```
function  absorbing(Q, R)
%
% This is a MATLAB function that calculates important characteristics of an↙
 absorbing Markov chain.
%
% The full Markov chain transition matrix P is assumed to be (k+m) by (k+m)↙
.
% The first k states are absorbing states; the last m states are transient ↙
states.
% Once the chain gets to an absorbing state, it cannot leave it.
% The chain eventually leaves the transient states.
%
% The matrix P is divided into 4 submatrices: k by k, k by m, m by k and m ↙
by m.
% The upper left submatrix is a k by k identity matrix (1's on the diagonal↙
; 0's elsewhere).
% The upper right submatrix is a k by m matrix of all 0's.
% The lower left submatrix is an m by k matrix R.
% The lower right submatrix is an m by m matrix Q.
% We input the matrices Q and R as data to the function.
% The matrix Q gives the transition probabilities among the m transient sta↙
tes.
% The matrix R gives the one-step transition probabilities from the m trans↙
ient states to the k absorbing states.
%
% Given the absorbing Markov chain characterized by the two matrices Q and ↙
R,
% we calculate three matrices describing the behavior of the Markov chain.
% We call these new matrices N, m and B.
%
% We first echo the input:
Q
R
%
%
% The first matrix we calculate is the fundamental matrix N.
% The entry N(i,j) gives the expected number of visits to transient state j↙
 before absorption, starting in transient state i.
% The fundamental matrix N depends only on the square sub-transition matrix↙
 Q.
% It is not difficult to see that the total expected number of visits is th↙
e sum of the expected number of
% visits on the different steps, and that the expected number of visits to ↙
j on the n^th step, starting in i
% is just Q(i,j).  Thus in matrix form, we have
% N = I + Q + Q^2 + Q^3 + Q^4 + ...
%
% We now obtain a relatively simple formula for the fundamental matrix N.
% To do so, we observe that,
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% if we multiply the left and right side by (I - Q), we obtain the equation↙
 N*(I-Q) = I
% because there is cancellation on the right side (it telescopes).
% Thus N = (I-Q)^{-1} = inv(I-Q),
% where inv is the matrix inverse function.
% Since Q^n goes to the zero matrix as n increases, it is possible to show ↙
that the inverse of I-Q always exists.
%
%
s = size(Q);
n = s(1);
I = eye(n);
N = inv(I - Q)
%
% Now we calculate the expected number of transitions before absorption, st↙
arting from each transient state.
% We let m(i) be this expected number of transitions before absorption, sta↙
rting in state i.
% Thus m is an m by 1 column vector.  Clearly, m is made up of the row sums↙
 of N.
%
w = ones(n,1);
m = N*w
%
% Now we calculate the probability of absorption in each of the k absorbing↙
 states, starting from each transient state.
% Let B(i,j) be the probability of being absorbed in absorbing state j star↙
ting in transient state i.
% Note that B(i,j) = R(i,j) + (Q*R)(i,j) + (Q^2*R)(i,j) + (Q^3*R)(i,j) + ..↙
.
% so that B = (I + Q + Q^2 + Q^3 + ...)*R = N*R.
%
B = N*R
```