You Incomplete Me: Safety Nets and Insurance

Tse-Ling Teh and Christopher Woolnough

Abstract

The existence or expectation of assistance following a loss can modify insurance demand and the type of insurance contracts demanded. This paper examines what type of insurance contracts are offered when insurance exists alongside ex-post safety net assistance. The optimal contract from the perspective of the assistance provider and the insurance consumer both exclude large losses, but not at the same level of exclusion. In a realistic market where any type of insurance contract is available and the assistance provider does not have dictatorial power, we demonstrate that subsidisation by an assistance provider can improve welfare outcomes. Further benefits arise from mandating a set of insurance contracts to be made available. Our findings are reflected in markets of natural disaster insurance and health insurance.

1 Introduction

The coexistence of financial assistance and insurance is observable in markets as diverse as natural disaster insurance to retirement savings. When loss occurs, governments and donors may provide ex-post safety net assistance. An example is the Federal Emergency Management Agency (FEMA) disaster assistance in the United States. This is available to individuals or households after a disaster declaration for losses not already covered by insurance.[1] In this sense FEMA disaster assistance is ex-post safety net assistance, since it is determined after a loss occurs and acquired only if the net loss level reduces welfare

[1] *Robert T. Stafford Disaster Relief and Emergency Assistance Act* (2003), P.L. 93-288 as amended, Federal Emergency Management Agency.

sufficiently. Even if ex-post assistance is not explicitly available, any previous assistance can lead to an expectation of assistance that can affect demand for insurance (van Asseldonk et al., 2002).

When individuals at risk of loss rely on ex-post assistance rather than purchasing insurance, there is a possible crowding out of insurance. The crowd out of insurance demand by assistance has been examined in theory (Coate 1995) and empirically, with mixed results (Disney 2000, Raschky et al. 2013). Nonetheless, crowd out can be minimised by the design of insurance contracts. Incomplete insurance contracts, that is contracts that do not cover all circumstances of loss, lead to increased insurance demand minimising crowd out (Teh 2017). However, within the spectrum of incomplete contracts some incomplete products may improve welfare more that others. What type of incomplete insurance contract is optimal has not been investigated. This paper addresses this gap by examining what type of incomplete contracts are preferred by the insurance provider, assistance provider and the insurance consumer.

Typically, incompleteness arises from an exclusion in an insurance contract. Exclusions are prevalent in the insurance market and exist in contracts such as travel insurance and health insurance. A common incomplete contract is a contract that excludes risks based upon the size of loss. For example, a deductible, as often found in theft insurance, restricts the insurance contract to cover claims only above a certain size. A maximum claim size, as often found in dental insurance, limits the contract to cover claims below a certain size. Incomplete contracts of this sort are easily verifiable, common and contractable. For these reasons, this type of contract is the main focus of our paper.

We examine how the availability of safety net assistance after a loss affects the design of insurance contracts that exclude risks by size of loss. Safety net assistance refers to a baseline level of wealth transfer following a loss. Beyond FEMA disaster assistance, there are also examples in health care and social assistance. Public health care in Australia provides a baseline level of health care for those without private insurance. Whilst in the Netherlands, "[s]ocial assistance is also regarded as our safety net. The Dutch system guarantees a minimum income for people who are not able to support themselves independently." (Work and Social Assistance Act 2004, Blommesteijn and Mallee April 2009).

We demonstrate that when safety net assistance is available, individuals at risk prefer to purchase contracts that exclude large losses and rely on the safety net in the case of large losses. For the safety net provider there is also a preference for large losses to be excluded.

However, the safety net provider would prefer less claims excluded than the individual at risk. The reason for this is that the safety net provider would prefer a market with the most complete insurance contract that the consumer would purchase. Whereas, the consumer still prefers (relatively) to rely more on safety net assistance than insurance.

A second finding of our paper is that richer individuals are less likely to rely on the safety net and more likely to purchase insurance contracts with fewer exclusions. This is due to the little value that a safety net provides for individuals of high wealth compared to insurance. These findings are discussed in section four and are reflected in health insurance markets in Australia and Flood insurance markets in the United States.

Our finding that large losses are excluded in insurance contracts where assistance is available, is reflected in insurance contracts for natural disasters. For example, in the National Flood Insurance Program in the United States where damages due to mud flows (small losses) are covered but damages due to landslides (large losses) are not (Hartwig and Wilkinson, 2005). Similarly, in the Netherlands homeowners insurance includes flood if due exclusively to heavy local rainfall, but not otherwise (Kok et al., 2002).

We establish that in the presence of safety net assistance, insurance markets tend to offer products with too many exclusions. This is because the market designs the contract for the potential consumer and does not take into account the externality placed on the safety net provider. Ex-post assistance drives demand towards incomplete insurance and away from full complete insurance. As a result, there are welfare gains to be made through regulation and subidisation.

Regulation that forces all individuals at risk to purchase full, complete insurance would achieve the maximal welfare for society. Since it removes any incentives for ex-post assistance and fully insures all individuals at a fair rate of premium. However, practically this is highly unrealistic since it is costly to the regulator and likely to be politically unacceptable. Rather, we focus on a more realistic setting where there is a market of diverse insurance contracts including ex-post safety net assistance (which can be considered free insurance). In this environment we show that regulation, subsidisation and a combination of the two can achieve varying levels of improvements in welfare beyond the unregulated market.

Section two of the paper introduces the model of the interaction between the safety net provider, the individual at risk and insurance provider. Section three provides the results for the optimal insurance contract and shows that the contract is incomplete and excludes large losses. Section four examines how demand for incomplete insurance differs based on wealth levels of the individual at risk. Section five discusses how subsidisation and regulation can be implemented within a realistic market structure, and Section six concludes.

2 Model of the Interaction Between Safety net provider and Individual at risk

Following in the tradition of insurance models (Rothschild and Stiglitz 1976, Coate 1995, Teh 2017), this model measures welfare in an expected-utility framework. The model consists of an individual at risk, who is the insurance consumer (denoted by the subscript i), a safety-net assistance provider (denoted by the subscript s) and a competitive insurance market. The individual at risk is risk averse, able to purchase insurance and receive assistance.[2] The safety net provider is empathetic towards the individual at risk, caring about both their own welfare and the welfare of the individual at risk. The safety net provider cannot commit to not provide assistance to the individual at risk. The assumption of being unable to commit is reasonable given empirical evidence of government behaviour (Eisensee and Strömberg, 2007) and individual expectations (van Asseldonk et al., 2002). The provider will provide assistance if it is of benefit to them and it is assumed that it is not of benefit to provide assistance if the risk does not materialise.

We consider a setting where the individual at risk faces a risk of incurring a loss L. A loss can occur from many potential causes, and the size of the loss can vary. To model this, the probability some loss occurs is given by π and, conditional on a loss occurring, the size of the loss is between $\underline{L}$ and $\bar{L}$, and drawn from the continuous distribution with cdf and pdf given by $F(l)$ and $f(l)$ respectively. The expected loss is given by $E(L) = \pi \int_{\underline{L}}^{\bar{L}} l f(l) dl$. Let Z represent the insurance contract, where $Z(l) \geq 0$ represents the payout if the loss is of size l.

