IEOR 6711: Stochastic Models I

Fall 2012, Professor Whitt

Homework Assignment 13, Tuesday, November 27

Chapter 5: Continuous-Time Markov Chains

Due on Tuesday, December 4.

Problems from Chapter 5 of *Stochastic Processes*, second edition, by Sheldon Ross.

Problem 5.12

Problem 5.13

Problem 5.14 (answer in back)

Problem 5.19

Problem 5.22

Problem 5.23 (answer in back)

Problem 5.24

Problem 5.25

Problem 5.26

Problem 5.28

Problem 5.34 (answer in back)

Easy Extra computational problem: Use the matlab program MMsrM.m (together with the matlab functions inversion2.m and laplace2.m) – which you can find on the computational tools web page – to solve for steady-state characteristics of the Erlang A model, i.e., the $M/M/s/r+M$ model, having s servers, r extra waiting spaces, a Poisson arrival process, IID exponential service times and IID exponential abandon times. Suppose that the arrival rate is $\lambda = 100$, the number of servers is $s = 100$, the size of the finite waiting room is $r = 100$, the mean individual service time is $1/\mu = 1$, the individual mean time to abandon is $1/\alpha = 1$. (The program MMsrM is already set with those parameters. You will also need to put the matlab functions inversion2.m and laplace2.m in your work space.) Use the matlab program to calculate:

(a) the mean and variance of the steady-state number of customers in the queue (waiting),

(b) the abandonment rate,

(c) the probability that an arrival can enter service immediately without waiting in queue,

(d) the expected conditional response time given that a customer is eventually served, i.e., the expected time from arrival to service completion, given that the customer is eventually served.

(e) The conditional probability that the waiting time is less than 10% of the mean service time, given that the customer is eventually served.

(f) How can you make a simple check on the validity of the answer in (c)?

(g) How can you use part (a) to produce an alternative calculation of (b)?