A General Golf Course Simulation Tool

Keeping Delays Down and Throughput Up

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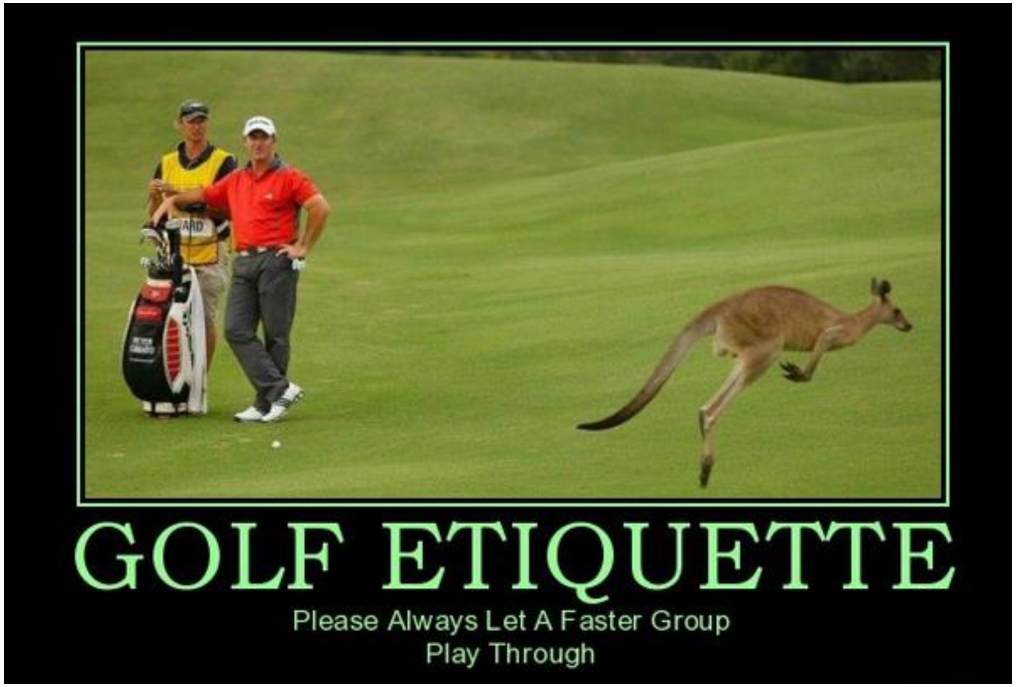


Objectives

- View golf course as a queueing model (series of 18 queues).
- Construct a simulation model applicable for many golf courses.
    - Use the simulation to develop strategies for a golf course manager to **maximize the throughput** while **keeping sojourn times low.**
        - ***Throughput:*** the number of groups that can play on a course in one day
        - ***Sojourn Time***: the time required for a group to play the course (from starting play on the first hole to completing play on the last hole)

Previous work

- **Kimes and Schruben (2002)** developed a simulation model in the paper, *Studying Tee Time Intervals.*

- **Tiger and Salzer (2004)** developed an interactive spreadsheet golf simulation to teach the concept of “throughput” and “bottleneck” in operations management to undergraduate students.

- **Riccio (2012)** showed how to improve the pace of play by applying principles in “factory physics”.

- **Whitt (2014)** and **Fu and Whitt (2015)** developed a new stochastic queueing model (with new group stage playing time primitives) and approximation formulas.
  - We now develop the simulation tool and apply it.

Description of a golf course

- A golf course consists of **18 holes**, each being one of three types:
  - (1) Par-3 (2) Par-4 and (3) Par-5.
- The par number is the target number of golf strokes.
- A higher par number means that it typically longer for a group to play.
- A typical golf course has
  - 12 Par-4 holes, 3 par-3 holes and 3 Par-5 holes.
- Many different arrangements occur, but it is rare for two Par-3 holes or two Par-5 to appear consecutively.
- There are **precedence constraints**: A group cannot begin play until the group in front has moved far enough ahead (***to not hit people!***).

Analyzing the bottlenecks on actual courses

- In practice, it has been found that **Par-3 Holes tend to be the bottleneck holes.**

  - Each group has to wait until the previous group **finishes** playing the entire hole before starting to play.

  - Alternative policy for par-3 hole: **Par-3 with “Wave-Up” (P3WU)**
    - If the next group is ready to start, a group waits near the green to let the next group hit its initial tee shots.
    - That allows two groups to play on the hole simultaneously.
    - But, if the next group is not ready yet, then the group does not wait.
    - That policy leads to more variance in playing and waiting times.

Analyzing the bottlenecks: balanced courses

- The **capacity of each hole** can be defined as the average cycle time (interval between successive group departures) from a fully loaded hole (with new groups always available to start play).

- Formulas for the capacities of each hole were derived in Whitt (2015), but **simulation is needed in general.**

- The bottleneck analysis leads to two kinds of course models:
  - **Balanced courses** (the capacities of all holes are the same)
  - **Unbalanced courses** (the capacities of all holes are not the same)

- We have primarily focused on balanced courses. **Simulation was used to choose parameters that make a balanced course.**
  - Our main models use P3WU holes instead of ordinary Par-3 holes.

Model Group Play: Example of Par-4

5 steps of group play combined into **3 stages**

$$(T, W_1) \rightarrow F \ \rightarrow (W_2, G)$$

referred to as $S_1 \rightarrow S_2 \rightarrow S_3$

***Basic Parameters** for **group n** on the hole:*

$S_{i,n}$ = playing time on stage *i* (**model primitive**)
A_n = time group **Arrives,** ready to begin play
B_n = time group actually **Begins** play
T_n = time group completes **Tee** shots
F_n = time group completes **Fairway** shots
G_n = time group clears **Green** (completes play on the hole)

Basic recursion for a Par-4 hole

- Below is a representation of a group play on a par-4 hole, with precedence constraints. (can be seen from $a \vee b = \max(a, b)$)

- Par−4 Hole:

  For $n \geq 1$, and with $F_0 \equiv G_0 \equiv 0$,

  (i) $B_n \equiv A_n \vee F_{n-1}$ (ii) $T_n \equiv B_n + S_{1,n}$

  (iii) $F_n \equiv (T_n \vee G_{n-1}) + S_{2,n}$ (iv) $G_n \equiv F_n + S_{3,n}$

- Performance Measures:
  - Waiting time: $W_n = B_n - A_n$
  - Playing time: $X_n = G_n - B_n$
  - Sojourn time: $X_n + W_n = (G_n - B_n) + (B_n - A_n) = G_n - A_n$
  - Cycle time: $C_n \equiv G_n - G_{n-1},\ n \geq 1$
    - Consequently, $\overline{C_n} = \frac{1}{n}\sum_{k=1}^{n} C_k = \frac{G_n}{n}$ indicates the average cycle time.