The insurance market is assumed to be competitive and offer actuarially fair insurance. The insurance contract charges a premium in the case of ***no*** loss. When a loss $L = l$ occurs, the

[2]Risk aversion is necessary to ensure that the individual has an interest in purchasing insurance.

contract pays out $Z(l)$.[3] The actuarially fair premium is calculated as:

$$P_Z = \frac{\pi}{1-\pi}\int_{\underline{L}}^{\bar{L}} Z(l) f(l) dl \tag{2.1}$$

The focus of this paper is the incompleteness of insurance. We say an insurance contract is incomplete when there is no payout for some loss amount l, that is $Z(l) = 0$. The size of a loss and the exclusions γ are now linked through the function Z. The exclusions, γ are defined by:

$$\gamma = 1 - \int_{\underline{L}}^{\bar{L}} \mathbf{1}\left(Z(l) > 0\right) dF(l) \tag{2.2}$$

The order of decisions is important in determining the outcome and optimal contract. Here, we will assume that the timing of decisions is as follows:

1. An insurance contract, or contracts, are offered in the market.

2. The individual at risk chooses whether to insure with one of the contracts or not.

3. The risk materialises (probability π) and the size of loss is determined (L).

4. The safety net provider chooses how much assistance to provide (τ).

One of the main points of this sequence is that assistance is provided ex-post, at the last stage. That is, the provider decides on the amount of assistance when the level of the individual's income and the state of the world is known. However, the individual at risk chooses their level of insurance when they are unaware of the future state of the world. The level of assistance depends on the individual's loss, insurance payout, and empathy of the safety net provider. This implies an endogenous form of limited liability, thereby taking into account a range of assistance levels.

In stage three, the risk may materialise. Here loss has been interpreted as a two step process, firstly the risk materialises or not. Secondly, if the risk materialises then the size of loss is drawn from a distribution. The two step interpretation of loss enables a simplification of

[3] Charging the premium only when there is no loss ensures that the safety net provider does not directly subsidise the insurance contract by bailing out the premium under large losses. This type of subsidisation leads to some technical results that are not the focus of this paper. For further discussion see Teh (2017), who describes the contract we use as the net-of-premium contract.

the mathematics and the avoidance of mass points at no loss (loss size of zero). This is not material to the results but aids in comprehension and interpretation of the model. Hence, there are two probabilities of importance. The first probability is the probability of loss, denoted $\pi \in (0,1)$, and the second, the size of the loss L which is continuously distributed over $\left[\underline{L}, \bar{L}\right]$.

2.1 Objective function of individual at risk and safety net provider

Let the individual at risk have an income level y_i and the safety net provider have an income level y_s. The individual at risk has the standard von-Neuman Morgenstein risk averse expected utility function $u(.)$, where $u'(.) > 0$ and $u"(.) < 0$.

The utility of the safety net provider is given by:

$$U_s = y_s - \tau_{l,z} + \delta u(w) \tag{2.3}$$

where w is the net wealth of the individual at risk, and $\tau_{l,z}$ is the transfer from the safety net provider to the individual at risk (which can depend on the loss incurred and the insurance payout). The individual at risk's welfare affects the safety net provider's welfare at an empathy weighting of δ, where $\delta \in (0, \infty)$. We assume that $\delta u'\left(y_i - \bar{L}\right) > 1$ and $\delta u'\left(y_i - E\left(L\right)\right) < 1$.[4] This means that, all else equal, the safety net provider would prefer to provide assistance when the individual at risk experiences a loss large enough and is not insured. While, when the individual at risk is fully insured the safety net provider would prefer not to provide assistance.

As the transfer occurs after the loss is incurred, the safety net provider can condition the transfer on the size of the loss and the insurance payout. Let $\tau_{l,z}$ be the transfer for a loss $L = l$ and a payout of $Z\left(l\right) = z$, and specifically $\tau_{0,0}$ be the transfer when there is no loss. The expected utility of the individual at risk with an insurance product Z is given by

$$E\left[U_i\right] = \left(1 - \pi\right) u\left(y_i + \tau_{0,0} - P_Z\right) + \pi \int_{\underline{L}}^{\bar{L}} u\left(y_i - l + Z(l) + \tau_{l,Z(l)}\right) f(l) dl$$

[4]The expected loss is calculated as $E\left[L\right] = \pi \int_{\underline{L}}^{\bar{L}} l dF(l)$.

The expected utility of the safety net provider is given by

$$E[U_s] = y_s - E[\tau] + \delta E[U_i]$$

where $E[\tau] = (1-\pi)\,\tau_{0,0} - \pi \int_{\underline{L}}^{\bar{L}} \tau_{l,z(l)} f(l)dl$.

3 Optimal incompleteness by size of loss exclusion

Since the safety net provider is unable to commit to not providing assistance, the individual at risk anticipates the availability of assistance. The altruistic welfare function of the safety net provider determines that the provider has a fixed target wealth level for the individual at risk. If the individual falls below this level, assistance will be provided, leading to the term safety net.

Proposition 1. *The safety net provider ensures a target wealth level, w^*, for the individual at risk and provides top-up assistance, $\tau_{l,z}$, to meet this level.*

Proof. Having observed l and z, the donor's welfare function is $U_s = y_s - \tau_{l,z} + \delta u\,(y_i - l + z + \tau)$. The level of assistance provided by the donor, will match the marginal cost of assistance with the marginal benefit of assistance. That is $\delta u_i'(w^*) = 1$, where w^* is the target wealth level. This means that $\tau_{l,Z(l)} = \max[w^* - y_i + l - Z(l),\ 0]$. □

From this, the expected utility of an insurance contract is given by

$$E[U_i] = (1-\pi)\,u\,(y_i - P_Z) + \pi \int_{\underline{L}}^{\bar{L}} u\,(\max[y_i - l + Z(l),\ w^*])\,f(l)dl \tag{3.1}$$

While the utility of no insurance is given by

$$U_i^{NI} = (1-\pi)\,u\,(y_i) + \pi \int_{\underline{L}}^{\bar{L}} u\,(\max[y_i - l,\ w^*])\,f(l)dl$$

The at risk individual will pick the insurance product which maximizes their expected utility $E[U_i]$, or choose no insurance if this provides a higher utility.

When

$$(1-\pi)\, u\,(y_i) + \pi \int_{\underline{L}}^{\bar{L}} u\,(\max\,[y_i - l,\ w^*])\, f(l)dl \geq u\,(y_i - E\,[L]) \tag{3.2}$$

the at risk individual will always choose no insurance over complete insurance.

In the next two subsections we will consider the optimal insurance contract from the perspective of the individual at risk first and secondly from the perspective of the safety net provider.

3.1 Individual's optimal insurance contract

We now examine what the optimal insurance contract is from the perspective of the individual at risk. This is akin to a competitive market, where insurers provide the insurance contract to match demand. The individual at risk will pick $Z(l)$ so as to maximize expected utility (Equation3.1). The level of coverage determines which types of losses are covered by the insurance contract.

