A General Golf Course Simulation Tool
Keeping Delays Down and Throughput Up

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Objectives

- View golf course as a queueing model (series of 18 queues).

- Construct a simulation model applicable for many golf courses.

  - Use the simulation to develop strategies for a golf course manager to **maximize the throughput** while **keeping sojourn times low**.

  - **Throughput**: the number of groups that can play on a course in one day
  
  - **Sojourn Time**: the time required for a group to play the course (from starting play on the first hole to completing play on the last hole)
Previous work

- **Kimes and Schruben (2002)** developed a simulation model in the paper, *Studying Tee Time Intervals*.

- **Tiger and Salzer (2004)** developed an interactive spreadsheet golf simulation to teach the concept of “throughput” and “bottleneck” in operations management to undergraduate students.

- **Riccio (2012)** showed how to improve the pace of play by applying principles in “factory physics”.

- **Whitt (2014) and Fu and Whitt (2015)** developed a new stochastic queueing model (with new group stage playing time primitives) and approximation formulas.
  - We now develop the simulation tool and apply it.
A golf course consists of **18 holes**, each being one of three types:

- (1) Par-3
- (2) Par-4
- (3) Par-5.

The par number is the target number of golf strokes.

A higher par number means that it typically longer for a group to play.

A typical golf course has

- 12 Par-4 holes,
- 3 par-3 holes
- and 3 Par-5 holes.

Many different arrangements occur, but it is rare for two Par-3 holes or two Par-5 to appear consecutively.

There are **precedence constraints**: A group cannot begin play until the group in front has moved far enough ahead (*to not hit people!*).
In practice, it has been found that **Par-3 Holes tend to be the bottleneck holes.**

- Each group has to wait until the previous group **finishes** playing the entire hole before starting to play.

- **Alternative policy for par-3 hole:** **Par-3 with “Wave-Up” (P3WU)**
  - If the next group is ready to start, a group waits near the green to let the next group hit its initial tee shots.
  - That allows two groups to play on the hole simultaneously.
  - But, if the next group is not ready yet, then the group does not wait.
  - That policy leads to more variance in playing and waiting times.
The capacity of each hole can be defined as the average cycle time (interval between successive group departures) from a fully loaded hole (with new groups always available to start play).

Formulas for the capacities of each hole were derived in Whitt (2015), but simulation is needed in general.

The bottleneck analysis leads to two kinds of course models:
- **Balanced courses** (the capacities of all holes are the same)
- **Unbalanced courses** (the capacities of all holes are not the same)

We have primarily focused on balanced courses. Simulation was used to choose parameters that make a balanced course.
- Our main models use P3WU holes instead of ordinary Par-3 holes.
Model Group Play: Example of Par-4

5 steps of group play combined into 3 stages

\[(T, W_1) \rightarrow F \rightarrow (W_2, G)\]

referred to as \(S_1 \rightarrow S_2 \rightarrow S_3\)

**Basic Parameters for group \(n\) on the hole:**

\(S_{i,n}\) = playing time on stage \(i\) (model primitive)
\(A_n\) = time group Arrives, ready to begin play
\(B_n\) = time group actually Begins play
\(T_n\) = time group completes **Tee** shots
\(F_n\) = time group completes **Fairway** shots
\(G_n\) = time group clears **Green** (completes play on the hole)
Basic recursion for a Par-4 hole

- Below is a representation of a group play on a par-4 hole, with precedence constraints. (can be seen from \( a \lor b = \max(a, b) \))

- Par-4 Hole:
  
  For \( n \geq 1 \), and with \( F_0 = G_0 = 0 \),
  
  (i) \( B_n \equiv A_n \lor F_{n-1} \)
  
  (ii) \( T_n \equiv B_n + S_{1,n} \)
  
  (iii) \( F_n \equiv (T_n \lor G_{n-1}) + S_{2,n} \)
  
  (iv) \( G_n \equiv F_n + S_{3,n} \)

- Performance Measures:
  - Waiting time: \( W_n = B_n - A_n \)
  - Playing time: \( X_n = G_n - B_n \)
  - Sojourn time: \( X_n + W_n = (G_n - B_n) + (B_n - A_n) = G_n - A_n \)
  - Cycle time: \( C_n \equiv G_n - G_{n-1}, \ n \geq 1 \)

