

B Supplemental Appendix: Online

B.1 Market Equilibrium: Detailed Comparative Statics

To save on notation, we keep $\Delta(\rho|x^+) = \rho(s^+|x^+) - \rho(s^-|x^+) - \rho(s^+) + \rho(s^-)$ for the following equations. For an overview of all selected comparative statics see Table 3.

B.1.1 Equilibrium Equations for Interior Solution

Clients per firm: $\frac{\partial G(\cdot)}{\partial n^l} - \frac{PA\pi}{C} = 0.$

Lobbying service fee: $\frac{\partial G(\cdot)}{\partial n^l} - k = 0.$

Lobbyists: $C - n^l L = 0.$

Political access per firm: $a^l - \frac{PA}{L}.$

Verification per firm: $\frac{\partial H(\cdot)}{\partial m^l} - \frac{\rho(x^+)s}{\alpha T} \Delta(\rho|x^+) = 0.$

Client portfolio per firm: $n^l - m^l - u^l - r^l = 0.$

Presentation portfolio per firm: $a^l - \rho(x^+)m^l - u^l = 0.$

Financial contribution per firm: $f^l - n^l k + H(m^l) + G(n^l) = 0.$

Entrepreneurs: $T - \bar{P} - \bar{C} - L - J = 0.$

B.1.2 Determinant of Jacobian

The determinant of the Jacobian for the system above is nonzero:

$$|J| = n^l \frac{\partial^2 G(\cdot)}{\partial n^{l2}} \frac{\partial^2 H(\cdot)}{\partial m^{l2}} > 0.$$

B.1.3 Selected Nonzero Results

1. Dishonesty/Effectiveness of Financial Contributions on

(a) Verification per Firm: $\frac{\partial m^l}{\partial \alpha} = -\frac{s\rho(x^+)\Delta(\rho|x^+)}{\alpha^2 TH''(\cdot)} < 0.$

(b) Financial Contributions per Firm: $\frac{\partial f^l}{\partial \alpha} = \frac{s\rho(x^+)\Delta(\rho|x^+)H'(\cdot)}{\alpha^2 TH''(\cdot)} > 0.$

(c) Unverified Presented Proposals per Firm: $\frac{\partial u^l}{\partial \alpha} = \frac{s\rho(x^+)^2\Delta(\rho|x^+)}{\alpha^2 TH''(\cdot)} > 0.$

2. Magnitude of Spillovers on

(a) Verification per Firm: $\frac{\partial m^l}{\partial s} = \frac{\rho(x^+)\Delta(\rho|x^+)}{\alpha TH''(\cdot)} > 0.$

- (b) Financial Contributions per Firm: $\frac{\partial f^l}{\partial s} = -\frac{\rho(x^+)\Delta(\rho|x^+)H'(\cdot)}{\alpha TH''(\cdot)} < 0$.
- (c) Unverified Presented Proposals per Firm: $\frac{\partial u^l}{\partial s} = -\frac{\rho(x^+)^2\Delta(\rho|x^+)}{\alpha TH''(\cdot)} < 0$.

3. Information Gains from Verification on

- (a) Verification per Firm: $\frac{\partial m^l}{\partial \Delta(\rho|x^+)} = \frac{\rho(x^+)s}{\alpha TH''(\cdot)} > 0$.
- (b) Financial Contributions per Firm: $\frac{\partial f^l}{\partial \Delta(\rho|x^+)} = -\frac{\rho(x^+)sH'(\cdot)}{\alpha TH''(\cdot)} < 0$.
- (c) Unverified Presented Proposals per Firm: $\frac{\partial u^l}{\partial \Delta(\rho|x^+)} = -\frac{\rho(x^+)^2s}{\alpha TH''(\cdot)} < 0$.

4. Magnitude of Private Benefits on

- (a) Financial Contributions per Firm: $\frac{\partial f^l}{\partial \pi} = \frac{PAn}{C} > 0$.
- (b) Unverified Presented Proposals per Firm: $\frac{\partial u^l}{\partial \pi} = \frac{A^2P^2}{C^2G''(\cdot)} > 0$.
- (c) Number of Clients per Firm: $\frac{\partial n^l}{\partial \pi} = \frac{AP}{CG''(\cdot)} > 0$.
- (d) Lobbyists: $\frac{\partial L}{\partial \pi} = -\frac{AP}{n^l{}^2G''(\cdot)} < 0$.