Stage playing time distributions (1)

- Starting point: symmetric **triangular distribution for $\boldsymbol{S_i}$**
  - Simple representation with mean m_i and variability a_i
  - Specifically, we let the stage playing times S_i be:
    - $S_i = m_i - a_i + 2a_iT$
    - where $T \equiv T[0,1]$ is a random variable with a symmetric triangular distribution on [0, 1] with density
    - $f(t) = 4t$, $0 \leq t \leq 0.5$ and $4 - 4t$, where $0.5 \leq t \leq 1$
    - $E[S_i] = m_i$ and $Var[S_i] = {a_i}^2/6$

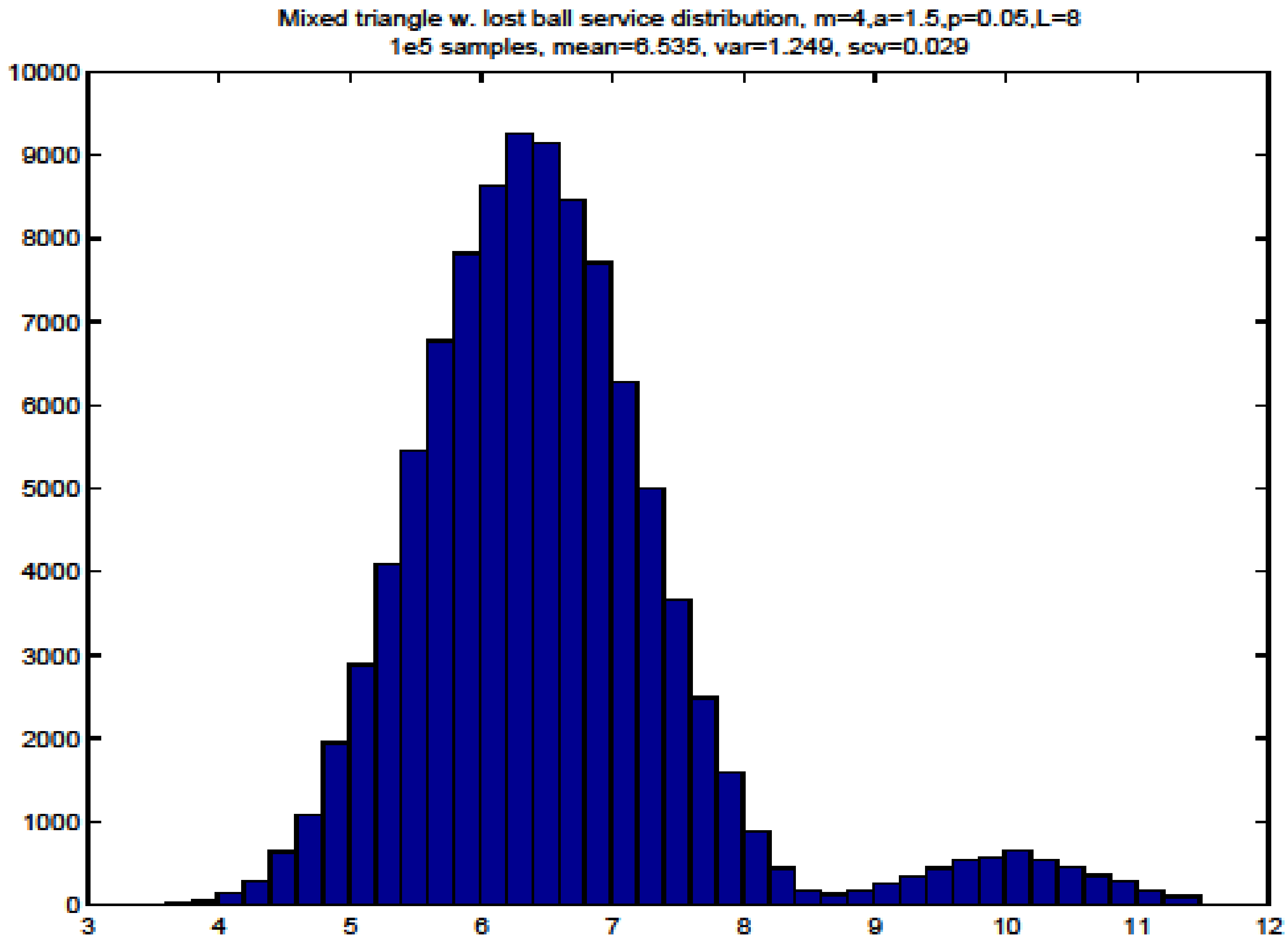
- Simplification: parameter reduction for Par-4:
  - **(*m,a,r*) = (4.0,1.5, 0.5)**
    - $m_1 = m_3 = m$, $m_2 = rm$ (mean parameters, m = 4.0, r = 0.5)
    - $a_1 = a_2 = a_3 = a$ (variability parameters, a = 1.5)

Stage playing time distributions (2)

- Allow for **lost ball on the initial (tee) shots**
  - Two parameters (*p*,*L*), taken to be (0.05, 8.0)
  - Probability of lost ball = *p*
    - Stage playing time S_1 as above if no lost ball
    - Stage playing time $S_1 = L$ if lost ball
- Overall **5-parameter model for Par-4**:
  - **(*m*,*a*,*r*,*p*,*L*) = (4.0,1.5, 0.5, 0.05, 8.0)**
    - $m_1 = m_3 = m$, $m_2 = r\,m$ (mean parameters, m = 4.0, r = 0.5)
    - $a_1 = a_2 = a_3 = a$ (variability parameters, a = 1.5)
    - $S_i = m_i - a_i + 2a_iT$ (resulting random variables)
    - T = random variable with triangular distribution on [0,1]
    - lost ball with (*p*,*L*) = (0.05, 8.0)
- Cycle time on a fully loaded Par-4 hole:

