IEOR 4701: Stochastic Models in Financial Engineering

Summer 2007, Professor Whitt

Homework Assignment 1: Tuesday, July 10, 2007

Due on Thursday, July 12 before class (10:00am); to be discussed at the recitation on Wednesday evening, July 11, 7:30-9:00pm in 633 Mudd.

Probability Review: Read Chapters 1 and 2 in the textbook, *Introduction to Probability Models*, ninth edition, by Sheldon Ross. Please do the six problems marked with an asterisk and turn them in on Thursday. These problems are written out in case you do not yet have the textbook (available in the Columbia Bookstore). Extra problems are provided in case you need extra review. Solutions will be provided for all the following problems.

CHAPTER 1

*Problem 1.3.

A coin is to be tossed until a head appears twice in a row. What is the sample space for this experiment? If the coin is fair, what is the probability that it will be tossed exactly four times?

*Problem 1.21

Suppose that 5% of men and 0.25% of women are color-blind. A color-blind person is chosen at random. What is the probability of this person being male? (Assume that there are an equal number of males and females.)

Problem 1.22

Two players, A and B, play a succession of games, winning or losing one point in each game. They continue playing until one has two points more than the other. Assuming that each point is independently won by player A with probability p, what is the probability that they will play a total of $2n$ games (points)? What is the probability that A will win?

Problem 1.31

Two dice are tossed, one green and one red. What is the conditional probability that the number on the green die is 6, given that the sum on the two dice is 7?

Extra part: What is the conditional probability that the number on the green die is 6, given that the sum on the two dice is 10?

*Problem 1.43

Suppose we have 10 special coins, which are such that if the i^{th} coin is flipped, the heads will appear with probability $i/10$, for $1 \leq i \leq 10$. Suppose that one of the ten coins is selected at random, with each one equally likely to be selected. Suppose that this randomly selected coin is flipped. What is the conditional probability that the randomly flipped coin is coin $i = 5$ given that the randomly selected coin showed heads?

CHAPTER 2

Problem 2.9

Problem 2.20

*Problem 2.32

Suppose that you buy a lottery ticket in 50 different lotteries, in each of which your chance of winning a prize is 1/100. What is the approximate probability that you will win a prize (a) at least once, (b) exactly once, (c) at least twice? (Give numerical answers if possible.)

Problem 2.33

*Problem 2.34

Let the probability density function (pdf) of a random variable X be $f(x) = c(4x - 2x^2)$ for $0 < x < 2$, where c is some constant, with $f(x) = 0$ otherwise. (a) What is the value of c? (b) What is $P(1/2 < X < 3/2)$?

Problem 2.39

*Problem 2.43

An urn contains $n+m$ balls, of which n are red and m are black. These balls are withdrawn from the urn, one at a time, and without replacement. Let X be the number of red balls removed before the first black ball is chosen. We are interested in determining $E[X]$. To obtain this quantity, number the red balls from 1 to n. Now define the (indicator) random variables X_i for $i = 1, \ldots, n$ by letting $X_i = 1$ if red ball i is chosen before any black ball is chosen, and let $X_i = 0$ otherwise. (a) Express X in terms of the random variables X_i; (b) Find $E[X]$.

Problem 2.48