A Robust Queueing Network Analyzer Based on Indices of Dispersion

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Motivation

- Many complex service systems can be modeled as open queueing networks (OQN)
- The estimation of performance measures
  - important in many applications;
  - theoretical analysis is limited;
  - approximation remains an important tool.
- In this work we propose a fast and accurate Robust Queueing Network Analyzer (RQNA) to approximation performance measures in single-server OQN.
Decomposition approximation methods
- Motivated by product-form solutions of Jackson Networks.
- Treat stations as independent single-server queues.

Examples
- The Queueing Network Analyzer (QNA) by Whitt (1983),
  - approximates each station by a GI/GI/1 queue.
- Kim (2011a, 2011b)
  - approximate each station by a MMPP(2)/GI/1 queue (Markov-Modulated Poisson Process);
Approximations using **Reflected Brownian Motion (RBM)**

- Approximate the steady-state queue length distribution by the stationary distribution of the limiting RBM;
- numerically calculate the steady-state mean of the RBM.

**Examples**

- **QNET** by Harrison and Nguyen (1990) for OQNs and by Dai and Harrison (1993) for CQNs;
- **SBD** by Dai, Nguyen and Reiman (1994).
Recent Developments

- Interpolation method (IR) by Wu and McGinnis (2014).
- (Parametric) Robust Queueing (RQ) by Bandi et al. (2015).
- (Non-parametric) RQ by Whitt and You (2018a).

In this talk,

- non-parametric Robust Queueing Network Analyzer (RQNA) for open queueing networks.
Dependence in Queues

**Figure:** A three-station example.

**Dependence** rises naturally in queueing network:

- Dependence within/between the flows¹:
  - introduced by departure, splitting and superposition;
  - also by customer feedback.

¹arrival processes, departure process, etc.
Dependence in Queues

Dependence has **significant impact** on performance measures.

- Dependence can have complicated **temporal structure**.
- The **level of impact** will depend on both the temporal structure and the traffic intensity.
- Parametric methods (QNA, QNET, parametric RQ) **using first two moments** to describe variability may fail.
3 Stations with Feedback

Poisson, $\lambda_{0,1} = 0.225$

Queue 1

Queue 2

Queue 3

$H_2, c^2 = 8$

$p_{2,3} = 0.5$

$p_{2,1} = 0.5$

$E_2, c^2 = 0.25$

$r = 0.5$

Table: The steady-state mean waiting time.

<table>
<thead>
<tr>
<th>Queue</th>
<th>$\rho$</th>
<th>Simu</th>
<th>QNET</th>
<th>SBD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.9</td>
<td>31.22</td>
<td>35.9 (15%)</td>
<td>26.0 (-17%)</td>
</tr>
<tr>
<td>2</td>
<td>0.675</td>
<td>8.32</td>
<td>10.2 (23%)</td>
<td>11.1 (33%)</td>
</tr>
<tr>
<td>3</td>
<td>0.45</td>
<td>2.00</td>
<td>1.89 (5.5%)</td>
<td>1.94 (3%)</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>138.7</td>
<td>161.3 (16%)</td>
<td>135.3 (-2.5%)</td>
</tr>
</tbody>
</table>

$r = 0.99$

<table>
<thead>
<tr>
<th>Queue</th>
<th>$\rho$</th>
<th>Simu</th>
<th>QNET</th>
<th>SBD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.9</td>
<td>27.67</td>
<td>35.9 (30%)</td>
<td>26.0 (-6.0%)</td>
</tr>
<tr>
<td>2</td>
<td>0.675</td>
<td>2.67</td>
<td>10.2 (282%)</td>
<td>11.1 (316%)</td>
</tr>
<tr>
<td>3</td>
<td>0.45</td>
<td>0.56</td>
<td>1.89 (236%)</td>
<td>1.94 (245%)</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>103.8</td>
<td>161.3 (55%)</td>
<td>135.3 (30%)</td>
</tr>
</tbody>
</table>
Indices of dispersion can describe the temporal structure.

- Fendick and Whitt (1989) first applied it to queueing approximation.