  E[*C*] = 6.535, Var[*C*] = 1.249

The cycle time on a Par-4 hole: histogram



Constructing the simulation code

Inputs

- # of playing groups (*N*)
- Golf hole sequence (array)
- Arrival distributions (deterministic)
- Playing time distributions (triangular)
  - Also includes a “lost ball” factor
- # of simulation replications (*N_Sim*)

Outputs

(for each hole and for entire course)

- Playing time data *X* for each group
- Waiting time data *W* for each group
- With above two →
  - Sojourn time for each group
  - Departure time for each group
  - “Inter-departure time” (which can also be referred as cycle time) for each group

Inputs for golf course simulation

- (1) Here is the list of the stage playing time mean values m_i for Par-3, Par-4 and Par-5: (note: all the numbers are in *minutes*)
  - Par-3 = $(3.5^T \rightarrow 2^G \rightarrow \frac{8}{3}^G)$
  - Par-4 = $(4^T \rightarrow 2^F \rightarrow 4^G)$
  - Par-5 = $(4^T \rightarrow 2^F \rightarrow 2^F \rightarrow \frac{4}{3}^G \rightarrow 4^G)$
    - Superscripts: T = Tee, F=Fairway, G=Green
    - *a = 1.5*, so *Var*(S_i) = $a^2/6$ = *0.375* (before lost ball)

- (2) **Lost ball** factor on the first hole: **(p, L) = (0.05, 8.0)**
  - With a probability *p* = 0.05, a player may lose a golf ball (e.g. fall in the bunker or water area) during the first stage
  - A lost ball increases the playing time on the first stage to L = 8

Data and simulation structure

Defining a golf course (1 x 18 array)

1

4	5	4	3	4	4	3	4	4	5	4	3	4	4	4	5	5	4

18

Data and simulation structure

Defining a golf course (1 x 18 array)

Define *N* : # Groups

1

4	5	4	3	4	4	3	4	4	5	4	3	4	4	4	5	5	4

18

e.g. *N* = 1000

Data and simulation structure

Defining a golf course (1 x 18 array)

1

4	5	4	3	4	4	3	4	4	5	4	3	4	4	4	5	5	4

18

Define N : # Groups

e.g. N = 1000

Generate an arrival matrix

$$A = \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \\ n_6 \\ \vdots \\ n_N \end{bmatrix}$$

N = 1, 2, … , 1000

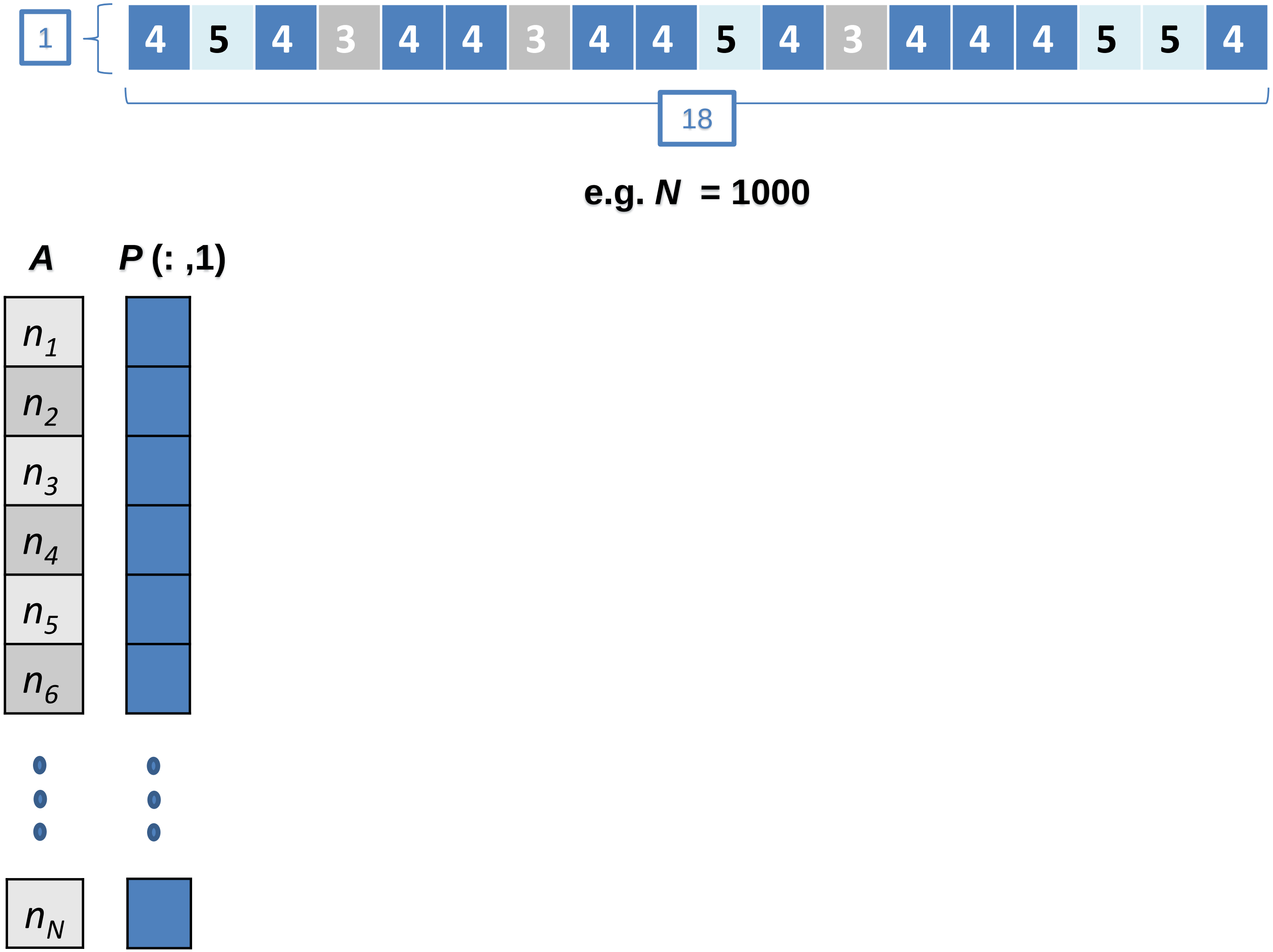
Data and simulation structure

Defining a golf course (1 x 18 array)