  • Consequently, \( \bar{C}_n = \frac{1}{n} \sum_{k=1}^{n} C_k = \frac{G_n}{n} \) indicates the average cycle time.
Stage playing time distributions (1)

- Starting point: symmetric **triangular distribution for** $S_i$
  - Simple representation with mean $m_i$ and variability $a_i$
  - Specifically, we let the stage playing times $S_i$ be:
    - $S_i = m_i - a_i + 2a_i T$
    - where $T \equiv T[0,1]$ is a random variable with a symmetric triangular distribution on $[0, 1]$ with density
    - $f(t) = 4t, \ 0 \leq t \leq 0.5$ and $4 - 4t, \ 0.5 \leq t \leq 1$
    - $E[S_i] = m_i$ and $Var[S_i] = a_i^2/6$

- Simplification: parameter reduction for Par-4:
  - $(m, a, r) = (4.0, 1.5, 0.5)$
    - $m_1 = m_3 = m, \ m_2 = rm$ (mean parameters, $m = 4.0, r = 0.5$)
    - $a_1 = a_2 = a_3 = a$ (variability parameters, $a = 1.5$)
Allow for **lost ball on the initial (tee) shots**
- Two parameters \((p, L)\), taken to be \((0.05, 8.0)\)
- Probability of lost ball = \(p\)
  - Stage playing time \(S_1\) as above if no lost ball
  - Stage playing time \(S_1 = L\) if lost ball

**Overall 5-parameter model for Par-4:**
- \((m, a, r, p, L) = (4.0, 1.5, 0.5, 0.05, 8.0)\)
  - \(m_1 = m_3 = m, m_2 = r m\) (mean parameters, \(m = 4.0, r = 0.5\))
  - \(a_1 = a_2 = a_3 = a\) (variability parameters, \(a = 1.5\))
  - \(S_i = m_i - a_i + 2a_i T\) (resulting random variables)
  - \(T = \text{random variable with triangular distribution on } [0,1]\)
  - lost ball with \((p, L) = (0.05, 8.0)\)

**Cycle time on a fully loaded Par-4 hole:**
\[E[C] = 6.535, \text{ Var}[C] = 1.249\]
The cycle time on a Par-4 hole: histogram

Mixed triangle w. lost ball service distribution, m=4, a=1.5, p=0.05, L=8
1e5 samples, mean=6.535, var=1.249, scv=0.029
Constructing the simulation code

**Inputs**

- # of playing groups \( (N) \)
- Golf hole sequence (array)
- Arrival distributions (deterministic)
- Playing time distributions (triangular)
  - Also includes a “lost ball” factor
- # of simulation replications \( (N_{Sim}) \)

**Outputs**

(for each hole and for entire course)

- Playing time data \( X \) for each group
- Waiting time data \( W \) for each group
- With above two →
  - Sojourn time for each group
  - Departure time for each group
  - “Inter-departure time” (which can also be referred as cycle time) for each group
(1) Here is the list of the stage playing time mean values $m_i$ for Par-3, Par-4 and Par-5: (note: all the numbers are in minutes)

- Par-3 $= (3.5^T \rightarrow 2^G \rightarrow \frac{8^G}{3})$
- Par-4 $= (4^T \rightarrow 2^F \rightarrow 4^G)$
- Par-5 $= (4^T \rightarrow 2^F \rightarrow 2^F \rightarrow \frac{4^G}{3} \rightarrow 4^G)$
  
  - Superscripts: $T = $ Tee, $F = $ Fairway, $G = $ Green
  - $a = 1.5$, so $\text{Var}(S_i) = a^2 / 6 = 0.375$ (before lost ball)

(2) **Lost ball** factor on the first hole: $(p, L) = (0.05, 8.0)$

- With a probability $p = 0.05$, a player may lose a golf ball (e.g. fall in the bunker or water area) during the first stage
- A lost ball increases the playing time on the first stage to $L = 8$
### Data and simulation structure

<table>
<thead>
<tr>
<th>Defining a golf course (1 x 18 array)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>18</td>
</tr>
</tbody>
</table>
Data and simulation structure

Defining a golf course (1 x 18 array)

Define $N$ : # Groups

1

[1 4 5 4 3 4 4 3 4 4 5 4 3 4 4 4 5 5 4]

