Statistical Analysis of Service and Patience Data

From Service System Data to Performance Models

IEOR 4615, Service Engineering, Professor Whitt

Lecture 19, April 14, 2015
What do we anticipate?

- We anticipate an Erlang A \((M/M/s + M)\) Model.

From last week: a NHPP Arrival Process.

However, daily totals may not be Poisson.

- over-dispersion in call centers
- under-dispersion with appointment systems (outpatient clinics)

We may be able to reduce impact by forecasting:

- We estimate the arrival rate function.

For staffing, we assume a piecewise-constant arrival rate function.

Focus on individual hours or half hours: stationary model.
What do we anticipate?

- We anticipate an **Erlang A \((M/M/s + M)\)** Model.

Today consider **service-time and patience-time distributions**.

- Assuming that these are i.i.d. exponential variables,
- we **estimate mean service time** \(1/\mu\)
- and we **estimate abandonment rate** \(\theta\). (censored data).

Assuming more general distributions: more complicated
Estimating the Service and Patience Parameters

- Estimate **mean service time** $1/\mu$ of each customer.
  - Observe $n$ service times $S_1, S_2, \ldots$; let $1/\hat{\mu} = n^{-1} \sum_{k=1}^{n} S_k$.

- Estimate **abandonment rate** $\theta$ for each customer from queue.
  - Problem: **censored data**. (We do not observe most patience times.)
  - Use $P(\text{Ab}|\text{Wait} > 0) = \theta E[\text{Wait}|\text{Wait} > 0]$ (explained on next slide).
  - Observe outcomes for $n$ arrivals that must wait (join the queue).
  - Observe positive waiting times $W_1, W_2, W_3, \ldots$.
    Estimate $E[\text{Wait}|\text{Wait} > 0]$ by $\bar{W}_n \equiv n^{-1} \sum_{k=1}^{n} W_k$.
  - Observe number $N_{ab}(n)$ of these that abandon.
    Estimate $P(\text{Ab}|\text{Wait} > 0)$ by $\bar{N}_{ab} \equiv N_{ab}(n)/n$.
  - Estimate $\theta$ by $\hat{\theta} = \bar{N}_{ab}/\bar{W}_n$,
    using $\theta = P(\text{Ab}|\text{Wait} > 0)/E[\text{Wait}|\text{Wait} > 0]$ from above.
Justifying the Abandonment Rate Formula

Why $P(Ab|\text{Wait} > 0) = \theta E[\text{Wait}|\text{Wait} > 0]$ in the Erlang-A model?

Use structure of the Erlang A model.

Apply Little’s Law.

Let $Q$ and $W$ be steady-state number in queue and waiting time.

$P(Ab) = \frac{\text{abandonment rate}}{\text{arrival rate}} = \sum_{k=1}^{\infty} \frac{P(Q=k)k\theta}{\lambda} = \frac{E[Q]\theta}{\lambda} = \theta E[W].$

Extend to conditioning upon positive wait:

1. $P(Ab) = P(Ab|W > 0)P(W > 0)$ and $E[W] = E[W|W > 0]P(W > 0),$

2. So $P(Ab|W > 0)P(W > 0) = P(Ab) = \theta E[W] = \theta E[W|W > 0]P(W > 0).$

3. Then divide by $P(W > 0)$ on both sides.
Apply Regression to Multiple Samples

![Graph showing the relationship between \( P\{\text{Abandon} | \text{Wait} > 0\} \) and \( E[\text{Wait} | \text{Wait} > 0] \). The equation is \( y = 0.0167x + 4 \times 10^{-5} \) with \( R^2 = 1 \).]

and we get

\[
P\{\text{Abandon} | \text{Wait} > 0\} = 4.35 \cdot 10^{-3} + 0.01666E[\text{Wait} | \text{Wait} > 0]
\]

\[
\approx 0.01666E[\text{Wait} | \text{Wait} > 0]
\]

\[
\Rightarrow \quad \text{Average Patience} = \frac{1}{0.01666} = 60.00817 \text{ sec}
\]
What Do We See in the Data?

Histogram of Banking Call Center Service Times

Average = 182 sec
St. dev. = 199 sec
Is Logarithm Normally Distributed?

Histogram of Logarithm of Banking Call Center Service Times

- **Average = 2.10**
- **St. dev. = 0.37**
Lognormal Distribution

- $X$ has a lognormal distribution if $\log_e X$ has a normal distribution:
  - $P(\log_e X \leq y) = P(N(\mu, \sigma^2) \leq y)$ or $X$ is distributed as $e^{N(\mu, \sigma^2)}$.
  - cdf: $P(X \leq x) = P(\log_e X \leq \log_e x) = P(N(\mu, \sigma^2) \leq \log_e x)$.