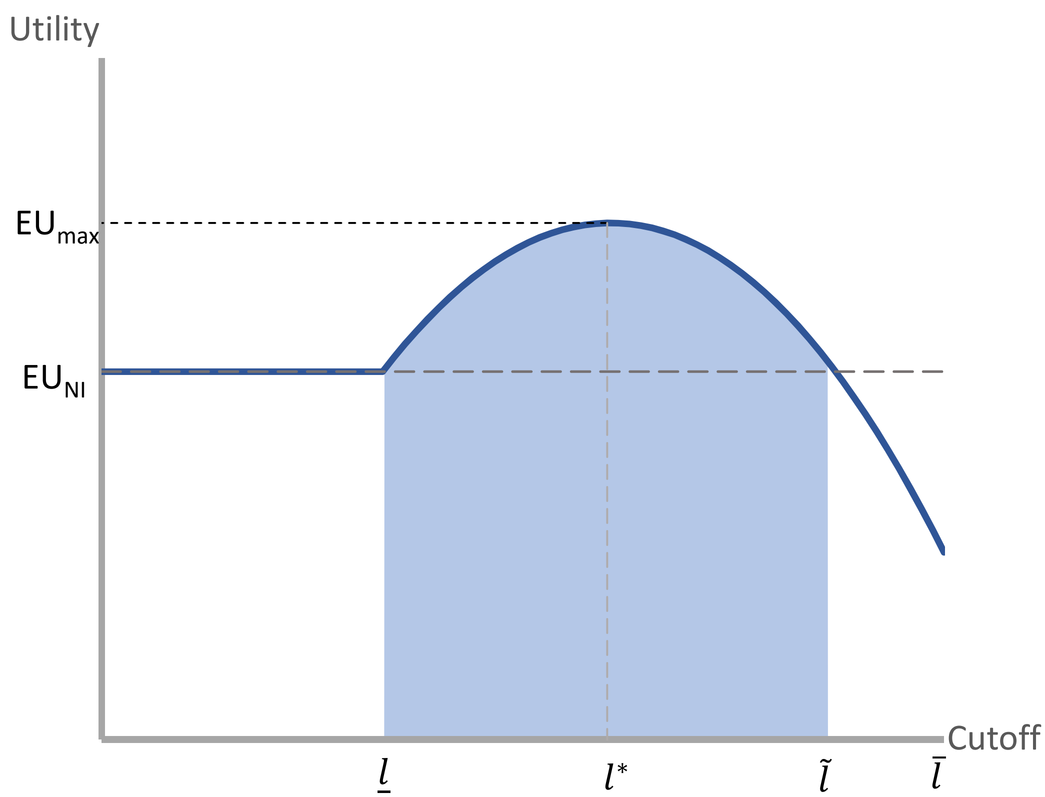
Proposition 2. *It is optimal for the individual at risk to fully insure small losses and exclude large losses from the insurance contract. That is, there exists an* $l^* \in \left[\underline{L}, \bar{L}\right]$ *such that for all* $l \leq l^*$ *the contract sets* $Z(l) = l - P^*$*, and* $Z(l) = 0$ *for all* $l > l^*$*, with* $P^* = \frac{\pi}{1-\pi} \int_{\underline{L}}^{l^*} l f(l)dl$*.*

Proof. See Appendix. □

Proposition 2 demonstrates that the individual, if given the option, would exclude large losses from the insurance contract. This can be explained by considering the value of insurance for the individual. The availability of assistance represents a form of free insurance up until the target wealth level, w^*. The additional benefit an individual receives from an insurance payment is $u(y_i - P) - u(w^*)$. Although an insurance premium covers the entire loss, in practical terms it also covers the assistance that would have been free. For a large loss, the purchase of insurance crowds out a large amount of assistance. Whereas in the case of a small loss, the purchase of insurance crowds out only a small amount of assistance. Thereby, in terms of expected utility, the value of the same premium is larger for small losses than large losses. This demonstrates that it is optimal for the at risk individual to exclude large losses.

Figure 3.1 illustrates Proposition 2 graphically. The curve shows the expected utility of the individual at risk across a range of cutoff based insurance contracts. The losses covered by insurance contracts stretches between $\underline{l}$ and $\bar{l}$. The point at which the expected utility from no insurance contract, EU_{NI} is equal to the expected utility from an insurance contract is shown by $\tilde{l}$. That is an insurance contract that covers losses up to $\tilde{l}$ has the same utility as not purchasing insurance. The shaded region illustrates the range of insurance contracts that are preferred over no insurance. The maximum expected utility, $EU_{max}$is shown by the peak of the expected utility curve. This is the contract that sets $Z(l) = l - P^*$ for all $l \leq l^*$and $Z(l) = 0$ otherwise.

Figure 3.1: Expected utility of individual at risk



The individual at risk's optimal utility is then

$$U_i^* = (1 - \pi (1 - \gamma^*)) u (y_i - P^*) + \pi \gamma^* u (w^*)$$

where γ^* is the probability that, conditional on a loss occurring, that it is greater l^*. That is $\gamma^* = \int_{l^*}^{\bar{L}} dF(l)$.

3.2 Safety net provider's optimal insurance contract

As a comparison, consider the optimal contract from the perspective of the safety net provider. In practical terms, the level of exclusions can be controlled by the safety net provider if there is regulation requiring it or the provider and insurer are branches of the same entity. An example, is a government that cannot commit to not providing assistance, but offers insurance contracts. This is the case for flood disasters in the United States, where the Federal Emergency Management Agency manages both disaster assistance and the national flood insurance program. In these situations, an optimal contract design may stem from the safety net provider's optimization.

The provider's optimization problem is to choose a type of insurance product that maximizes their own welfare. The purchase of insurance by the individual is welfare improving for the provider as it reduces the burden of costly ex-post assistance. Because of this, the donor will choose an optimal size of loss exclusion that encourages insurance demand by the individual, whilst maximizing their own welfare.

Given our assumptions, the safety net provider would always prefer the individual at risk to take out full and complete insurance. However, we will assume that the safety net provider has some limit to its power in the sense that the individual at risk can opt-out of the insurance (but still receive assistance payments ex-post). Thus, the safety net provider must offer an insurance at least as good as the individual at risks outside option. We will denote this outside option as an insurance product $\tilde{Z}$, which provides the individual at risk with expected utility $\tilde{u}$. Usually we will consider this outside option to be the no-insurance contract, where $\tilde{Z}(l) = 0$ for all l and $\tilde{u} = U_i^{NI}$. However, for the analysis we will leave this open to also consider cases where other insurance products not controlled by the safety net provider are also on the market.

The provider's ex-ante expected welfare is given by

$$W_s = y_s - E\left[\tau\right] + \delta E\left[u_i\right]$$

where the expectation of τ for an insurance product is given by

$$E\left[\tau\right] = \pi \int_{\underline{L}}^{\bar{L}} max\left[w^* - y_i + l - Z(l), 0\right] dF(l)$$

The provider's problem can be stated as

$$max_{g(L)}\, y_s - E[\tau] + \delta E[U_i]$$

subject to the condition that the individual purchases insurance,

$$E[U_i] \geq \tilde{u}$$

Proposition 3. *If the individual at risk purchases incomplete insurance, it is optimal for the safety net provider to include small losses and exclude large losses from the insurance contract.*

That is, there exists an $\tilde{L}_{\tilde{u}} \in \left[\underline{L}, \bar{L}\right]$ such that for all $l \leq \tilde{L}_{\tilde{u}}$ the contract sets $Z(l) = l - \tilde{P}$, and for all $l > \tilde{L}_{\tilde{u}}$ the contract sets $Z(l) = 0$, with $\tilde{P} = \pi \int_{\underline{L}}^{l^*} l f(l) dl$.

Proof. See Appendix. □

Proposition 3 illustrates that from the provider's perspective when there is demand for incomplete insurance, it is optimal to exclude large losses. Large losses require a higher ex-post transfer from the safety net provider, but are also the most costly to convince the at risk individual to insure. Thus, when designing insurance, the safety net provider has a trade-off: include a few large losses in the product, or include more small losses. Proposition 3 indicates that for the safety net provider it is better to include more of the small losses, and provide the safety net in the case of large losses. Thus, both the safety net provider and the individual at risk prefer contracts where large losses are excluded. However, the next proposition shows that there is disagreement on the size of exclusions.

Proposition 4. *The safety net provider's optimal insurance contract excludes less than the individual at risk's optimal contract. That is $\tilde{L}_{\tilde{u}} \geq L^*$, with equality only when $\tilde{u} = U_i^*$.*

Proof. See Appendix. □

Proposition 4 indicates that although the direction of exclusions is the same for both the individual at risk and the safety net provider, the extent of exclusions differs. The individual prefers to have more exclusions in their optimal contract than the provider would find optimal. This is because any additional exclusions to the contract beyond those required to encourage the individual to purchase insurance, decrease the welfare of the provider.

