IEOR 4701: Stochastic Models for Financial Engineering

Professor Whitt

Sample Entertainment

The following problems may seem frivolous, featherbrained, foolish, fanciful, fatuous or all of the above, but if you understand them, you may be able to approach the world with a better understanding of randomness and uncertainty.

1. **The Game Show**

Suppose you are on a game show, and you are given the choice of three doors. You win what is behind the door you choose. Behind one door is a new car; behind the other two doors are goats. You pick a door, say door number 1. Afterwards, the game show host, who knows what is behind all the doors, opens another door, say door number 3, and shows you a goat. He says to you, "Do you want to change your pick to door number 2?"

Is it to your advantage to switch your choice of doors? Why?

2. **Cars and Trucks**

Three out of every four trucks on the road are followed by a car, while only one out of every five cars is followed by a truck. What proportion of vehicles on the road are trucks?

3. **The Knight Errant (The Random Knight)**

A knight is placed alone on one of the corner squares of a chessboard (having $8 \times 8 = 64$ squares). What is the expected total number of moves for the knight to return to its initial position, if we assume that the knight moves randomly, taking each of its legal moves in each step with equal probability?

4. **Driving Back and Forth**

A truck driver regularly drives round trips between Atlanta (A) and Boston (B). Each time he drives from A to B, he drives at a fixed speed that (in miles per hour) is uniformly distributed between 40 and 60. Each time he drives from B to A, he drives at a fixed speed that (in miles per hour) is equally likely to be either 40 or 60.

In the long-run, what proportion of his driving time is spent going from A to B?

5. **Patterns**

Consider successive independent flips of a biased coin. On each flip, the coin comes up heads (H) with probability p or tails (T) with probability $q = 1 - p$, where $0 < p < 1$. A given segment of finitely many consecutive outcomes is called a *pattern*. The pattern is said to occur at flip n if the pattern is completed at flip n. For example, the pattern $A \equiv HTHTHT$ occurs at flips 8 and 10 in the sequence $TTHTHTHTHTTTTHHHT\ldots$ and at no other times among the first 17 flips.

What is the expected number of flips until the pattern HHH first appears?

6. **Hitting Times for Brownian Motion**

Let T_a be the first time standard Brownian motion hits the state a. Compute

$$P(T_1 < T_{-1} < T_2)?$$

7. **Arbitrage**

The Columbia hockey team is playing a game against Cornell: it will either win, lose or draw. A gambler offers you the following three payoffs, each for a \$1.00 bet.

	win	*lose*	*draw*
*bet*1	0	1	1.5
*bet*2	2	2	0.0
*bet*3	0.5	1.5	1.0

Assume that you are able to buy any amounts of these bets (even negative). Is there an arbitrage opportunity?