- Other characteristics:
  - pdf: $f_X(x) = \frac{d}{dx}P(X \leq x) = \frac{d}{dx}P(N(\mu, \sigma^2) \leq \log_e x)$
    \[ f_N(\mu, \sigma^2)(\log_e x) \frac{1}{x} = \frac{1}{\sqrt{2\pi\sigma^2}x} e^{-(\log_e x - \mu)^2/2\sigma^2}. \]
  - moments: $E[X^k] = E[e^{kN(\mu, \sigma^2)}]$, but mgf: $E[e^{tN(\mu, \sigma^2)}] = e^{\mu t + \sigma^2 t^2/2}$ (p. 64 of Ross 2010) so that first two moments are $E[X] = e^{\mu + \sigma^2/2}$ and $E[X^2] = e^{2\mu + 2\sigma^2}$, so that $c_X^2 = \frac{E[X^2]}{E[X]^2} - 1 = e^{\sigma^2} - 1$.
  - median = $e^\mu$, mode = $e^{\mu - \sigma^2}$, so mean > median > mode, somewhat heavy tail.
Median of Lognormal Distribution

- \( P(\log_e X \leq y) = P(N(\mu, \sigma^2) \leq y) \).

- median = \( e^\mu \), Proof:

\[
P(X \leq m) = 1/2 \quad \text{for} \quad m = \text{median,}
\]

\[
P(\log_e X \leq \log_e m) = 1/2
\]

\[
P(N(\mu, \sigma^2) \leq \log_e m) = 1/2
\]

so that \( \log_e m = \mu \) and thus \( m = e^\mu \).
Mode of the Lognormal Distribution

- **cdf:** \( P(X \leq x) = P(\log_e X \leq \log_e x) = P(N(\mu, \sigma^2) \leq \log_e x) \).

- **pdf:** 
  \[
  f_X(x) = \frac{d}{dx} P(X \leq x) = \frac{d}{dx} P(N(\mu, \sigma^2) \leq \log_e x) \\
  = \frac{f_{N(\mu, \sigma^2)}(\log_e x)}{x} = \frac{1}{\sqrt{2\pi}\sigma^2 x} e^{-(\log_e x - \mu)^2/2\sigma^2}.
  \]

- **mode** = \( e^{\mu - \sigma^2} \), Proof: 
  \[
  \frac{d}{dx} f_X(x) = 0 \quad \text{for} \quad f_X(x) = \frac{A e^{-u(x)}}{x} \quad \text{and} \quad u(x) = \frac{(\log_e(x) - \mu)^2}{2\sigma^2}
  \]
  
  \[
  0 = \frac{-Ae^{-u(x)}}{x^2} + \frac{-A\dot{u}(x)e^{-u(x)}}{x} \\
  = 1 + \dot{u}(x) = 1 + \frac{\log_e(x) - \mu}{\sigma^2}
  \]
  or \( \log_e(x) - \mu = -\sigma^2 \)
Length-of-Stay (LOS) at the Internal Wards of a Hospital: Mixture (of Skewed-Normals), in *Hours*
Length-of-Stay (LOS) at the Internal Wards of a Hospital: LogNormality, in *Days*
Strange Service Times: Israeli Telecom

IL Telecom: Dynamics of the distribution of agent service time for Private calls

Agent service time Private
Week days

Median = 106.00 (Sept-04)
Median = 107.00 (Sept-05)
Mean = 179.66 (Sept-04)
Mean =175.13 (Sept-05)
Std = 215.05 (Sept-04)
Std =201.66 (Sept-05)
### Diagnosis: Look at Individual Agents

