positive due to $a^1_{mD} > a^1_{R}$, $E(X)$ is convex, and (A.49) is positive. So lower values of a^e_{mD} will make the surface of $E(X)$ more concave, again implying that $\mathcal{U}_{mD}$ becomes concave on S^2. If $\delta < 0$, then $\frac{\partial^2 E(X)}{\partial N^2_{mDk}}$ is negative, $E(X)$ is concave, and (A.49) is ambiguous. However, low values of a^e_{mD} will not alter much the concavity of $E(X)$, and the concavity of T_{ijk} will dominate.

A.10 Minority Power and Total Benefits



(a) $\pi_{mD} = 4$, $\pi_{nD} = 3$, $\pi_R = 1$ (b) $\pi_{mD} = 150$, $\pi_{nD} = 3$, $\pi_R = 1$

Figure 8: Total Minority Benefits – Minority Power, closed primaries.



(a) $\pi_{mD} = 4$, $\pi_{nD} = 3$, $\pi_R = 1$ (b) $\pi_{mD} = 150$, $\pi_{nD} = 3$, $\pi_R = 1$

Figure 9: Total Minority Benefits – Minority Power, open primaries.

B Online Appendix: Simulations

B.1 Minority Distributive Payoffs and Voter Distribution – Group Power

Group Power – **Minority Power**					
π_{mD}	π_{nD}	π_R	Total	Average	$R(\mathbf{D})$
1	3	1	0.750	0.25	75%
3	3	1	1.167	0.39	39%
5	3	1	1.426	0.48	15%
7	3	1	1.654	0.55	5%
10	3	1	1.899	0.63	3%
1	3	2	0.600	0.20	75%
3	3	2	0.912	0.30	42%
5	3	2	1.208	0.40	11%
7	3	2	1.450	0.48	4%
10	3	2	1.713	0.57	1%
1	3	3	0.500	0.17	75%
3	3	3	0.750	0.25	1%
5	3	3	1.071	0.36	0%
7	3	3	1.313	0.44	0%
10	3	3	1.579	0.53	0%
1	3	4	0.500	0.17	75%
3	3	4	0.750	0.25	75%
5	3	4	0.995	0.33	28%
7	3	4	1.217	0.41	9%
10	3	4	1.477	0.49	3%
1	3	5	0.500	0.17	75%
3	3	5	0.750	0.25	75%
5	3	5	0.987	0.33	38%
7	3	5	1.159	0.39	26%
10	3	5	1.399	0.47	9%

Group Power – **Large Differences**					
π_{mD}	π_{nD}	π_R	Total	Average	$R(\mathbf{D})$
1	5	1	0.750	0.25	75%
3	5	1	1.087	0.36	42%
5	5	1	1.318	0.44	38%
7	5	1	1.458	0.49	35%
10	5	1	1.669	0.56	9%
1	5	3	0.500	0.17	75%
3	5	3	0.750	0.25	75%
5	5	3	0.953	0.32	41%
7	5	3	1.129	0.38	23%
10	5	3	1.373	0.46	8%
1	5	5	0.375	0.13	75%
3	5	5	0.643	0.21	75%
5	5	5	0.750	0.25	0%
7	5	5	0.955	0.32	0%
10	5	7	1.098	0.37	18%
1	5	7	0.375	0.13	75%
3	5	7	0.643	0.21	75%
5	5	7	0.750	0.25	75%
7	5	7	0.905	0.30	39%
10	5	7	1.098	0.37	18%
1	5	10	0.375	0.13	75%
3	5	10	0.643	0.21	75%
5	5	10	0.750	0.25	75%
7	5	10	0.895	0.30	40%
10	5	10	1.072	0.36	38%

Group Power – **Homogeneous Nonminority Power**					
π_{mD}	π_{nD}	π_R	Total	Average	$R(\mathbf{D})$
1	1	1	0.750	0.25	0%
2	1	1	1.200	0.40	0%
3	1	1	1.500	0.50	0%
4	1	1	1.714	0.57	0%
5	1	1	1.875	0.63	0%
1	3	3	0.500	0.17	75%
2	3	3	0.667	0.22	75%
3	3	3	0.750	0.25	1%
4	3	3	0.923	0.31	0%
5	3	3	1.071	0.36	0%
1	5	5	0.375	0.13	75%
2	5	5	0.545	0.18	75%
3	5	5	0.643	0.21	75%
4	5	5	0.706	0.24	75%
5	5	5	0.750	0.25	0%
1	7	7	0.300	0.10	75%
2	7	7	0.462	0.15	75%
3	7	7	0.563	0.19	75%
4	7	7	0.632	0.21	75%
5	7	7	0.682	0.23	75%
1	10	10	0.231	0.08	75%
2	10	10	0.375	0.13	75%
3	10	10	0.474	0.16	75%
4	10	10	0.545	0.18	75%
5	10	10	0.600	0.20	75%

Group Power – **Heterogeneous Nonminority Power**					
π_{mD}	π_{nD}	π_R	Total	Average	$R(\mathbf{D})$
1	5	1	0.750	0.25	75%
2	5	1	0.911	0.30	52%
3	5	1	1.087	0.36	42%
4	5	1	1.219	0.41	39%
5	5	1	1.318	0.44	38%
1	5	3	0.500	0.17	75%
2	5	3	0.667	0.22	75%
3	5	3	0.750	0.25	75%
4	5	3	0.846	0.28	46%
5	5	3	0.953	0.32	41%
1	5	5	0.375	0.13	75%
2	5	5	0.545	0.18	75%
3	5	5	0.643	0.21	75%
4	5	5	0.706	0.24	75%
5	5	7	0.750	0.25	75%
1	5	7	0.375	0.13	75%
2	5	7	0.545	0.18	75%
3	5	7	0.643	0.21	75%
4	5	7	0.706	0.24	75%
5	5	7	0.750	0.25	75%
1	5	10	0.375	0.13	75%
2	5	10	0.545	0.18	75%
3	5	10	0.643	0.21	75%
4	5	10	0.706	0.24	75%
5	5	10	0.750	0.25	75%

Table 9: Districting Plans Maximizing Minority Distributive Benefits – Group Power, $N_{mD} = 25\%$, $N_{nD} = 40\%$, and $N_R = 35\%$.

B.2 Minority Distributive Payoffs and Voter Distribution – Demographics

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	0%	75%	25%	4%	0%	96%	86%	0%	14%	0.900	0.300	86%
2	3	1	0%	75%	25%	45%	0%	55%	45%	0%	55%	1.241	0.414	45%
3	3	1	38%	0%	62%	38%	0%	62%	14%	75%	11%	1.447	0.482	24%
4	3	1	34%	0%	66%	34%	0%	66%	23%	75%	2%	1.625	0.542	11%
5	3	1	33%	0%	67%	32%	0%	68%	25%	75%	0%	1.770	0.590	8%
1	3	2	0%	4%	96%	0%	71%	29%	90%	0%	10%	0.818	0.273	90%
2	3	2	0%	75%	25%	8%	0%	92%	82%	0%	18%	0.900	0.300	82%
3	3	2	41%	0%	59%	41%	0%	59%	8%	75%	17%	1.106	0.369	33%
4	3	2	35%	0%	65%	35%	0%	65%	21%	75%	4%	1.291	0.430	14%
5	3	2	33%	0%	67%	33%	0%	67%	25%	75%	0%	1.450	0.483	8%
1	3	3	0%	35%	65%	0%	35%	65%	90%	4%	6%	0.750	0.250	90%
2	3	3	0%	35%	65%	0%	35%	65%	90%	5%	5%	0.857	0.286	90%
3	3	3	30%	25%	45%	30%	25%	45%	30%	25%	45%	0.900	0.300	0%
4	3	3	30%	27%	43%	30%	25%	45%	30%	23%	47%	1.091	0.364	0%
5	3	3	30%	11%	59%	30%	11%	59%	30%	54%	16%	1.250	0.417	0%
1	3	4	0%	33%	67%	0%	32%	68%	90%	10%	0%	0.750	0.250	90%
2	3	4	0%	62%	38%	0%	3%	97%	90%	10%	0%	0.857	0.286	90%
3	3	4	0%	63%	37%	0%	2%	98%	90%	10%	0%	0.900	0.300	90%
4	3	4	53%	47%	0%	37%	28%	35%	0%	0%	100%	0.998	0.333	53%
5	3	4	40%	60%	0%	29%	15%	57%	22%	0%	78%	1.126	0.375	18%
1	3	5	0%	32%	68%	0%	33%	67%	90%	10%	0%	0.750	0.250	90%
2	3	5	0%	43%	57%	0%	22%	78%	90%	10%	0%	0.857	0.286	90%
3	3	5	0%	35%	65%	0%	30%	70%	90%	10%	0%	0.900	0.300	90%
4	3	5	60%	40%	0%	30%	35%	35%	0%	0%	100%	0.967	0.322	60%
5	3	5	51%	49%	0%	39%	26%	35%	0%	0%	100%	1.070	0.357	51%

Intermediate Minority Population: $N_{mD} = 0.3,\ N_{nD} = 0.25,\ N_R = 0.45$.