Figure 3.2: Ratio of change in individual's utility over provider's utility

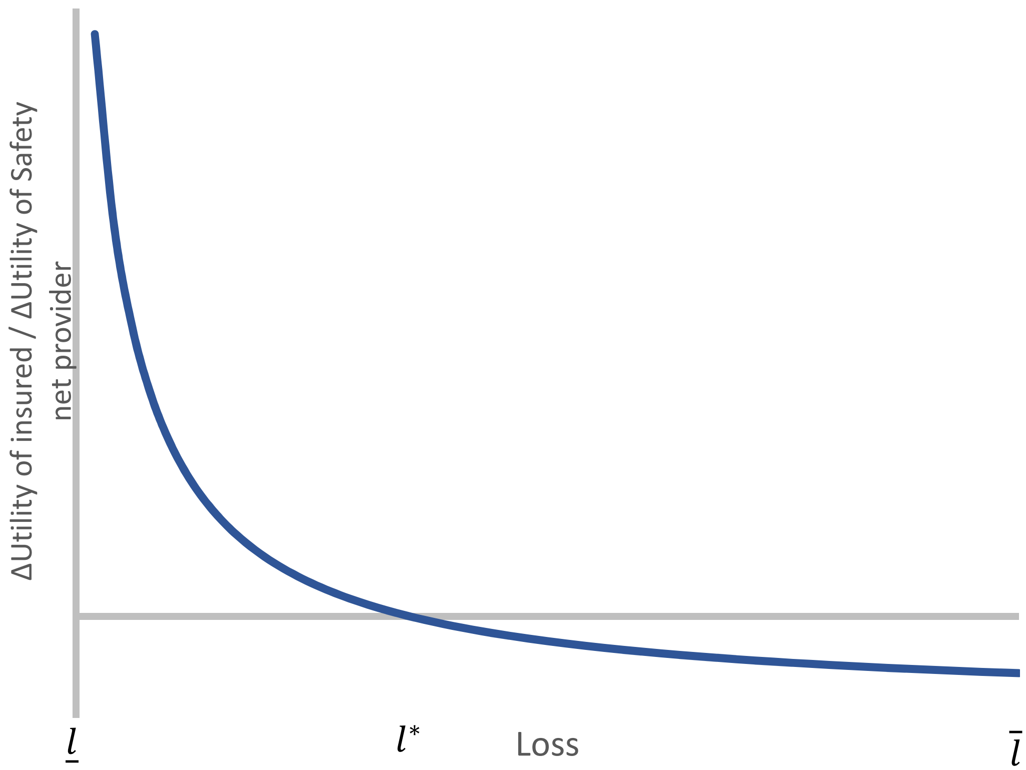


Figure 3.2 displays the ratio of the change in utility of the individual at risk divided by the change in utility of the safety net provider at discrete loss levels. The change is measured from the optimal contract for the individual at risk when each discrete loss level is added to the insurance contract. Thereby, l^* indicates the optimal insurance contract where the change in utility for the individual at risk is zero. The change in the utility of the safety net provider is always positive in this graph since the purchase of more complete insurance by the individual at risk results in positive utility for the safety net provider. The figure shows that as the insurance contract includes loss sizes of larger amounts the ratio decreases and after $l^*$the change in utility for the individual at risk becomes negative. This indicates that although the change in utility for the safety net provider is still positive, that of the individual at risk is negative. That is, the safety net provider's utility increases with higher loss levels included and therefore has an optimal contract that excludes less than the individual at risk.

Proposition 2 and 3 result in optimal contracts are contrary to Arrow's theorem of the deductible. The inclusion of safety net assistance reverses the optimal contract to one excluding large losses and including small losses, rather than that of Arrow's theorem that excludes small losses. Proposition 2 and 3 suggest that the availability of safety net assistance is an important consideration when designing insurance contracts and failure to do so may result in sub-optimal outcomes.

This Section has demonstrated that in the presence of ex-post safety net assistance, the optimal insurance contract is one that excludes large losses. This is consistent from both the perspective of the safety net provider and the individual at risk. Nonetheless, although the direction of exclusions is consistent, the extent of optimal exclusions differs between the safety net provider and the individual at risk. Specifically, the safety net provider would prefer that the individual choose a contracts with less exclusions that is optimal for the individual. The reason for this is that the more complete the insurance product, the less the safety net provider needs to contribute to the individual in the event of a risk materialising.

4 The effect of wealth on the optimal contracts

The income of the individual at risk will have a large impact on the optimal insurance contract. Within the safety net framework, the target wealth level is not affected by a change in the individual's income. The safety net provider maintains the same target wealth level. Nonetheless, a change in income affects the individual's preference for insurance through two aspects driven by the risk averse utility of the individual. Firstly, when incomes increase, the utility of ex-post assistance decreases. Secondly, when incomes increase, the relative cost of insurance decreases.

Proposition 5 relates to a fixed target wealth level, that is where assistance decreases as a proportion of pre-loss income as incomes rise.

Proposition 5. *As the income of the individual at risk rises, their optimal level of incompleteness tends to zero.*

Figure 4.1: Expected utility of individual at risk (varying exclusions γ)

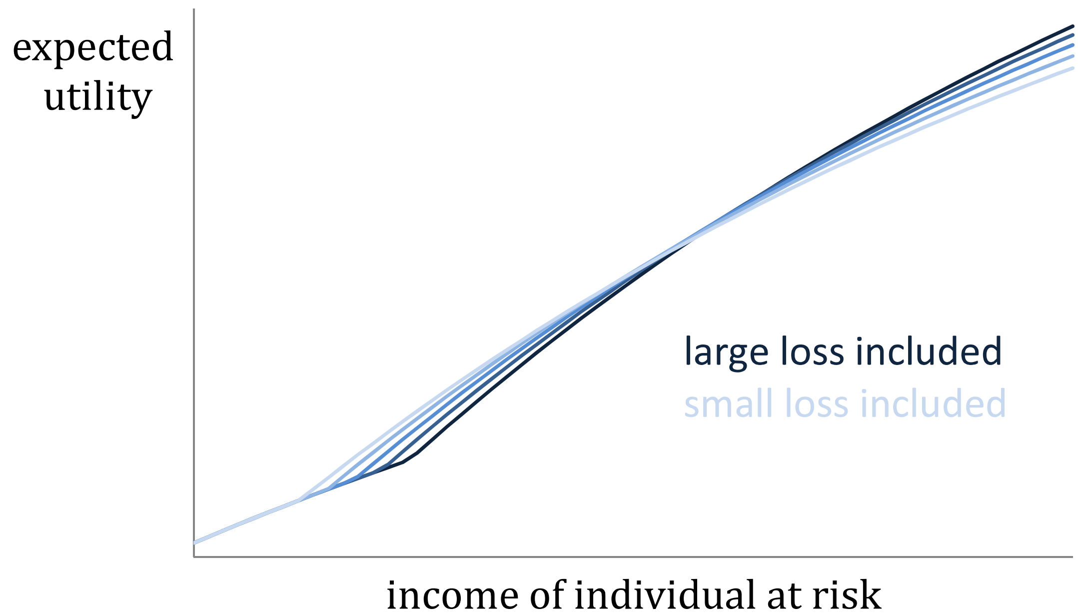


Figure 4.1 illustrates how the optimal contract for an individual at risk changes as income changes. The darker the curves the larger the loss that is included in the insurance contract. Initially at low income levels no insurance is purchased, however as incomes rise more levels of loss are included in the insurance product starting with smaller losses. Eventually a turning point is reached and the large losses are also included in the insurance contract.

Proposition 6. *As income of the individual at risk rises, the set of insurance products that would be purchased increases.*

The assistance provider once satisfied that the at risk individual has adequate utility, would improve their welfare if they could commit to no assistance, thereby encouraging the at risk individual to purchase complete insurance. On the other hand, the at risk individual enjoys higher utility with a positive gamma for longer than the assistance provider would wish. This result illustrates the mismatched incentives between the vulnerable individual and the assistance provider caused by the Samaritan's Dilemma (Buchanan, 1975).

