IEOR 3106: Introduction to Operations Research: Stochastic Models

Fall 2013, Professor Whitt

Homework Assignment 1: Tuesday, September 3, 2013

Due on Tuesday, September 10 before class; to be discussed at the recitation on Sunday, September 8, 4:00-6:00pm, in 303 Mudd.

Probability Review: Read Chapters 1 and 2 in the textbook, *Introduction to Probability Models*, tenth edition, by Sheldon Ross. (It is also OK to use the ninth edition.) Please do the six problems marked with an asterisk and turn them in next Tuesday. These problems are written out in case you do not yet have the textbook (available in the Columbia Bookstore). Extra problems are provided in case you need extra review. Solutions will be provided for all the following problems.

CHAPTER 1

*Problem 1.3.

A coin is to be tossed until a head appears twice in a row. What is the sample space for this experiment? If the coin is fair, what is the probability that it will be tossed exactly four times?

*Problem 1.21

Suppose that 5% of men and 0.25% of women are color-blind. A color-blind person is chosen at random. What is the probability of this person being male? (Assume that there are an equal number of males and females.)

Problem 1.22

Two players, A and B, play a succession of games, winning or losing one point in each game. They continue playing until one has two points more than the other. Assuming that each point is independently won by player A with probability p, what is the probability that they will play a total of $2n$ games (points)? What is the probability that A will win?

Problem 1.31

Two dice are tossed, one green and one red. What is the conditional probability that the number on the green die is 6, given that the sum on the two dice is 7?

Extra part: What is the conditional probability that the number on the green die is 6, given that the sum on the two dice is 10?

*Problem 1.43

Suppose we have 10 special coins, which are such that if the i^{th} coin is flipped, the heads will appear with probability $i/10$, for $1 \leq i \leq 10$. Suppose that one of the ten coins is selected at random, with each one equally likely to be selected. Suppose that this randomly selected coin is flipped. What is the conditional probability that the randomly flipped coin is coin $i = 5$ given that the randomly selected coin showed heads?

CHAPTER 2

Problem 2.9

Suppose that the distribution function F is given by:

$$
\begin{aligned}
F(b) &= 0, \quad b < 0, \\
F(b) &= 1/2, \quad 0 \leq b < 1, \\
F(b) &= 3/5, \quad 1 \leq b < 2, \\
F(b) &= 4/5, \quad 2 \leq b < 3, \\
F(b) &= 9/10, \quad 3 \leq b < 3.5, \\
F(b) &= 1, \quad 3.5 \leq b.
\end{aligned} \tag{1}
$$

Calculate the associated probability mass function.

Problem 2.20

A television store owner figures that 50% of the customers entering his store will purchase an ordinary (black and white) television set, 20% will purchase a color television, and 30% will just be browsing (and thus not purchase anything). If 5 customers enter his store on a certain day, what is the probability that 2 customers purchase color sets, 1 purchases an ordinary set, and 2 customers purchase nothing?

*Problem 2.32

Suppose that you buy a lottery ticket in 50 different lotteries, in each of which your chance of winning a prize is 1/100. What is the approximate probability that you will win a prize (a) at least once, (b) exactly once, (c) at least twice? (Give numerical answers if possible.)

Problem 2.33

Let X be a random variable with probability density

$$f(x) = c(1 - x^2), \quad -1 < x < 1,$$

and $f(x) = 0$ otherwise.

(a) What is the value of c?

(b) What is the cumulative distribution function (cdf) of X?

*Problem 2.34

Let the probability density function (pdf) of a random variable X be $f(x) = c(4x - 2x^2)$ for $0 < x < 2$, where c is some constant, with $f(x) = 0$ otherwise. (a) What is the value of c? (b) What is $P(1/2 < X < 3/2)$?

Problem 2.39

The random variable X has the following probability mass function: $P(X = 1) = 1/2$, $P(X = 2) = 1/3$, and $P(X = 24) = 1/6$. Calculate $E[X]$.

*Problem 2.43

An urn contains $n+m$ balls, of which n are red and m are black. These balls are withdrawn from the urn, one at a time, and without replacement. Let X be the number of red balls removed before the first black ball is chosen. We are interested in determining $E[X]$. To obtain this quantity, number the red balls from 1 to n. Now define the (indicator) random variables X_i for $i = 1, \ldots, n$ by letting $X_i = 1$ if red ball i is chosen before any black ball is chosen, and let $X_i = 0$ otherwise. (a) Express X in terms of the random variables X_i; (b) Find $E[X]$.

Problem 2.48 (in 9^{th} edition, the intended problem, no answer in back)

Suppose that X is uniformly distributed over $[0, 1]$. Calculate $E[X^2]$.

Problem 2.48 (in 10^{th} edition, harder than intended, answer in back)

Suppose that X is a nonnegative random variable with pdf f, and g is a differentiable function with $g(0) = 0$. Show that

$$E[g(X)] = \int_0^\infty P(X > t) g'(t)\, dt.$$