Optimal Battery Purchasing and Charging at an Electric Vehicle Battery Swap Station

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A full paper entitled "*Optimal Battery Purchasing and Charging at an Electric Vehicle Battery Swap Station*" is available at http://www.columbia.edu/ xs2235/OptEVBSS.pdf.

Why EV Usage Is Likely to Expand Rapidly?

- No tailpipe emissions; can be refueled using renewable energy (solar, thermal and wind power).
- Strong government support; e.g., Beijing waives license plate lottery for the EV users; EV owners in Ontario can travel in HOV and HOT lanes.

Issues with EV Adoption

1. Range anxieties

2. Long charging times

Modes of Refueling

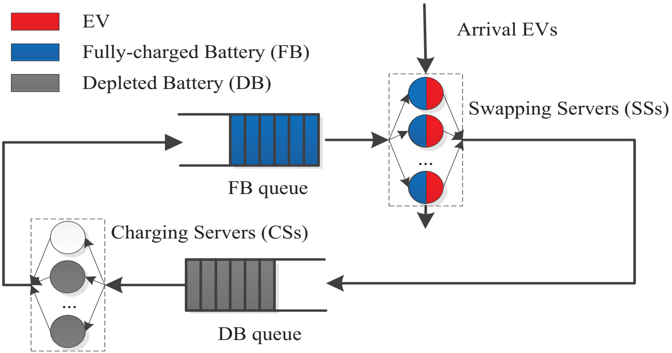
Rapid charging

Battery swap

Pros and Cons of Batter Swap vs. Rapid Charging

- Pros
  - Lower charge voltages prolong battery life; batteries suffer from stress when exposed to heat.
  - Ability to use grid electricity when it is off-peak, cheapest, or when some green energy generation is available.
  - Provides a more rapid way of refueling the EV; enable EVs to travel nonstop on long road trips.
- Cons
  - Ownership issue - consumers (especially private users) would like to buy the vehicle together with the battery.
- Conclusion
  - Battery swap is most likely to thrive in companies with fleet vehicles (e.g., city taxis and electric power trucks) and future Mobility Systems with self-drive EVs.

A Battery Swap Station



- Two jobs
  - It provides battery swap service for EVs (uncontrollable).
  - It recharges DBs so as to produce FBs for future use (controllable).
- Two types of resource constraints
  - The number of charging bays (model parameter)
  - The number of batteries in circulation (a decision variable)

Objectives

- Long-term decision on the number of charging bays
- Medium-term decision on the number of batteries to be purchased
- Short-term decisions on when and how many batteries to recharge

Primarily focus on medium-term and short-term decision making.

Existing Work and Our Contributions

- Literature review
  - Optimizing BSS operations
    Schneider et al. (2017); Sun et al. (2017); Widrick et al. (2018)
    All follow an MDP approach and can be computationally expensive, especially for large scale problems.
  - Fluid-model analysis
    Maglaras and Meissner (2006); Whitt (2006); Dai et al. (2018)
    Focus on different application domains.
- Our contribution:
  - Propose a fluid-based formulation that allows for easy implementation of large-scale systems.
  - Obtain managerial insights for optimizing BSS operations under non-stationary demand and energy price.
  - In the event of high service levels, propose a robust formulation to account for demand uncertainty.

Fluid-based Optimization - I

- System parameters
  - Demand function $\lambda \equiv \{\lambda(t); t \geq 0\}$
  - Per-charger charging rate μ
  - Total number of batteries b
  - Total number of charging bays κ
- Cost parameters
  - Amortized battery purchasing cost per unit time γ
  - Day-ahead electricity price $p \equiv \{p(t); t \geq 0\}$
  - Waiting cost per unit time c
- State $x \equiv \{x(t); t \geq 0\}$ representing the number of FBs
- Control $m \equiv \{m(t); t \geq 0\}$ representing the number of (depleted) batteries being recharged