18

e.g. $N = 1000$
Data and simulation structure

Defining a golf course (1 x 18 array)

Define $N$: # Groups

Generate an arrival matrix

$A$

$n_1$

$n_2$

$n_3$

$n_4$

$n_5$

$n_6$

$\vdots$

$\vdots$

$\vdots$

$n_N$

$N = 1, 2, \ldots, 1000$

e.g. $N = 1000$
Data and simulation structure

1. Defining a golf course (1 x 18 array)

2. Define $N$: # Groups

3. Generate an arrival matrix

4. Calculate various outputs for the first hole

Define $N$: $N = 1000$

- $A$
- $P(:,1)$

$n_1$
$n_2$
$n_3$
$n_4$
$n_5$
$n_6$

...$

n_N$
Data and simulation structure

1. Defining a golf course (1 x 18 array)
2. Define $N$: # Groups
3. Generate an arrival matrix
4. Calculate various outputs for the first hole
5. Repeat for the next 17 holes

For example, $N = 1000$

$A$ matrix (1000 by 18)

$P$ matrix (1000 by 18)
# Data and simulation structure

1. **Defining a golf course** (1 x 18 array)
2. **Define** $N$: # of Groups
3. **Generate an arrival matrix**
4. **Calculate various outputs for the first hole**
5. **Repeat for the next 17 holes**
6. **Repeat for the next $NSim$ replications**

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### Example

- **$N = 1000$**

### $A$ matrix

<table>
<thead>
<tr>
<th>$n_1$</th>
<th>$n_2$</th>
<th>$n_3$</th>
<th>$n_4$</th>
<th>$n_5$</th>
<th>$n_6$</th>
<th>$n_N$</th>
</tr>
</thead>
</table>

### $P$ matrix (1000 by 18)
Find the maximum number of groups that can play each day subject to two constraints:

- The total time available is $\tau$ minutes.
- The expected sojourn time for all groups is no more than $\gamma$ minutes.
  - For example, $\tau = 840$ minutes would be 14 hours, while $\gamma = 240$ minutes would be 4 hours.
Running the simulation

- Sample golf courses:
  - Base Case:
  - Spread-out case:
  - Concentrated P3 case (beg):
  - Concentrated P3 case (end):
  - Concentrated P5 case:

- Again, we have the following hole parameters:
  - Par-3 = \((3.5^T \rightarrow 2^G \rightarrow \frac{8^G}{3})\)
  - Par-4 = \((4^T \rightarrow 2^F \rightarrow 4^G)\)
  - Par-5 = \((4^T \rightarrow 2^F \rightarrow 2^F \rightarrow \frac{4^G}{3} \rightarrow 4^G)\)

- The simulation was conducted by running 2000 replications of 100 groups playing on various 18-hole courses.
Playing, waiting and sojourn times

Base Case: E[V], E[X], E[W] averaged over NSim=2000

- Playing Time
- Waiting Time
- Sojourn Time

Spreadout Case: E[V], E[X], E[W] averaged over NSim=2000

- Playing Time
- Waiting Time
- Sojourn Time

P3-concentrated-begin: E[V], E[X], E[W] averaged over NSim=2000

- Playing Time
- Waiting Time
- Sojourn Time

P3-concentrated-end: E[V], E[X], E[W] averaged over NSim=2000

- Playing Time
- Waiting Time
- Sojourn Time

A General Golf Course Simulation Tool
Throughput analysis

- Ultimately, how many customers can play per day (840 minutes)?
- Note that the max throughput occurs when the course is under-loaded (i.e. $\rho \approx \frac{6.53}{7.15} \approx 0.91$).