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	20%	0%	80%	100%	0%	0%	0%	75%	25%	1.200	0.400	100%
2	3	1	59%	0%	41%	59%	0%	41%	2%	75%	23%	1.500	0.500	58%
3	3	1	48%	0%	53%	48%	0%	53%	25%	75%	0%	1.712	0.571	23%
4	3	1	47%	0%	53%	48%	0%	53%	25%	75%	0%	1.875	0.625	23%
5	3	1	45%	0%	55%	29%	71%	0%	46%	4%	50%	1.996	0.665	17%
1	3	2	20%	0%	80%	100%	0%	0%	0%	75%	25%	1.111	0.370	100%
2	3	2	0%	75%	25%	79%	0%	21%	41%	0%	59%	1.200	0.400	79%
3	3	2	51%	0%	49%	51%	0%	49%	19%	75%	6%	1.403	0.468	32%
4	3	2	25%	75%	0%	47%	0%	53%	47%	0%	53%	1.596	0.532	22%
5	3	2	43%	9%	48%	43%	0%	57%	34%	66%	0%	1.752	0.584	9%
1	3	3	100%	0%	0%	0%	42%	58%	20%	33%	47%	1.077	0.359	100%
2	3	3	20%	38%	42%	100%	0%	0%	0%	37%	63%	1.143	0.381	100%
3	3	3	43%	24%	34%	39%	26%	36%	39%	26%	36%	1.200	0.400	4%
4	3	3	40%	23%	37%	40%	25%	35%	40%	27%	33%	1.412	0.471	0%
5	3	3	40%	23%	37%	40%	21%	39%	40%	30%	30%	1.579	0.526	0%
1	3	4	100%	0%	0%	20%	75%	5%	0%	0%	100%	1.075	0.358	100%
2	3	4	0%	0%	100%	20%	75%	5%	100%	0%	0%	1.140	0.380	100%
3	3	4	100%	0%	0%	0%	0%	100%	20%	75%	5%	1.197	0.399	100%
4	3	4	59%	36%	5%	61%	39%	0%	0%	0%	100%	1.324	0.441	61%
5	3	4	31%	0%	69%	41%	23%	36%	48%	52%	0%	1.456	0.485	17%
1	3	5	100%	0%	0%	20%	75%	5%	0%	0%	100%	1.074	0.358	100%
2	3	5	20%	75%	5%	100%	0%	0%	0%	0%	100%	1.138	0.379	100%
3	3	5	100%	0%	0%	20%	75%	5%	0%	0%	100%	1.194	0.398	100%
4	3	5	62%	38%	0%	0%	0%	100%	58%	37%	5%	1.316	0.439	62%
5	3	5	61%	39%	0%	59%	36%	5%	0%	0%	100%	1.412	0.471	61%

Large Minority Population: $N_{mD} = 0.4,\ N_{nD} = 0.25,\ N_R = 0.35$.

Table 10: Districting Plans Maximizing Minority Distributive Benefits – State Demographics.

B.3 Minority Distributive Payoffs and Voter Distribution – Number of Districts

5 Districts – $N_{mD} = 0.25,\ N_{nD} = .4,\ N_R = 0.35$					
π_{mD}	π_{nD}	π_R	Total	Average	$R(\mathbf{D})$
1	3	1	1.250	0.25	88%
2	3	1	1.765	0.35	42%
3	3	1	2.045	0.41	42%
4	3	1	2.222	0.44	42%
5	3	1	2.404	0.48	15%
1	3	2	1.143	0.23	100%
2	3	2	1.250	0.25	63%
3	3	2	1.552	0.31	42%
4	3	2	1.778	0.36	25%
5	3	2	2.020	0.40	11%
1	3	3	1.100	0.22	100%
2	3	3	1.182	0.24	100%
3	3	3	1.250	0.25	0%
4	3	3	1.538	0.31	0%
5	3	3	1.786	0.36	0%
1	3	4	1.100	0.22	100%
2	3	4	1.182	0.24	100%
3	3	4	1.250	0.25	63%
4	3	4	1.464	0.29	40%
5	3	4	1.667	0.33	21%
1	3	5	1.100	0.22	100%
2	3	5	1.182	0.24	100%
3	3	5	1.250	0.25	63%
4	3	5	1.463	0.29	42%
5	3	5	1.634	0.38	39%

12 Districts – $N_{mD} = 0.25,\ N_{nD} = .4,\ N_R = 0.35$					
π_{mD}	π_{nD}	π_R	Total	Average	$R(\mathbf{D})$
1	3	1	3.000	0.25	94%
2	3	1	4.200	0.35	43%
3	3	1	4.868	0.41	41%
4	3	1	5.354	0.45	36%
5	3	1	5.856	0.49	10%
1	3	2	3.000	0.25	100%
2	3	2	3.000	0.25	100%
3	3	2	3.708	0.31	42%
4	3	2	4.281	0.36	21%
5	3	2	4.868	0.41	10%
1	3	3	3.000	0.25	100%
2	3	3	3.000	0.25	100%
3	3	3	3.000	0.25	42%
4	3	3	3.692	0.31	0%
5	3	3	4.286	0.36	0%
1	3	4	2.470	0.21	100%
2	3	4	3.000	0.25	100%
3	3	4	3.000	0.25	78%
4	3	4	3.533	0.29	39%
5	3	4	3.995	0.33	20%
1	3	5	3.000	0.25	100%
2	3	5	3.000	0.25	100%
3	3	5	3.000	0.25	94%
4	3	5	3.516	0.29	40%
5	3	5	3.955	0.33	38%

Table 11: Districting Plans Maximizing Minority Distributive Benefits – Number of Districts.

B.4 Minority Total Payoffs and Voter Distribution – Group Power

Group Power – **Minority Power**

π_{mD}	π_{nD}	π_R	Total	Average	$R(\mathbf{D})$
1	3	1	2.172	0.72	75%
3	3	1	2.562	0.85	40%
5	3	1	2.795	0.93	28%
7	3	1	3.007	1.00	8%
10	3	1	3.248	1.08	3%
1	3	2	2.022	0.67	75%
3	3	2	2.308	0.77	44%
5	3	2	2.571	0.86	23%
7	3	2	2.804	0.93	9%
10	3	2	3.064	1.02	4%
1	3	3	1.922	0.64	75%
3	3	3	2.172	0.72	75%
5	3	3	2.424	0.81	11%
7	3	3	2.663	0.89	6%
10	3	3	2.929	0.98	4%
1	3	4	1.883	0.63	75%
3	3	4	2.133	0.71	75%
5	3	4	2.331	0.78	12%
7	3	4	2.562	0.85	2%
10	3	4	2.825	0.94	1%
1	3	5	1.883	0.63	75%
3	3	5	2.128	0.71	75%
5	3	5	2.312	0.77	38%
7	3	5	2.493	0.83	17%
10	3	5	2.743	0.91	4%

Group Power – **Large Differences**

π_{mD}	π_{nD}	π_R	Total	Average	$R(\mathbf{D})$
1	5	1	2.172	0.72	75%
3	5	1	2.483	0.83	43%
5	5	1	2.713	0.90	38%
7	5	1	2.851	0.95	39%
10	5	1	3.023	1.01	14%
1	5	3	1.922	0.64	75%
3	5	3	2.172	0.72	75%
5	5	3	2.349	0.78	43%
7	5	3	2.514	0.84	40%
10	5	3	2.729	0.91	14%
1	5	5	1.797	0.60	75%
3	5	5	2.065	0.69	75%
5	5	5	2.172	0.72	75%
7	5	5	2.311	0.77	25%
10	5	7	2.438	0.81	8%
1	5	7	1.758	0.59	75%
3	5	7	2.025	0.68	75%
5	5	7	2.133	0.71	75%
7	5	7	2.230	0.74	38%
10	5	7	2.438	0.81	8%
1	5	10	1.758	0.59	75%
3	5	10	2.025	0.68	75%
5	5	10	2.133	0.71	75%
7	5	10	2.190	0.73	75%
10	5	10	2.397	0.80	38%

Group Power – **Homogeneous Nonminority Power**

π_{mD}	π_{nD}	π_R	Total	Average	$R(\mathbf{D})$
1	1	1	2.172	0.72	75%
2	1	1	2.551	0.85	7%
3	1	1	2.850	0.95	4%
4	1	1	3.064	1.02	3%
5	1	1	3.225	1.07	3%
1	3	3	1.922	0.64	75%
2	3	3	2.089	0.70	75%
3	3	3	2.172	0.72	75%
4	3	3	2.284	0.76	40%
5	3	3	2.424	0.81	11%
1	5	5	1.797	0.60	75%
2	5	5	1.967	0.66	75%
3	5	5	2.065	0.69	75%
4	5	5	2.128	0.71	75%
5	5	5	2.172	0.72	75%
1	7	7	1.722	0.57	75%
2	7	7	1.884	0.63	75%
3	7	7	1.984	0.66	75%
4	7	7	2.054	0.68	75%
5	7	7	2.104	0.70	75%
1	10	10	1.653	0.55	75%
2	10	10	1.797	0.60	75%
3	10	10	1.896	0.63	75%
4	10	10	1.967	0.66	75%
5	10	10	2.022	0.67	75%

Group Power – **Heterogeneous Nonminority Power**

π_{mD}	π_{nD}	π_R	Total	Average	$R(\mathbf{D})$
1	5	1	2.172	0.72	75%
2	5	1	2.311	0.77	54%
3	5	1	2.483	0.83	43%
4	5	1	2.614	0.87	39%
5	5	1	2.713	0.90	38%
1	5	3	1.922	0.64	75%
2	5	3	2.089	0.70	75%
3	5	3	2.172	0.72	75%
4	5	3	2.244	0.75	50%
5	5	3	2.349	0.78	43%
1	5	5	1.797	0.60	75%
2	5	5	1.967	0.66	75%
3	5	5	2.065	0.69	75%
4	5	5	2.128	0.71	75%
5	5	7	2.133	0.71	75%
1	5	7	1.758	0.59	75%
2	5	7	1.928	0.64	75%
3	5	7	2.025	0.68	75%
4	5	7	2.089	0.70	75%
5	5	7	2.133	0.71	75%
1	5	10	1.758	0.59	75%
2	5	10	1.928	0.64	75%
3	5	10	2.025	0.68	75%
4	5	10	2.089	0.70	75%
5	5	10	2.133	0.71	75%

Table 12: Districting Plans Maximizing Minority Total Benefits – Group Power, $N_{mD} = 25\%$, $N_{nD} = 40\%$, $N_R = 35\%$, $K = 3$, closed primaries.