The result that completeness increases with income can be observed in disaster and health insurance. One example, is flood insurance in the United States. In the United States, flood insurance is offered primarily through the National Flood Insurance Program (NFIP).[5] Homeowner's insurance excludes flood damage, but the purchase of flood insurance can override this exclusion. Within this model, flood insurance can be considered as increasing the completeness of one's insurance. If this is the case, then by the comparative statics discussed above, individuals at risk with higher levels of income are likely to purchase greater amounts of flood insurance.

Several papers have concluded that income demand for flood insurance rises as income rises (Browne and Hoyt 2000, Landry and Jahan-Parvar 2011). Halpin (2013) finds at the time of Hurricane Sandy in 2012, 64% of FEMA individual assistance recipients in New Jersey did not have flood insurance. However, when the sample is limited to low income households this figure rises to 90% without flood insurance. Indicating that incompleteness is more evident in lower income households in line with Proposition 5.

A similar pattern has been observed in health insurance where different insurance products are available. Cameron et al. (1988) finds significant differences in types of insurance held by individuals in Australia by income group when public health care is available. Health care is provided in three types: the first is free public health care, the second levied public health care and the third is private health care.[6] With respect to the model, the first reflects assistance, the second incomplete insurance and the third complete insurance. As income rises the percentage with complete insurance also rises. Of low income households 27.5% have complete insurance, middle income 38.2% and high income 59.1%.

5 Policy interventions: regulation and subsidies

Propositions 2 and 3 have shown that when safety net assistance is available the optimal contract is a contract with exclusions, in other words an incomplete contract. The existence of safety net assistance is equivalent to free insurance that distorts the market and is a form of the Samaritan's Dilemma.

In a competitive insurance market, competition among insurance providers drives insurance

[5] Around 20% of premiums are subsidized, with the other 80% at a market rate incorporating risk and administrative expenses (FEMA 2013).

[6] Free public health care is only available to eligible groups e.g. pensioners, low income individuals. The levied health care is equivalent to the free public health but there is charge for those ineligible.

contracts to be tailored to provide maximum benefits to consumers. In this case, the individual at risk. If there were no market externalities, this would lead to an efficient market. However, as the individual at risk does not take into account the welfare of the safety net provider when choosing insurance, the market will provide too little coverage as it will offer the optimal contract from the perspective of the individual at risk. With only this type of contract available, the externality is not internalised and leads to a reduction in overall welfare. In this Section we consider how two types of policy intervention: regulation and subsidies, can alleviate this issue.

5.1 Regulation

In a trivial case, regulators could ban ex-post assistance. With actuarially fair premiums in a competitive market this would result in the individual at risk purchasing full complete insurance. Although this maximises societal welfare, such an intervention is highly unrealistic and it would be difficult, if not impossible, to regulate such a rule.[7] There is ample indication that such a rule would not work politically nor practically (Eisensee and Strömberg, 2007, van Asseldonk et al., 2002). Leaving aside this unlikely scenario, regulation could instead regulate the type of insurance contracts that are on the market. For example, in the UK compulsory third party insurance for car owners requires all car owners to purchase third party insurance with a minimum standard contract. In the US, the 2008 Paul Wellstone and Pete Domenici Mental Health Parity and Addiction Equity Act (MHPAEA) ensured equal coverage of treatment for mental illness and addiction as other chronic health issues. In 2013 this act was implemented with Federal rules that required all health care insurers to provide care for mental illnesses.

It follows from Proposition 4 that when a safety net provider is able to determine the insurance products available in the market[8], then it should offer an insurance product with the smallest level of exclusions that would still be better than no insurance. In the case where $U_i^{NI} \leq u\left(y_i - E\left[L\right]\right)$, the safety net provider can obtain its first best by mandating insurance companies provide only complete insurance. When this condition does not hold, the best for the safety net provider is to mandate that insurance contacts cover at least losses up to size L, where $L = \tilde{l}$ determined by $\tilde{u} = U_i^{NI}$.

[7] For example, it would also require banning of private transfers such as between a parent and child after a risk materialises.

[8] But not force individuals to buy insurance.

5.2 Subsidies

Another policy alternative that the safety net provider has is offering a subsidy for buying certain insurance contracts. Subsidies, such as tax credits on insurance premiums, reduce the cost of premiums for insurance consumers. This stimulates demand by making insurance products cheaper, but also impacts individual perspectives on risk level and can be hard to remove once in place. In cases where the insurance and assistance markets are not controlled by the government, they will have little ability to regulate and mandate insurance and assistance. This means that alternative insurance products will be available on the market that are attractive to the individual at risk. Considering an insurance market not controlled by the safety net provider, with a wide variety of alternative insurance products, we show that a subsidy can still improve the welfare of the safety net provider. Assuming that the set of outside options is fixed, the safety net provider can increase their welfare by offering a subsidy to the at risk individual for the purchase of complete insurance.

A subsidized contract takes the form $Z(L)$ with $P_S = \pi \int_{\underline{L}}^{\bar{L}} Z(l)dF(l) - \frac{1}{1-\pi}S$, where S is the subsidy given. We assume the subsidy is provided upfront by the safety net provider. As such, the welfare of the safety net provider under this contract is:

$$W_s = y_s - E[\tau] - S + \delta E[U_i]$$

With

$$E[\tau] = \pi \int_{\underline{L}}^{\bar{L}} max[w^* - y_i + l - Z(l), 0]\, dF(l)$$

$$E[U_i] = (1-\pi)\, u(y_i - P_S) + \pi \int_{\underline{L}}^{\bar{L}} u(\max[y_i - l + Z(l),\ w^*])\, f(l)dl$$

For the moment we assume the individual at risk can opt-out and purchase some other insurance product $\tilde{Z}(L)$ with utility $\tilde{u}$.

Proposition 7. *Fixing the outside option of the individual at risk to $\tilde{Z}(L)$; the safety net provider can improve their welfare, as compared to both the outside option and best incomplete contract, by offering a complete insurance contract with a subsidy. The optimal subsidy is given $S = max\{u^{-1}(\tilde{u}) + E[L] - y_i, 0\}$.*

Proof. See Appendix. □

Proposition 7 shows that a safety net provider can improve welfare by providing an upfront subsidy to improve the welfare of the at risk individual. A similar result was shown by Coate (1995) in the specific case when losses are binary.