<table>
<thead>
<tr>
<th>Tee Interval</th>
<th>Sojourn Constraint (&lt; 240 minutes)</th>
<th>Departure Constraint (&lt; 840 minutes)</th>
<th>Overall Throughput Level</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>“4 5 4” “5 5 5”</td>
<td>“4 5 4” “5 5 5”</td>
<td>“4 5 4” “5 5 5”</td>
</tr>
<tr>
<td>5.00</td>
<td>10 10</td>
<td>86 86</td>
<td>10 10</td>
</tr>
<tr>
<td>5.50</td>
<td>12 12</td>
<td>86 86</td>
<td>12 12</td>
</tr>
<tr>
<td>6.00</td>
<td>15 15</td>
<td>86 86</td>
<td>15 15</td>
</tr>
<tr>
<td>6.50</td>
<td>21 21</td>
<td>86 86</td>
<td>21 21</td>
</tr>
<tr>
<td>7.00</td>
<td>42 42</td>
<td>85 85</td>
<td>42 42</td>
</tr>
<tr>
<td>7.50</td>
<td>100 100</td>
<td>82 82</td>
<td>82 82</td>
</tr>
<tr>
<td>8.00</td>
<td>100 100</td>
<td>79 78</td>
<td>79 78</td>
</tr>
<tr>
<td>8.50</td>
<td>100 100</td>
<td>75 75</td>
<td>75 75</td>
</tr>
<tr>
<td>9.00</td>
<td>100 100</td>
<td>71 71</td>
<td>71 71</td>
</tr>
<tr>
<td>9.50</td>
<td>100 100</td>
<td>68 68</td>
<td>68 68</td>
</tr>
<tr>
<td>10.00</td>
<td>100 100</td>
<td>64 64</td>
<td>64 64</td>
</tr>
</tbody>
</table>

Figure 1.1 Comparison of throughput levels between two balanced golf courses, with a tee interval ranging from 5.00 to 10.00.

<table>
<thead>
<tr>
<th>Tee Interval</th>
<th>Sojourn Constraint (&lt; 240 minutes)</th>
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<tr>
<td></td>
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<td>“4 5 4” “5 5 5”</td>
</tr>
<tr>
<td>7.00</td>
<td>42 42</td>
<td>85 85</td>
<td>42 42</td>
</tr>
<tr>
<td>7.05</td>
<td>50 49</td>
<td>85 85</td>
<td>50 49</td>
</tr>
<tr>
<td>7.10</td>
<td>69 63</td>
<td>85 85</td>
<td>69 63</td>
</tr>
<tr>
<td>7.15</td>
<td>100 100</td>
<td>85 85</td>
<td>85 85</td>
</tr>
<tr>
<td>7.20</td>
<td>100 100</td>
<td>84 84</td>
<td>84 84</td>
</tr>
<tr>
<td>7.25</td>
<td>100 100</td>
<td>84 84</td>
<td>84 84</td>
</tr>
<tr>
<td>7.30</td>
<td>100 100</td>
<td>83 83</td>
<td>83 83</td>
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<tr>
<td>7.35</td>
<td>100 100</td>
<td>83 83</td>
<td>83 83</td>
</tr>
<tr>
<td>7.40</td>
<td>100 100</td>
<td>83 83</td>
<td>83 83</td>
</tr>
<tr>
<td>7.45</td>
<td>100 100</td>
<td>83 83</td>
<td>83 83</td>
</tr>
<tr>
<td>7.50</td>
<td>100 100</td>
<td>82 82</td>
<td>82 82</td>
</tr>
</tbody>
</table>

Figure 1.2 “Zoomed-in” comparison of throughput levels between two balanced golf courses, with a tee interval ranging from 7.00 to 7.50.
Below graph shows throughput alters with varying inter-arrival times.

Throughput Level Comparison

- Throughput Level for (4 5 4) course
- Throughput Level for (5 5 5) course
- Throughput Level (Concentrated in Beginning)
- Throughput Level (Concentrated in the Middle)
- Throughput Level (Concentrated in the End)
Sojourn time comparison

Sojourn times among underloaded cases

Sojourn Time (minutes)

Playing Group

- rho=7.00
- rho=7.05
- rho=7.10
- rho=7.15
- rho=7.20
- rho=7.25
- rho=7.30
- rho=7.35
- rho=7.40
Insights from the graphs

- There is a much greater penalty from making the inter-arrival time (tee interval) too small, than for making it too large.
  - The optimal interval is 7.15 minutes yielding 85 groups.
  - An interval of 7.00 minutes yields about 40 groups.
  - An interval of 7.30 minutes yields 84 groups.
  - An interval of 8.00 minutes yields 80 groups.

- As long as the course is balanced, the hole sequence does not affect the maximum throughput rate.

- The sojourn time converges faster to a steady-state value as the traffic intensity $\rho$ decreases. (familiar property of queues)
 Evaluate how **unbalanced courses** are different from balanced courses.