Group Power – **Minority Power**					
π_{mD}	π_{nD}	π_R	Total	Average	$R(\mathbf{D})$
1	3	1	2.069	0.69	75%
3	3	1	2.370	0.79	38%
5	3	1	2.637	0.88	11%
7	3	1	2.874	0.96	5%
10	3	1	3.125	1.04	7%
1	3	2	1.919	0.64	75%
3	3	2	2.137	0.71	75%
5	3	2	2.422	0.81	2%
7	3	2	2.669	0.89	3%
10	3	2	2.935	0.98	5%
1	3	3	1.880	0.63	75%
3	3	3	2.130	0.71	75%
5	3	3	2.317	0.77	36%
7	3	3	2.540	0.85	12%
10	3	3	2.803	0.93	7%
1	3	4	1.880	0.63	75%
3	3	4	2.130	0.71	75%
5	3	4	2.310	0.77	39%
7	3	4	2.477	0.83	37%
10	3	4	2.708	0.90	14%
1	3	5	1.880	0.63	75%
3	3	5	2.130	0.71	75%
5	3	5	2.304	0.77	40%
7	3	5	2.470	0.82	38%
10	3	5	2.648	0.88	26%

Group Power – **Large Differences**					
π_{mD}	π_{nD}	π_R	Total	Average	$R(\mathbf{D})$
1	5	1	2.069	0.69	75%
3	5	1	2.290	0.76	43%
5	5	1	2.524	0.84	41%
7	5	1	2.666	0.89	38%
10	5	1	2.890	0.96	8%
1	5	3	1.819	0.61	75%
3	5	3	2.069	0.69	75%
5	5	3	2.157	0.72	50%
7	5	3	2.335	0.78	13%
10	5	3	2.589	0.86	2%
1	5	5	1.755	0.58	75%
3	5	5	2.023	0.67	75%
5	5	5	2.130	0.71	75%
7	5	5	2.230	0.74	39%
10	5	7	2.400	0.80	38%
1	5	7	1.755	0.58	75%
3	5	7	2.023	0.67	75%
5	5	7	2.130	0.71	75%
7	5	7	2.222	0.74	42%
10	5	7	2.400	0.80	38%
1	5	10	1.755	0.58	75%
3	5	10	2.023	0.67	75%
5	5	10	2.130	0.71	75%
7	5	10	2.213	0.74	46%
10	5	10	2.388	0.80	39%

Group Power – **Homogeneous Nonminority Power**					
π_{mD}	π_{nD}	π_R	Total	Average	$R(\mathbf{D})$
1	1	1	2.130	0.71	75%
2	1	1	2.431	0.81	18%
3	1	1	2.725	0.91	8%
4	1	1	2.938	0.98	6%
5	1	1	3.098	1.03	5%
1	3	3	1.880	0.63	75%
2	3	3	2.046	0.68	75%
3	3	3	2.130	0.71	75%
4	3	3	2.205	0.74	40%
5	3	3	2.317	0.77	36%
1	5	5	1.755	0.58	75%
2	5	5	1.925	0.64	75%
3	5	5	2.023	0.67	75%
4	5	5	2.086	0.70	75%
5	5	5	2.130	0.71	75%
1	7	7	1.680	0.56	75%
2	7	7	1.841	0.61	75%
3	7	7	1.942	0.65	75%
4	7	7	2.011	0.67	75%
5	7	7	2.062	0.69	75%
1	10	10	1.611	0.54	75%
2	10	10	1.755	0.58	75%
3	10	10	1.853	0.62	75%
4	10	10	1.925	0.64	75%
5	10	10	1.980	0.66	75%

Group Power – **Heterogeneous Nonminority Power**					
π_{mD}	π_{nD}	π_R	Total	Average	$R(\mathbf{D})$
1	5	1	2.069	0.69	75%
2	5	1	2.176	0.73	75%
3	5	1	2.290	0.76	43%
4	5	1	2.423	0.81	39%
5	5	1	2.524	0.84	41%
1	5	3	1.819	0.61	75%
2	5	3	1.985	0.66	75%
3	5	3	2.069	0.69	75%
4	5	3	2.119	0.71	75%
5	5	3	2.157	0.72	50%
1	5	5	1.755	0.58	75%
2	5	5	1.925	0.64	75%
3	5	5	2.023	0.67	75%
4	5	5	2.086	0.70	75%
5	5	7	2.130	0.71	75%
1	5	7	1.755	0.58	75%
2	5	7	1.925	0.64	75%
3	5	7	2.023	0.67	75%
4	5	7	2.086	0.70	75%
5	5	7	2.130	0.71	75%
1	5	10	1.755	0.58	75%
2	5	10	1.925	0.64	75%
3	5	10	2.023	0.67	75%
4	5	10	2.086	0.70	75%
5	5	10	2.130	0.71	75%

Table 13: Districting Plans Maximizing Minority Total Benefits – Group Power, $N_{mD} = 25\%$, $N_{nD} = 40\%$, $N_R = 35\%$, $K = 3$, open primaries.

B.5 Minority Total Payoffs and Voter Distribution – State Demographics

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	0%	8%	92%	0%	67%	33%	90%	0%	10%	2.220	0.74	90%
2	3	1	45%	0%	55%	0%	75%	25%	45%	0%	55%	2.561	0.85	45%
3	3	1	41%	0%	59%	7%	75%	18%	41%	0%	59%	2.747	0.92	34%
4	3	1	36%	0%	64%	19%	75%	6%	36%	0%	64%	2.911	0.97	17%
5	3	1	33%	0%	67%	24%	75%	1%	33%	0%	67%	3.051	1.02	9%
1	3	2	0%	38%	62%	0%	37%	63%	90%	0%	10%	2.138	0.71	90%
2	3	2	0%	0%	100%	0%	75%	25%	90%	0%	10%	2.220	0.74	90%
3	3	2	45%	0%	55%	0%	75%	25%	45%	0%	55%	2.422	0.81	45%
4	3	2	38%	0%	62%	14%	75%	11%	38%	0%	62%	2.581	0.86	24%
5	3	2	35%	0%	65%	21%	75%	4%	35%	0%	65%	2.733	0.91	14%
1	3	3	0%	0%	100%	0%	75%	25%	90%	0%	10%	2.070	0.69	90%
2	3	3	0%	0%	100%	0%	75%	25%	90%	0%	10%	2.177	0.73	90%
3	3	3	45%	0%	55%	0%	75%	25%	45%	0%	55%	2.220	0.74	45%
4	3	3	37%	0%	63%	16%	75%	9%	37%	0%	63%	2.373	0.79	21%
5	3	3	24%	75%	1%	33%	0%	67%	33%	0%	67%	2.528	0.84	9%
1	3	4	0%	58%	42%	0%	7%	93%	90%	10%	0%	2.051	0.68	90%
2	3	4	0%	50%	50%	0%	15%	85%	90%	10%	0%	2.158	0.72	90%
3	3	4	0%	36%	64%	0%	29%	71%	90%	10%	0%	2.201	0.73	90%
4	3	4	25%	75%	0%	4%	0%	96%	61%	0%	39%	2.238	0.75	57%
5	3	4	25%	75%	0%	32%	0%	68%	33%	0%	67%	2.389	0.80	8%
1	3	5	0%	59%	41%	0%	6%	94%	90%	10%	0%	2.051	0.68	90%
2	3	5	0%	57%	43%	0%	8%	92%	90%	10%	0%	2.158	0.72	90%
3	3	5	90%	10%	0%	0%	0%	100%	0%	65%	35%	2.201	0.73	90%
4	3	5	90%	10%	0%	0%	0%	100%	0%	65%	35%	2.224	0.74	90%
5	3	5	46%	54%	0%	0%	0%	100%	44%	21%	35%	2.305	0.77	46%

Intermediate Minority Population - D Majority: $N_{mD} = 0.3,\ N_{nD} = 0.25,\ N_R = 0.45.$

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	47%	0%	53%	0%	45%	55%	43%	0%	57%	2.052	0.68	47%
2	3	1	45%	0%	55%	45%	0%	55%	0%	45%	55%	2.393	0.80	45%
3	3	1	36%	0%	64%	36%	0%	64%	18%	45%	37%	2.620	0.87	18%
4	3	1	33%	0%	67%	33%	0%	67%	24%	45%	31%	2.812	0.94	9%
5	3	1	31%	0%	69%	31%	0%	69%	27%	45%	28%	2.963	0.99	4%
1	3	2	0%	39%	61%	90%	0%	10%	0%	6%	94%	1.970	0.66	90%
2	3	2	24%	0%	76%	66%	0%	34%	0%	45%	55%	2.052	0.68	66%
3	3	2	44%	0%	56%	44%	0%	56%	3%	45%	52%	2.254	0.75	41%
4	3	2	35%	0%	65%	35%	0%	65%	20%	45%	35%	2.441	0.81	14%
5	3	2	33%	0%	67%	33%	0%	67%	24%	45%	31%	2.601	0.87	8%
1	3	3	0%	25%	75%	90%	0%	10%	0%	20%	80%	1.902	0.63	90%
2	3	3	0%	0%	100%	90%	0%	10%	0%	45%	55%	2.009	0.67	90%
3	3	3	45%	0%	55%	45%	0%	55%	0%	45%	55%	2.052	0.68	45%
4	3	3	35%	0%	65%	35%	0%	65%	20%	45%	35%	2.209	0.74	15%
5	3	3	32%	0%	68%	32%	0%	68%	26%	45%	29%	2.366	0.79	6%
1	3	4	0%	18%	82%	0%	17%	83%	90%	10%	0%	1.883	0.63	90%
2	3	4	0%	0%	100%	90%	10%	0%	0%	35%	65%	1.990	0.66	90%
3	3	4	0%	22%	78%	90%	10%	0%	0%	13%	87%	2.033	0.68	90%
4	3	4	0%	0%	100%	35%	0%	65%	55%	45%	0%	2.070	0.69	55%
5	3	4	27%	0%	73%	27%	0%	73%	36%	45%	19%	2.203	0.73	9%
1	3	5	0%	0%	100%	0%	35%	65%	90%	10%	0%	1.883	0.63	90%
2	3	5	0%	0%	100%	90%	10%	0%	0%	35%	65%	1.990	0.66	90%
3	3	5	0%	25%	75%	90%	10%	0%	0%	10%	90%	2.033	0.68	90%
4	3	5	0%	27%	73%	90%	10%	0%	0%	8%	92%	2.056	0.69	90%
5	3	5	0%	0%	100%	35%	0%	65%	55%	45%	0%	2.121	0.71	55%

Intermediate Minority Population - R Majority: $N_{mD} = 0.3,\ N_{nD} = 0.15,\ N_R = 0.55.$

Table 14: Districting Plans Maximizing Minority Total Benefits – State Demographics, closed primaries.