Figure 5.1: Expected utility with subsidy

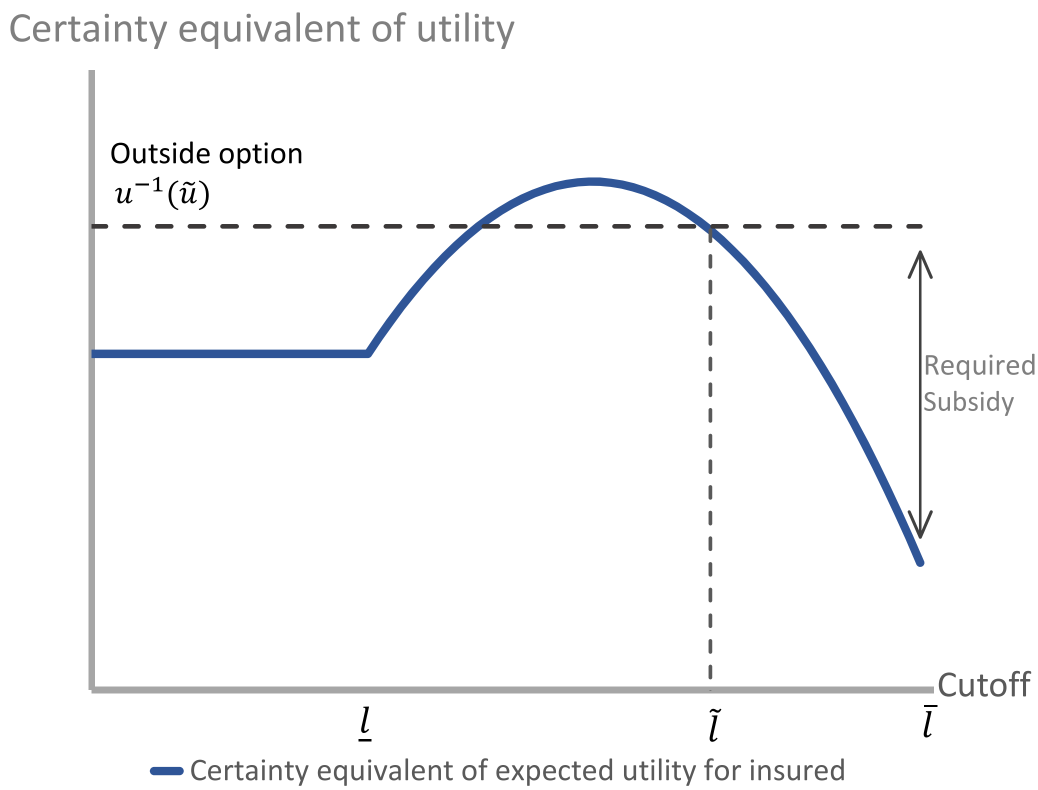


Figure 5.1 demonstrates the level of subsidy to encourage the individual at risk to purchase a complete contract over the outside option of an incomplete contract. Insurance contracts are on the horizontal axis with each cutoff representing what is excluded from the insurance contract. Each loss cutoff represents a different insurance contract where the contract is $Z(l) = l - P$ for $l < l_{cutoff}$ and zero otherwise. A complete contract ensures that the safety net provider will not need to provide a payout if a risk materialises. The required subsidy is shown as the gap between the outside option and the certainty equivalent of utility at the complete insurance contract, $Z(l) = l - P_{\bar{l}}$ for $l < \bar{l}$. The subsidy is cheaper for the safety net provider than the ex-post assistance because of the curvature of the utlity of the individual at risk via Jensen's inequality.

Our findings demonstrate that policy makers have two tools at their disposal to improve insurance coverage in spite of ex-post assistance. When $U_i^{NI} \leq u\left(y_i - E\left[L\right]\right)$, the policy maker can place a mandate on the level of coverage, without the need for subsidies, to result in the individual at risk choosing the full complete insurance contract. However, when this is not the case, offering an upfront subsidy can improve the safety net provider's welfare.

Where possible, these subsides should be combined with mandates for the level of exclusions. The logic behind this is simple. The cost of providing a subsidy depends on the outside options of the individual at risk. The better the outside insurance policy, the higher the subsidy required to move the individual at risk across to a complete insurance contract. Thus, if the market provides the individual at risk with their optimal incomplete contract $Z^*(L)$, then a large subsidy is required to move the individual at risk to complete insurance. By mandating that insurance contracts have a higher level of coverage (less exclusions), then the outside options of the individual are limited, reducing the required subsidy for complete insurance.

Corollary 1. *If the market offers the individual at risk the contract $Z^*(L)$, an optimal response for the safety net provider is to mandate that insurance contracts cover losses up to size L, where $L = \tilde{L}$ determined by $\tilde{u} = U_i^{NI}$, and offer a subsidy S for complete insurance, where $S = u^{-1}\left(U_i^{NI}\right) - y_i + E[L]$.*

Corollary 1 establishes that often the optimal policy intervention for safety net providers involves a combination of subsidies and regulation on the types of insurance that insurance companies can offer. Such an intervention encourages insurance demand for full complete insurance internalising the externality placed on the safety net provider.

Figure 5.2: Expected utility with subsidy and regulation

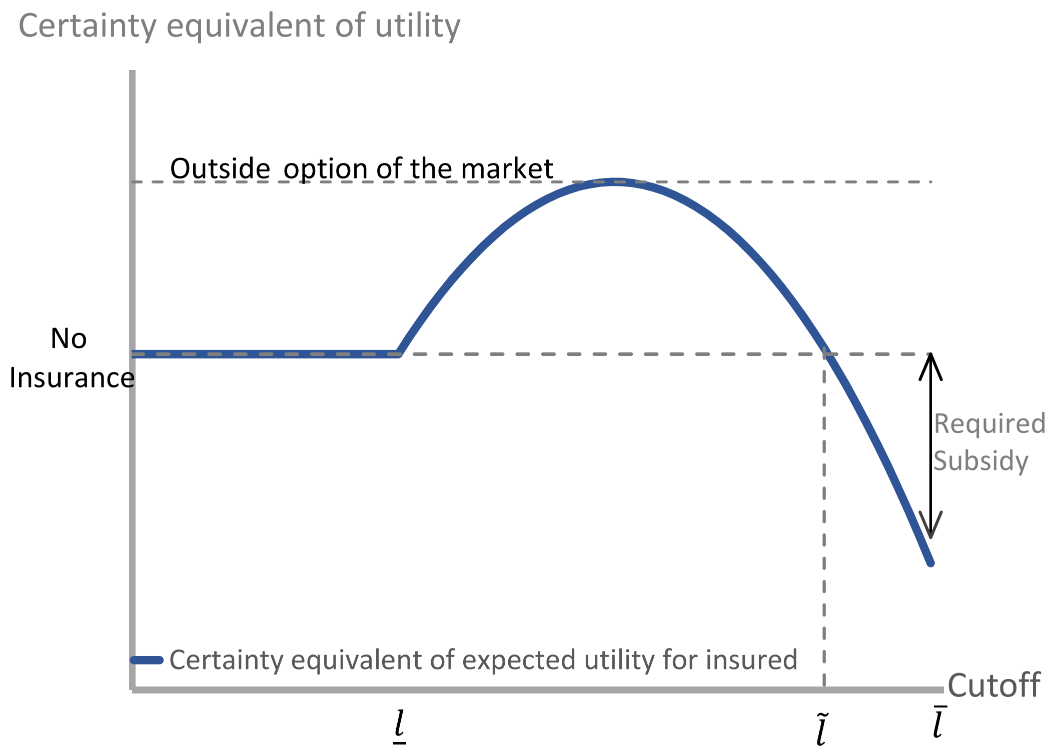


Figure 5.2 illustrates the certainty equivalent of the expected utility of the individual at risk under different insurance contracts. Each loss cutoff represents a different insurance contract where the contract is $Z(l) = l - P$ for $l < l_{cutoff}$ and zero otherwise. In comparison to Figure 5.1, the required subsidy is now smaller. This is because with regulation it is possible to limit the range of outside options. In the figure, regulation has removed all outside options and limited the market to choice to having no insurance. Thus, the safety net provider can offer a smaller subsidy that encourages the individual at risk to purchase the complete insurance product. Thereby, reducing the distortions caused by the safety net.

6 Conclusion

Insurance markets and ex-post safety net assistance both exist to manage risk. The interaction of these leads to surprising results in terms of insurance contract design. Our findings demonstrate that in the presence of safety nets, individuals demand contracts that exclude large losses. In a competitive market, this is also the type of contract that will be provided since other contracts will have less demand. As a result the provider of safety net assistance contributes more in assistance than is optimal. In addition, as income levels of the individual at risk increase, less reliance is placed on the safety net and more complete insurance is optimal.

These results have implications for the nature of insurance markets when competing with ex-post assistance. In many markets, providers of ex-post assistance are unable to commit to not providing assistance prior to a risk materialising. For example, in disaster assistance, health assistance, and financial bail outs. Notwithstanding the supply side reasons for incomplete contracts, this paper has shown that there is a demand side reason for large losses to be excluded from insurance contracts. This trend is observable in the prevalence of contracts limiting the maximum liability insurance for disasters and maximum claim amounts in health care. In contrast, when safety net assistance is not available contracts excluding large losses are rare. For example, in car insurance where ex-post assistance is unlikely to be available and contracts have less size of loss exclusions.