5. Number of Policymakers on

- (a) Financial Contributions per Firm: $\frac{\partial f^l}{\partial P} = \frac{A\pi}{L} > 0$.
- (b) Unverified Presented Proposals per Firm: $\frac{\partial u^l}{\partial P} = \frac{A\left(1+\frac{AP\pi}{CnG''(\cdot)}\right)}{L} > 0$.
- (c) Number of Clients per Firm: $\frac{\partial n^l}{\partial P} = \frac{A\pi}{CG''(\cdot)} > 0$.
- (d) Lobbyists: $\frac{\partial L}{\partial P} = -\frac{A\pi}{n^l{}^2G''(\cdot)} < 0$.

B.2 Comparison of Social Optimum and Market Equilibrium: Detailed Comparative Statics

First, we describe the detailed comparative statics of the socially optimal values. Then we proceed with a detailed description of the comparative statics for the differences between socially optimal values and market equilibrium values.

B.2.1 Social Optimum: Detailed Comparative Statics

To save on notation, we keep $\Delta(\rho|x^+) = \rho(s^+|x^+) - \rho(s^-|x^+) - \rho(s^+) + \rho(s^-)$ for the following equations. For an overview of all selected comparative statics see Table 3.

Equilibrium Equations for Interior Solution The equations of interest are the following.

Verification per firm: $\frac{\partial H(.)}{\partial m^l} + \frac{\partial G(.)}{\partial m^l} - \rho(x^+)s\Delta(\rho|x^+) = 0.$

Presentation portfolio per firm: $g_2 = a^l - \rho(x^+)m^l = 0.$

Access to policymakers: $La^l + A^c - PA^p = 0.$

Citizen-clients: $C - Lm^l - A^c = 0.$

Population: $C + L + P - T = 0.$

Note that $u^* = r^* = En^* = 0$, $n^* = m^*$, k is not determined, and $f^* = 0$ if $\alpha < 1$ but $f^* \geq 0$ if $\alpha = 1$.

Determinant of Jacobian The determinant of the Jacobian for the system above is nonzero:

$$|J| = (A^p + a^l - m^l) \left(\frac{\partial^2 G(.)}{\partial m^{l2}} + \frac{\partial^2 H(.)}{\partial m^{l2}} \right) \neq 0 \text{ iff } A^p \neq \rho(x^-)m^l.$$

Selected Nonzero Results

1. Magnitude of Spillovers on

- (a) Verification per Firm: $\frac{\partial m^l}{\partial s} = \frac{\rho(x^+)\Delta(\rho|x^+)}{G''(.)+H''(.)} > 0.$
- (b) Direct Access for Citizens: $\frac{\partial A^c}{\partial s} = \frac{-A^p L \rho(x^+) \Delta(\rho|x^+)}{(A^p - \rho(x^-)m^*)(G''(.)+H''(.))} \geq 0.$
- (c) Number of Lobbyists: $\frac{\partial L}{\partial s} = \frac{\rho(x^+)\rho(x^-)\Delta(\rho|x^+)L}{(A^p - \rho(x^-)m^*)(G''(.)+H''(.))} \geq 0.$
- (d) Number of Policymakers: $\frac{\partial P}{\partial s} = \frac{-\rho(x^+)\rho(x^-)\Delta(\rho|x^+)L}{(A^p - \rho(x^-)m^*)(G''(.)+H''(.))} \leq 0.$

