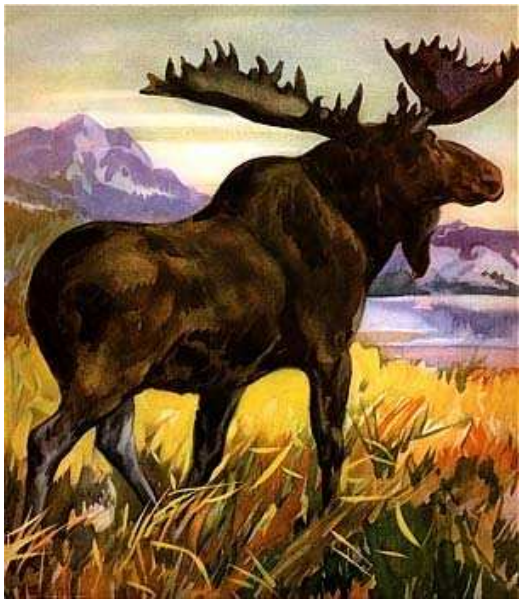
IEOR 3106: Introduction to Operations Research: Stochastic Models

Part I of First Midterm Exam, Chapters 1-3, September 30, 2008

You moose show your work.

Figure 1: a moose: Alces alces



From Wikipedia, the free encyclopedia: The moose (North America) or elk (Europe), Alces alces, is the largest extant species of the deer family. Moose are distinguished by the palmate antlers of the males, as depicted in Figure 1; other members of the deer family have antlers with a "twig-like" configuration.

1. The Moose-Recognition Test. (15 points)

In an animal-recognition test, 99% of Alaskan high school students can properly identify a moose from a picture, but only 10% of other American high school students can properly identify a moose from a picture. Alaskan high school students make up approximately 2% of the American high-school student population.

(a) (5 points) What is the probability that a randomly selected American high school student can properly identify a moose from a picture?

(b) (10 points) What is the conditional probability that a randomly selected American high school student is from Alaska, given that the student can properly identify a moose from a picture?

2. The Great Moose Weigh-Off. (15 points)

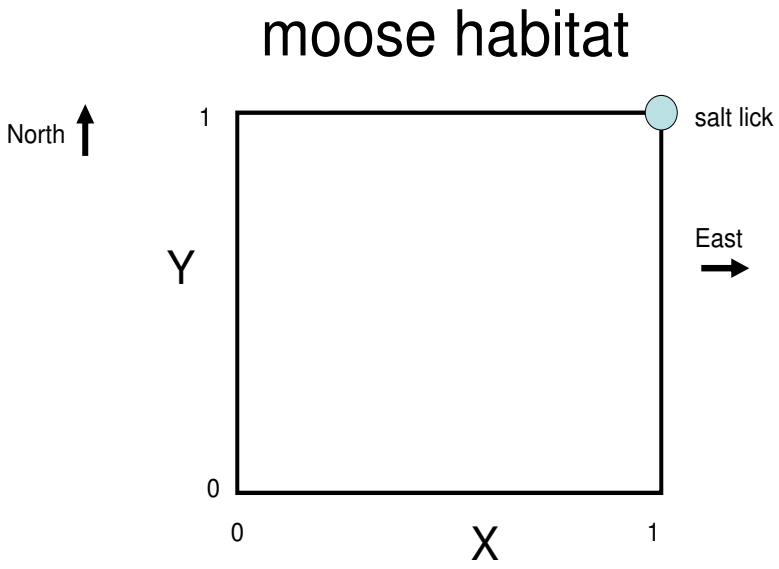
The male moose is an impressively large animal. The male moose is so big that it has been claimed that 100 randomly selected male moose together would weigh more than a diesel locomotive (train engine). A mayor of a small Alaskan town has decided to test that claim by having *The Great Moose Weigh-Off*. Suppose that the selected diesel locomotive weighs $52,000$ kilograms (kgs.). Suppose that a random male moose has a weight that is distributed according to a gamma distribution, having mean 500 kgs. and standard deviation 100 kgs.

(a) (10 points) Under these assumptions, what is the approximate probability that 100 randomly and independently selected male moose together weigh more than the selected diesel locomotive?

(b) (5 points) Explain the mathematical basis for your answer in part (a).

3. A Moose Habitat (20 points)

An old grey moose has settled down to live in a comfortable swampy habitat, which happens to occupy a one-mile by one-mile square region, as depicted in the figure below. There is a salt lick in the northeast corner of the habitat, which the moose likes to visit.



Let the random vector (X, Y) represent the random location of the old grey moose at any time. Let the random vector have the probability density function

$$f_{X,Y}(x, y) = x + y, \quad 0 \leq x \leq 1, \quad 0 \leq y \leq 1.$$

Determine the following quantities:

(a) (3 points) the probability density function of X,

(b) (3 points) the cumulative distribution function of X

(c) (3 points) $E[X]$,

(d) (3 points) $Var(X)$, the variance of X,

(e) (4 points) $Cov(X, Y)$, the covariance of X and Y,

(f) (4 points) $P(X \geq 1/2, Y \geq 1/2)$ (Is the moose likely to be near the salt lick?)

Honor Code: Students are expected to behave honorably, following the accepted code of academic honesty. After completing your exam, please affirm that you have done so by writing "I have neither given not received improper help on this examination," on your examination booklet and sign your name. You may keep the exam itself. Solutions will eventually be posted on line.