**Table 52: Number of calls handled by an agent**

<table>
<thead>
<tr>
<th></th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>AVI</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1117</td>
<td>2208</td>
<td>2019</td>
<td>2789</td>
<td>2710</td>
<td>1417</td>
<td>2026</td>
<td>2523</td>
<td>2395</td>
</tr>
<tr>
<td>AVNI</td>
<td>1493</td>
<td>1736</td>
<td>642</td>
<td>539</td>
<td>1786</td>
<td>2219</td>
<td>2092</td>
<td>2392</td>
<td>1156</td>
<td>1888</td>
<td>1988</td>
<td>2136</td>
</tr>
<tr>
<td>BASCH</td>
<td>999</td>
<td>1164</td>
<td>1708</td>
<td>1155</td>
<td>982</td>
<td>906</td>
<td>858</td>
<td>2185</td>
<td>1973</td>
<td>1055</td>
<td>1326</td>
<td>1242</td>
</tr>
<tr>
<td>BENSON</td>
<td>1283</td>
<td>1135</td>
<td>0</td>
<td>1053</td>
<td>1108</td>
<td>1016</td>
<td>1682</td>
<td>1298</td>
<td>1076</td>
<td>1303</td>
<td>1546</td>
<td>1176</td>
</tr>
<tr>
<td>DARMON</td>
<td>309</td>
<td>515</td>
<td>633</td>
<td>519</td>
<td>577</td>
<td>436</td>
<td>309</td>
<td>370</td>
<td>297</td>
<td>194</td>
<td>425</td>
<td>128</td>
</tr>
<tr>
<td>DORIT</td>
<td>696</td>
<td>1047</td>
<td>0</td>
<td>811</td>
<td>546</td>
<td>862</td>
<td>750</td>
<td>2228</td>
<td>1319</td>
<td>1384</td>
<td>1640</td>
<td>1605</td>
</tr>
<tr>
<td>ELI</td>
<td>387</td>
<td>508</td>
<td>777</td>
<td>447</td>
<td>560</td>
<td>436</td>
<td>395</td>
<td>458</td>
<td>416</td>
<td>363</td>
<td>502</td>
<td>352</td>
</tr>
<tr>
<td>GELBER</td>
<td>333</td>
<td>143</td>
<td>510</td>
<td>427</td>
<td>859</td>
<td>281</td>
<td>386</td>
<td>332</td>
<td>67</td>
<td>179</td>
<td>165</td>
<td>269</td>
</tr>
<tr>
<td>GILI</td>
<td>658</td>
<td>614</td>
<td>1155</td>
<td>803</td>
<td>1108</td>
<td>974</td>
<td>418</td>
<td>0</td>
<td>355</td>
<td>456</td>
<td>412</td>
<td>298</td>
</tr>
<tr>
<td>KAZAV</td>
<td>1995</td>
<td>1693</td>
<td>1240</td>
<td>1451</td>
<td>1731</td>
<td>2251</td>
<td>1737</td>
<td>1168</td>
<td>729</td>
<td>1570</td>
<td>1047</td>
<td>2038</td>
</tr>
<tr>
<td>MEIR</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>127</td>
<td>344</td>
<td>318</td>
<td>280</td>
<td>406</td>
<td>454</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>MORIAH</td>
<td>1360</td>
<td>1223</td>
<td>1591</td>
<td>1351</td>
<td>1866</td>
<td>1980</td>
<td>2416</td>
<td>2152</td>
<td>1526</td>
<td>1940</td>
<td>1793</td>
<td>515</td>
</tr>
<tr>
<td>PINHAS</td>
<td>79</td>
<td>40</td>
<td>359</td>
<td>244</td>
<td>31</td>
<td>311</td>
<td>422</td>
<td>241</td>
<td>143</td>
<td>105</td>
<td>51</td>
<td>63</td>
</tr>
<tr>
<td>ROTH</td>
<td>0</td>
<td>0</td>
<td>397</td>
<td>1292</td>
<td>1928</td>
<td>1967</td>
<td>1831</td>
<td>1749</td>
<td>1625</td>
<td>1914</td>
<td>1458</td>
<td>1038</td>
</tr>
<tr>
<td>SHARON</td>
<td>1985</td>
<td>1674</td>
<td>2780</td>
<td>1938</td>
<td>2563</td>
<td>2657</td>
<td>2537</td>
<td>2875</td>
<td>1803</td>
<td>1935</td>
<td>2532</td>
<td>2140</td>
</tr>
<tr>
<td>STEREN</td>
<td>0</td>
<td>1043</td>
<td>2294</td>
<td>1516</td>
<td>2163</td>
<td>2231</td>
<td>1423</td>
<td>2455</td>
<td>1672</td>
<td>709</td>
<td>2375</td>
<td>2568</td>
</tr>
<tr>
<td>TOVA</td>
<td>1923</td>
<td>1679</td>
<td>1562</td>
<td>1059</td>
<td>1464</td>
<td>1389</td>
<td>1890</td>
<td>1811</td>
<td>1361</td>
<td>1971</td>
<td>941</td>
<td>0</td>
</tr>
<tr>
<td>VICKY</td>
<td>895</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1006</td>
<td>1378</td>
<td>1415</td>
<td>1674</td>
<td>1472</td>
<td>1582</td>
<td>1641</td>
<td>1990</td>
</tr>
<tr>
<td>YIFAT</td>
<td>1312</td>
<td>1901</td>
<td>1745</td>
<td>1305</td>
<td>1464</td>
<td>1076</td>
<td>780</td>
<td>90</td>
<td>1137</td>
<td>1315</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>YITZ</td>
<td>1771</td>
<td>1791</td>
<td>1402</td>
<td>1203</td>
<td>1355</td>
<td>1367</td>
<td>1009</td>
<td>69</td>
<td>705</td>
<td>1743</td>
<td>2420</td>
<td>2353</td>
</tr>
<tr>
<td>ZOHARI</td>
<td>891</td>
<td>1144</td>
<td>1398</td>
<td>1148</td>
<td>1479</td>
<td>1450</td>
<td>980</td>
<td>1494</td>
<td>1423</td>
<td>1359</td>
<td>1504</td>
<td>1094</td>
</tr>
<tr>
<td>Z2ARIE</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>56</td>
<td>225</td>
<td>315</td>
<td>432</td>
</tr>
<tr>
<td>Z2ELINOR</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>45</td>
<td>352</td>
<td>288</td>
<td>222</td>
</tr>
<tr>
<td>Z2EYAL</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>95</td>
<td>331</td>
<td>428</td>
<td>579</td>
</tr>
<tr>
<td>Z2IFAT</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>94</td>
<td>260</td>
<td>314</td>
<td>215</td>
</tr>
<tr>
<td>Z2LOR</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>84</td>
<td>250</td>
<td>136</td>
<td>126</td>
</tr>
<tr>
<td>Z2NIRIT</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>116</td>
<td>327</td>
<td>474</td>
<td>387</td>
</tr>
<tr>
<td>Z2OFERZ</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>71</td>
<td>311</td>
<td>260</td>
<td>242</td>
</tr>
<tr>
<td>Z2SPIEGEL</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>71</td>
<td>311</td>
<td>260</td>
<td>153</td>
</tr>
</tbody>
</table>
## Diagnosis: Look at Individual Agents