Define N : # Groups

Generate an arrival matrix

Calculate various outputs for the first hole



Data and simulation structure

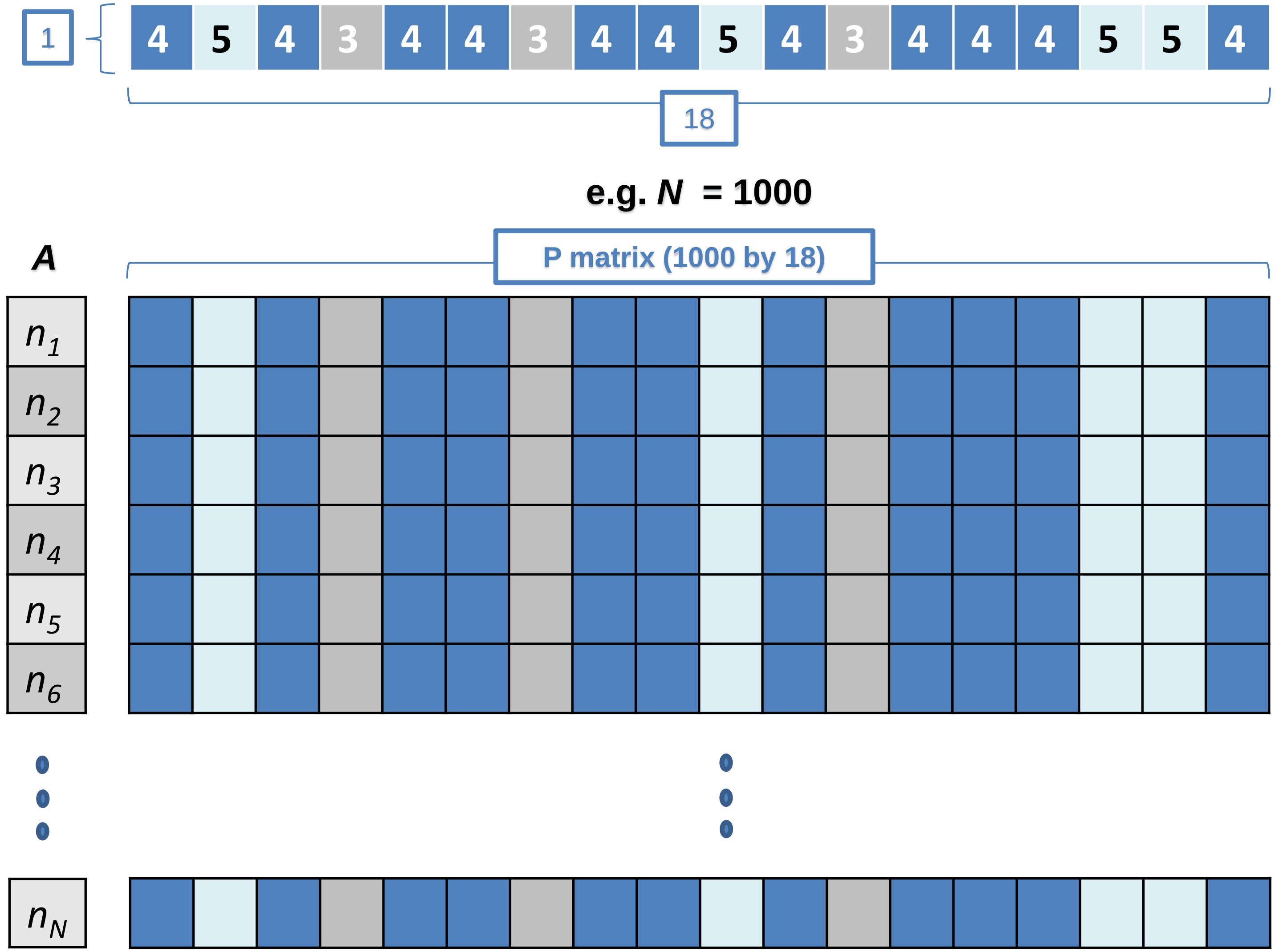
Defining a golf course (1 x 18 array)

Define *N* : # Groups

Generate an arrival matrix

Calculate various outputs for the first hole

Repeat for the next 17 holes



Data and simulation structure

Defining a golf course (1 x 18 array)

