Correction

Correction note on $L = \lambda W$

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The purpose of this note is to point out and correct a careless error in the proofs of theorems 2.1 and 2.2 in Whitt [6], which also appeared in Glyn and Whitt [1-3]. In particular, as pointed out by S. Stidham, Jr., the first inequality in (2.1) in [6] is not correct unless the service discipline is first-in-first-out (FIFO). Of course, the relation $L = \lambda W$ does not actually depend on the FIFO condition. To obtain what was intended, simply replace $D(k)$ in both (2.1) and the statement of theorem 2.1 of [6] by $D'(k)$, where $D'(k)$ counts the number of $k$ such that $D_j \leq t$ with $D_j = \max(D_j : 1 \leq j \leq k)$ as in (2.6) of [6]. Note that the complication of non-FIFO disciplines is accounted for in (2.6); this modification does the same for (2.1). Indeed, a variant of this modification is used in the more general setting in Glyn and Whitt [4]; see remark 5 and lemma 3 on p. 640 there. Moreover, proper modifications of (2.1) routinely appear elsewhere, such as in Stidham [5] and Wolff [7]. The use of $D'_j$ and $D'(k)$ is a conceptually simple approach.

The indicated modification of (2.1) in [6] is also needed in the proof of theorem 2.2 in [6] (which shows that the condition on $D'(k)$ in the new statement of theorem 2.1 in [6] is actually not needed). This modification works because, first, $A_k \leq D_k \leq D'_k$ and $D'(k) \leq D(k) \leq A(k)$ and, second, $t^{-1}D'(k) \to \lambda$ as $t \to \infty$ if and only if $k^{-1}D'_k \to \lambda^{-1}$ as $k \to \infty$. (We use the fact that $D'_k$ is nondecreasing in $k$ here. In contrast, as noted in theorem 3.8(d) of [1], in general (without FIFO) the limit $k^{-1}D_k \to \lambda^{-1}$ as $k \to \infty$ implies, but is not implied by, the limit $t^{-1}D(k) \to \lambda$ as $t \to \infty$. The failure of the limit $t^{-1}D(k) \to \lambda$ as $t \to \infty$ is the key reason for the asymmetry in theorem 2.2 of [6]; the second sentence of remark 2.2 in [6] confuses the issue.)

Unfortunately, even though this oversight concerning (2.1) in [6] does not appear in [4], it does appear in previous papers. This same error appears in the first inequality in theorem 1a, p. 196, of Glyn and Whitt [1], but not in the remainder term $R(k)$ there; the proof of theorem 20(f) on p. 686 of Glyn and Whitt [2]; and (4.2) in the proof of (1.16) in theorem 3 on p. 704 of Glyn and Whitt [5].
Whitt [3]. Fortunately, the error is easily corrected by the argument above in each case.

The relation among the relevant limits seems to be well summarized by theorem 2 of [1], with the understanding that part (b) should be augmented by the equivalent limit \( t^{-2}D(t) \to 0 \) as \( t \to 0^+ \). However, in [1] the assumption that the limits \( \omega \) and \( q \) be finite should be stated. Indeed, if \( \omega = 0^+ \), then \( \varphi (x) \) does not imply \( \omega \) there, and theorem 1(a) is incorrect. (See remark 2.3 of [6].) Moreover, the last sentence on p. 199 of [1] should read: "The implication (viii) \( \to \) (a) is provided by applying theorem 1(a) plus (b) and (d) above." Finally, \( D^{(1)}(t) \) should appear instead of \( O(t) \) on top of p. 200, as well as in theorem 2(a) of [1].

References