REAL-TIME DELAY ESTIMATION IN OVERLOADED MULTISERVER QUEUES WITH ABANDONMENTS SUPPLEMENTARY MATERIAL

by

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Abstract from main paper

We use heavy-traffic limits and computer simulation to study the performance of alternative real-time delay estimators in the overloaded $GI/GI/s+GI$ multiserver queueing model, allowing customer abandonment. These delay estimates may be used to make delay announcements in call centers and related service systems. We characterize performance by the expected mean squared error in steady state. We exploit established approximations for performance measures with a non-exponential abandonment-time distribution to obtain new delay estimators that effectively cope with non-exponential abandonment-time distributions.

Keywords: delay estimation; delay announcements; many-server queues; call centers; simulation; heavy traffic.

June 13, 2008

Contents

1. Introduction

We present additional supporting material in this supplement to the main paper. In §2, we present detailed simulation results for the $GI/M/s+M$ model, with alternative values of the abandonment rate α. We present additional simulation results for non-exponential service-time distributions in §3. In §4, we consider different combinations of service and abandonment-time distributions, to study the effect of low variability in the service and abandonment times. Finally, in §5, we present the relevant tables and figures. Throughout this supplement, we refer to equations as numbered in the main paper.

2. Simulation Results for the $GI/M/s+M$ Model

In this section, we present tables of simulation results (point and 95% confidence interval estimates) quantifying the performance of the alternative delay estimators in the $GI/M/s+M$ model. For the interarrival-time distribution, we consider M (exponential), D (deterministic), and H_2 (hyperexponential with SCV $c_a^2 = 4$ and balanced means). We consider the same values of s as before: $s = 100, 300, 500, 700$, and 1000. We let the service rate be $\mu = 1.0$, and consider three different values of the abandonment rate, $\alpha = 1.0, 5.0$, and 0.2. We vary the arrival rate λ to get a fixed value of ρ for alternative values of s, $\rho = 1.4$. In this model, QL_{ap} coincides with QL_m so we do not include separate results for it. The results of this section correspond to Tables 1 - 54 of §5.

2.1. The $M/M/s+M$ Model with $\alpha = 1.0$

The results in this subsection correspond to Tables 1-6 of §5. With exponential interarrival times, Table 1 shows that, consistent with theory, QL_m is the best possible delay estimator, under the MSE criterion. The QL_r^m and QL_r estimators are nearly identical, with QL_r^m slightly outperforming QL_r. They are both nearly as efficient as QL_m. The RRASE for QL_m ranges from about 14% for $s = 100$ to about 4% when $s = 1000$. We see that the accuracy of this estimator improves as the number of servers increases. Note that all estimators are relatively accurate for this model, with the possible exception of QL. For example, the RRASE of LES ranges from about 22% for $s = 100$ to about 7% for $s = 1000$.

Table 5 shows that, consistent with equation (35), the LES estimator performs worse than QL_m, but not greatly so: The relative error (RE) between the simulation estimates for $\mathrm{ASE(LES)}/\mathrm{ASE}(\mathrm{QL}_m)$ and the numerical value, 2.0, given by (35) is less than 1% throughout. Table 6 shows that, consistent with equation (26), the NI estimator is less efficient than QL_m (and LES): The RE between the simulation estimates for $\mathrm{ASE(NI)}/\mathrm{ASE}(\mathrm{QL}_m)$ and the numerical value, 3.5, given by (26) is less than 1% throughout. The QL estimator performs significantly worse than all the other estimators, particularly for large values of s. The ratio $\mathrm{ASE(QL)}/\mathrm{ASE}(\mathrm{QL}_m)$ ranges from about 3 when $s = 100$ to nearly 16 when $s = 1000$.

Tables 2, 3, and 4 show that the ASE's of QL_m, LES, and NI are consistent with the analytical formulas for the expected MSE's given in (22), (33), and (25), respectively. These formulas are remarkably accurate: The RE's reported are less than 2% throughout.

2.2. The $M/M/s + M$ Model with $\alpha = 5.0$ and $\alpha = 0.2$

The results in this subsection correspond to Tables 7-18 of §5. We now consider different abandonment rates; specifically we let $\alpha = 5.0$ and $\alpha = 0.2$. As indicated by formulas (3) and (7), the queue length and delay tend to be inversely proportional to α. Thus, changing α from 1.0 to 5.0 or 0.2 tends to change congestion by a factor of 5. The system is very heavily loaded when $\alpha = 0.2$, but relatively lightly loaded when $\alpha = 5.0$.

Table 7 compares the efficiencies of the alternative estimators with $\alpha = 5.0$, which makes the model more lightly loaded. In this more lightly loaded setting, the ASE's of all the estimators are relatively low, being smaller than for the $M/M/s + M$ model with $\alpha = 1.0$, in Table 1, by a factor of about 5.

However, the lighter loading makes the ED approximations less appropriate. Tables 8, 9, and 10 compare the ASE's of QL_m, LES, and NI to the expected MSE's in formulas (22), (33), and (25), respectively; the RE's reported are higher than with $\alpha = 1.0$, especially when the number of servers is small (e.g., in Table 9, with $s = 100$, the RE reported exceeds 20%). But, the formulas are much more accurate with a larger number of servers (e.g., the RE's are close to 1 or 2%, with $s = 1000$).