2. Information Gains from Verification on

- (a) Verification per Firm: $\frac{\partial m^l}{\partial \Delta(\rho|x^+)} = \frac{\rho(x^+)s}{G''(.)+H''(.)} > 0.$
- (b) Direct Access for Citizens: $\frac{\partial A^c}{\partial \Delta(\rho|x^+)} = \frac{-A^p}{(A^p - \rho(x^-)m^*)(G''(.)+H''(.))} \leq 0.$
- (c) Number of Lobbyists: $\frac{\partial L}{\partial \Delta(\rho|x^+)} = \frac{\rho(x^+)\rho(x^-)sL}{(A^p - \rho(x^-)m^*)(G''(.)+H''(.))} \geq 0.$
- (d) Number of Policymakers: $\frac{\partial P}{\partial \Delta(\rho|x^+)} = \frac{-\rho(x^+)\rho(x^-)\Delta(\rho|x^+)L}{(A^p - \rho(x^-)m^*)(G''(.)+H''(.))} \leq 0.$

B.2.2 Comparison: Detailed Comparative Statics

Here, we list the not so obvious nonzero comparative statics summarize all selected comparative statics. For an overview of all selected comparative statics see Table 4.

B.2.3 Selected Nonzero Results

1. Dishonesty/Effectiveness of Financial Contributions on

- (a) Verification per Firm: $\frac{\partial(m^*-m^{**})}{\partial\alpha} = 0 + \frac{s\rho(x^+)\Delta(\rho|x^+)}{\alpha^2 TH''(.)} \gtrless 0$ for $m^* \gtrless m^{**}$.
- (b) Financial Contributions per Firm: $\frac{\partial(f^*-f^{**})}{\partial\alpha} = 0 + \frac{s\rho(x^+)\Delta(\rho|x^+)H'(.)}{\alpha^2 TH''(.)} > 0$ for $f^* \leq f^{**}$.
- (c) Unverified Presented Proposals per Firm: $\frac{\partial(u^*-u^{**})}{\partial\alpha} = 0 - \frac{s\rho(x^+)^2\Delta(\rho|x^+)}{\alpha^2 TH''(.)} > 0$ for $u^* \leq u^{**}$.

2. Magnitude of Spillovers on

- (a) Verification per Firm: $\frac{\partial(m^*-m^{**})}{\partial s} = \frac{\rho(x^+)\Delta(\rho|x^+)}{G''(m^*)+H''(m^*)} - \frac{\rho(x^+)\Delta(\rho|x^+)}{\alpha TH''(m^{**})} \gtrless 0$ for $m^* \gtrless m^{**}$ and $a = \alpha TH''(m^{**}) \gtrless G''(m^*) + H''(m^*) = b$.
 - $\frac{\partial(m^*-m^{**})}{\partial s} > 0$ if $\{m^* > m^{**}; a > b\}$ or $\{m^* < m^{**}; a < b\}$.
 - $\frac{\partial(m^*-m^{**})}{\partial s} < 0$ if $\{m^* > m^{**}; a < b\}$ or $\{m^* < m^{**}; a > b\}$.
- (b) Financial Contributions per Firm: $\frac{\partial(f^*-f^{**})}{\partial s} = 0 - \frac{\rho(x^+)\Delta(\rho|x^+)H'(.)}{\alpha TH''(.)} \leq 0$ for $f^* \leq f^{**}$.
- (c) Unverified Presented Proposals per Firm: $\frac{\partial(u^*-u^{**})}{\partial s} = 0 - \frac{\rho(x^+)^2\Delta(\rho|x^+)}{\alpha TH''(.)} \leq 0$ for $u^* \leq u^{**}$.
- (d) Clients per Firm: $\frac{\partial(n^*-n^{**})}{\partial s} = \frac{\rho(x^+)\Delta(\rho|x^+)}{G''(.)+H''(.)} - 0 \gtrless 0$ for $n^* = m^*$ and $n^* \geq n^{**}$.
- (e) Direct Access for Citizens: $\frac{\partial(A^c-A^{c**})}{\partial s} = \frac{-A^p L \rho(x^+)\Delta(\rho|x^+)}{(A^p - \rho(x^-)m^*)(G''(.)+H''(.))} - 0 \lesseqgtr 0$ for $A^p \gtrless \rho(x^-)m^*$.
- (f) Number of Lobbyists: $\frac{\partial(L^*-L^{**})}{\partial s} = \frac{\rho(x^+)\rho(x^-)\Delta(\rho|x^+)L}{(A^p - \rho(x^-)m^*)(G''(.)+H''(.))} - 0 \gtrless 0$ for $L^* \gtrless L^{**}$ and $A^p \gtrless \rho(x^-)m^*$.
 - $\frac{\partial(L^*-L^{**})}{\partial s} > 0$ if $\{L^* > L^{**}; A^p > \rho(x^-)m^*\}$ or $\{L^* < L^{**}; A^p < \rho(x^-)m^*\}$.
 - $\frac{\partial(L^*-L^{**})}{\partial s} < 0$ if $\{L^* > L^{**}; A^p < \rho(x^-)m^*\}$ or $\{L^* < L^{**}; A^p > \rho(x^-)m^*\}$.