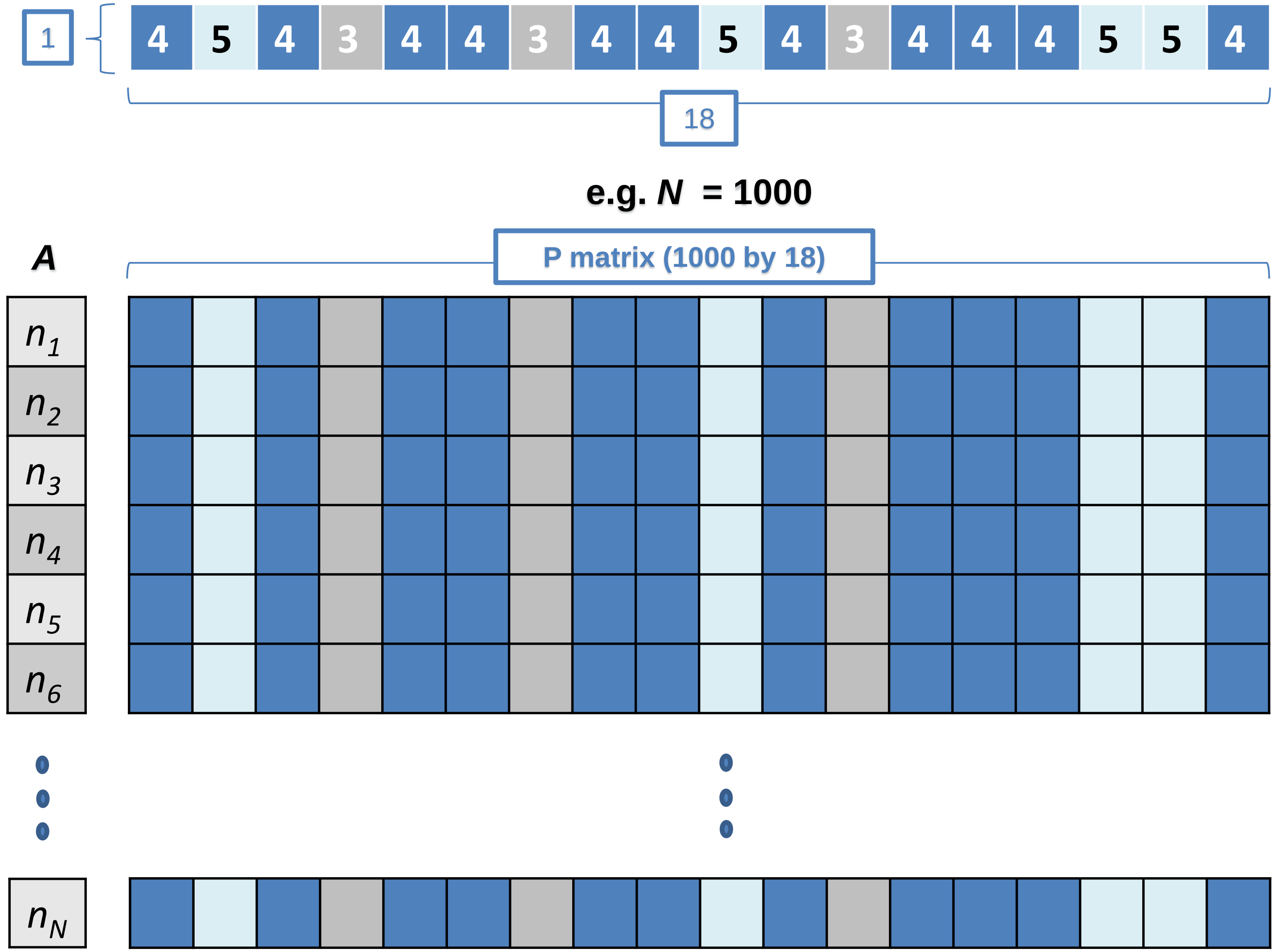
Define *N* : # Groups

Generate an arrival matrix

Calculate various outputs for the first hole

Repeat for the next 17 holes

Repeat for the next *NSim* replications



An Optimization Problem

- Find the maximum number of groups that can play each day subject to two constraints:
  - The total time available is τ minutes.
  - The expected sojourn time for all groups is no more than γ minutes.
    - For example, τ = 840 minutes would be 14 hours, while γ = 240 minutes would be 4 hours.

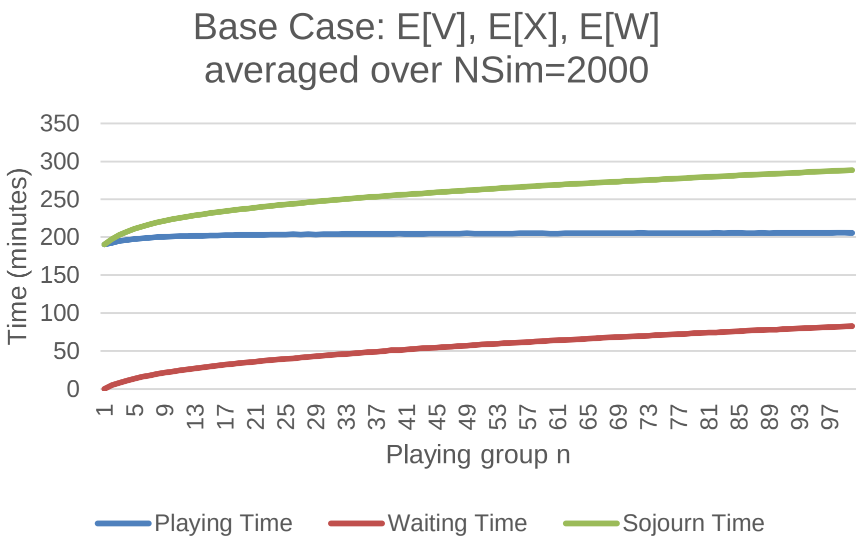
Running the simulation

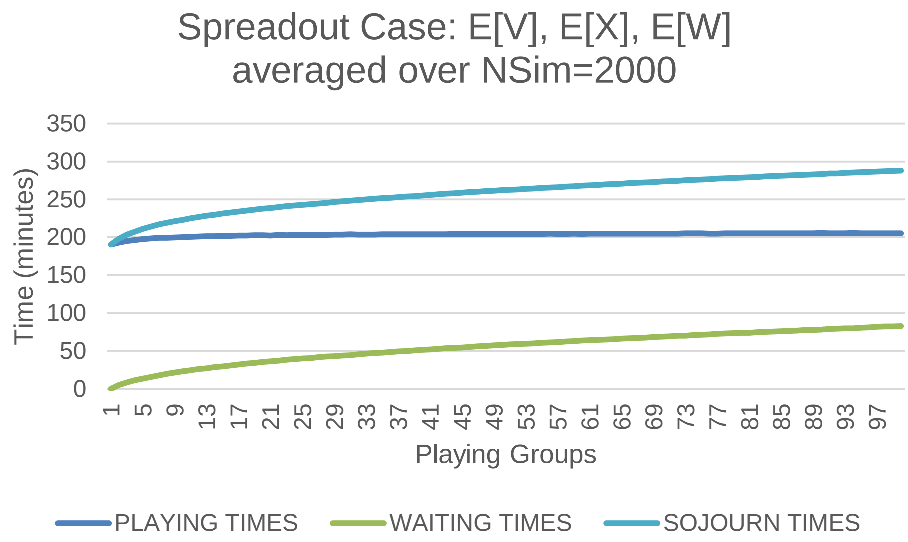
- Sample golf courses:
  - Base Case:
  - Spread-out case:
  - Concentrated P3 case (beg):
  - Concentrated P3 case (end):
  - Concentrated P5 case:

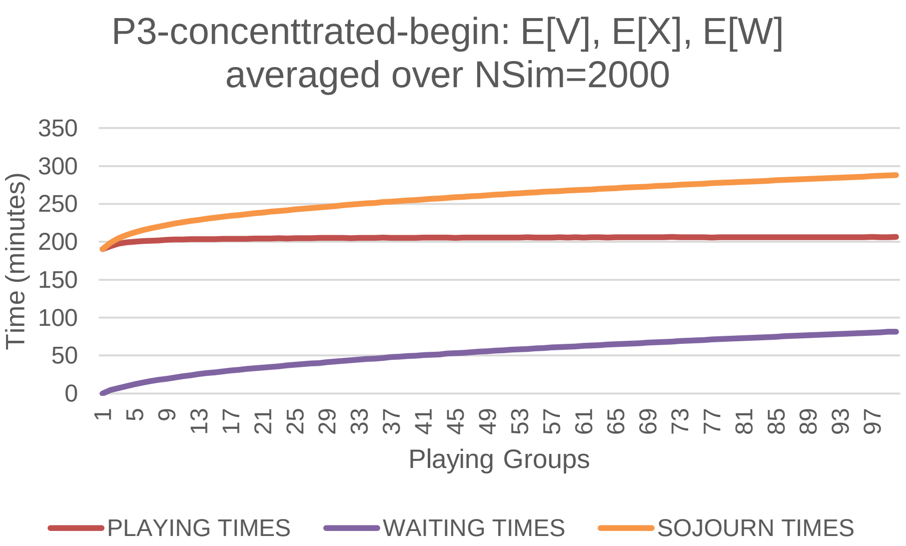
4	4	3	4	5	4	4	5	4	3	4	4	3	4	5	4	4	4
4	3	4	4	5	4	4	3	4	4	5	4	4	3	4	4	5	4
3	3	3	4	5	4	4	4	4	4	5	4	4	4	4	4	5	4
4	5	4	4	4	4	4	5	4	4	4	4	4	5	4	3	3	3
5	5	5	3	4	3	4	3	4	4	4	4	4	4	4	4	4	4

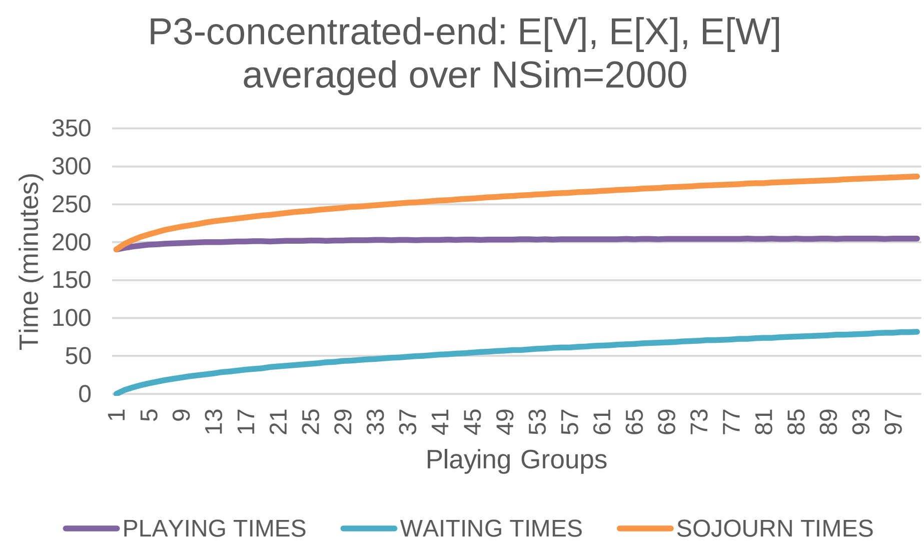
- Again, we have the following hole parameters:
  - Par-3 = $(3.5^T \rightarrow 2^G \rightarrow \frac{8}{3}^G)$
  - Par-4 = $(4^T \rightarrow 2^F \rightarrow 4^G)$
  - Par-5 = $(4^T \rightarrow 2^F \rightarrow 2^F \rightarrow \frac{4}{3}^G \rightarrow 4^G)$
- The simulation was conducted by running 2000 replications of 100 groups playing on various 18-hole courses.

Playing, waiting and sojourn times









Throughput analysis

- Ultimately, how many customers can play per day (840 minutes)?
- Note that the max throughput occurs when the course is under-loaded (i.e. $\rho \approx \frac{6.53}{7.15} \approx 0.91$).

Throughput Comparison (tee ranges from 5 to 10 with 0.5 incr)						
Tee Interval	Sojourn Constraint (< 240 minutes)		Departure Constraint (< 840 minutes)		Overall Throughput Level	
	"4 5 4"	"5 5 5"	"4 5 4"	"5 5 5"	"4 5 4"	"5 5 5"
5.00	10	10	86	86	**10**	**10**
5.50	12	12	86	86	**12**	**12**
6.00	15	15	86	86	**15**	**15**
6.50	21	21	86	86	**21**	**21**
7.00	42	42	85	85	**42**	**42**
7.50	100	100	82	82	**82**	**82**
8.00	100	100	79	78	**79**	**78**
8.50	100	100	75	75	**75**	**75**
9.00	100	100	71	71	**71**	**71**
9.50	100	100	68	68	**68**	**68**
10.00	100	100	64	64	**64**	**64**

