IEOR 3106: Introduction to Operations Research: Stochastic Models

Professor Whitt, Fall, 2012

Problem for Discussion, Tuesday, October 9

Trip to the Post Office

Five students from IEOR 3106 – Siddhant Bhatt (B), Ratnam Jain (J), Ethan Kochav (K), Juan Carlos Mendoza (M), and Daniel O'Leary (O) – simultaneously enter an empty post office, where there are three clerks ready to serve them. Siddhant (B), Ratnam (J) and Ethan (K) begin to receive service immediately, while Juan Carlos (M) and Daniel (O) wait in a single line, ready to be served by the first free clerk, with Juan Carlos (M) at the head of the line (to be served first when a server becomes free), and Daniel (O) after Juan Carlos (M). Suppose that the service times of the three clerks (for all customers) are independent exponential random variables, each with mean 2 minutes.

(a) What is the expected time (from the moment the students enter the post office) until Ethan (K) completes service?

(b) What is the probability that Ethan (K) is still in service after 6 minutes?

(c) What is the *conditional* probability that Ethan (K) is still in service after 10 minutes, given that Ethan (K) has not yet been served after 4 minutes?

(d) What is the *conditional* probability that Ethan (K) is still in service after 10 minutes, given that Siddhant (B) has not yet been served after 4 minutes?

(e) What is the probability that Siddhant (B) is the first to complete service?

(f) What is the expected time (from the moment the students enter the post office) until the first student completes service?

(g) What is the variance of the time (from the moment the students enter the post office) until the first student completes service?

(h) What is the expected time (from the moment the students enter the post office) until Juan Carlos (M) completes service?

(i) What is the expected time (again since entering the post office) until *all* five students finish service?

(j) What is the variance of the time until *all* five students finish service?

(k) What is the probability that Juan Carlos (M) is the *third* student to finish service?

(l) Suppose that you wanted to calculate the probability that the time required for all five students to complete service will exceed 10 minutes. What computational tool makes that calculation easy to perform? Briefly explain why.

Allison Lim's Flashlight

Allison Lim has a flashlight. Allison's flashlight needs two batteries to be operational. Suppose that, in addition to his (empty) flashlight, Allison has a set of 12 functioning batteries, called battery 1, battery 2, and so forth. Initially, Allison puts batteries 1 and 2 into his flashlight, so that it starts working. Then batteries fail one by one. Whenever a battery in the flashlight fails, the flashlight stops working. Allison then tests the two batteries in the flashlight to see which one had failed, and he removes that battery. He then puts in the next available unused battery with the remaining working battery, so that the flashlight is again working. Suppose that the batteries remain like new until installed in the flashlight. Suppose that the lifetimes of the different batteries (in use in the flashlight) are independent random variables, each with an exponential distribution having a mean of 4 months. Let T be the time that the flashlight ceases to work, i.e., the time that the flashlight fails and Allison's supply of batteries is exhausted. At that moment, exactly one of the original 12 batteries will still be working. Let that last remaining working battery be battery N. Note that N is a random variable taking values in the set $\{1, 2, \ldots, 12\}$. (It will be the number of the one remaining working battery in the flashlight.)

(a) What is the expected value of T?

(b) What is $P(N = 12)$?

(c) What is $P(N = 1)$?