3. Information Gains from Verification on

- (a) Verification per Firm: $\frac{\partial(m^*-m^{**})}{\partial\Delta(\rho|x^+)} = \frac{\rho(x^+)s}{G''(m^*)+H''(m^*)} - \frac{\rho(x^+)s}{\alpha TH''(m^{**})} \gtrless 0$ for $m^* \gtrless m^{**}$
and $a = \alpha TH''(m^{**}) \gtrless G''(m^*) + H''(m^*) = b$.
- $\frac{\partial(m^*-m^{**})}{\partial\Delta(\rho|x^+)} > 0$ if $\{m^* > m^{**}; a > b\}$ or $\{m^* < m^{**}; a < b\}$.
 - $\frac{\partial(m^*-m^{**})}{\partial\Delta(\rho|x^+)} < 0$ if $\{m^* > m^{**}; a < b\}$ or $\{m^* < m^{**}; a > b\}$.
- (b) Financial Contributions per Firm: $\frac{\partial(f^*-f^{**})}{\partial\Delta(\rho|x^+)} = 0 - \frac{\rho(x^+)sH'(\cdot)}{\alpha TH''(\cdot)} \leq 0$ for $f^* \leq f^{**}$.
- (c) Unverified Presented Proposals per Firm: $\frac{\partial(u^*-u^{**})}{\partial\Delta(\rho|x^+)} = 0 - \frac{\rho(x^+)^2s}{\alpha TH''(\cdot)} \leq 0$ for $u^* \leq u^{**}$.
- (d) Clients per Firm: $\frac{\partial(A^{c*}-A^{c**})}{\partial\Delta(\rho|x^+)} = \frac{\rho(x^+)s}{G''(\cdot)+H''(\cdot)} - 0$ for $n^* = m^*$ and $n^* \geq n^{**}$.
- (e) Direct Access for Citizens: $\frac{\partial(A^{c*}-A^{c**})}{\partial\Delta(\rho|x^+)} = \frac{-A^p}{(A^p-\rho(x^-)m^*)(G''(\cdot)+H''(\cdot))} + 0 \gtrless 0$ for $A^p \gtrless \rho(x^-)m^*$.
- (f) Number of Lobbyists: $\frac{\partial(L^*-L^{**})}{\partial\Delta(\rho|x^+)} = \frac{\rho(x^+)\rho(x^-)sL}{(A^p-\rho(x^-)m^*)(G''(\cdot)+H''(\cdot))} - 0 \gtrless 0$ for $L^* \gtrless L^{**}$
and $A^p \gtrless \rho(x^-)m^*$.
- $\frac{\partial(L^*-L^{**})}{\partial s} > 0$ if $\{L^* > L^{**}; A^p > \rho(x^-)m^*\}$ or $\{L^* < L^{**}; A^p < \rho(x^-)m^*\}$.
 - $\frac{\partial(L^*-L^{**})}{\partial s} < 0$ if $\{L^* > L^{**}; A^p < \rho(x^-)m^*\}$ or $\{L^* < L^{**}; A^p > \rho(x^-)m^*\}$.