Definition from Cox and Lewis (1966)

\[ I_a(t) \equiv \frac{\text{Var}(A(t))}{E[A(t)]}, \quad t \geq 0, \]

where \( A(t) \) is any stationary point process.
Indices of Dispersion for Counts (IDC)

Theorem (renewal process characterization theorem)

A renewal process $A(t)$ with positive rate $\lambda$ is fully characterized by the IDC of its equilibrium (stationary) version $A_e(t)$:

$$I_a(t) \equiv \frac{\text{Var}(A_e(t))}{E[A_e(t)]}.$$ 

- RQ-IDC, and so RQNA-IDC, utilize much more information of the underlying distribution;
- potentially more accurate and adaptive to complex distributions.
Let \( Z \) be the workload (virtual waiting time) of a single-server queue.

RQ for the workload in *Whitt and You (2018a)*

\[
Z \approx Z^* \equiv \sup_{N\in\mathcal{U}} \sup_{0\leq s\leq \infty} \{N(s)\},
\]

where

\[
\mathcal{U} = \left\{ N : N(s) \leq -(1 - \rho)s + \sqrt{2\rho s(l_a(s) + c_s^2)}/\mu, \ s \geq 0 \right\}.
\]
Robust Queueing for continuous-time workload

Equivalent to

\[ Z^* = \sup_{0 \leq s \leq \infty} \left\{ -(1 - \rho)s + \sqrt{2\rho s (I_a(s) + c_s^2) / \mu} \right\}. \quad \text{(RQ-IDC)} \]

- Requires IDC \( I_a \) as model input;
- \( I_a \) is defined for the \textit{stationary} arrival process;
- \( I_a \) can be
  - \textbf{calculated} in special cases\(^2\) (e.g. renewal process);
  - \textbf{estimated} by simulation or from data; or
  - \textbf{approximated} by RQNA.

\(^2\)by numerically inverting the Laplace Transform
Generalization to RQNA

To extend RQ to RQNA, we need to

- (for external flows\(^3\)) calculate/estimate the IDC from distribution or data;
- (for internal flows\(^4\)) approximate internal arrival IDC at any queue in a open queueing network;

\(^3\)Service processes, external arrival processes.

\(^4\)Internal arrival processes, departure processes.
Generalization to RQNA: Internal Flows

The **total arrival process** at any queue:
- **superposition** of external arrival and **splittings** of **departure** processes.

**Figure:** A three-station example.
The IDC Equations

Notations

- $I_{a,i}$: IDC of the total arrival process at station $i$;
- $I_{s,i}$: IDC of the service process at station $i$;
- $I_{d,i}$: IDC of the total departure process at station $i$;

The Departure Equation

$$I_{d,i}(t) \approx w_i(t)I_{a,i}(t) + (1 - w_i(t))I_{s,i}(t), \quad \text{(Dep)}$$

where $w_i$ is a weight function with explicit expression.

- Departure IDC is a **convex combination**;
- Supported by Heavy-traffic (HT) limit for the **stationary departure process** $\Rightarrow$ **asymptotically exact**.
The IDC Equations

One more notation

- $I_{a,i,j}$: IDC of the flow from station $i$ to station $j$;

The **Splitting and Superposition** Equation

$$I_{a,i,j}(t) \approx p_{i,j}l_{d,i}(t) + (1 - p_{i,j}) + \alpha_{i,j}(t) \quad (\text{Spl})$$

$$I_{a,i}(t) \approx \sum_{j=0}^{K} (\lambda_{j,i}/\lambda_i) I_{a,j,i}(t) + \beta_{i}(t) \quad (\text{Sup})$$

where $\alpha_{i,j}$ and $\beta_{i}$ are correction term with explicit expression and $\lambda_{j,i} = p_{j,i} \lambda_j$ is the rate of the flow from $i$ to $j$.