Fluid-based Optimization - II

We formulate the BSS battery purchasing and charging problem as

$$\min_{b} \quad \underbrace{\gamma\tau b}_{\text{battery cost}} + \underbrace{V(b)}_{\text{operating cost}} \qquad \textbf{(first-stage)}$$

where the **second-stage** problem is given by

$$V(b) \equiv \min_{(x_0,m)\in\mathcal{X}(b)} \underbrace{\int_0^\tau p(t)m(t)dt}_{\text{charging cost}} + \underbrace{c\int_0^\tau x^-(t)dt}_{\text{waiting cost}}$$

and the decision region for the recourse variables

$$\mathcal{X}(b) \equiv \{x_0 \le b, m : \dot{x}(t) = \mu m(t) - \lambda(t), \quad 0 \le m(t) \le \kappa, \quad m(t) + x^+(t) \le b, \quad x(0) = x(\tau) = x_0.\}$$

Structural Properties

Theorem

(I) There exists at least one optimal solution (x_0^*, m^*) to the second-stage problem. (II) The optimal value function V(b) is convex in b. In addition, there exists a **cutoff value** *b^* such that any number of batteries beyond this threshold b^* will not improve the operating cost $V(b)$.*

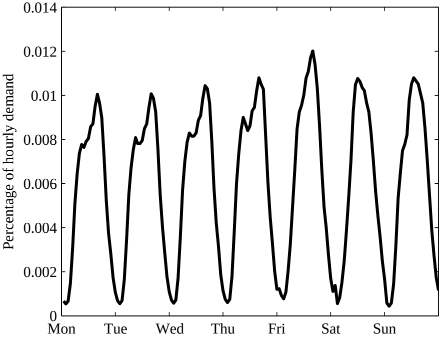
- b^* is the minimum number of batteries that guarantees zero wait and the lowest charging cost.
- b^* permits an explicit representation.

Example

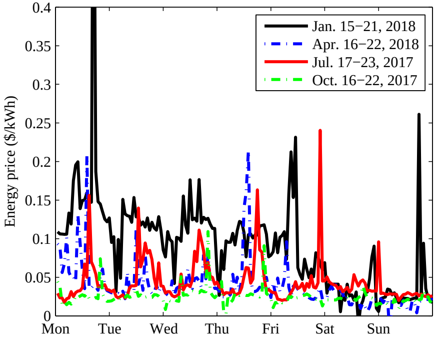
Suppose $p(t) = \bar{p} + A_p \sin(2\pi t/\tau)$ and $\lambda(t) = \bar{\lambda} + A_\lambda \sin(2\pi(t - \psi)/\tau)$. In addition, $\mu = 1$ and $\kappa = 2\bar{\lambda}$. Then

$$b^* = \kappa + \bar{\lambda}\tau - \int_{\tau/2}^{\tau} \lambda(t)dt = 2\bar{\lambda} + \frac{\bar{\lambda}\tau}{2} + \frac{A_\lambda \tau}{\pi} \cos(2\pi\psi/\tau).$$

Numerical Studies



(a) Demand for battery swap



(b) Electricity price

Figure 1: Illustrating the battery-swapping demand and the energy price.