4. Magnitude of Private Benefits on

- (a) Financial Contributions per Firm: $\frac{\partial(f^*-f^{**})}{\partial\pi} = 0 + \frac{PA^pn}{C} > 0$ for $f^* \leq f^{**}$.
- (b) Unverified Presented Proposals per Firm: $\frac{\partial(u^*-u^{**})}{\partial\pi} = 0 + \frac{A^pP}{CG''(\cdot)} > 0 \leq 0$ for $u^* \leq u^{**}$.
- (c) Clients per Firm: $\frac{\partial(n^*-n^{**})}{\partial\pi} = 0 - \frac{A^pP}{CG''(\cdot)} \leq 0$ for $n^* \gtrless n^{**}$.
- (d) Number of Lobbyists: $\frac{\partial(L^*-L^{**})}{\partial\pi} = 0 + \frac{A^pP}{n^2G''(\cdot)} \gtrless 0$ if $L^* \gtrless L^{**}$.

5. Number of Policymakers on

- (a) Financial Contributions per Firm: $\frac{\partial(f^*-f^{**})}{\partial P} = 0 + \frac{A^p\pi}{L} > 0 > 0$ for $f^* \leq f^{**}$.
- (b) Unverified Presented Proposals per Firm: $\frac{\partial(u^*-u^{**})}{\partial P} = 0 + \frac{A^p(1+\frac{A^pP\pi}{CnG''(\cdot)})}{L} > 0 > 0$
for $u^* \leq u^{**}$.
- (c) Clients per Firm: $\frac{\partial(n^*-n^{**})}{\partial P} = 0 - \frac{A^p\pi}{CG''(\cdot)} \leq 0$ for $n^* \gtrless n^{**}$.

B.2.4 Overview: All Selected Comparative Statics

Comp. Statics Market Outcome						Comp. Statics Social Optimum for $\alpha < 1$						
						$f^* = 0 \quad u^* = 0 \quad n^* = m^*$						
	dm^{**}	df^{**}	du^{**}	dn^{**}	dL^{**}	dm^*	df^*	du^*	dn^*	dA^{c*}	dL^*	dP^*
$d\alpha$	-	+	+	0	0	0	0	0	0	0	0	0
ds	+	-	-	0	0	+	0	0	+	-/+	+/-	-/+
$d\Delta(\rho x^+)$	+	-	-	0	0	+	0	0	+	-/+	+/-	-/+
$d\pi$	0	+	+	+	-	0	0	0	0	0	0	0
dP	0	+	+	+	-	0	0	0	0	NA	NA	NA
						Assumption: $A^p \neq \rho(x^-)m^*$. 3: if $A^p > \rho(x^-)m^*$ / if $A^p < \rho(x^-)m^*$.						

Table 3: Selected Comparative Statics for Market and Socially Optimal Values.

Comp. Statics Differences Socially Optimal and Market Values for $\alpha < 1$						
	$m^* \gtrless m^{**}$	$f^{**} \geq f^*$	$u^{**} \geq u^*$	$n^* \gtrless n^{**}$	$A^{c*} \geq A^{c**} = 0$	$L^* \gtrless L^{**}$
	$d(m^* - m^{**})$	$d(f^* - f^{**})$	$d(u^* - u^{**})$	$d(n^* - n^{**})$	$d(A^{c*} - A^{c**})$	$d(L^* - L^{**})$
$d\alpha$	+/- ¹	+	+	0	0	0
ds	+/- ^{1,2}	-	-	+/- ³	-/+ ⁴	+/- ^{4,5}
$d\Delta(\rho x^+)$	+/- ^{1,2}	-	-	+/- ³	-/+ ⁴	+/- ^{4,5}
$d\pi$	0	+	+	-/+ ³	0	+/- ⁵
dP	0	+	+	-/+ ³	NA	NA
1: if $m^* > m^{**}$ / if $m^* < m^{**}$; 2: if $\alpha TH''(m^{**}) \gtrless G''(m^*) + H''(m^*)$; 3: if $n^* > n^{**}$ / if $n^* < n^{**}$; 4: if $A^p > \rho(x^-)m^*$ / if $A^p < \rho(x^-)m^*$; 5: if $L^* > L^{**}$ / if $L^* < L^{**}$.						

Table 4: Selected Comparative Statics for Comparison of Market and Socially Optimal Values.