

IEOR 4106: Introductions to Operations Research: Stochastic Models

Spring 2006, Professor Whitt

A Markov Chain Example in Credit Risk Modelling

This is not an assignment; you do not need to do anything. This is a concrete example of a Markov chain from finance. Specifically, this come from p.626-627 of Hull's **Options, Futures, and Other Derivatives**, 5th edition.

Background.

Over time, bonds are liable to move from one rating category to another. This is sometimes referred to as *credit ratings migration*. Rating agencies produce from historical data a *ratings transition matrix*. This matrix show the probabilitiy of a bond moving from one rating to another during a certain period of time. Usually the period of time is one year.

Below is a table giving a rating transition matrix produced from historical data by Standard and Poor's (S&P).

Table 1: One-year transition probabilities matrix.

Initial ratings	Ratings at year-end							
	AAA	AA	A	BBB	BB	B	CCC	Default
AAA	0.9366	0.0583	0.0040	0.0009	0.0002	0	0	0
AA	0.0066	0.9172	0.0694	0.0049	0.0006	0.0009	0.0002	0.0002
A	0.0007	0.0225	0.9176	0.0518	0.0049	0.0020	0.0001	0.0004
BBB	0.0003	0.0026	0.0483	0.8924	0.0444	0.0081	0.0016	0.0023
BB	0.0003	0.0006	0.0044	0.0666	0.8323	0.0746	0.0105	0.0107
B	0	0.0010	0.0032	0.0046	0.0572	0.8362	0.0384	0.0594
CCC	0.0015	0	0.0029	0.0088	0.0191	0.1028	0.6123	0.2526
Default	0	0	0	0	0	0	0	1.0000

Source: Standard & Poor's, January 2001

In reality, this transition matrix is updated every year. However if we assume no significant change in the transition matrix in the future, then we can use the transition matrix to predict what will happen over several years in the future. In particular, we can regard the transition matrix as a specification of a Markov chain model.

A Markov Chain Model

Henceforth, regard the table as a specification of a Markov chain model.

- How should the entries of the matrix above be estimated from data?
- Classify the states of ratings (transient, null recurrent, positive recurrent). What subsets of ratings are irreducible closed sets? What ratings are absorbing states?

- What is the probability that a currently AAA rated bond becomes default after 2 years?

Now consider what will happen in the long run, assuming that the transition matrix above operates every year. In the long run,

- what fraction of bonds are in 'AAA'?
- what fraction of bonds are in default?

Do you think these conclusions from the mathematical model are realistic? What are missed in the above modelling?

To answer our own question: Yes, there are some things missing from the model above. One glaring omission is that the model doesn't consider the introduction of newly issued bonds.

So now we consider what happens if we introduce new bonds each year. Please note that this model modification is made up, so it is very artificial. It is not based on real observations. Hopefully it captures the spirit of what might be reasonable.

Suppose that each year new bonds are issued and their ratings are 'AAA', 'AA', 'A', or 'BBB' with equal chance. Suppose that the number of new bonds issued each year is the same as the number of bonds that default in that year. So we now assume that the number of bonds (non-defaulted bonds) is constant over time.

- Under the new modelling assumption, what should be the transition matrix?
- How would you determine what fraction of bonds are in AAA in the long run?
- How would you determine what is the expected fraction of bonds that default each year in the long run?
- How would you determine the expected number of time periods before a 'BBB' rated bond defaults?