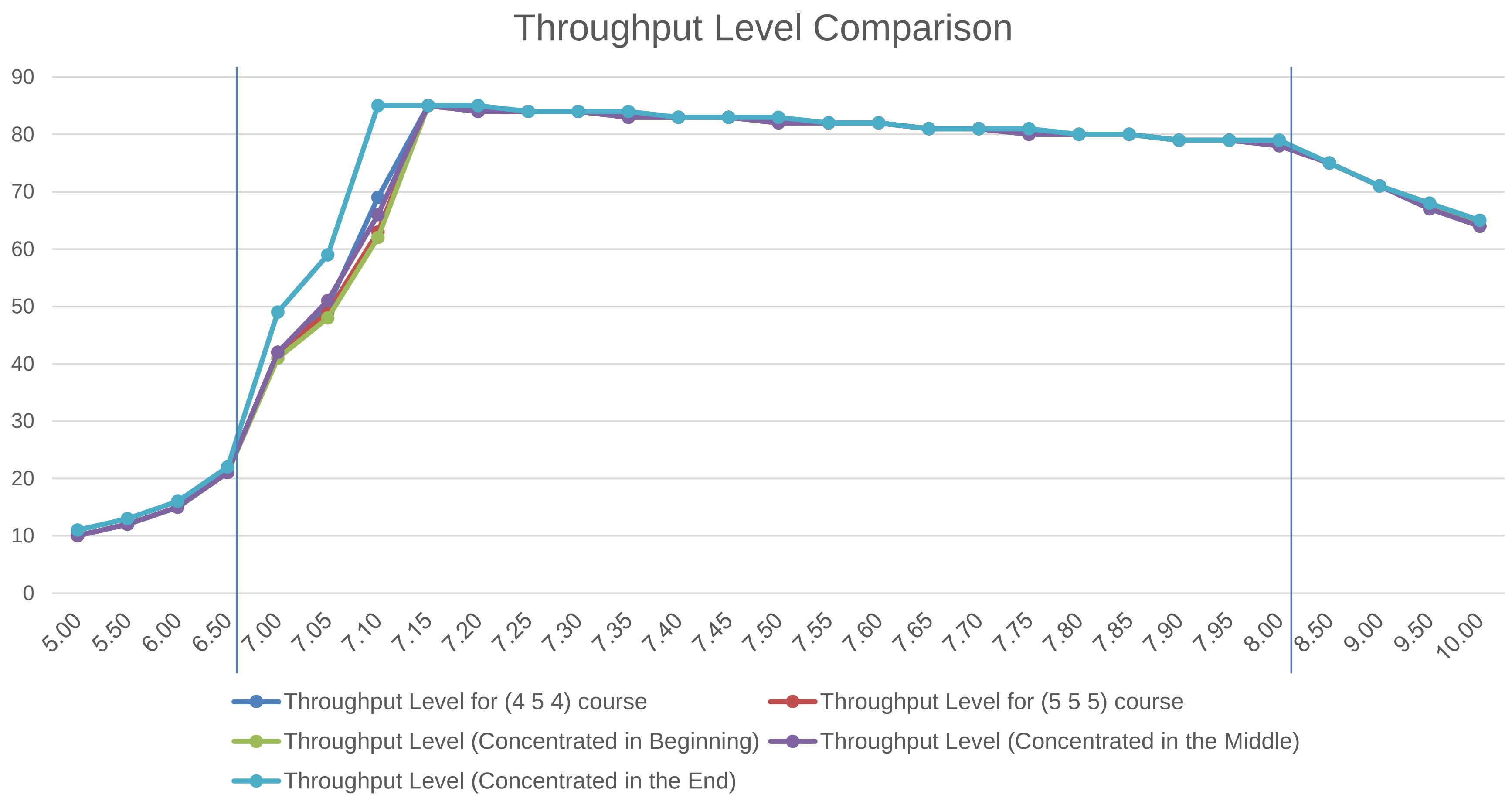
Figure 1.1 Comparison of throughput levels between two balanced golf courses, with a tee interval ranging from 5.00 to 10.00

Throughput Comparison (tee ranges from 7.00 to 7.50 with 0.05 increment)						
Tee Interval	Sojourn Constraint (< 240 minutes)		Departure Constraint (< 840 minutes)		Overall Throughput Level	
	"4 5 4"	"5 5 5"	"4 5 4"	"5 5 5"	"4 5 4"	"5 5 5"
7.00	42	42	85	85	**42**	**42**
7.05	50	49	85	85	**50**	**49**
7.10	69	63	85	85	**69**	**63**
7.15	100	100	85	85	**85**	**85**
7.20	100	100	84	84	**84**	**84**
7.25	100	100	84	84	**84**	**84**
7.30	100	100	84	84	**84**	**84**
7.35	100	100	83	83	**83**	**83**
7.40	100	100	83	83	**83**	**83**
7.45	100	100	83	83	**83**	**83**
7.50	100	100	82	82	**82**	**82**

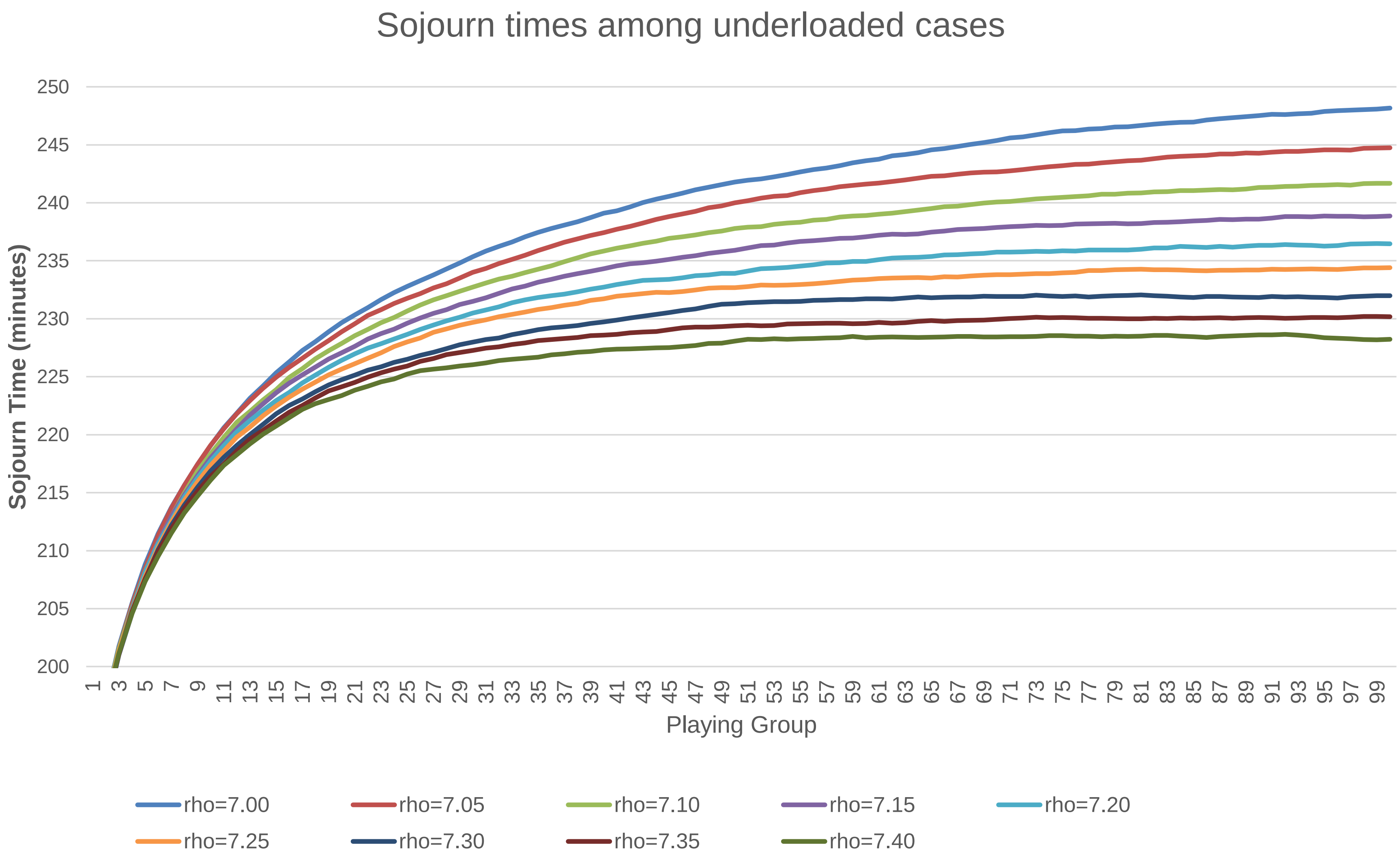
Figure 1.2 "Zoomed-in" comparison of throughput levels between two balanced golf courses, with a tee interval ranging from 7.00 to 7.50

