

# IEOR 8100: Topics in OR: Asymptotic Methods in Queueing Theory

Fall 2009, Professor Whitt

Class Lecture Notes: Monday, September 28.

## many-server heavy-traffic limits

### Topics Covered

1. The three regimes: (i) the underloaded or quality driven (QD) regime, (ii) the critically loaded, “Halfin-Whitt” or quality-and-efficiency-driven (QED) regime, (iii) the overloaded or efficiency-driven (ED) regime. The Markov case:  $M/M/s + M$  and  $G/M/s + M$ .

#### References

- i. S. Halfin, W. Whitt. Heavy-Traffic Limits for Queues with Many Exponential Servers. *Operations Research*, vol. 29, No. 3, May-June 1981, pp. 567–588.
- ii. O. Garnett, A. Mandelbaum, M. Reiman. Designing a Call Center with Impatient Customers. *Manufacturing and Service Operations Management*, vol. 4, 2002, pp. 208–227.
- iii. G. Pang, R. Talreja, W. Whitt. Martingale Proofs of Many-Server Heavy-Traffic Limits for Markovian Queues. *Probability Surveys*, vol. 4, 2007, pp. 193-267.

2. Deterministic Fluid Approximations for the  $G_t/GI/s_t + GI$  Model

#### References

- i. W. Whitt. Fluid Models for Multiserver Queues with Abandonments. *Operations Research*, vol. 54, No. 1, 2006, pp. 37–54.
- ii. Y. Liu, W. Whitt. A Fluid Model for the  $G_t/M/s_t + GI$  Many-Server Queue with Time-Varying Arrivals. in preparation.

3. FWLLN’s and FCLT’s for many-server queue with non-exponential distributions

#### References

- i. G. Pang, W. Whitt. Two-Parameter Heavy-Traffic Limits for Infinite-Server Queues. Submitted to *Queueing Systems*.
- ii. J. Reed, R. Talreja. Distribution-Valued Heavy Traffic Limits for the  $G/GI/\infty$  Queue. Available at: <http://www.columbia.edu/~rt2146>.