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	90%	0%	10%	0%	75%	25%	0%	0%	100%	2.164	0.72	90%
2	3	1	45%	0%	55%	0%	75%	25%	45%	0%	55%	2.303	0.77	45%
3	3	1	37%	0%	63%	16%	75%	9%	37%	0%	63%	2.497	0.83	20%
4	3	1	33%	0%	67%	25%	75%	0%	32%	0%	68%	2.685	0.90	8%
5	3	1	25%	75%	0%	32%	0%	68%	33%	0%	67%	2.831	0.94	8%
1	3	2	0%	75%	25%	0%	0%	100%	90%	0%	10%	2.082	0.69	90%
2	3	2	0%	0%	100%	0%	75%	25%	90%	0%	10%	2.164	0.72	90%
3	3	2	0%	0%	100%	0%	75%	25%	90%	0%	10%	2.195	0.73	90%
4	3	2	25%	75%	0%	32%	0%	68%	33%	0%	67%	2.350	0.78	8%
5	3	2	27%	73%	0%	31%	2%	66%	31%	0%	69%	2.511	0.84	4%
1	3	3	0%	65%	35%	0%	0%	100%	90%	10%	0%	2.037	0.68	90%
2	3	3	0%	0%	100%	0%	65%	35%	90%	10%	0%	2.145	0.71	90%
3	3	3	0%	65%	35%	0%	0%	100%	90%	10%	0%	2.187	0.73	90%
4	3	3	69%	31%	0%	0%	0%	100%	21%	44%	35%	2.219	0.74	69%
5	3	3	46%	54%	0%	15%	0%	85%	30%	21%	50%	2.332	0.78	31%
1	3	4	0%	0%	100%	0%	65%	35%	90%	10%	0%	2.037	0.68	90%
2	3	4	0%	65%	35%	0%	0%	100%	90%	10%	0%	2.145	0.71	90%
3	3	4	90%	10%	0%	0%	0%	100%	0%	65%	35%	2.187	0.73	90%
4	3	4	90%	10%	0%	0%	0%	100%	0%	65%	35%	2.211	0.74	90%
5	3	4	58%	42%	0%	0%	0%	100%	32%	33%	35%	2.282	0.76	58%
1	3	5	0%	65%	35%	0%	0%	100%	90%	10%	0%	2.037	0.68	90%
2	3	5	90%	10%	0%	0%	0%	100%	0%	65%	35%	2.145	0.71	90%
3	3	5	90%	10%	0%	0%	0%	100%	0%	65%	35%	2.187	0.73	90%
4	3	5	90%	10%	0%	0%	0%	100%	0%	65%	35%	2.211	0.74	90%
5	3	5	66%	34%	0%	0%	0%	100%	24%	41%	35%	2.253	0.75	66%

Intermediate Minority Population - D Majority: $N_{mD} = 0.3,\ N_{nD} = 0.25,\ N_R = 0.45.$

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	0%	0%	100%	90%	0%	10%	0%	45%	55%	1.992	0.66	90%
2	3	1	45%	0%	55%	45%	0%	55%	0%	45%	55%	2.131	0.71	45%
3	3	1	33%	0%	67%	33%	0%	67%	23%	45%	32%	2.358	0.79	10%
4	3	1	30%	0%	70%	30%	0%	70%	31%	45%	24%	2.561	0.85	1%
5	3	1	28%	0%	72%	28%	0%	72%	34%	45%	21%	2.719	0.91	6%
1	3	2	0%	0%	100%	90%	0%	10%	0%	45%	55%	1.910	0.64	90%
2	3	2	0%	0%	100%	90%	0%	10%	0%	45%	55%	1.992	0.66	90%
3	3	2	0%	0%	100%	90%	0%	10%	0%	45%	55%	2.023	0.67	90%
4	3	2	28%	0%	72%	28%	0%	72%	34%	45%	21%	2.186	0.73	5%
5	3	2	28%	0%	72%	28%	0%	72%	34%	45%	21%	2.353	0.78	6%
1	3	3	0%	0%	100%	90%	10%	0%	0%	35%	65%	1.869	0.62	90%
2	3	3	0%	0%	100%	90%	10%	0%	0%	35%	65%	1.977	0.66	90%
3	3	3	0%	0%	100%	90%	10%	0%	0%	35%	65%	2.019	0.67	90%
4	3	3	0%	0%	100%	86%	14%	0%	4%	31%	65%	2.043	0.68	86%
5	3	3	18%	0%	82%	17%	0%	83%	55%	45%	0%	2.149	0.72	38%
1	3	4	0%	0%	100%	90%	10%	0%	0%	35%	65%	1.869	0.62	90%
2	3	4	0%	0%	100%	90%	10%	0%	0%	35%	65%	1.977	0.66	90%
3	3	4	0%	0%	100%	90%	10%	0%	0%	35%	65%	2.019	0.67	90%
4	3	4	0%	0%	100%	90%	10%	0%	0%	35%	65%	2.043	0.68	90%
5	3	4	0%	0%	100%	72%	28%	0%	18%	17%	65%	2.073	0.69	72%
1	3	5	0%	0%	100%	90%	10%	0%	0%	35%	65%	1.869	0.62	90%
2	3	5	0%	35%	65%	90%	10%	0%	0%	0%	100%	1.977	0.66	90%
3	3	5	0%	35%	65%	90%	10%	0%	0%	0%	100%	2.019	0.67	90%
4	3	5	0%	0%	100%	90%	10%	0%	0%	35%	65%	2.043	0.68	90%
5	3	5	0%	0%	100%	90%	10%	0%	0%	35%	65%	2.057	0.69	90%

Intermediate Minority Population - R Majority: $N_{mD} = 0.3,\ N_{nD} = 0.15,\ N_R = 0.55.$

Table 15: Districting Plans Maximizing Minority Total Benefits – State Demographics, open primaries.

B.6 Minority Total Payoffs and Voter Distribution – Ideological Benefits

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	0%	39%	61%	0%	81%	19%	75%	0%	25%	1.752	0.58	75%
2	3	1	46%	0%	54%	0%	100%	0%	29%	20%	51%	1.950	0.65	46%
3	3	1	40%	0%	60%	0%	100%	0%	35%	20%	45%	2.142	0.71	40%
4	3	1	37%	20%	43%	0%	100%	0%	38%	0%	62%	2.275	0.76	38%
5	3	1	35%	0%	65%	6%	94%	0%	34%	26%	40%	2.375	0.79	28%
1	3	2	0%	60%	40%	0%	60%	40%	75%	0%	25%	1.602	0.53	75%
2	3	2	75%	0%	25%	0%	60%	40%	0%	60%	40%	1.752	0.58	75%
3	3	2	31%	20%	49%	0%	100%	0%	44%	0%	56%	1.888	0.63	44%
4	3	2	40%	0%	60%	0%	100%	0%	35%	20%	45%	2.029	0.68	40%
5	3	2	35%	0%	65%	12%	88%	0%	29%	32%	40%	2.151	0.72	23%
1	3	3	0%	100%	0%	0%	20%	80%	75%	0%	25%	1.502	0.50	75%
2	3	3	0%	98%	2%	0%	22%	78%	75%	0%	25%	1.669	0.56	75%
3	3	3	0%	54%	46%	0%	66%	34%	75%	0%	25%	1.752	0.58	75%
4	3	3	40%	0%	60%	0%	100%	0%	35%	20%	45%	1.864	0.62	40%
5	3	3	25%	39%	36%	19%	81%	0%	31%	0%	69%	2.004	0.67	11%
1	3	4	0%	48%	52%	0%	47%	53%	75%	25%	0%	1.463	0.49	75%
2	3	4	0%	95%	5%	0%	0%	100%	75%	25%	0%	2.696	0.90	75%
3	3	4	0%	81%	19%	0%	14%	86%	75%	25%	0%	1.713	0.57	75%
4	3	4	0%	0%	100%	36%	59%	5%	39%	61%	0%	1.788	0.60	39%
5	3	4	30%	70%	0%	27%	50%	23%	18%	0%	82%	1.911	0.64	12%
1	3	5	0%	11%	89%	0%	84%	16%	75%	25%	0%	1.463	0.49	75%
2	3	5	0%	53%	47%	0%	42%	58%	75%	25%	0%	1.629	0.54	75%
3	3	5	0%	17%	83%	0%	78%	22%	75%	25%	0%	1.713	0.57	75%
4	3	5	0%	48%	52%	0%	47%	53%	75%	25%	0%	1.763	0.59	75%
5	3	5	34%	66%	0%	28%	54%	18%	13%	0%	87%	1.865	0.62	20%

Low Minority Benefit from Majority Democrat Candidate: $\beta = 0$.