Our findings have provided an analysis of incomplete contracts under the aspect of size of loss exclusions. We have shown that without intervention insurance contracts will have too many exclusions when safety nets are available. However, subsidies and regulation can remedy this situation. Although our analysis focuses on size of loss exclusions, our findings are equally applicable to contracts that cover a subset of risks rather than all risks. This could be the case in home insurance that covers fire damage but not flooding damage. Analogously, it is more likely that assistance is available after a flood than after a fire, thus a logical demand driven exclusion from the insurance contract.

Our analysis focuses on safety and their implication for existence of incomplete insurance. However, there also other well known causes of incomplete insurance driven by supply side factors, such as the cost of covering correlated losses, and asymmetric information. Both of these issues are absent from our analysis, and a better understanding of how these factors interact forms an important area of further research. Our research would benefit from testing in an experimental setting or through an analysis of demand for insurance contracts, however this is left to further research.

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A Appendix

Proposition 2. *It is optimal for the individual at risk to include small losses and exclude large losses from the insurance contract. That is, there exists an* $L^* \in \left[\underline{L}, \bar{L}\right]$ *such that for all* $l \leq L^*$ *the contract sets* $Z(l) = l - P^*$*, and for all* $l > L^*$ *the contract sets* $Z(l) = 0$*, with* $P^* = \frac{\pi}{1-\pi} \int_{\underline{L}}^{l^*} l f(l) dl$*.*

Proof. The individual at risk seeks to maximise

$$E\left[U_i\right] = (1-\pi)\, u\left(y_i - P_Z\right) + \pi \int_{\underline{L}}^{\bar{L}} u\left(\max\left[y_i - l + Z(l),\ w^*\right]\right) f(l) dl$$

where $P_Z = \frac{\pi}{1-\pi} \int_{\underline{L}}^{\bar{L}} Z(l) f(l) dl$

First, we can show that at any level of loss L the individual at risk would like to either pick $Z(l) = l - P$ or $Z(l) = 0$. Differentiating with respect to $Z(L)$ where L is a particular level of loss, we get

$$\frac{E\left[U_i\right]}{dZ\left(l\right)}=\begin{cases}-\pi f(l)u'\left(y_i-P_Z\right) & \text{for } Z(l)<w^*-y_i+l\\ \pi f(l)\left[u'\left(y_i-l+Z(l)\right)-u'\left(y_i-P_Z\right)\right] & \text{for } Z(l)>w^*-y_i+l\end{cases}$$

Between 0 and $l-P$ this is decrease then increasing. So the maximum is achieved either at $Z(l)=0$ or $Z(l)=l-P$. Now consider the problem of choosing $g(l)\in[0,1]$, with

$$\begin{aligned}E\left[U_i\right]&=\left(1-\pi\right)u\left(y_i-P\right)+\pi\int_{\underline{L}}^{\bar{L}}\left[g(l)u\left(y_i-P\right)+\left(1-g(l)\right)u\left(w^*\right)\right]f(l)dl\\&=\left(1-\pi\left(1-\gamma\right)\right)u\left(y_i-P\right)+\gamma\pi u\left(w^*\right)\end{aligned}$$

where $\gamma=\pi\int_{\underline{L}}^{\bar{L}}\left(1-g(l)\right)f(l)dl$ and $P=\frac{\pi}{1-\gamma\pi}\int_{\underline{L}}^{\bar{L}}lg(l)f(l)dl$

Taking the first order condition with respect to $g(L)$, where L is a particular level of loss,

$$\frac{dE[U_i]}{dg(l)}=-\pi u(y_i-P)\frac{d\gamma}{dg(l)}+\pi u(w^*)\frac{d\gamma}{dg(l)}-\left(1-\pi\gamma\right)u'\left(y_i-p\right)\frac{dp}{dg(l)}\qquad\text{(A.1)}$$

Derivatives of γ and p are given by

$$\frac{d\gamma}{dg(l)}=-f(l)$$

$$\frac{dP}{dg(l)}=\frac{\pi\left(l-P\right)f(l)}{\left(1-\pi\gamma\right)}$$

Filling these in,

$$\frac{dE[u_i]}{dg(l)}=\pi f(l)\left[u(y_i-p)-u(w^*)-u'\left(y_i-P\right)\left(l-P\right)\right]\qquad\text{(A.2)}$$

From this, it can be seen that $\frac{dE[u_i]}{dg(l)}$ does not depend on $g(l)$ directly.

First consider the extremes. If $g(l)=0$ and $\frac{dE[u_i]}{dg(l)}\leq 0$ for all l, then it is optimal to not insure at all. Alternatively, if $g(l)=1$ and $\frac{dE[u_i]}{dg(l)}\geq 0$ for all l, then it is optimal to completely insure (and purchase sufficient insurance).

The most interesting case arises when there is an interior solution. As $\frac{1}{\pi f(l)} \frac{dE[u_i]}{dg(l)}$ is decreasing in l, it must be that there is a some $\tilde{l}$ such that $g(l) = 1$ for $l < \tilde{l}$ and $g(l) = 0$ for $l > \tilde{l}$. So the at risk individual chooses to include small losses and rely on assistance for large losses. $\square$

Proposition 3. *If the individual at risk purchases incomplete insurance, it is optimal for the safety net provider to include small losses and exclude large losses from the insurance contract.*

That is, there exists an $\tilde{L}_{\tilde{u}} \in \left[\underline{L}, \bar{L}\right]$ *such that for all* $l \leq \tilde{L}_{\tilde{u}}$ *the contract sets* $Z(l) = l - \tilde{P}$*, and for all* $l > \tilde{L}_{\tilde{u}}$ *the contract sets* $Z(l) = 0$*, with* $\tilde{P} = \pi \int_{\underline{L}}^{l^*} l f(l) dl$*.*

Proof. The safety net provider seeks to maximize

$$E\left[U_s\right] = y_s - E\left[\tau\right] + \delta E\left[U_i\right]$$

where

$$\begin{aligned} E\left[\tau\right] &= \pi \int_{\underline{L}}^{\bar{L}} \max\left[w^* - y_i + l - Z(l),\ 0\right] f(l) dl \\ E\left[U_i\right] &= (1-\pi)\, u\left(y_i - P\right) + \pi \int_{\underline{L}}^{\bar{L}} u\left(\max\left[y_i - l + Z(l),\ w^*\right]\right) f(l) dl \\ P &= \frac{\pi}{1-\pi} \int_{\underline{L}}^{\bar{L}} Z\left(l\right) f(l) dl \end{aligned}$$

First consider the case when the constraint that $E\left[U_i\right] \geq \tilde{u}$ is non-binding.

When $Z(l) > w^* - y_i + l$ then $\frac{dE[\tau]}{dZ(l)} = 0$, so

$$\frac{E\left[U_s\right]}{dZ(l)} = \delta \pi f(l) \left[u'\left(y_i - l + Z(l)\right) - u'\left(y_i - P\right)\right]$$

which is greater than zero if $Z(l) < l - P$ and less than zero when $Z(l) > l - P$. This means that $Z(l) \leq l - P$

When $Z(l) < w^* - y_i + l$ then $\frac{dE[\tau]}{dZ(l)} = -\pi f(l)$ and $\frac{E[U_i]}{dZ(l)} = -(1-\pi)\, u'(y_i - P)\, \frac{dP}{dZ(l)}$, so

$$\frac{E[U_s]}{dZ(l)} = \pi f(l) \left[1 - \delta u'(y_i - P)\right]$$

which is greater than zero as $Z(l) \leq l - P$ and we have assumed that $\delta u'(y_i - E[L]) < 1$. Therefore we can conclude that it is optimal to set $Z(l) = l - E(L)$ for all $l \in \left[\underline{L}, \overline{L}\right]$.

