IEOR 8100S21: PhD Seminar on Queueing Theory

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January 11 and 13, 2021

Plan: Research Course

This will be a research course in which the participants will read research papers, conduct individual research projects and make presentations to the class. Students can choose their own topics, in consultation with the instructor. Students already engaged in research can present their own recent research results as well as important background papers and new related research papers in the same area. Other students can present their own new research as well as research papers by others. The topics should be broadly related to queueing theory and its applications, as illustrated by my completed research (available for downloading on my web page).

Sample of the Wide Range of Topics

1. **Forecasting Arrivals and Occupancy Levels in an Emergency Department**, Operations Research for Health Care, vol. 21, 2019, pp. 1-18. (WW & Xiaopei Zhang) [**neural network models and statistics**]
2. **Using Simulation to Help Manage the Pace of Play in Golf**, International Journal of Golf Science, vol. 6, No. 2, 2017, pp. 85-117. (MoonSoo Choi, Qi Fu & WW).
3. **Epidemic models with varying infectivity** and **Estimating the state of the Covid-19 epidemic in France using a non-Markovian model**, 2020, by Rafael Forien, Guodong Pang and Etienne Pardoux.

Logistics

1. Course material will be made available on a link on my home page.
2. There will be no required homework or exams. The grades will be based on participation and the presentations.
3. The course will initially be given over Zoom, but that may change.
4. The first lecture will be by the instructor on Monday, January 18.
5. Please send an email to introduce yourself. Briefly describe your background and interests.

More Logistics

1. To make allowances for doctoral student Yan Chen who is initially in China, the first lecture on **Monday, January 18** will be given by zoom at **10:10am-11:25am** instead of the usual time of 2:40pm-3:55pm. We intend to revert back to the initial time after he comes back to New York. That is planned for the end of the month.
2. We have been assigned a small seminar room, Northwest Corner Building 502, which we will need to change if we have a larger class and we decide to hav in-person meetings. (That seminar room is accessed by going up the stairs by the library entrance.)

The Planned Lecture on January 18

Source: **"Queues with Path-Dependent Arrival Processes,"** Journal of Applied Probability, 2021, forthcoming, with **Kerry W. Fendick**. See http://www.columbia.edu/∼ww2040/allpapers.html

1. The generalized Polya process (GPP) and path-dependent behavior
2. Stability Properties of the $\Psi - GPP/GI/1$ Queue
3. Heavy-Traffic Limits for the $\sum_{i=1}^{n} P_i/GI/1$ queue

   (tractable approximations for *transient* queue-length distribution)

Asymptotic Loss of Memory

- The standard notion of steady state for a stochastic Process:

$$X(t) \Rightarrow X(\infty) \quad \text{as} \quad t \to \infty,$$

  where $X(\infty)$ is independent of $X(t)$ for any fixed t.

- For time-varying model, **asymptotic loss of memory (ALOM):**

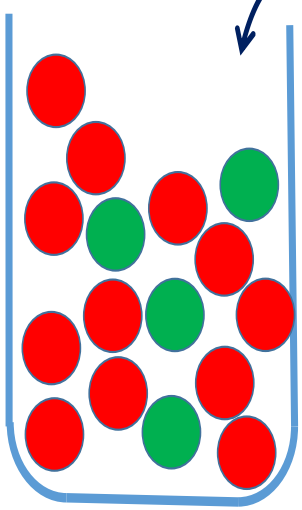
  Large-Time Asymptotics for the $G_t/M_t/s_t + GI_t$ Many-Server Fluid Queue with Abandonment, Queueing Systems, 67 (2011) 145-182 (with Yunan Liu).

- **Now we consider processes where ALOM does NOT hold.**

A Classic Urn Model from Feller, Volume I

Polya urn model

Start with:
r red balls **and**
g green balls



Step 1. Take one ball out of the urn picked at random.

Step 2. Return that ball plus one more of the same color.

Step 3. Repeat. Let X_n be the proportion of red balls in the urn after n steps.

As $n \to \infty$, $X_n \to$ ***Beta*(r,*g*)**

Generalized Polya Process (GPP)

- Definition. A **GPP** $N \equiv \{N(t) : t \geq 0\}$ is a Markov point process with stochastic intensity (defined in terms of the internal histories $\mathcal{H}_t$ by)

$$\lambda(t) \equiv \lambda(t|\mathcal{H}_t) \equiv (\gamma N(t-) + \beta)\kappa(t),$$

where $N(0) = 0$, γ and β are positive constants and $\kappa(t)$ is a positive integrable deterministic real-valued function.

- Definition. A **Polya point process** is the special case in which $\beta = 1$ and

$$\kappa(t) = \frac{1}{\gamma t + 1} \qquad \text{so that} \quad \lambda(t) = \frac{\gamma N(t-) + 1}{\gamma t + 1}.$$