

IEOR 6711: Stochastic Models I

Fall 2012, Professor Whitt

Homework Assignment 7, Tuesday, October 16

Chapter 3: Renewal Theory

Due on Tuesday, October 23.

Problems from Chapter 3 of *Stochastic Processes*, second edition, by Sheldon Ross.

Problem 3.12 (Hint: Just find an appropriate function h .)

Problem 3.13

Problem 3.14

Problem 3.15

Problem 3.16

Problem 3.17 (answer in back)

Problem 3.18

Problem 3.22 (Hint: See Example 3.5A on page 125.)

Problem 3.23 (Hint: See Example 3.5A on page 125.)

Problem 3.21 (Hint: Use Example 3.5A plus Wald's equation.)

Problem 3.24 (answer in back)

Problem 3.25 (Hints: (a) There are two “standard” approaches: The first standard approach is to condition on the time of the *first renewal* and uncondition. That produces a *renewal equation* (an integral equation) of the form

$$g(t) = h(t) + \int_0^t g(t-x) dF(x) ,$$

which we then show has the unique solution

$$g(t) = h(t) + \int_0^t h(t-x) dM(x) ,$$

where M is the renewal function associated with the cdf F , i.e.,

$$M(t) \equiv E[N(t)] = \sum_{n=1}^{\infty} P(S_n \leq t) = \sum_{n=1}^{\infty} F_n(t) ,$$

and F_n is the cdf of $X_1 + \dots + X_n$, with X_i being IID with cdf F . For the second step, we can use Laplace transforms. In that step, observe that

$$\hat{M}(s) \equiv \int_0^{\infty} e^{-sx} M(x) dx$$

is not the same as

$$\hat{m}(s) \equiv \int_0^{\infty} e^{-sx} dM(x)$$

Indeed,

$$\hat{M}(s) = \hat{m}(s)/s.$$

The second approach is to condition on the time of the *last renewal* before time t and directly obtain the solution above, as is done in the proof of Lemma 3.4.3 on page 113.

(b) Apply the key renewal theorem.

Problem 3.27 (answer in back)

Problem 3.28