- Red terms recovers **independent** splitting.
- Blue term models **dependence** in the splitting or superposition operation.
- Supported by Heavy-traffic (HT) limit for the **stationary flows** in OQN.
The IDC Equations

In summary, the **IDC equations** are

\[
I_{d,i}(t) = w_i(t)I_{a,i}(t) + (1 - w_i(t))I_{s,i}(\rho t), \quad \text{(Dep)}
\]
\[
I_{a,i,j}(t) = p_{i,j}I_{d,i}(t) + (1 - p_{i,j}) + \alpha_{i,j}(t), \quad \text{(Spl)}
\]
\[
I_{a,i}(t) = \sum_{j=0}^{K} (\lambda_{j,i}/\lambda_i)I_{a,j,i}(t) + +\beta_i(t). \quad \text{(Sup)}
\]

In matrix notation, we have

\[
I(t) = M(t)I(t) + b(t).
\]

- For each fixed \( t \), the IDC equations form a system of **linear equations**;
- The IDC equations have **unique solution** if every customer eventually leave the system.
3 Stations with Feedback

\[ \lambda_{0,1} = 0.225 \]

\[ p_{2,1} = 0.5 \]

\[ p_{2,3} = 0.5 \]

\[ p_{3,2} = 0.5 \]

**Figure:** A three-station example.

**Table:** Traffic intensity.

<table>
<thead>
<tr>
<th>Case</th>
<th>( \rho_1 )</th>
<th>( \rho_2 )</th>
<th>( \rho_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.675</td>
<td>0.900</td>
<td>0.450</td>
</tr>
<tr>
<td>2</td>
<td>0.900</td>
<td>0.675</td>
<td>0.900</td>
</tr>
<tr>
<td>3</td>
<td>0.900</td>
<td>0.675</td>
<td>0.450</td>
</tr>
<tr>
<td>4</td>
<td>0.900</td>
<td>0.675</td>
<td>0.675</td>
</tr>
</tbody>
</table>

**Table:** Variability of the service distributions.

<table>
<thead>
<tr>
<th>Case</th>
<th>( c_{s,1}^2 )</th>
<th>( c_{s,2}^2 )</th>
<th>( c_{s,3}^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>B</td>
<td>2.25</td>
<td>0.00</td>
<td>0.25</td>
</tr>
<tr>
<td>C</td>
<td>0.25</td>
<td>0.25</td>
<td>2.25</td>
</tr>
<tr>
<td>D</td>
<td>0.00</td>
<td>2.25</td>
<td>2.25</td>
</tr>
<tr>
<td>E</td>
<td>8.00</td>
<td>8.00</td>
<td>0.25</td>
</tr>
</tbody>
</table>
# 3 Stations with Feedback

