# IEOR 3106: A DTMC Example

Consider a Markov chain on the eight states $\{1, 2, \cdots, 8\}$ with transition matrix $P$ given by

$$P = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array} \begin{pmatrix} 0.7 & 0.0 & 0.0 & 0.0 & 0.0 & 0.3 & 0.0 & 0.0 \\ 0.1 & 0.2 & 0.3 & 0.0 & 0.1 & 0.1 & 0.2 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.1 & 0.2 & 0.0 & 0.0 & 0.2 & 0.2 & 0.3 \\ 0.3 & 0.0 & 0.0 & 0.0 & 0.0 & 0.7 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 \\ 0.0 & 0.0 & 0.0 & 0.5 & 0.0 & 0.0 & 0.5 & 0.0 \end{pmatrix}$$

**Note that we are numbering the rows in the natural order 1, 2, ... , 8, with the columns labeled the same as the rows.

1. How can we understand this Markov chain?
2. What happens in the long run if we start in state 3?
3. What happens in the long run if we start in state 1?
4. What happens in the long run if we start in state 4?
5. What happens in the long run if we start in state 2?

Put the transition matrix in canonical form (showing the original states in their new positions).

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We reorder the states, putting the recurrent states (states in closed communication classes) first, keeping the states in the same communication class together. We order the communication classes by size, putting the smallest ones first. We then put the transient states (states in open communication classes) last. We order the open communication classes, putting the ones that can be reached from other open classes above those, if there are such. Here there are three closed communication classes, $\{3\}$, $\{1, 6\}$ and $\{4, 7, 8\}$, and only one open communication class, $\{2, 5\}$. The canonical form is

$$P = \begin{matrix} 3 \\ 1 \\ 6 \\ 4 \\ 7 \\ 8 \\ 2 \\ 5 \end{matrix} \begin{pmatrix} 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.7 & 0.3 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.3 & 0.7 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.5 & 0.5 & 0.0 & 0.0 & 0.0 \\ 0.3 & 0.1 & 0.1 & 0.0 & 0.2 & 0.0 & 0.2 & 0.1 \\ 0.2 & 0.0 & 0.2 & 0.0 & 0.2 & 0.3 & 0.1 & 0.0 \end{pmatrix}$$

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