IEOR 3106: Professor Whitt

Lecture Notes, Thursday, September 26, 2006

Bonus-Malus, Credit Risk Models, and Gambler's Ruin

We discussed three Markov chain examples:

1. The Bonus-Malus Insurance Problem

This is Example 4.7 on pages 184-5 in the textbook. This is a slight simplification of a realistic application of Markov chains. We construct the model by constructing the transition matrix, given here on top of p. 185. The elements of the matrix are probabilities. The row sums necessarily are 1. The probability entries a_k here are in this case computed as Poisson probabilities, as indicated on the bottom of page 184. It is natural to use the Poisson distribution to describe the probability a person makes some number of claims. The parameter λ is the mean of that distribution. The parameter λ might well depend on the driver or type of driver. We then use the model to describe the evolving state of the drivers (as a function of λ) and the long-run distribution of premiums, given the premiums charged per state. To appreciate what you can do with the model, you need to learn what you can do to describe performance, after you have built the model. We will be talking about that. You will be reading about that. For example, here you might consider what appropriate premiums in each state should be. This depends on the goals of course, but also on the way the Markov chain evolves.

2. Credit Risk Modelling

I distributed an example of a Markov chain used in bond rating. This example is also posted on line with these lecture notes. The particular Markov chain is an absorbing Markov chain, because the probability of defaulting eventually in the long run is 1. But we could modify the model by introducing new bonds each period.

3. Gambler's Ruin Problem

We talked about making successive 1 dollar bets on red or black in roulette. We talked about the probability of winning \$5 before losing \$10, making \$1 bets each spin of the wheel. This Markov chain is also an absorbing Markov chain. It is discussed by Ross in Section 4.5.1 on page 213. Because the model (transition matrix) really depends on a single parameter p, it is possible to give explicit formulas for key performance probabilities, as shown on pages 214-5. Because of the special numbers 0 and possibly 00 (colored green) on the roulette wheel, the probability of red (or black) is actually 18/37 or 18/38 instead of 18/36, assuming that 18 numbers are red and 18 numbers are black.