<table>
<thead>
<tr>
<th>Case</th>
<th>Simu</th>
<th>QNA</th>
<th>QNET</th>
<th>SBD</th>
<th>RQNA</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>40.39</td>
<td>20.5</td>
<td>diverging</td>
<td>43.0 (6.4%)</td>
<td>44.8 (11.0%)</td>
</tr>
<tr>
<td>2</td>
<td>59.58</td>
<td>36.0</td>
<td>56.7 (-4.9%)</td>
<td>58.2 (-2.4%)</td>
<td>69.3 (16.4%)</td>
</tr>
<tr>
<td>3</td>
<td>40.72</td>
<td>24.0</td>
<td>38.7 (-5.0%)</td>
<td>40.2 (-1.3%)</td>
<td>43.3 (6.3%)</td>
</tr>
<tr>
<td>4</td>
<td>42.12</td>
<td>26.2</td>
<td>41.8 (-0.7%)</td>
<td>42.7 (1.3%)</td>
<td>41.2 (-2.2%)</td>
</tr>
<tr>
<td>B</td>
<td>52.40</td>
<td>42.0</td>
<td>52.6 (0.4%)</td>
<td>50.2 (-4.2%)</td>
<td>53.1 (1.4%)</td>
</tr>
<tr>
<td>2</td>
<td>91.52</td>
<td>94.1 (2.8%)</td>
<td>83.7 (-8.5%)</td>
<td>95.3 (4.1%)</td>
<td>94.5 (3.2%)</td>
</tr>
<tr>
<td>3</td>
<td>61.68</td>
<td>72.2 (17%)</td>
<td>61.9 (0.4%)</td>
<td>60.9 (-1.3%)</td>
<td>60.5 (-1.9%)</td>
</tr>
<tr>
<td>4</td>
<td>63.34</td>
<td>75.8 (20%)</td>
<td>64.1 (1.3%)</td>
<td>64.7 (2.1%)</td>
<td>62.4 (-1.4%)</td>
</tr>
<tr>
<td>C</td>
<td>44.24</td>
<td>31.3</td>
<td>37.0 (-16%)</td>
<td>47.1 (6.4%)</td>
<td>42.1 (-4.8%)</td>
</tr>
<tr>
<td>2</td>
<td>92.42</td>
<td>87.4</td>
<td>91.2 (-1.4%)</td>
<td>91.6 (-0.8%)</td>
<td>96.0 (3.8%)</td>
</tr>
<tr>
<td>3</td>
<td>44.26</td>
<td>33.2</td>
<td>44.0 (-0.7%)</td>
<td>45.0 (1.7%)</td>
<td>44.0 (-0.6%)</td>
</tr>
<tr>
<td>4</td>
<td>50.20</td>
<td>41.4</td>
<td>51.1 (1.7%)</td>
<td>52.2 (4.0%)</td>
<td>45.9 (-8.6%)</td>
</tr>
<tr>
<td>E</td>
<td>134.4</td>
<td>265 (97%)</td>
<td>155 (15%)</td>
<td>116 (-14%)</td>
<td>120 (-11%)</td>
</tr>
<tr>
<td>2</td>
<td>213.1</td>
<td>308 (45%)</td>
<td>228 (7.1%)</td>
<td>206 (-3.3%)</td>
<td>173 (-19%)</td>
</tr>
<tr>
<td>3</td>
<td>138.7</td>
<td>244 (76%)</td>
<td>161 (16%)</td>
<td>135 (-2.5%)</td>
<td>136 (-2.0%)</td>
</tr>
<tr>
<td>4</td>
<td>155.1</td>
<td>252 (63%)</td>
<td>168 (8.2%)</td>
<td>147 (-5.0%)</td>
<td>148 (-4.8%)</td>
</tr>
</tbody>
</table>

**Table:** A comparison of four approximation methods to simulation for the total sojourn time in the three-station example.
3 Stations with Feedback

**Table:** A comparison of six approximation methods to simulation for the sojourn time at each station of the three-station example.

<table>
<thead>
<tr>
<th>Queue</th>
<th>Simu</th>
<th>QNET</th>
<th>SBD</th>
<th>RQNA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>31.22</td>
<td>35.9 (15%)</td>
<td>26.0 (-17%)</td>
<td>26.0 (-17%)</td>
</tr>
<tr>
<td>2</td>
<td>8.32</td>
<td>10.2 (23%)</td>
<td>11.1 (33%)</td>
<td>11.8 (42%)</td>
</tr>
<tr>
<td>3</td>
<td>2.00</td>
<td>1.89 (5.5%)</td>
<td>1.94 (3%)</td>
<td>0.93 (-54%)</td>
</tr>
<tr>
<td>Sum</td>
<td>138.7</td>
<td>161.3 (16%)</td>
<td>135.3 (-2.5%)</td>
<td>136.1 (-1.9%)</td>
</tr>
</tbody>
</table>

**Case E3, r = 0.99**

<table>
<thead>
<tr>
<th>Queue</th>
<th>Simu</th>
<th>QNET</th>
<th>SBD</th>
<th>RQNA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>27.67</td>
<td>35.9 (30%)</td>
<td>26.0 (-6.0%)</td>
<td>26.0 (-6.0%)</td>
</tr>
<tr>
<td>2</td>
<td>2.67</td>
<td>10.2 (282%)</td>
<td>11.1 (316%)</td>
<td>6.03 (125%)</td>
</tr>
<tr>
<td>3</td>
<td>0.56</td>
<td>1.89 (236%)</td>
<td>1.94 (245%)</td>
<td>0.50 (-11%)</td>
</tr>
<tr>
<td>Sum</td>
<td>103.8</td>
<td>161.3 (55%)</td>
<td>135.3 (30%)</td>
<td>112.1 (8%)</td>
</tr>
</tbody>
</table>
References on Robust Queueing:


References on queueing network approximations:


References

References on HT limits:


References on departure processes:


Other Performance Measures

\[ Z_\rho^* = \sup_{0 \leq s \leq \infty} \left\{ -(1 - \rho)s + \sqrt{2\rho s l_w(s)/\mu} \right\}. \]