Now consider the case where the constraint is binding. That is $E[U_i] = \tilde{u}$. Setting up the a Lagrange we have

$$max_{Z(l)}\ \bar{\mathcal{L}} = y_s - E[\tau] + \delta E[U_i] + \bar{\lambda}\left(E[U_i] - \tilde{u}\right)$$

where $\lambda > 0$. From this, if $Z(l) > w^* - y_i + l$ then

$$\frac{d\bar{\mathcal{L}}}{dz(l)} = \left(\delta + \bar{\lambda}\right) \pi f(l) \left[u'(y_i - l + Z(l)) - u'(y_i - P)\right]$$

which again is greater than zero if $Z(l) < l - P$ and less than zero when $Z(l) > l - P$.

When $Z(l) < w^* - y_i + l$ then

$$\frac{d\bar{\mathcal{L}}}{dz(l)} = \pi f(l) \left[1 - \left(\delta + \bar{\lambda}\right) u'(y_i - P)\right]$$

The sign of this does not depend on l, so we can conclude from this that it must be less than zero, other wise full insurance is optimal and the constraint is not binding.

This means that at specific value of l the optimal must have either full insurance or no insurance.

As in the proof of proposition **2,** we can again restate the problem as choosing $g(l) \in [0, 1]$, with

$$\begin{aligned} E[U_i] &= (1-\pi)\, u(y_i - P) + \pi \int_{\underline{L}}^{\bar{L}} \left[g(l) u(y_i - P) + (1 - g(l))\, u(w^*)\right] f(l) dl \\ &= (1 - \pi(1-\gamma))\, u(y_i - P) + \gamma \pi u(w^*) \end{aligned}$$

where $\gamma = \pi \int_{\underline{L}}^{\bar{L}} (1 - g(l)) f(l) dl$, $P = \frac{\pi}{1-\gamma\pi} \int_{\underline{L}}^{\bar{L}} lg(l)f(l)dl$, and $E[\tau] = \pi \int_{\underline{L}}^{\bar{L}} g(l) (w^* - y_i + l) f(l)dl$

From this we can restate the problem as

$$min_{g(l)}\ E[\tau]$$

subject to $E[U_i] \geq \tilde{u}$.

Setting up the Lagrange gives

$$\mathcal{L} = E[\tau] - \lambda (E[U_i] - \tilde{u})$$

The first order condition with respect to $g(l)$ is given by

$$\frac{d\mathcal{L}}{dg(l)} = -\pi f(l) (w^* - y_i + l) - \lambda \frac{dE[U_i]}{dg(l)}$$

where $\lambda > 0$.

Rearranging the first order condition provides:

$$\begin{aligned} \frac{1}{\pi f(L)} \frac{d\mathcal{L}}{dg(L)} &= -(w^* - y_i + l) - \lambda [u(y_i - P) - u(w^*) - u'(y_i - P)(l - P)] \\ &= (\lambda u'(y_i - P) - 1) l + (y_i - w^*) - \lambda [u(y_i - P) - u(w^*) + pu'(y_i - P)] \end{aligned}$$

In order to determine whether small or large losses are optimal to exclude, it is necessary to determine whether $\frac{1}{\pi f(L)} \frac{d\mathcal{L}}{g(L)}$ is increasing or decreasing in l. In order to determine this, the coefficient of l, $\lambda u'(y_i - p) - 1$ is examined.

Consider an interior solution and set $\frac{1}{\pi f(L)} \frac{d\mathcal{L}}{g(L)} = 0$ for some l. Then

$$0 = (w^* - y_i + L) + \lambda [u(y_i - p) - u(w^*) - u'(y_i - p)(L - p)]$$

By risk aversion, $u(y_i - p) - u(w^*) > u'(y_i - p)(y_i - p - w^*)$.

From this it follows that

$$\begin{aligned}
0 &> (w^* - y_i + l) + \lambda \left\{ u'(y_i - p) \left[(y_i - p - w^*) - (l - p) \right] \right\} \\
0 &> (w^* - y_i + l) - \lambda u'(y_i - p) (w^* - y_i + l) \\
\lambda u'(y_i - p) (w^* - y_i + l) &> (w^* - y_i + l) \\
\lambda u'(y_i - p) &> 1
\end{aligned}$$

Since the coefficient on l is positive, $\frac{1}{\pi f(L)} \frac{d\mathcal{L}}{dg(L)}$ is increasing in l. This means that for a large loss, l greater than some threshold $\tilde{l}$, $\frac{1}{\pi f(l)} \frac{d\mathcal{L}}{g(l)} > 0$ and consequently, the expected transfer is minimized by excluding these losses and setting $g(l) = 0$. And, for a smaller loss, l smaller than some threshold $\tilde{l}$, $\frac{1}{\pi f(l)} \frac{d\mathcal{L}}{g(l)} < 0$, so it is optimal to cover the loss and set $g(l) = 1$. Thus, from the safety net provider's perspective it is also optimal to exclude large losses. □

Proposition 4. *The safety net provider's optimal insurance contract excludes less than the individual at risk's optimal contract. That is $\tilde{L}_{\tilde{u}} \geq L^*$, with equality only when $\tilde{u} = U_i^*$.*

Proof. From proposition **2** and **3** we have that the optimal in both cases is to exclude large loses. First, it cannot be that $\tilde{L}_{\tilde{u}} < L^*$ as the safety net provider could always offer the contract Z^* which would simultaneously decrease the expected transfer and increase the welfare of the individual at risk. This would increase the safety net providers welfare. This means that $\tilde{L}_{\tilde{u}} \geq L^*$.

If $\tilde{u} < U_i^*$ then, as shown in the proof of proposition , setting $Z(l) = l - P$ increases the safety net provider's welfare at any l. So it is possible to increase $\tilde{L}$ a from L^* so that the safety net providers welfare increases and the incentive compatibility constraint still holds. □

Proposition 7. *Fixing the outside option of the individual at risk at $\tilde{Z}(L)$; the safety net provider can improve their welfare, as compared to both the outside option and best incomplete contract, by offering a complete insurance contract with a subsidy. The optimal subsidy is given $S = max \left\{ u^{-1}(\tilde{u}) + E[L] - y_i, 0 \right\}$.*

Proof. Let $\tilde{u}$ be the outside option of the individual at risk. If $u(y_i - E[L]) \geq \tilde{u}$ then the individual at risk will be willing to accept full insurance without needing a subsidy.

Otherwise, from proposition 3 we have the optimal incomplete contract $\tilde{Z}$. Now let $\hat{S}$ equal the expected transfer, $E[\tilde{\tau}]$, under the contract $\tilde{Z}$. Now offer the individual at risk a complete insurance product with subsidy $\hat{S}$. That is $\hat{P} = E(L) - \frac{1}{1-\pi}\hat{P}$ and $Z(l) = l - \hat{P}$.

The expected utility of the individual at risk is $E[U_i] = u\left(y_i - \hat{P}\right)$. Further, the expected wealth of the individual at risk is the same as it is under the contract $\tilde{Z}$ and the subsidy, so by Jensen's inequality $u\left(y_i - \hat{P}\right) > E\left[\tilde{U}_i\right] = E[\tilde{u}]$ (where $E\left[\tilde{U}_i\right]$ is the expected utility under contract $\tilde{Z}$). This means that

$$\hat{U}_s = y_s - \hat{S} + \delta u\left(y_i - \hat{P}\right) > y_s - E[\tilde{\tau}] + \delta E\left[\tilde{U}_i\right]$$

Therefore this improves the welfare of the assistance provider and is incentive compatible for the individual at risk.

The doner can of course do better, by offer a subsidy $S = u^{-1}(\tilde{u}) + E[L] - y_i$, the gets utility

$$U_s = y_s - S + \delta u(y_i - P_s)$$

where $P_s = E[L] - S$. This increases utility as

$$U_s - \hat{U}_s = \left(\hat{S} - S\right) - \delta\left[u\left(y_i - E[L] + \hat{S}\right) - u(y_i - E[L] + S)\right]$$

which is greater than zero as $\delta u'(y_i - E[L]) < 1$. □