### Table 53: Number of calls with short service time

<table>
<thead>
<tr>
<th></th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>MORIAH</td>
<td>233</td>
<td>230</td>
<td>356</td>
<td>290</td>
<td>614</td>
<td>695</td>
<td>865</td>
<td>597</td>
<td>490</td>
<td>455</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>AVI</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>47</td>
<td>111</td>
<td>144</td>
<td>295</td>
<td>221</td>
<td>121</td>
<td>76</td>
<td>35</td>
<td>26</td>
</tr>
<tr>
<td>AVNI</td>
<td>11</td>
<td>13</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>25</td>
<td>16</td>
<td>18</td>
<td>4</td>
<td>8</td>
<td>8</td>
<td>11</td>
</tr>
<tr>
<td>DARMON</td>
<td>2</td>
<td>11</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>7</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>ELI</td>
<td>9</td>
<td>7</td>
<td>10</td>
<td>12</td>
<td>22</td>
<td>18</td>
<td>15</td>
<td>4</td>
<td>8</td>
<td>3</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>KAZAV</td>
<td>57</td>
<td>40</td>
<td>48</td>
<td>44</td>
<td>48</td>
<td>63</td>
<td>40</td>
<td>27</td>
<td>15</td>
<td>18</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>MEIR</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>8</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PINHAS</td>
<td>3</td>
<td>0</td>
<td>58</td>
<td>25</td>
<td>4</td>
<td>14</td>
<td>11</td>
<td>6</td>
<td>8</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>ROTH</td>
<td>0</td>
<td>0</td>
<td>10</td>
<td>10</td>
<td>36</td>
<td>21</td>
<td>43</td>
<td>25</td>
<td>32</td>
<td>31</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>SHARON</td>
<td>58</td>
<td>49</td>
<td>86</td>
<td>52</td>
<td>67</td>
<td>78</td>
<td>66</td>
<td>63</td>
<td>38</td>
<td>23</td>
<td>43</td>
<td>49</td>
</tr>
<tr>
<td>TOVA</td>
<td>52</td>
<td>163</td>
<td>269</td>
<td>132</td>
<td>231</td>
<td>193</td>
<td>100</td>
<td>109</td>
<td>207</td>
<td>190</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>ZOHARI</td>
<td>4</td>
<td>8</td>
<td>12</td>
<td>22</td>
<td>17</td>
<td>20</td>
<td>9</td>
<td>14</td>
<td>5</td>
<td>7</td>
<td>10</td>
<td>7</td>
</tr>
</tbody>
</table>
Kolmogorov-Smirnov (KS) Tests for Service Times

- **H0**: \( \{S_k\} \) iid sequence with \( P(S_k \leq x) = F(x) \) with \( F \) continuous.