- Preliminary work shows that groups experience large delays at bottleneck holes on unbalanced courses.

Derive **approximation formulas** for sojourn and playing times:

- Heavy-traffic case \((\rho \geq 1)\) → Done
- Under-loaded case \((\rho < 1)\) → using QNA in Whitt (1983)

Study **alternative non-constant tee intervals** on first hole.

- Non-constant schedules can be better.

Study impact of **slow groups** (take longer to play on all holes).

- Dramatic: One slow group can cause huge delays!
References


Hole designs: Par-3

- Par-3 is the bottleneck hole for the golf course: a playing group cannot play until the preceding group departs the hole
  - Alternative Policy: Par-3 with Wave-Up ("P3WU")
  - Two playing groups play along the hole simultaneously → more variance in playing and waiting times
- Event $E_n$ defines whether the preceding group needs to wait for the succeeding group before moving on
  - $E_n \equiv \{A_n \leq W_{n-1} \lor G_{n-2} < T_n\}, n \geq 1$ and
  - $E_n^c \equiv \{A_n > W_{n-1} \lor G_{n-2}\} \cup \{T_n \leq W_{n-1} \lor G_{n-2}\}$
  - The recursive formulas are as follows:
    - (i) $B_n \equiv (W_{n-1} \lor G_{n-2})1_{E_n} + (A_n \lor G_{n-1})1_{E_n^c}$ $(n \geq 2)$, $B_1 \equiv A_1$
    - (ii) $T_n \equiv B_n + S_{1,n}$, (iii) $W_n \equiv T_n + S_{2,n}$ $(n \geq 1)$
    - (iv) $G_n \equiv \left[(W_n \lor G_{n-1})1_{E_{n+1}} + T_{n+1}1_{E_{n+1}}\right] + S_{3,n}$
      \[
      \equiv (W_n \lor G_{n-1}) + S_{1,n+1}1_{E_{n+1}} + S_{3,n}
      \]
Hole designs: Par-4, Par-5

- As seen in slide 4, each hole has a set of **precedent constraints**

- For Par-4 and Par-5 holes, each playing group can only play after the preceding group has completed two stages before them

- **Par-4 Hole:**
  
  For $n \geq 1$, and with $F_0 \equiv G_0 \equiv 0$,
  
  (i) $B_n \equiv A_n \lor F_{n-1}$  
  (ii) $T_n \equiv B_n + S_{1,n}$
  
  (iii) $F_n \equiv (T_n \lor G_{n-1}) + S_{2,n}$  
  (iv) $G_n \equiv F_n + S_{3,n}$

- **Par-5 Hole:**
  
  (i) $B_n \equiv A_n \lor F_{1,n-1}$  
  (ii) $T_n \equiv B_n + S_{1,n}$
  
  (iii) $F_{1,n} \equiv (T_n \lor G_{n-1}) + S_{2,n}$  
  (iv) $W_{2,n} \equiv F_{1,n} + S_{3,n}$
  
  (v) $F_{2,n} \equiv (W_{2,n} \lor G_{n-1}) + S_{4,n}$  
  and  
  (vi) $G_n \equiv F_{2,n} + S_{3,n}$
Queueing analysis: balanced courses

- Let $\overline{\theta}_1, \overline{\theta}_2, \ldots, \overline{\theta}_{18}$ be the throughput rate for holes 1, 2, \ldots, 18
  - $\overline{\theta}_i^{-1} = \frac{1}{E[Y_i]}$ → the reciprocal of throughput rate is cycle time

- If $\overline{\theta}_1 \approx \overline{\theta}_2 \approx \cdots \approx \overline{\theta}_{18}$, then the max throughput $\overline{\theta}$ for the entire course will also be approx. cycle time of an individual hole

- Max throughput rate remains the same for all hole sequences if $\overline{\theta}_1 \approx \overline{\theta}_2 \approx \cdots \approx \overline{\theta}_{18}$
  - The four graphs on the next slide illustrates this point

- With the playing time parameters we have in slide 11 –
  - (3.5, 2, 8/3), (4, 2, 4) and (4, 2, 2, 4/3, 4) for Par-3, Par-4 and Par-5, the cycle time $\frac{1}{E[Y_i]}$ is approximately 6.53