Quiz Sheet 6

Instructions: Turn in responses to TWO questions of your choice. This quiz will be timed for 15 minutes.

Question 1. Do all of the following:

(a) Let f be a k-tensor on V and let g be an l-tensor on V. Define what it means for a function $f \otimes g$ to be the *tensor product* of f and g, and specify the domain and the range of $f \otimes g$.

(b) Let f and g be the following tensors on $\mathbb{R}^4$:

$$f(x, y, z) = 2x_1y_2z_2 - x_2y_3z_1$$
$$g = \phi_{2,1} - 5\phi_{3,1}$$

Express $f \otimes g$ as a linear combination of elementary 5-tensors.

(c) Express $(f \otimes g)(x, y, z, u, v)$ as a function.

Question 2. Do all of the following:

(a) Given a symmetric group (also denoted as a permutation group) S_n define what it means for an element e_i, $1 \leq i < k$, of S_n to be an *elementary permutation*.

(b) Define what it means for a k-tensor f on V to be an *alternating tensor*.

(c) Which of the following are alternating tensors in $\mathbb{R}^4$?

$$f(x, y) = x_1y_2 - x_2y_1 + x_1y_1$$
$$g(x, y) = x_1y_3 - x_3y_2$$
$$h(x, y) = (x_1)^3(y_2)^3 - (x_2)^3(y_1)^3$$

Question 3. Do all of the following:

(a) Let V be a vector space with basis $a_1, ..., a_n$. Let $I = (i_1, ..., i_k)$ be an ascending k-tuple from the set $\{1, ..., n\}$. Define what means for the tensor ψ_I on V to be an *elementary alternating k-tensor* on V corresponding to the basis $a_1, ..., a_n$ for V.

(b) Let V be a vector space, and $f \in A^k(V)$ and $g \in A^l(V)$. Define what means for a tensor $f \wedge g$ be the *wedge product* of f and g.

(c) Show that $f \wedge g \wedge f = 0$ where f and g are the following alternating tensors in $\mathbb{R}^2$:

$$f(x, y) = x_1 y_2 - x_2 y_1$$
$$g(x, y) = -x_1 y_2 + x_2 y_1$$

Hint: As an alternative to a direct computation, you can determine the space containing this wedge product and apply the relevant result from the homework submitted last week (Homework Sheet 6).