- Use ECDF
  \[ \hat{F}_n(x) = n^{-1} \sum_{k=1}^{n} 1\{S_k \leq x\} \] and
  \[ \hat{D}_n = \sup_x |\hat{F}_n(x) - F(x)|. \]

- Observe \( n \) service times \( S_k \). Under H0, \( Y_k = F(S_k) \) are iid uniforms on \([0, 1]\).
  \[ P(F(S_k) \leq x) = P(S_k \leq F^{-1}(x)) = F(F^{-1}(x)). \]

- Under H0, \( Z_k = -\log(1 - Y_K) \) iid exponentials. (same proof)

**Alternative KS Tests** with **H0**: \( \{Y_k\} \) iid uniform on \([0, 1]\), \( F(x) = x \).

- **standard KS**: Reject if \( P(D_n > \hat{D}_n|H0) > \alpha = 0.05. \)

- **Durbin**: Sort to get ordered uniforms, do as before (last class).

- **Exp+CU**: With exponentials \( Z_k \) and \( S_k = Z_1 + \cdots + Z_k \), apply Conditional Uniform (CU) to get \( S_k/S_n \) ordered uniforms on \([0, 1]\).

- **Exp+CU+Durbin** (*Kim&W 2015*): Do Durbin after Exp+CU, as in Lewis.
**Example: Different KS Tests Applied to an Alternative Simulation Experiment:** Apply the KS test of iid mean-1 exponential service times to the **alternative:** iid mean-1 $H_2$ (hyperexponential, mixture of two exponentials) service times with $c_X^2 = \frac{\text{Var}(X)}{(E[X])^2} = 2.0$.

**Table:** **Performance of alternative KS tests** of i.i.d. mean-1 exponential variables for the sample size $n = 200$ with significance level $\alpha = 0.05$: the case of i.i.d. $H_2$ interarrival times having $EX = 1$ and $c_X^2 = 2$, based on $10^4$ replications.

<table>
<thead>
<tr>
<th>KS test</th>
<th>Exp+CU+Durbin</th>
<th>Standard</th>
<th>Exp+CU</th>
<th>Durbin</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power</td>
<td>0.93</td>
<td>0.64</td>
<td>0.28</td>
<td>0.14</td>
</tr>
<tr>
<td>Average $p$ value</td>
<td>0.02</td>
<td>0.09</td>
<td>0.24</td>
<td>0.40</td>
</tr>
</tbody>
</table>
Simulation Experiment: Apply the KS test to the alternative: a non-Poisson renewal process with interarrival times having an $H_2$ (hyperexponential) CDF (mixture of two exponentials) with a squared coefficient of variation $c_X^2 = \frac{\text{Var}(X)}{(E[X])^2} = 2.0$.

Table: Performance of alternative KS tests of a rate-1 Poisson process for the time interval $[0, 200]$ with significance level $\alpha = 0.05$: the case of a renewal process with $H_2$ interarrival times having $c_X^2 = 2$, based on $10^4$ replications.

<table>
<thead>
<tr>
<th>KS test</th>
<th>Lewis</th>
<th>Standard</th>
<th>Log</th>
<th>CU</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power</td>
<td>0.94</td>
<td>0.63</td>
<td>0.51</td>
<td>0.28</td>
</tr>
<tr>
<td>Average p value</td>
<td>0.01</td>
<td>0.10</td>
<td>0.13</td>
<td>0.23</td>
</tr>
</tbody>
</table>
References


Exponential Distribution:

Density: \( f(x) = \lambda e^{-\lambda x}, \quad x \geq 0 \),
Mean: \( E[X] = \lambda^{-1} \),
Variance: \( Var(X) = \lambda^{-2} \),
Coefficient of Variance: \( C_v = \frac{SDV(X)}{E[X]} = 1 \),
Median: \( \lambda^{-1} \ln 2 \).
Check that the Parallel Server Model Makes Sense

One Queue with Multiple Servers Working in Parallel

Customer arrival process

Queue of waiting customers

Service facility with servers

Customer departure process