IEOR 4701: Stochastic Models in Financial Engineering

Summer 2007, Professor Whitt

Homework Assignment 8: Arbitrage

Due on Monday, August 13 (not Wednesday)

Read Sections 10.4.1 and 10.4.2 in Ross.

Do the following exercises at the end of Chapter 10.

1. Exercise 10.12. corrections: We must add the strike price for the option: Suppose that the option gives you the opportunity, but not the obligation, to purchase a share of stock at time 1 for $K = 125$ dollars per share, again in present value dollars). That was left out of the question by mistake. You have the opportunity to either buy or sell any amount of the stock or the option at time 0. You can buy one and sell the other if you wish. As stated, the option costs C dollars per share.

(e) **extra part of problem**: How does the analysis for parts (a) and (d) change if we do not measure returns in present value dollars? Instead, let the interest rate be given as $r = 0.25$ per year, with the time period being one year. Moreover, assume that the stated returns at the end of the year are expressed as dollars at time 1. Remember that we buy the stock and the option at time 0, but if we elect to exercise the option, then we make that stock purchase at time 1. (It is a European option.)

2. Exercise 10.13.

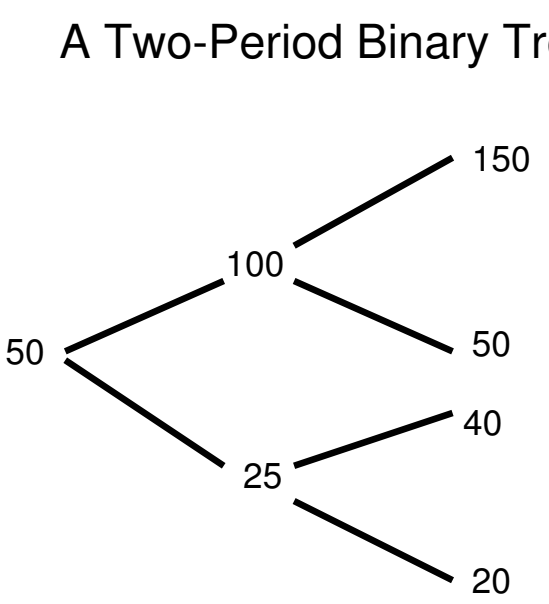
3. Exercise 10.14. Unlike in problem 10.12 above, we are given the strike price of the option: k dollars per share, with k specified in the parts below. With the option, we have the opportunity, but not the obligation, to buy the stock at time 1 for the present value, which is 100. As in problem 10.12, assume that the returns in period 1 are expressed in present value dollars, so that we do not separately need to consider the interest rate. Note: This problem introduces an important phenomenon: When we have more than two possible outcomes in this single-stage tree, the absence of arbitrage no longer determines a unique arbitrage-free cost for the option.

4. A Two-Period Binary Tree

Consider a stock that is initially priced at 50 dollars per share. Let the stock price evolve randomly over time according to the two-period binary tree shown in Figure .

We assume that, after one time period, the stock will either go up to 100 or drop to 25; these are the only two possibilities. If the stock does go up to 100 at then end of the first period, then we assume that the stock will either go up further to 150 or drop to 50 at the end of the second period. On the other hand, if instead the stock drops to 25 at the end of the first period, then we assume that the stock will either go up to 40 or drop further to 20 at the end of the second period. As above, we assume that all prices are in present value dollars, so that we can ignore interest.

Suppose now that you are offered a *European call option* with *strike price* 90 dollars per



share and *expiry* 2. That is, you are offered the option to purchase the stock at the end of the second period for 90. That is, the option gives you the opportunity, but not the obligation, to buy shares of the stock at the end of period 2 for 90 dollars per share.

(a) What is the appropriate (unique arbitrage-free) price for the option at time 0?

(b) Find a hedging strategy that allows us to replicate the option through buying and selling the stock at times 0 and 1.

5. The Put-Call Option Parity Formula

Let C be the price of a **call option** that enables its holder to buy one share of a stock at the exercise price K at the expiration time t. Also let P be the price of a **put option** that enables its holder to sell one share of a stock for the amount K at the expiration time t. Let S be the price of the stock at time 0. Assume that interest is continuously discounted at the nominal rate r. Show that the unique arbitrage-free price P of the put option can be expressed in terms of the other variables by

$$P = C + Ke^{-rt} - S \ .$$