IEOR 3106: Introduction to Operations Research: Stochastic Models

Professor Whitt, Fall, 2013

Problem for Discussion, Tuesday, October 8

Trip to the Post Office

Five students from IEOR 3106 – Alexander Frazer (A), Yunhe Wang (Y), Bohao Zhou (B), Zhenlun Zhou (Z), and Chaitanya Kanitkar (C) – simultaneously enter an empty post office, where there are three clerks ready to serve them. Alexander (A), Yunhe (Y), Bohao (B) begin to receive service immediately, while Zhenlun (Z) and Chaitanya (C) wait in a single line, ready to be served by the first free clerk, with Zhenlun (Z) at the head of the line (to be served first when a server becomes free), and Chaitanya (C) after Zhenlun (Z). Suppose that the service times of the three clerks (for all customers) are independent exponential random variables, each with mean 2 minutes.

(a) What is the expected time (from the moment the students enter the post office) until Bohao (B) completes service?

(b) What is the probability that Bohao (B) is still in service after 6 minutes?

(c) What is the *conditional* probability that Bohao (B) is still in service after 10 minutes, given that Bohao (B) has not yet been served after 4 minutes?

(d) What is the *conditional* probability that Bohao (B) is still in service after 10 minutes, given that Alexander (A) has not yet been served after 4 minutes?

(e) What is the probability that Alexander (A) is the first to complete service?

(f) What is the expected time (from the moment the students enter the post office) until the first student completes service?

(g) What is the variance of the time (from the moment the students enter the post office) until the first student completes service?

(h) What is the expected time (from the moment the students enter the post office) until Zhenlun (Z) completes service?

(i) What is the expected time (again since entering the post office) until *all* five students finish service?

(j) What is the variance of the time until *all* five students finish service?

(k) What is the probability that Zhenlun (Z) is the *third* student to finish service?

(l) Suppose that you wanted to calculate the probability that the time required for all five students to complete service will exceed 10 minutes. What computational tool makes that calculation easy to perform? Briefly explain why.

Sean Curran's Flashlight

Sean Curran has a flashlight. Sean's flashlight needs two batteries to be operational. Suppose that, in addition to his (empty) flashlight, Sean has a set of 12 functioning batteries, called battery 1, battery 2, and so forth. Initially, Sean puts batteries 1 and 2 into his flashlight, so that it starts working. Then batteries fail one by one. Whenever a battery in the flashlight fails, the flashlight stops working. Sean then tests the two batteries in the flashlight to see which one had failed, and he removes that battery. He then puts in the next available unused battery with the remaining working battery, so that the flashlight is again working. Suppose that the batteries remain like new until installed in the flashlight. Suppose that the lifetimes of the different batteries (in use in the flashlight) are independent random variables, each with an exponential distribution having a mean of 4 months. Let T be the time that the flashlight ceases to work, i.e., the time that the flashlight fails and Sean's supply of batteries is exhausted. At that moment, exactly one of the original 12 batteries will still be working. Let that last remaining working battery be battery N. Note that N is a random variable taking values in the set $\{1, 2, \ldots, 12\}$. (It will be the number of the one remaining working battery in the flashlight.)

(a) What is the expected value of T?

(b) What is $P(N = 12)$?

(c) What is $P(N = 1)$?