

```
function v = stationary(P)
%
% This is a MATLAB function that calculates the stationary probability vector
or v
% of a Markov chain transition matrix P, i.e., we solve  $v = vP$  .
% We assume the existence of a unique stationary vector.
% For a finite-state Markov chain, the condition is that the chain be irreducible.
%
% To solve the system of n equations in n unknowns, we use the matrix inverse function inv.
% For a square matrix A,  $inv(A)*A = I$ , where I is the identity matrix (1's on the diagonal, 0's elsewhere).
%
% We input the matrix P when we call the function.
% First find the number n of rows in the transition matrix P.
s = size(P);
n = s(1);
% There is one redundant equation in the n equations  $v = vP$ .
% We fill gap by using the fact that  $v(1) + \dots + v(n) = 1$ .
% We eliminate redundant equation by replacing last column of P with ones.
PP = P;
PP(:,end) = [];
w = ones(n,1);
PP = [PP w];
% PP is the matrix P with the last column replaced by a column of 1's.
%
% Note that for the desired v,  $v*PP$  equals v except the last element is 1.
% We thus need to modify the equation.
% For that purpose, introduce auxiliary matrices I and L.
% I is the identity matrix.
% L is all zeros except a 1 in the bottom right.
I = eye(n);
f = [zeros(1, n-1) 1];
L=diag(f);
% Now  $v = vP$  for prob vector v becomes:  $v*(PP) = v + f*(1-v(n))$ 
% Or, equivalently,  $v*(PP) = v*I - v*L + f$ 
% Or  $v*(PP - I + L) = f$ 
%
% We want to solve  $v*R = f$ , where  $R = PP-I+L$ .
%
R = PP-I+L;
%
% We can solve  $v*R = f$  in three ways:
% First, we can write  $v = f/R$ , understanding v and f to be row vectors.
% Second, we can take transposes and work with column vectors.
% We get  $(v*R)' = R'*v' = f'$ .
% We then write  $v' = R'\backslash f'$ .
% Third, we can solve for v by directly inverting the matrix  $R = PP-I+L$ .
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% The solution is  $v = f*RR$ , where RR is the inverse of R.  
%  
RR= inv(R);  
v = f*RR;  
%  
% The desired v is the last row of the matrix RR.  
% By multiplying RR by f, we get the last row of RR.
```