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	75%	0%	25%	0%	60%	40%	0%	60%	40%	1.962	0.65	75%
2	3	1	29%	20%	51%	0%	100%	0%	46%	0%	54%	2.160	0.72	46%
3	3	1	35%	20%	45%	0%	100%	0%	40%	0%	60%	2.352	0.78	40%
4	3	1	38%	0%	62%	0%	100%	0%	37%	20%	43%	2.485	0.83	38%
5	3	1	35%	0%	65%	6%	94%	0%	34%	26%	40%	2.585	0.86	28%
1	3	2	0%	60%	40%	0%	60%	40%	75%	0%	25%	1.812	0.60	75%
2	3	2	75%	0%	25%	0%	60%	40%	0%	60%	40%	1.962	0.65	75%
3	3	2	31%	20%	49%	0%	100%	0%	44%	0%	56%	2.098	0.70	44%
4	3	2	40%	0%	60%	0%	100%	0%	35%	20%	45%	2.239	0.75	40%
5	3	2	29%	32%	40%	12%	88%	0%	35%	0%	65%	2.361	0.79	23%
1	3	3	0%	100%	0%	0%	20%	80%	75%	0%	25%	1.712	0.57	75%
2	3	3	0%	100%	0%	0%	20%	80%	75%	0%	25%	1.879	0.63	75%
3	3	3	0%	43%	57%	0%	77%	23%	75%	0%	25%	1.962	0.65	75%
4	3	3	40%	0%	60%	0%	100%	0%	35%	20%	45%	2.074	0.69	40%
5	3	3	25%	39%	36%	19%	81%	0%	31%	0%	69%	2.214	0.74	11%
1	3	4	0%	0%	100%	0%	95%	5%	75%	25%	0%	1.673	0.56	75%
2	3	4	0%	95%	5%	0%	0%	100%	75%	25%	0%	1.839	0.61	75%
3	3	4	0%	95%	5%	0%	0%	100%	75%	25%	0%	1.923	0.64	75%
4	3	4	0%	58%	42%	38%	62%	0%	37%	0%	63%	1.982	0.66	38%
5	3	4	30%	70%	0%	27%	50%	23%	18%	0%	82%	2.121	0.71	12%
1	3	5	0%	0%	100%	0%	95%	5%	75%	25%	0%	1.673	0.56	75%
2	3	5	0%	0%	100%	0%	95%	5%	75%	25%	0%	1.839	0.61	75%
3	3	5	0%	65%	35%	0%	30%	70%	75%	25%	0%	1.923	0.64	75%
4	3	5	0%	0%	100%	8%	87%	5%	67%	33%	0%	1.981	0.66	67%
5	3	5	37%	58%	5%	38%	62%	0%	0%	0%	100%	2.102	0.70	38%

Low Minority Benefit from Majority Democrat Candidate: $\beta = 0.25$.

Table 16: Districting Plans Maximizing Minority Total Benefits – Low nD Benefit, closed primaries.

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	0%	60%	40%	75%	0%	25%	0%	60%	40%	2.382	0.79	75%
2	3	1	0%	100%	0%	46%	0%	54%	29%	20%	51%	2.580	0.86	46%
3	3	1	0%	100%	0%	40%	0%	60%	35%	20%	45%	2.772	0.92	40%
4	3	1	0%	100%	0%	38%	0%	62%	37%	20%	43%	2.905	0.97	38%
5	3	1	6%	94%	0%	35%	0%	65%	34%	26%	40%	3.005	1.00	28%
1	3	2	0%	84%	16%	75%	0%	25%	0%	36%	64%	2.232	0.74	75%
2	3	2	0%	60%	40%	75%	0%	25%	0%	60%	40%	2.382	0.79	75%
3	3	2	0%	100%	0%	44%	0%	56%	31%	20%	49%	2.518	0.84	44%
4	3	2	0%	100%	0%	40%	0%	60%	35%	20%	45%	2.659	0.89	40%
5	3	2	12%	88%	0%	35%	0%	65%	29%	32%	40%	2.781	0.93	23%
1	3	3	0%	100%	0%	75%	0%	25%	0%	20%	80%	2.132	0.71	75%
2	3	3	0%	100%	0%	75%	0%	25%	0%	20%	80%	2.299	0.77	75%
3	3	3	0%	20%	80%	75%	0%	25%	0%	100%	0%	2.382	0.79	75%
4	3	3	0%	100%	0%	40%	0%	60%	35%	20%	45%	2.494	0.83	40%
5	3	3	19%	81%	0%	31%	0%	69%	25%	39%	36%	2.634	0.88	11%
1	3	4	0%	40%	60%	75%	25%	0%	0%	55%	45%	2.093	0.70	75%
2	3	4	0%	0%	100%	75%	25%	0%	0%	95%	5%	2.259	0.75	75%
3	3	4	0%	0%	100%	75%	25%	0%	0%	95%	5%	2.343	0.78	75%
4	3	4	0%	0%	100%	39%	61%	0%	36%	59%	5%	2.418	0.81	39%
5	3	4	30%	70%	0%	18%	0%	82%	27%	50%	23%	2.541	0.85	12%
1	3	5	0%	0%	100%	75%	25%	0%	0%	95%	5%	2.093	0.70	75%
2	3	5	0%	0%	100%	75%	25%	0%	0%	95%	5%	2.259	0.75	75%
3	3	5	0%	0%	100%	75%	25%	0%	0%	95%	5%	2.343	0.78	75%
4	3	5	43%	57%	0%	32%	63%	5%	0%	0%	100%	2.412	0.80	43%
5	3	5	0%	0%	100%	37%	58%	5%	38%	62%	0%	2.522	0.84	38%

Low Minority Benefit from Majority Democrat Candidate: $\beta = 0.75$.

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	75%	0%	25%	0%	60%	40%	0%	60%	40%	2.592	0.86	75%
2	3	1	29%	20%	51%	0%	100%	0%	46%	0%	54%	2.790	0.93	46%
3	3	1	40%	0%	60%	0%	100%	0%	35%	20%	45%	2.982	0.99	40%
4	3	1	37%	20%	43%	0%	100%	0%	38%	0%	62%	3.115	1.04	38%
5	3	1	35%	0%	65%	6%	94%	0%	34%	26%	40%	3.215	1.07	28%
1	3	2	0%	100%	0%	75%	0%	25%	0%	20%	80%	2.442	0.81	75%
2	3	2	0%	60%	40%	0%	60%	40%	75%	0%	25%	2.592	0.86	75%
3	3	2	31%	20%	49%	0%	100%	0%	44%	0%	56%	2.728	0.91	44%
4	3	2	35%	20%	45%	0%	100%	0%	40%	0%	60%	2.869	0.96	40%
5	3	2	29%	32%	40%	12%	88%	0%	35%	0%	65%	2.991	1.00	23%
1	3	3	0%	53%	47%	0%	67%	33%	75%	0%	25%	2.342	0.78	75%
2	3	3	0%	100%	0%	0%	20%	80%	75%	0%	25%	2.509	0.84	75%
3	3	3	0%	57%	43%	0%	63%	37%	75%	0%	25%	2.592	0.86	75%
4	3	3	40%	0%	60%	0%	100%	0%	35%	20%	45%	2.704	0.90	40%
5	3	3	25%	39%	36%	19%	81%	0%	31%	0%	69%	2.844	0.95	11%
1	3	4	0%	95%	5%	0%	0%	100%	75%	25%	0%	2.303	0.77	75%
2	3	4	0%	0%	100%	0%	95%	5%	75%	25%	0%	2.469	0.82	75%
3	3	4	0%	95%	5%	0%	0%	100%	75%	25%	0%	2.638	0.88	75%
4	3	4	0%	0%	100%	39%	61%	0%	36%	59%	5%	2.628	0.88	39%
5	3	4	30%	70%	0%	27%	50%	23%	18%	0%	82%	2.751	0.92	12%
1	3	5	0%	0%	100%	0%	95%	5%	75%	25%	0%	2.303	0.77	75%
2	3	5	0%	95%	5%	0%	0%	100%	75%	25%	0%	2.469	0.82	75%
3	3	5	0%	73%	27%	0%	22%	78%	75%	25%	0%	2.553	0.85	75%
4	3	5	32%	63%	5%	0%	0%	100%	43%	57%	0%	2.622	0.87	43%
5	3	5	38%	62%	0%	37%	58%	5%	0%	0%	100%	2.732	0.91	38%

Low Minority Benefit from Majority Democrat Candidate: $\beta = 1$.

Table 17: Districting Plans Maximizing Minority Total Benefits – High nD Benefit, closed primaries.

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	0%	100%	0%	75%	0%	25%	0%	20%	80%	1.531	0.51	75%
2	3	1	0%	100%	0%	75%	0%	25%	0%	20%	80%	1.638	0.55	75%
3	3	1	18%	0%	82%	57%	20%	23%	0%	100%	0%	1.760	0.59	57%
4	3	1	24%	76%	0%	19%	0%	81%	32%	44%	24%	1.912	0.64	13%
5	3	1	29%	71%	0%	18%	0%	82%	29%	49%	23%	2.074	0.69	11%
1	3	2	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.453	0.48	75%
2	3	2	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.619	0.54	75%
3	3	2	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.703	0.57	75%
4	3	2	0%	0%	100%	34%	61%	5%	41%	59%	0%	1.779	0.59	41%
5	3	2	37%	63%	0%	6%	0%	94%	32%	57%	11%	1.896	0.63	31%
1	3	3	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.453	0.48	75%
2	3	3	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.619	0.54	75%
3	3	3	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.703	0.57	75%
4	3	3	0%	0%	100%	30%	65%	5%	45%	55%	0%	1.773	0.59	45%
5	3	3	35%	60%	5%	0%	0%	100%	40%	60%	0%	1.883	0.63	40%
1	3	4	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.453	0.48	75%
2	3	4	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.619	0.54	75%
3	3	4	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.703	0.57	75%
4	3	4	0%	0%	100%	27%	68%	5%	48%	52%	0%	1.767	0.59	48%
5	3	4	0%	0%	100%	35%	60%	5%	40%	60%	0%	1.877	0.63	40%
1	3	5	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.453	0.48	75%
2	3	5	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.619	0.54	75%
3	3	5	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.703	0.57	75%
4	3	5	0%	0%	100%	23%	72%	5%	52%	48%	0%	1.763	0.59	52%
5	3	5	0%	0%	100%	41%	59%	0%	34%	61%	5%	1.871	0.62	41%

Low Minority Benefit from Majority Democrat Candidate: $\beta = 0$.