Balancing Battery Purchasing Cost and Operating Cost - I

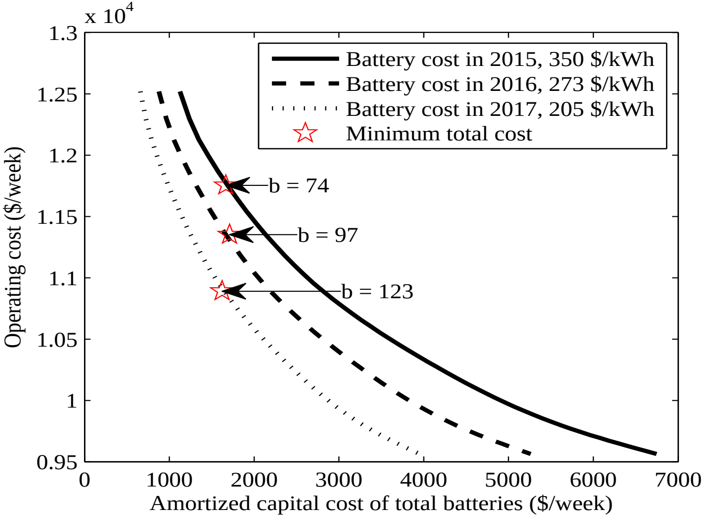


Figure 2: Impact of the battery capital price

Balancing Battery Purchasing Cost and Operating Cost - II

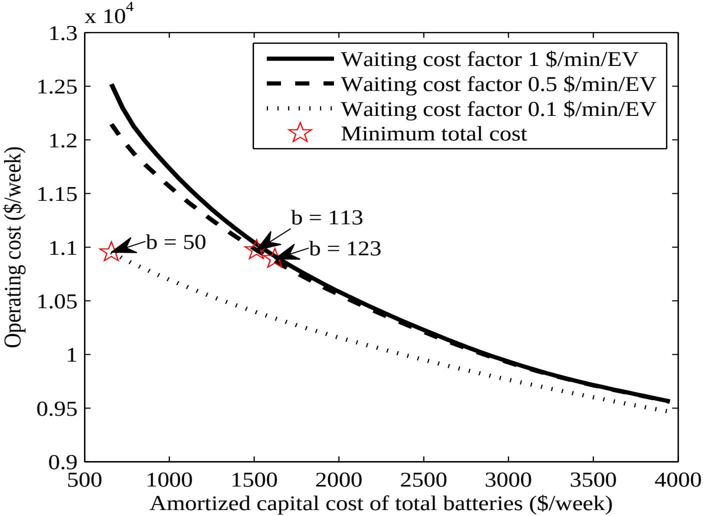


Figure 3: Impact of service level

High Service Level

In the event of high service levels, one would want to avoid negative FB inventory entirely. In that case the second-stage problem becomes

$$V(b) \equiv \min_{(x_0, m) \in \mathcal{X}(b)} \underbrace{\int_0^\tau p(t) m(t) dt}_{\text{charging cost}}$$

where the decision region $\mathcal{X}(b)$ for the recourse variables

$$\begin{aligned} \mathcal{X}(b) \equiv \{ & x_0 \le b, m : \dot{x}(t) = \mu m(t) - \lambda(t), \quad 0 \le m(t) \le \kappa, \\ & x(t) \ge 0, \quad m(t) + x(t) \le b, \quad x(0) = x(\tau) = x_0. \} \end{aligned} \tag{1}$$

A Robust Formulation - I

- The idea is to model demand uncertainty via uncertainty set. Let λ and $\tilde{\lambda}$ to denote the nominal and realized demand, respectively, and $\hat{\lambda}$ be such that

$$0 < \hat{\lambda}(t) < \lambda(t)/2 \quad \text{and} \quad \left|\tilde{\lambda}(t) - \lambda(t)\right| \leq \hat{\lambda}(t).$$

- Introduce a *budget-of-uncertainty function* $\Gamma(\cdot)$, and stipulate

$$\int_0^t \left|\tilde{\lambda}(u) - \lambda(u)\right| \Big/ \hat{\lambda}(u) du \leq \Gamma(t) \quad \text{for all} \quad t \in [0, \tau].$$

- Let $\mathcal{F}$ be the uncertainty set. Then the robust cost minimization problem

$$\min_{(x_0, m, \chi) \in \mathcal{X}(b)} \quad \chi$$

subject to $\dot{x}(t) = \mu m(t) - \tilde{\lambda}(t)$, $0 \leq m(t) \leq \kappa$, $x(t) \geq 0, m(t) + x(t) \leq b$, $x(0) = x(\tau) = x_0$, and $\chi \geq \int_0^\tau p(t) m(t) dt$ for all $\tilde{\lambda} \in \mathcal{F}$.

A Robust Formulation - II

- It turns out that preceding robust formulation can be simplified, yielding an equivalent form as specified below:

$$\min_{(x_0,m)\in\mathcal{X}(b)} \quad \int_0^{\tau} p(t)m(t)dt$$

where the decision region $\mathcal{X}(b)$ for the recourse variables

$$\begin{aligned}\mathcal{X}(b) \equiv & \{x_0 \leq b, m : \dot{x}(t) = \mu m(t) - \lambda(t), \quad 0 \leq m(t) \leq \kappa, \\ & x(t) \geq \eta(t), \quad m(t) + x(t) \leq b - \eta(t), \quad x(0) = x(\tau) = x_0\},\end{aligned} \tag{2}$$

and the function η *only depends on the budget-of-uncertainty function* Γ.

