## IEOR 4615: Service Engineering, Professor Whitt

## Midterm Exam, March 31, 2015.

## Please explain your reasoning; show your work.

### 1. Average Performance of a Hospital Emergency Room (25 points)

A hospital emergency room (ER) is organized so that all patients register through an initial check-in process. At his/her turn, each patient first registers, then is seen by an ER doctor and then exits the ER, either followed immediately by admission to the hospital or departing altogether. Currently, 40 people per hour arrive at the ER, 20% of whom are admitted to the hospital after seeing an ER doctor. On average, 20 people are waiting to be registered and 40 are registered and waiting to see an ER doctor. The registration process takes, on average, 3 minutes per patient. Among patients who ultimately are admitted to the hospital the average time spent with a doctor is 40 minutes. Among those not admitted to the hospital, the average time is 5 minutes.

(a) On average, how long does a patient stay in the ER?

(b) On average, how many patients are being examined by doctors at any one time?

(c) On average, how many patients are in the ER?

### 2. Staffing the Emergency Room (25 points)

Again consider the emergency Room (ER) introduced in Problem 1, with the parameters specified there, but for this problem we focus only on the doctors, because the doctors are reasonably judged to be the critical resource. In this problem we want to estimate how many doctors we need in the emergency room.

(a) We start with a stationary model, which has a constant patient arrival rate over each day, as specified in problem 1. We assume that each patient is seen by a single doctor. For this constant arrival rate model, what is the offered load for the doctors?

(b) In the setting of part (a), what is an approximate number of doctors required at each time to provide reasonable performance in the ER? (Explain your assumptions and reasoning.)

(c) Suppose that the arrival rate to the emergency room varies by time of day, as it does in practice. In particular, for simplicity, assume that, each day, the arrival rate is 10 per hour in the interval $[0, 8]$ (between midnight and 8 am), 60 per hour in the interval $[8, 16]$ (between 8 am and 4 pm) and 50 per hour in the interval $[16, 24] = [16, 0]$ (between 4 pm and midnight). Assume that the distribution of the length of time each patient spends with a doctor does not change (provided that there are adequately many doctors). What are the approximate offered loads at (i) 4 am and at (ii) 12 noon, and what are consistent approximately appropriate time-varying staffing levels for these two times?

(d) How is the general time-varying offered load defined for this system with the specified time-varying arrival rate (under the assumptions of part (c))?

(e) Suppose that we wanted to staff in a way to achieve roughly a constant level of performance for all times in the day. Consider the three times: (i) $0 : 05 = 12 : 05$ am (right after midnight), (ii) $8 : 05$ am (right after 8 am), and (iii) $16 : 05 = 4.05$ pm (right after 4 pm). How should the staffing levels at these three times compare to achieve our goal of stable performance? That is, how should the staffing levels at these three times be ranked (ordered); i.e., when should the staffing level be highest, next highest and then lowest? Justify your answers.

**3. Using Data to Estimate the Expected Time Spent in the ER (25 points)**

Suppose that you have collected data on the performance of the emergency room (ER) in problem 1 above. You observe the time each successive arriving patient spends in the ER for all arrivals in a common two-hour period on each of 10 Mondays. Let $X_{i,j}$ be the time spent in the ER on day $i$ by patient $j$, with $j$ indexing the order of arrival within the interval on day $i$. Let $n_i$ be the number of patients observed on day $i$. (The ten numbers $n_i$ all fall in the interval $[50, 120]$.) You compute the following statistics:

$$\bar{X}_i \equiv \frac{1}{n_i}\sum_{j=1}^{n_i} X_{i,j} \quad \text{and} \quad \bar{S}_i^2 \equiv \sum_{j=1}^{n_i}(X_{i,j} - \bar{X}_i)^2, \quad 1 \le i \le 10,$$

$$\bar{X} \equiv \frac{1}{10}\sum_{j=1}^{10} \bar{X}_i \quad \text{and} \quad \bar{S}^2 \equiv \sum_{j=1}^{10}(\bar{X}_i - \bar{X})^2.$$

(a) Is it appropriate to assume, for each fixed $i$, that the $n_i$ random variables $X_{i,j}$, $1 \le j \le n_i$, are independent random variables? Why or why not?

(b) What is an appropriate estimate of the expected time for a patient to spend in the ER during this 2-hour period on a Monday based on the statistics above?

(c) How is your answer to part (b) affected by your answer to part (a)? Explain.

(d) Explain how you would use the statistics above to estimate a 95% two-sided confidence interval for the expected value estimated in part (b). Give a representation of the estimated confidence interval that is as explicit as possible (given the information above).

(e) What assumptions justify your answer in part (d)?

**4. A Small Medical Clinic (25 points)**

Consider a small clinic run by a single doctor. Patients arrive at this clinic according to a Poisson process at rate $\lambda = 1.6$ per hour, and all wait patiently until they are seen one at a time by the doctor. The expected time the doctor spends with each patient is 30 minutes.

(a) If the times patients spend with the doctor are i.i.d. exponential random variables, what is the steady-state distribution of the number of patients at the clinic?

(b) What is the expected time each patient spends at the clinic?

(c) Suppose, instead, that the clinic serves two kinds of patients: Type 1 patients have mean service times of 60 minutes, whereas Type 2 patients have mean service times of 15 minutes. Each arrival is type 1 with probability $1/3$. What is the mean and squared coefficient of variation (scv, variance divided by the square of the mean) of the service time of an arbitrary patient?

(d) Use a heavy-traffic approximation to estimate how much the answer in part (b) changes under the assumption of part (c).

(e) Suppose now that the clinic admits patients for 8 hours per day, starting at 8 am, but no new patients are admitted after the closing time of 4 pm. However, patients in the clinic at 4 pm will be seen by the doctor after 4 pm. The doctor also takes a lunch break from noon until 1 pm. Finally, now the arrival rate of patients between 8 am and 4 pm is 3.0 per hour, instead of the previously assumed 1.6 per hour. Use a fluid approximation to estimate when the doctor can leave the clinic.

(f) How would the answer in part (e) change if a second doctor were hired to work part time in the clinic, so that the schedule before 1 pm is unchanged, but two doctors both work in the clinic after 1 pm (until all patients have been seen)?