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	0%	100%	0%	0%	20%	80%	75%	0%	25%	1.800	0.60	75%
2	3	1	0%	20%	80%	0%	100%	0%	75%	0%	25%	1.907	0.64	75%
3	3	1	44%	20%	36%	0%	100%	0%	31%	0%	69%	2.058	0.69	44%
4	3	1	38%	30%	32%	10%	90%	0%	27%	0%	73%	2.198	0.73	27%
5	3	1	29%	44%	27%	24%	76%	0%	22%	0%	78%	2.350	0.78	7%
1	3	2	0%	100%	0%	0%	0%	100%	75%	20%	5%	1.668	0.56	75%
2	3	2	0%	95%	5%	0%	0%	100%	75%	25%	0%	1.833	0.61	75%
3	3	2	0%	0%	100%	0%	95%	5%	75%	25%	0%	1.916	0.64	75%
4	3	2	37%	63%	0%	32%	57%	11%	6%	0%	94%	1.999	0.67	30%
5	3	2	31%	69%	0%	28%	51%	20%	15%	0%	85%	2.146	0.72	16%
1	3	3	0%	95%	5%	0%	0%	100%	75%	25%	0%	1.666	0.56	75%
2	3	3	0%	95%	5%	0%	0%	100%	75%	25%	0%	1.833	0.61	75%
3	3	3	0%	95%	5%	0%	0%	100%	75%	25%	0%	1.916	0.64	75%
4	3	3	33%	62%	5%	0%	0%	100%	42%	58%	0%	1.989	0.66	42%
5	3	3	39%	61%	0%	36%	59%	5%	0%	0%	100%	2.100	0.70	39%
1	3	4	0%	95%	5%	0%	0%	100%	75%	25%	0%	1.666	0.56	75%
2	3	4	0%	95%	5%	0%	0%	100%	75%	25%	0%	1.833	0.61	75%
3	3	4	0%	95%	5%	0%	0%	100%	75%	25%	0%	1.916	0.64	75%
4	3	4	29%	66%	5%	0%	0%	100%	46%	54%	0%	1.983	0.66	46%
5	3	4	40%	60%	0%	0%	0%	100%	35%	60%	5%	2.093	0.70	40%
1	3	5	0%	0%	100%	0%	95%	5%	75%	25%	0%	1.666	0.56	75%
2	3	5	0%	95%	5%	0%	0%	100%	75%	25%	0%	1.833	0.61	75%
3	3	5	0%	95%	5%	0%	0%	100%	75%	25%	0%	1.916	0.64	75%
4	3	5	0%	0%	100%	25%	70%	5%	50%	50%	0%	1.979	0.66	50%
5	3	5	35%	60%	5%	0%	0%	100%	40%	60%	0%	2.087	0.70	40%

Low Minority Benefit from Majority Democrat Candidate: $\beta = 0.25$.

Table 18: Districting Plans Maximizing Minority Total Benefits – Low nD Benefit, open primaries.

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	0%	60%	40%	75%	0%	25%	0%	60%	40%	2.342	0.78	75%
2	3	1	0%	100%	0%	49%	0%	51%	26%	20%	54%	2.496	0.83	49%
3	3	1	0%	100%	0%	40%	0%	60%	35%	20%	45%	2.686	0.90	40%
4	3	1	0%	100%	0%	38%	0%	62%	37%	20%	43%	2.819	0.94	38%
5	3	1	31%	33%	36%	31%	0%	69%	13%	87%	0%	2.933	0.98	18%
1	3	2	0%	60%	40%	75%	0%	25%	0%	60%	40%	2.192	0.73	75%
2	3	2	0%	60%	40%	75%	0%	25%	0%	60%	40%	2.342	0.78	75%
3	3	2	0%	100%	0%	47%	0%	53%	28%	20%	52%	2.433	0.81	47%
4	3	2	0%	100%	0%	40%	0%	60%	35%	20%	45%	2.573	0.86	40%
5	3	2	31%	0%	69%	18%	82%	0%	26%	38%	36%	2.714	0.90	13%
1	3	3	0%	48%	52%	75%	25%	0%	0%	47%	53%	2.100	0.70	75%
2	3	3	0%	48%	52%	75%	25%	0%	0%	47%	53%	2.267	0.76	75%
3	3	3	0%	47%	53%	75%	25%	0%	0%	48%	52%	2.350	0.78	75%
4	3	3	25%	40%	35%	25%	40%	35%	25%	40%	35%	2.433	0.81	0%
5	3	3	25%	40%	35%	25%	40%	35%	25%	40%	35%	2.581	0.86	0%
1	3	4	0%	48%	52%	75%	25%	0%	0%	47%	53%	2.100	0.70	75%
2	3	4	0%	48%	52%	75%	25%	0%	0%	47%	53%	2.267	0.76	75%
3	3	4	0%	48%	52%	75%	25%	0%	0%	47%	53%	2.350	0.78	75%
4	3	4	33%	62%	5%	42%	58%	0%	0%	0%	100%	2.416	0.81	42%
5	3	4	0%	0%	100%	38%	62%	0%	37%	58%	5%	2.527	0.84	38%
1	3	5	0%	48%	52%	75%	25%	0%	0%	47%	53%	2.100	0.70	75%
2	3	5	0%	48%	52%	75%	25%	0%	0%	47%	53%	2.267	0.76	75%
3	3	5	0%	48%	52%	75%	25%	0%	0%	47%	53%	2.350	0.78	75%
4	3	5	0%	0%	100%	45%	55%	0%	30%	65%	5%	2.410	0.80	45%
5	3	5	0%	0%	100%	39%	61%	0%	36%	59%	5%	2.520	0.84	39%

Low Minority Benefit from Majority Democrat Candidate: $\beta = 0.75$.

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	0%	60%	40%	0%	60%	40%	75%	0%	25%	2.635	0.88	75%
2	3	1	28%	20%	52%	0%	100%	0%	47%	0%	53%	2.812	0.94	47%
3	3	1	34%	20%	46%	0%	100%	0%	41%	0%	59%	3.002	1.00	41%
4	3	1	39%	0%	61%	0%	100%	0%	36%	20%	44%	3.134	1.04	39%
5	3	1	35%	0%	65%	8%	92%	0%	32%	28%	40%	3.237	1.08	27%
1	3	2	0%	60%	40%	0%	60%	40%	75%	0%	25%	2.485	0.83	75%
2	3	2	0%	60%	40%	0%	60%	40%	75%	0%	25%	2.635	0.88	75%
3	3	2	45%	0%	55%	0%	100%	0%	30%	20%	50%	2.749	0.92	45%
4	3	2	34%	20%	46%	0%	100%	0%	41%	0%	59%	2.890	0.96	41%
5	3	2	28%	32%	40%	12%	88%	0%	35%	0%	65%	3.015	1.00	23%
1	3	3	0%	60%	40%	0%	60%	40%	75%	0%	25%	2.385	0.80	75%
2	3	3	0%	60%	40%	0%	60%	40%	75%	0%	25%	2.552	0.85	75%
3	3	3	0%	60%	40%	0%	60%	40%	75%	0%	25%	2.635	0.88	75%
4	3	3	25%	40%	35%	25%	40%	35%	25%	40%	35%	2.740	0.91	0%
5	3	3	25%	40%	35%	25%	40%	35%	25%	40%	35%	2.888	0.96	0%
1	3	4	0%	48%	52%	0%	47%	53%	75%	25%	0%	2.348	0.78	75%
2	3	4	0%	47%	53%	0%	48%	52%	75%	25%	0%	2.514	0.84	75%
3	3	4	0%	47%	53%	0%	48%	52%	75%	25%	0%	2.598	0.87	75%
4	3	4	17%	39%	43%	0%	38%	62%	58%	42%	0%	2.650	0.88	58%
5	3	4	25%	40%	35%	25%	40%	35%	25%	40%	35%	2.791	0.93	0%
1	3	5	0%	47%	53%	0%	48%	52%	75%	25%	0%	2.348	0.78	75%
2	3	5	0%	47%	53%	0%	48%	52%	75%	25%	0%	2.514	0.84	75%
3	3	5	0%	48%	52%	0%	47%	53%	75%	25%	0%	2.598	0.87	75%
4	3	5	0%	48%	52%	0%	47%	53%	75%	25%	0%	2.648	0.88	75%
5	3	5	36%	59%	5%	39%	61%	0%	0%	0%	100%	2.737	0.91	39%

Low Minority Benefit from Majority Democrat Candidate: $\beta = 1$.

Table 19: Districting Plans Maximizing Minority Total Benefits – High nD Benefit, open primaries.

B.7 Minority Total Payoffs and Voter Distribution – Crossover Rates

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{MD}	π_R	n_{mD1}	n_{MD1}	n_{R1}	n_{mD2}	n_{MD2}	n_{R2}	n_{mD3}	n_{MD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	0%	60%	40%	75%	0%	25%	0%	60%	40%	2.124	0.71	75%
2	3	1	0%	100%	0%	46%	0%	54%	29%	20%	51%	2.315	0.77	46%
3	3	1	0%	100%	0%	40%	0%	60%	35%	20%	45%	2.506	0.84	40%
4	3	1	0%	100%	0%	38%	0%	62%	37%	20%	43%	2.639	0.88	38%
5	3	1	2%	98%	0%	37%	0%	63%	36%	22%	42%	2.736	0.91	34%
1	3	2	0%	23%	77%	75%	0%	25%	0%	97%	3%	1.974	0.66	75%
2	3	2	0%	60%	40%	75%	0%	25%	0%	60%	40%	2.124	0.71	75%
3	3	2	0%	100%	0%	44%	0%	56%	31%	20%	49%	2.253	0.75	44%
4	3	2	0%	100%	0%	40%	0%	60%	35%	20%	45%	2.394	0.80	40%
5	3	2	8%	92%	0%	36%	0%	64%	31%	28%	41%	2.508	0.84	28%
1	3	3	0%	100%	0%	75%	0%	25%	0%	20%	80%	1.874	0.62	75%
2	3	3	0%	100%	0%	75%	0%	25%	0%	20%	80%	2.041	0.68	75%
3	3	3	0%	20%	80%	75%	0%	25%	0%	100%	0%	2.124	0.71	75%
4	3	3	0%	100%	0%	40%	0%	60%	35%	20%	45%	2.228	0.74	40%
5	3	3	17%	83%	0%	33%	0%	67%	25%	37%	38%	2.358	0.79	16%
1	3	4	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.823	0.61	75%
2	3	4	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.990	0.66	75%
3	3	4	0%	0%	100%	75%	25%	0%	0%	95%	5%	2.073	0.69	75%
4	3	4	0%	46%	54%	26%	74%	0%	49%	0%	51%	2.131	0.71	49%
5	3	4	25%	48%	27%	22%	0%	78%	28%	72%	0%	2.261	0.75	6%
1	3	5	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.823	0.61	75%
2	3	5	0%	45%	55%	75%	25%	0%	0%	50%	50%	1.990	0.66	75%
3	3	5	0%	43%	57%	75%	25%	0%	0%	52%	48%	2.073	0.69	75%
4	3	5	0%	0%	100%	52%	48%	0%	23%	72%	5%	2.127	0.71	52%
5	3	5	0%	0%	100%	37%	58%	5%	38%	62%	0%	2.236	0.75	38%

Low Majority Democrat Primary Crossover $a^1_{MD} = 0.1$.