This RQ formulation gives approximation of the mean steady-state workload. For other performance measures, we have

- Mean steady-state waiting time:
  \[ E[W] \approx \max\{0, Z_\rho^*/\rho - (c_s^2 + 1)/2\mu\}. \]
  - obtained by Brumelle's formula:
    \[ E[Z] = \rho E[W] + \rho \frac{E[V^2]}{2\mu} = \rho E[W] + \rho \frac{(c_s^2 + 1)}{2\mu}. \]

- Mean steady-state queue length, by Little's law,
  \[ E[Q] = \lambda E[W] = \rho E[W]. \]
3 Stations with Feedback

**Table:** A comparison of six approximation methods to simulation for the sojourn time at each station of the three-station example.

<table>
<thead>
<tr>
<th>Queue</th>
<th>Simu</th>
<th>QNA</th>
<th>QNET</th>
<th>SBD</th>
<th>RQNA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.478</td>
<td>1.24 (-16%)</td>
<td>1.48 (0.1%)</td>
<td>1.47 (-0.5%)</td>
<td>1.69 (14%)</td>
</tr>
<tr>
<td>2</td>
<td>10.22</td>
<td>13.9 (36%)</td>
<td>10.6 (3.7%)</td>
<td>10.4 (1.8%)</td>
<td>10.4 (1.8%)</td>
</tr>
<tr>
<td>3</td>
<td>1.563</td>
<td>1.53 (-2.1%)</td>
<td>1.54 (-1.5%)</td>
<td>1.59 (1.7%)</td>
<td>1.53 (-2.1%)</td>
</tr>
<tr>
<td>Sum</td>
<td>57.42</td>
<td>71.4 (24%)</td>
<td>58.8 (2.4%)</td>
<td>58.2 (1.4%)</td>
<td>58.7 (2.2%)</td>
</tr>
</tbody>
</table>

**Case D1, r = 0.99**

<table>
<thead>
<tr>
<th>Queue</th>
<th>Simu</th>
<th>QNA</th>
<th>QNET</th>
<th>SBD</th>
<th>RQNA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.145</td>
<td>1.24 (8.3%)</td>
<td>1.48 (29%)</td>
<td>1.47 (28%)</td>
<td>1.28 (12%)</td>
</tr>
<tr>
<td>2</td>
<td>10.15</td>
<td>13.9 (37%)</td>
<td>10.6 (4.4%)</td>
<td>10.4 (2.5%)</td>
<td>10.4 (2.5%)</td>
</tr>
<tr>
<td>3</td>
<td>1.119</td>
<td>1.53 (37%)</td>
<td>1.54 (38%)</td>
<td>1.59 (42%)</td>
<td>1.28 (14%)</td>
</tr>
<tr>
<td>Sum</td>
<td>55.26</td>
<td>71.4 (29%)</td>
<td>58.8 (6.4%)</td>
<td>58.2 (5.3%)</td>
<td>57.0 (3.1%)</td>
</tr>
</tbody>
</table>
# The Heavy-Traffic Bottleneck Phenomenon

\[ H_2(8) \rightarrow M, \rho_1 = 0.6 \rightarrow \cdots \rightarrow 8 \rightarrow M, \rho_1 = 0.6 \rightarrow 9 \]

\[ \lambda = 1 \]

**Figure:** The heavy-traffic bottleneck example in Suresh and Whitt (1990).

<table>
<thead>
<tr>
<th>Arrival Process</th>
<th>( H_2, c_a^2 = 8 ) &amp; ( r = 0.5 )</th>
<th>( H_2, c_a^2 = 8 ) &amp; ( r = 0.95 )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Queue 8</strong></td>
<td>Simulation</td>
<td>1.44</td>
</tr>
<tr>
<td>M/M/1</td>
<td>0.90 (-38%)</td>
<td>0.90 (-2.1%)</td>
</tr>
<tr>
<td>QNA</td>
<td>1.04 (-28%)</td>
<td>1.04 (13%)</td>
</tr>
<tr>
<td>SBD</td>
<td>1.01 (-29%)</td>
<td>1.01 (10%)</td>
</tr>
<tr>
<td><strong>Queue 9</strong></td>
<td>Simulation</td>
<td>29.15</td>
</tr>
<tr>
<td>M/M/1</td>
<td>8.1 (-72%)</td>
<td>8.1 (-9.4%)</td>
</tr>
<tr>
<td>QNA</td>
<td>8.9 (-69%)</td>
<td>8.9 (-0.4%)</td>
</tr>
<tr>
<td>SBD</td>
<td>36.5 (25%)</td>
<td>36.5 (308%)</td>
</tr>
</tbody>
</table>

