Quiz Sheet 8

Instructions: Turn in responses to TWO questions of your choice. This quiz will be timed for 15 minutes.

Question 1. Do all of the following:

(a) Given an open set A in $\mathbb{R}^n$, define what it means for a function f on A to be a *differential form of order zero* (or simply a 0-*form*).

(b) Define what it means for a function to be an *exterior derivative* of a 0-form.

(c) Find the exterior derivative $d\omega$ of ω given by $\omega(x) = x_1x_2$. Evaluate $df(x)((x, v))$ for $x = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $v = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$.

Question 2. Do all of the following:

(a) Write the representation formula for k-forms in Ω^k that allows us to define the exterior derivative operator.

(b) Define what it means for d to be the *exterior derivative operator* on $\Omega^k \in \mathbb{R}^n$ for $k \geq 1$.

(c) Find the exterior derivative $d\omega$ of ω given by $\omega(x) = x_1x_2dx$. Evaluate $d\omega(x)((x, v_1), (x, v_2))$ for $x = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $v_1 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, $v_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

Question 3. Do all of the following:

(a) If ω and η are forms of order k and l, respectively, state the 'product' rule for evaluating $d(\omega \wedge \eta)$.

(b) Consider the forms

$$\omega = xydx + 3dy - yzdz$$
$$\eta = xdx - yz^2dy + 2xdz$$

$\in \mathbb{R}^3$. Verify by direct computation that $d(d\omega) = 0$ and $d(w \wedge \eta) = (d\omega) \wedge \eta - \omega \wedge d\eta$.