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	0%	46%	54%	75%	0%	25%	0%	74%	26%	2.220	0.74	75%
2	3	1	0%	100%	0%	45%	0%	55%	30%	20%	50%	2.426	0.81	45%
3	3	1	0%	100%	0%	40%	0%	60%	35%	20%	45%	2.618	0.87	40%
4	3	1	0%	100%	0%	38%	0%	62%	37%	20%	43%	2.750	0.92	38%
5	3	1	9%	91%	0%	33%	0%	67%	32%	29%	38%	2.856	0.95	24%
1	3	2	0%	60%	40%	75%	0%	25%	0%	60%	40%	2.070	0.69	75%
2	3	2	0%	100%	0%	75%	0%	25%	0%	20%	80%	2.220	0.74	75%
3	3	2	0%	100%	0%	43%	0%	57%	32%	20%	48%	2.363	0.79	43%
4	3	2	0%	100%	0%	40%	0%	60%	35%	20%	45%	2.505	0.84	40%
5	3	2	33%	0%	67%	14%	86%	0%	28%	34%	38%	2.634	0.88	19%
1	3	3	0%	20%	80%	75%	0%	25%	0%	100%	0%	1.970	0.66	75%
2	3	3	0%	100%	0%	75%	0%	25%	0%	20%	80%	2.137	0.71	75%
3	3	3	0%	20%	80%	75%	0%	25%	0%	100%	0%	2.220	0.74	75%
4	3	3	26%	35%	39%	34%	0%	66%	15%	85%	0%	2.345	0.78	19%
5	3	3	25%	41%	34%	29%	0%	71%	21%	79%	0%	2.491	0.83	8%
1	3	4	0%	17%	83%	75%	25%	0%	0%	78%	22%	1.942	0.65	75%
2	3	4	0%	0%	100%	75%	25%	0%	0%	95%	5%	2.109	0.70	75%
3	3	4	0%	27%	73%	75%	25%	0%	0%	68%	32%	2.192	0.73	75%
4	3	4	0%	0%	100%	39%	61%	0%	36%	59%	5%	2.283	0.76	39%
5	3	4	32%	68%	0%	15%	0%	85%	29%	52%	20%	2.403	0.80	17%
1	3	5	0%	48%	52%	75%	25%	0%	0%	47%	53%	1.942	0.65	75%
2	3	5	0%	0%	100%	75%	25%	0%	0%	95%	5%	2.109	0.70	75%
3	3	5	0%	67%	33%	75%	25%	0%	0%	28%	72%	2.192	0.73	75%
4	3	5	0%	0%	100%	41%	59%	0%	34%	61%	5%	2.277	0.76	41%
5	3	5	0%	0%	100%	38%	62%	0%	37%	58%	5%	2.388	0.80	38%

High Majority Democrat Primary Crossover $a^1_{nD} = 0.5$.

Table 20: Districting Plans Maximizing Minority Total Benefits – Primary Crossover, closed primaries.

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{MD}	π_R	n_{mD1}	n_{MD1}	n_{R1}	n_{mD2}	n_{MD2}	n_{R2}	n_{mD3}	n_{MD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	0%	60%	40%	75%	0%	25%	0%	60%	40%	1.956	0.65	75%
2	3	1	0%	100%	0%	49%	0%	51%	26%	20%	54%	2.106	0.70	49%
3	3	1	0%	100%	0%	42%	0%	58%	33%	20%	47%	2.293	0.76	42%
4	3	1	0%	100%	0%	39%	0%	61%	36%	20%	44%	2.425	0.81	39%
5	3	1	0%	100%	0%	38%	0%	62%	37%	20%	43%	2.521	0.84	38%
1	3	2	0%	60%	40%	75%	0%	25%	0%	60%	40%	1.806	0.60	75%
2	3	2	0%	89%	11%	75%	0%	25%	0%	31%	69%	1.956	0.65	75%
3	3	2	0%	100%	0%	49%	0%	51%	26%	20%	54%	2.042	0.68	49%
4	3	2	0%	100%	0%	42%	0%	58%	33%	20%	47%	2.181	0.73	42%
5	3	2	0%	100%	0%	41%	0%	59%	34%	20%	46%	2.289	0.76	41%
1	3	3	0%	31%	69%	75%	0%	25%	0%	89%	11%	1.706	0.57	75%
2	3	3	0%	85%	15%	75%	0%	25%	0%	35%	65%	1.873	0.62	75%
3	3	3	0%	97%	3%	75%	0%	25%	0%	23%	77%	1.956	0.65	75%
4	3	3	0%	100%	0%	46%	0%	54%	29%	20%	51%	2.016	0.67	46%
5	3	3	0%	100%	0%	41%	0%	59%	34%	20%	46%	2.125	0.71	41%
1	3	4	0%	60%	40%	75%	0%	25%	0%	60%	40%	1.635	0.54	75%
2	3	4	0%	76%	24%	75%	0%	25%	0%	44%	56%	1.806	0.60	75%
3	3	4	0%	60%	40%	75%	0%	25%	0%	60%	40%	1.898	0.63	75%
4	3	4	0%	25%	75%	75%	0%	25%	0%	95%	5%	1.956	0.65	75%
5	3	4	14%	86%	0%	61%	0%	39%	0%	34%	66%	2.005	0.67	61%
1	3	5	0%	45%	55%	75%	25%	0%	0%	50%	50%	1.588	0.53	75%
2	3	5	0%	69%	31%	75%	25%	0%	0%	26%	74%	1.755	0.58	75%
3	3	5	0%	60%	40%	75%	0%	25%	0%	60%	40%	1.849	0.62	75%
4	3	5	0%	60%	40%	75%	0%	25%	0%	60%	40%	1.912	0.64	75%
5	3	5	4%	96%	0%	71%	0%	29%	0%	24%	76%	1.957	0.65	71%

Low Majority Democrat General Crossover $a^2_{MD} = 0.1$.

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	0%	60%	40%	75%	0%	25%	0%	60%	40%	2.100	0.70	75%
2	3	1	0%	100%	0%	47%	0%	53%	28%	20%	52%	2.282	0.76	47%
3	3	1	0%	100%	0%	41%	0%	59%	34%	20%	46%	2.472	0.82	41%
4	3	1	0%	100%	0%	39%	0%	61%	36%	20%	44%	2.605	0.87	39%
5	3	1	0%	100%	0%	38%	0%	62%	37%	20%	43%	2.701	0.90	38%
1	3	2	0%	60%	40%	75%	0%	25%	0%	60%	40%	1.950	0.65	75%
2	3	2	0%	60%	40%	75%	0%	25%	0%	60%	40%	2.100	0.70	75%
3	3	2	0%	100%	0%	45%	0%	55%	30%	20%	50%	2.219	0.74	45%
4	3	2	0%	100%	0%	41%	0%	59%	34%	20%	46%	2.360	0.79	41%
5	3	2	2%	98%	0%	39%	0%	61%	34%	22%	44%	2.469	0.82	37%
1	3	3	0%	92%	8%	75%	0%	25%	0%	28%	72%	1.850	0.62	75%
2	3	3	0%	63%	37%	75%	0%	25%	0%	57%	43%	2.017	0.67	75%
3	3	3	0%	71%	29%	75%	0%	25%	0%	49%	51%	2.100	0.70	75%
4	3	3	0%	100%	0%	41%	0%	59%	34%	20%	46%	2.194	0.73	41%
5	3	3	12%	88%	0%	36%	0%	64%	28%	32%	41%	2.312	0.77	24%
1	3	4	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.784	0.59	75%
2	3	4	0%	75%	25%	75%	0%	25%	0%	45%	55%	1.950	0.65	75%
3	3	4	0%	60%	40%	75%	0%	25%	0%	60%	40%	2.042	0.68	75%
4	3	4	0%	100%	0%	75%	0%	25%	0%	20%	80%	2.100	0.70	75%
5	3	4	25%	75%	0%	28%	0%	72%	22%	45%	33%	2.209	0.74	6%
1	3	5	0%	60%	40%	75%	25%	0%	0%	35%	65%	1.784	0.59	75%
2	3	5	0%	92%	8%	75%	25%	0%	0%	3%	97%	1.951	0.65	75%
3	3	5	0%	0%	100%	75%	25%	0%	0%	95%	5%	2.034	0.68	75%
4	3	5	0%	0%	100%	75%	25%	0%	0%	95%	5%	2.084	0.69	75%
5	3	5	0%	0%	100%	38%	62%	0%	37%	58%	5%	2.176	0.73	38%

High Majority Democrat General Crossover $a^2_{nD} = 0.5$.

Table 21: Districting Plans Maximizing Minority Total Benefits – General Crossover, closed primaries.