**Table:** Mean steady-state waiting times at Queue 8 and 9, compared with M/M/1 values and approximations.
The Heavy-traffic Bottleneck Phenomenon

\[ H_2(8) \]
\[ \lambda = 1 \]
\[ M, \rho_1 = 0.6 \]
\[ \cdots \]
\[ M, \rho_1 = 0.6 \]
\[ M, \rho_1 = 0.9 \]

<table>
<thead>
<tr>
<th>Arrival Process</th>
<th>[ H_2, c_a^2 = 8 ]</th>
<th>[ H_2, c_a^2 = 8 ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ r = 0.5 ]</td>
<td>0.90 (-38%)</td>
<td>0.90 (-2.1%)</td>
</tr>
<tr>
<td>[ r = 0.99 ]</td>
<td>1.04 (-28%)</td>
<td>1.04 (13%)</td>
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<table>
<thead>
<tr>
<th>Queue 8</th>
<th>Simulation</th>
<th>[ M/M/1 ]</th>
<th>QNA</th>
<th>SBD</th>
<th>IR</th>
<th>RQ</th>
</tr>
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<tbody>
<tr>
<td>M/M/1</td>
<td>0.90 (-38%)</td>
<td>1.04 (-28%)</td>
<td>1.01 (-29%)</td>
<td>1.20 (-17%)</td>
<td>1.27 (-12%)</td>
<td></td>
</tr>
<tr>
<td>QNA</td>
<td>1.04 (-28%)</td>
<td>1.04 (13%)</td>
<td>1.01 (10%)</td>
<td>1.20 (7.1%)</td>
<td>0.92 (-0.5%)</td>
<td></td>
</tr>
<tr>
<td>SBD</td>
<td>1.01 (-29%)</td>
<td>1.01 (10%)</td>
<td>1.01 (10%)</td>
<td>1.20 (7.1%)</td>
<td>0.92 (-0.5%)</td>
<td></td>
</tr>
<tr>
<td>IR</td>
<td>1.20 (-17%)</td>
<td>1.20 (7.1%)</td>
<td>1.20 (7.1%)</td>
<td>0.92 (-0.5%)</td>
<td>0.92 (-0.5%)</td>
<td></td>
</tr>
<tr>
<td>RQ</td>
<td>1.27 (-12%)</td>
<td>0.92 (-0.5%)</td>
<td>0.92 (-0.5%)</td>
<td>0.92 (-0.5%)</td>
<td>0.92 (-0.5%)</td>
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<td>M/M/1</td>
<td>8.1 (-72%)</td>
<td>8.9 (-69%)</td>
<td>36.5 (25%)</td>
<td>21.1 (-28%)</td>
<td>37.0 (27%)</td>
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<tr>
<td>QNA</td>
<td>8.9 (-69%)</td>
<td>8.9 (-69%)</td>
<td>36.5 (25%)</td>
<td>21.1 (-28%)</td>
<td>37.0 (27%)</td>
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<tr>
<td>SBD</td>
<td>36.5 (25%)</td>
<td>36.5 (25%)</td>
<td>36.5 (25%)</td>
<td>21.1 (136%)</td>
<td>16.5 (84%)</td>
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</tr>
<tr>
<td>IR</td>
<td>21.1 (-28%)</td>
<td>21.1 (136%)</td>
<td>21.1 (136%)</td>
<td>16.5 (84%)</td>
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<tr>
<td>RQ</td>
<td>37.0 (27%)</td>
<td>37.0 (27%)</td>
<td>37.0 (27%)</td>
<td>16.5 (84%)</td>
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