- Comparing (2) with (1), one immediately sees that Γ trades off between the level of conservatism of the robust solution and its performance.

References

- Maglaras, C., & Meissner, J. (2006). Dynamic pricing strategies for multiproduct revenue management problems. Manufacturing & Service Operations Management, 8(2), 136-148.
- Schneider, F., Thonemann, U. W., & Klabjan, D. (2017). Optimization of battery charging and purchasing at electric vehicle battery swap stations. Transportation Science.
- Sun, B., Tan, X., & Tsang, D. (2017). Optimal charging operation of battery swapping and charging stations with QoS guarantee. IEEE Transactions on Smart Grid.
- Whitt, W. (2006). Staffing a call center with uncertain arrival rate and absenteeism. Production and operations management, 15(1), 88.
- Widrick, R. S., Nurre, S. G., & Robbins, M. J. (2016). Optimal policies for the management of an electric vehicle battery swap station. Transportation Science, 52(1), 59-79.
- Dai, J. J., Kleywegt, A. J., & Xiao, Y. Network Revenue Management with Cancellations and No?shows. Production and Operations Management.

THANK YOU!

Optimal Control and State Trajectory

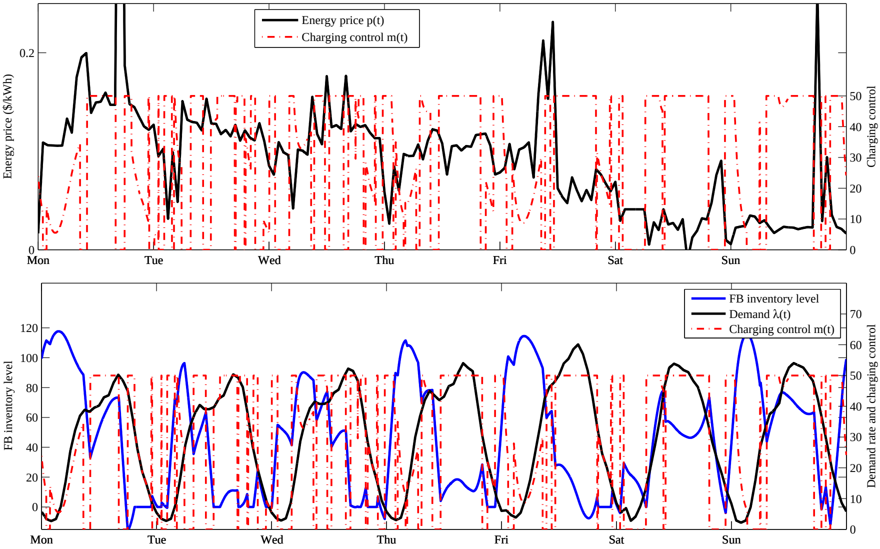
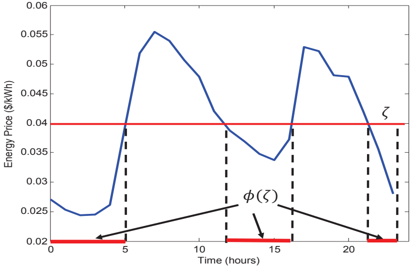


Figure 4: Illustration of the optimal charging control and state trajectory.

Deriving the Cutoff Value b^*

- Define the set-valued function $\phi(\zeta) \equiv \{0 \le t \le \tau : p(t) \le \zeta\}$.



- Optimal charging rule (run in full capacity when the energy price is among the lowest)

$$m^*(t) = \begin{cases} \kappa & \text{if} \quad t \in \phi(\zeta^*), \\ 0 & \text{if} \quad t \notin \phi(\zeta^*), \end{cases}$$

- Minimum initial FB inventory that leads to zero congestion:

$$x^*(0) \equiv \sup_{0 \le t \le \tau} \left[\Lambda(t) - \mu \int_0^t m^*(u) du \right]^+ .$$

- Minimum number of batteries that ensures the lowest operating cost:

$$b^* \equiv \sup_{t \le \tau} \{ m^*(t) + x^*(t) \}.$$