Queuing Analysis

- Below graph shows throughput alters with varying inter-arrival times.



Sojourn time comparison



Insights from the graphs

- There is a much greater penalty from making the inter-arrival time (tee interval) too small, than for making it too large.
  - The optimal interval is 7.15 minutes yielding 85 groups.
  - An interval of 7.00 minutes yields about 40 groups.
  - An interval of 7.30 minutes yields 84 groups.
  - An interval of 8.00 minutes yields 80 groups.

- As long as the course is balanced, the hole sequence does not affect the maximum throughput rate.

- The sojourn time converges faster to a steady-state value as the traffic intensity ρ decreases. (familiar property of queues)

Work in progress

- Evaluate how **unbalanced courses** are different from balanced courses.
  - Preliminary work shows that groups experience large delays at bottleneck holes on unbalanced courses.

- Derive **approximation formulas** for sojourn and playing times:
  - Heavy-traffic case ($\rho \geq 1$) → Done
  - Under-loaded case ($\rho < 1$) → using QNA in Whitt (1983)

- Study **alternative non-constant tee intervals** on first hole.
  - Non-constant schedules can be better.

- Study impact of **slow groups** (take longer to play on all holes).
  - Dramatic: One slow group can cause huge delays!

References

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Extra Backup Slides

Hole designs: Par-3

- Par-3 is the bottleneck hole for the golf course: a playing group cannot play until the preceding group **departs** the hole
  - Alternative Policy: **Par-3 with Wave-Up (“P3WU”)**
  - Two playing groups play along the hole simultaneously → more variance in playing and waiting times
- Event E_n defines whether the preceding group **needs to wait** for the succeeding group before moving on
- $E_n \equiv \{A_n \leq W_{n-1} \vee G_{n-2} < T_n\}, n \geq 1$ and
- $E_n^c \equiv \{A_n > W_{n-1} \vee G_{n-2}\} \cup \{T_n \leq W_{n-1} \vee G_{n-2}\}$
- The recursive formulas are as follows:
- (i) $B_n \equiv (W_{n-1} \vee G_{n-2})1_{E_n} + (A_n \vee G_{n-1})1_{E_n^c}$ $(n \geq 2)$, $B_1 \equiv A_1$
- (ii) $T_n \equiv B_n + S_{1,n}$, (iii) $W_n \equiv T_n + S_{2,n}$ $(n \geq 1)$
- (iv) $G_n \equiv \left[(W_n \vee G_{n-1})1_{E_{n+1}^C} + T_{n+1}1_{E_{n+1}}\right] + S_{3,n}$

$$\equiv (W_n \vee G_{n-1}) + S_{1,n+1}1_{E_{n+1}} + S_{3,n}$$

Hole designs: Par-4, Par-5

- As seen in slide 4, each hole has a set of **precedent constraints**

- For Par-4 and Par-5 holes, each playing group can only play after the preceding group has completed two stages before them

- Par−4 Hole:

  For $n \geq 1$, and with $F_0 \equiv G_0 \equiv 0$,

  (i) $B_n \equiv A_n \vee F_{n-1}$ (ii) $T_n \equiv B_n + S_{1,n}$

  (iii) $F_n \equiv (T_n \vee G_{n-1}) + S_{2,n}$ (iv) $G_n \equiv F_n + S_{3,n}$

- Par−5 Hole:

  (i) $B_n \equiv A_n \vee F_{1,n-1}$ (ii) $T_n \equiv B_n + S_{1,n}$

  (iii) $F_{1,n} \equiv (T_n \vee G_{n-1}) + S_{2,n}$ (iv) $W_{2,n} \equiv F_{1,n} + S_{3,n}$

  (v) $F_{2,n} \equiv \left(W_{2,n} \vee G_{n-1}\right) + S_{4,n}$ and (vi) $G_n \equiv F_{2,n} + S_{3,n}$

Queueing analysis: balanced courses

- Let $\bar{\theta}_1, \bar{\theta}_2, \dots, \bar{\theta}_{18}$ be the throughput rate for holes 1, 2, … , 18
  - $\bar{\theta}_i^{-1} = \frac{1}{E[Y_i]} \rightarrow$ the reciprocal of throughput rate is cycle time

- If $\bar{\theta}_1 \approx \bar{\theta}_2 \approx \cdots \approx \bar{\theta}_{18}$, then the max throughput $\bar{\theta}$ for the entire course will also be approx. cycle time of an individual hole

- Max throughput rate remains the same for all hole sequences if $\bar{\theta}_1 \approx \bar{\theta}_2 \approx \cdots \approx \bar{\theta}_{18}$
  - The four graphs on the next slide illustrates this point

- With the playing time parameters we have in slide 11 –

(3.5, 2, 8/3), (4, 2, 4) and (4, 2, 2, 4/3, 4) for Par-3, Par-4 and Par-5, the cycle time $\frac{1}{E[Y_i]}$ is approximately **6.53**