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	0%	100%	0%	75%	0%	25%	0%	20%	80%	2.027	0.68	75%
2	3	1	0%	100%	0%	63%	0%	37%	12%	20%	68%	2.138	0.71	63%
3	3	1	0%	100%	0%	39%	0%	61%	36%	20%	44%	2.321	0.77	39%
4	3	1	0%	100%	0%	36%	0%	64%	39%	20%	41%	2.455	0.82	39%
5	3	1	17%	83%	0%	28%	0%	72%	30%	37%	33%	2.578	0.86	14%
1	3	2	0%	100%	0%	75%	0%	25%	0%	20%	80%	1.877	0.63	75%
2	3	2	0%	100%	0%	75%	0%	25%	0%	20%	80%	2.027	0.68	75%
3	3	2	0%	100%	0%	75%	0%	25%	0%	20%	80%	2.095	0.70	75%
4	3	2	9%	91%	0%	33%	0%	67%	32%	29%	38%	2.211	0.74	24%
5	3	2	23%	77%	0%	26%	0%	74%	26%	43%	31%	2.362	0.79	4%
1	3	3	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.822	0.61	75%
2	3	3	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.989	0.66	75%
3	3	3	0%	0%	100%	75%	25%	0%	0%	95%	5%	2.072	0.69	75%
4	3	3	0%	0%	100%	31%	64%	5%	44%	56%	0%	2.132	0.71	44%
5	3	3	32%	56%	12%	7%	0%	93%	36%	64%	0%	2.245	0.75	29%
1	3	4	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.822	0.61	75%
2	3	4	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.989	0.66	75%
3	3	4	0%	0%	100%	75%	25%	0%	0%	95%	5%	2.072	0.69	75%
4	3	4	0%	0%	100%	53%	47%	0%	22%	73%	5%	2.127	0.71	53%
5	3	4	0%	0%	100%	36%	59%	5%	39%	61%	0%	2.236	0.75	39%
1	3	5	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.822	0.61	75%
2	3	5	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.989	0.66	75%
3	3	5	0%	0%	100%	75%	25%	0%	0%	95%	5%	2.072	0.69	75%
4	3	5	0%	0%	100%	63%	37%	0%	12%	83%	5%	2.124	0.71	63%
5	3	5	0%	0%	100%	40%	60%	0%	35%	60%	5%	2.230	0.74	40%

Low Majority Democrat Primary Crossover $a^1_{nD} = 0.1$.

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	0%	100%	0%	75%	0%	25%	0%	20%	80%	2.110	0.70	75%
2	3	1	0%	100%	0%	55%	0%	45%	20%	20%	60%	2.228	0.74	55%
3	3	1	0%	100%	0%	37%	0%	63%	38%	20%	42%	2.420	0.81	38%
4	3	1	33%	0%	67%	39%	23%	38%	3%	97%	0%	2.555	0.85	36%
5	3	1	25%	0%	75%	20%	80%	0%	30%	40%	30%	2.696	0.90	9%
1	3	2	0%	100%	0%	75%	0%	25%	0%	20%	80%	1.960	0.65	75%
2	3	2	0%	100%	0%	75%	0%	25%	0%	20%	80%	2.110	0.70	75%
3	3	2	0%	100%	0%	75%	20%	5%	0%	0%	100%	2.190	0.73	75%
4	3	2	23%	0%	77%	29%	43%	28%	23%	77%	0%	2.328	0.78	6%
5	3	2	22%	0%	78%	26%	74%	0%	27%	46%	27%	2.484	0.83	5%
1	3	3	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.937	0.65	75%
2	3	3	0%	0%	100%	75%	25%	0%	0%	95%	5%	2.104	0.70	75%
3	3	3	0%	0%	100%	75%	25%	0%	0%	95%	5%	2.187	0.73	75%
4	3	3	0%	0%	100%	36%	59%	5%	39%	61%	0%	2.279	0.76	39%
5	3	3	38%	62%	0%	0%	0%	100%	37%	58%	5%	2.390	0.80	38%
1	3	4	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.937	0.65	75%
2	3	4	0%	0%	100%	75%	25%	0%	0%	95%	5%	2.104	0.70	75%
3	3	4	0%	0%	100%	75%	25%	0%	0%	95%	5%	2.187	0.73	75%
4	3	4	0%	0%	100%	33%	62%	5%	42%	58%	0%	2.273	0.76	42%
5	3	4	0%	0%	100%	36%	59%	5%	39%	61%	0%	2.384	0.79	39%
1	3	5	0%	0%	100%	75%	25%	0%	0%	95%	5%	1.937	0.65	75%
2	3	5	0%	0%	100%	75%	25%	0%	0%	95%	5%	2.104	0.70	75%
3	3	5	0%	0%	100%	75%	25%	0%	0%	95%	5%	2.187	0.73	75%
4	3	5	0%	0%	100%	44%	56%	0%	31%	64%	5%	2.268	0.76	44%
5	3	5	0%	0%	100%	40%	60%	0%	35%	60%	5%	2.378	0.79	40%

High Majority Democrat Primary Crossover $a^1_{nD} = 0.5$.

Table 22: Districting Plans Maximizing Minority Total Benefits – Primary Crossover, open primaries.

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	0%	60%	40%	75%	0%	25%	0%	60%	40%	1.920	0.64	75%
2	3	1	0%	60%	40%	75%	0%	25%	0%	60%	40%	2.027	0.68	75%
3	3	1	0%	100%	0%	44%	0%	56%	31%	20%	49%	2.142	0.71	44%
4	3	1	0%	100%	0%	40%	0%	60%	35%	20%	45%	2.273	0.76	40%
5	3	1	0%	100%	0%	38%	0%	62%	37%	20%	43%	2.369	0.79	38%
1	3	2	0%	60%	40%	75%	0%	25%	0%	60%	40%	1.770	0.59	75%
2	3	2	0%	60%	40%	75%	0%	25%	0%	60%	40%	1.920	0.64	75%
3	3	2	0%	60%	40%	75%	0%	25%	0%	60%	40%	1.988	0.66	75%
4	3	2	0%	100%	0%	48%	0%	52%	27%	20%	53%	2.031	0.68	48%
5	3	2	0%	100%	0%	42%	0%	58%	33%	20%	47%	2.138	0.71	42%
1	3	3	0%	60%	40%	75%	0%	25%	0%	60%	40%	1.670	0.56	75%
2	3	3	0%	60%	40%	75%	0%	25%	0%	60%	40%	1.837	0.61	75%
3	3	3	0%	60%	40%	75%	0%	25%	0%	60%	40%	1.920	0.64	75%
4	3	3	0%	60%	40%	75%	0%	25%	0%	60%	40%	1.970	0.66	75%
5	3	3	25%	40%	35%	25%	40%	35%	25%	40%	35%	2.008	0.67	0%
1	3	4	0%	48%	52%	75%	25%	0%	0%	48%	52%	1.648	0.55	75%
2	3	4	0%	48%	52%	75%	25%	0%	0%	47%	53%	1.814	0.60	75%
3	3	4	0%	48%	52%	75%	25%	0%	0%	47%	53%	1.898	0.63	75%
4	3	4	0%	48%	52%	75%	25%	0%	0%	47%	53%	1.948	0.65	75%
5	3	4	0%	47%	53%	75%	25%	0%	0%	48%	52%	1.981	0.66	75%
1	3	5	0%	48%	52%	75%	25%	0%	0%	47%	53%	1.648	0.55	75%
2	3	5	0%	47%	53%	75%	25%	0%	0%	48%	52%	1.814	0.60	75%
3	3	5	0%	48%	52%	75%	25%	0%	0%	47%	53%	1.898	0.63	75%
4	3	5	0%	48%	52%	75%	25%	0%	0%	47%	53%	1.948	0.65	75%
5	3	5	0%	48%	52%	75%	25%	0%	0%	47%	53%	1.981	0.66	75%

Low Majority Democrat General Crossover $a^2_{nD} = 0.1$.

Group Power			District 1			District 2			District 3			Benefits		
π_{mD}	π_{nD}	π_R	n_{mD1}	n_{nD1}	n_{R1}	n_{mD2}	n_{nD2}	n_{R2}	n_{mD3}	n_{nD3}	n_{R3}	Total	Aver.	$R(\mathbf{D})$
1	3	1	0%	60%	40%	75%	0%	25%	0%	60%	40%	2.006	0.67	75%
2	3	1	0%	100%	0%	67%	0%	33%	8%	20%	72%	2.116	0.71	67%
3	3	1	0%	100%	0%	40%	0%	60%	35%	20%	45%	2.293	0.76	40%
4	3	1	0%	100%	0%	37%	0%	63%	38%	20%	42%	2.427	0.81	38%
5	3	1	30%	0%	70%	32%	33%	35%	13%	87%	0%	2.538	0.85	19%
1	3	2	0%	60%	40%	75%	0%	25%	0%	60%	40%	1.856	0.62	75%
2	3	2	0%	60%	40%	75%	0%	25%	0%	60%	40%	2.006	0.67	75%
3	3	2	0%	60%	40%	75%	0%	25%	0%	60%	40%	2.074	0.69	75%
4	3	2	0%	100%	0%	39%	0%	61%	36%	20%	44%	2.181	0.73	39%
5	3	2	29%	0%	71%	26%	39%	34%	19%	81%	0%	2.319	0.77	10%
1	3	3	0%	14%	86%	75%	25%	0%	0%	81%	19%	1.784	0.59	75%
2	3	3	0%	57%	43%	75%	25%	0%	0%	38%	62%	1.951	0.65	75%
3	3	3	0%	0%	100%	75%	25%	0%	0%	95%	5%	2.034	0.68	75%
4	3	3	0%	95%	5%	75%	25%	0%	0%	0%	100%	2.084	0.69	75%
5	3	3	33%	67%	0%	13%	0%	87%	29%	53%	18%	2.191	0.73	20%
1	3	4	0%	50%	50%	75%	25%	0%	0%	45%	55%	1.784	0.59	75%
2	3	4	0%	48%	52%	75%	25%	0%	0%	47%	53%	1.951	0.65	75%
3	3	4	0%	0%	100%	75%	25%	0%	0%	95%	5%	2.034	0.68	75%
4	3	4	0%	13%	87%	75%	25%	0%	0%	82%	18%	2.084	0.69	75%
5	3	4	0%	0%	100%	35%	60%	5%	40%	60%	0%	2.177	0.73	40%
1	3	5	0%	73%	27%	75%	25%	0%	0%	22%	78%	1.784	0.59	75%
2	3	5	75%	25%	0%	0%	48%	52%	0%	47%	53%	1.951	0.65	75%
3	3	5	0%	48%	52%	75%	25%	0%	0%	47%	53%	2.034	0.68	75%
4	3	5	0%	73%	27%	75%	25%	0%	0%	22%	78%	2.084	0.69	75%
5	3	5	0%	0%	100%	34%	61%	5%	41%	59%	0%	2.171	0.72	41%

High Majority Democrat General Crossover $a^2_{nD} = 0.5$.

Table 23: Districting Plans Maximizing Minority Total Benefits – General Crossover, open primaries.