Tables 11 and 12 compare the efficiencies of LES, QL_m, and NI. Table 11 shows that the ratio $\mathrm{ASE(LES)}/\mathrm{ASE}(\mathrm{QL}_m)$ is well approximated by the numerical value, 2.0, predicted by equation (35), except when the number of servers is small (e.g., with $s = 100$, RE $\approx$ 10%). Similarly, Table 12 shows that the ratio $\mathrm{ASE(NI)}/\mathrm{ASE}(\mathrm{QL}_m)$ is well approximated by the numerical value, 3.5, given by (26), except when s is small: The RE reported when $s = 100$ is close to 20%.

Table 13 compares the efficiencies of the alternative estimators with $\alpha = 0.2$. In this more heavily loaded setting, the ASE's of the alternative estimators are higher than with $\alpha = 1.0$, by a factor of about 5, especially when the number of servers is large.

Tables 14, 15, and 16 compare the ASE's of QL_m, LES, and NI to the expected MSE's in formulas (22), (33), and (25), respectively. These formulas are remarkably accurate: The RE's reported are less than 2% throughout (except with $s = 1000$ in Table 15, where the RE $\approx$ 5%, which is likely due to statistical error). Tables 17 and 18 compare the efficiencies of LES, QL_m, and NI. The ratios ASE(LES)/ASE(QL_m) and ASE(NI)/ASE(QL_m) agree closely with the values predicted by formulas (35) and (26): The RE's reported are less than 5% throughout.

2.3. The $D/M/s + M$ Model with $\alpha = 1.0$

The results in this subsection correspond to Tables 19-24 of §5. Table 19 compares the efficiencies of the estimators in the $D/M/s + M$ model with $\alpha = 1.0$. Consistent with theory, QL_m is the best possible delay estimator, under the MSE criterion. The RRASE of QL_m ranges from about 16% when $s = 100$ to about 5% when $s = 1000$. All estimators are relatively accurate as well; e.g., the RRASE of LES ranges from about 24% when $s = 100$ to about 6% when $s = 1000$. The QL_r estimator is nearly as efficient as QL_m.

Table 23 shows that, consistent with equation (36), the LES estimator performs slightly worse than QL_m: The RE between the simulation estimates for ASE(LES)/ASE(QL_m) and the numerical value, 1.286, given by (36) is less than 3% throughout. With a deterministic arrival process, the LES estimator performs better, compared to QL_m, than with a Poisson arrival process. Similarly, Table 24 shows that, consistent with equation (26), the NI estimator is less efficient than QL_m: The RE between the simulation estimates for ASE(NI)/ASE(QL_m) and the numerical value, 2.25, given by (26) is less than 4% throughout. The QL estimator is, once more, the least efficient estimator: The ratio ASE(QL)/ASE(QL_m) ranges from about 3 when $s = 100$ to about 15 when $s = 1000$.

Tables 20, 21, and 22 show that the ASE's of QL_m, LES, and NI are consistent with the analytical formulas for the expected MSE's given in (22), (33), and (25), respectively. These formulas are quite accurate: The RE's reported are less than 3% throughout, except when the number of servers is large (e.g., with $s = 1000$ in Table 21, RE $\approx$ 7%).

The observations made above for the $M/M/s + M$ model with $\alpha = 5.0$ and $\alpha = 0.2$ still apply, essentially, to the $D/M/s + M$ model (and the $H_2/M/s + M$ model) with these values of α, so we will not treat these cases separately. Tables 25-30 and 31-36 treat the $D/M/s + M$ model with $\alpha = 5.0$ and $\alpha = 0.2$, respectively.

2.4. The $H_2/M/s+M$ Model with $\alpha = 1.0$

The results in this subsection correspond to Tables 37-42 of §5. With hyperexponential interarrival times, Table 37 shows that, consistent with theory, QL_m is the best possible delay estimator, under the MSE criterion. The RRASE for QL_m ranges from about 16% for $s = 100$ to about 5% when $s = 1000$. The QL_r estimator is only slightly outperformed by QL_m.

The ED approximations are less accurate with highly variable interarrival times than with exponential interarrival times. Table 41 shows that, consistent with equation (37), the LES estimator performs worse than QL_m: The RE between the simulation estimates for $\mathrm{ASE(LES)}/\mathrm{ASE}(\mathrm{QL}_m)$ and the numerical value, 4.143, given by (37) ranges from about 6% when $s = 100$ to about 2% when $s = 1000$. The LES estimator performs worse, compared to QL_m, with hyperexponential interarrival times, than with exponential interarrival times. Table 42 shows that, consistent with equation (26), the NI estimator is significantly less efficient than QL_m (and LES): The RE between the simulation estimates for $\mathrm{ASE(NI)}/\mathrm{ASE}(\mathrm{QL}_m)$ and the numerical value, 7.25, given by (26) ranges from about -9.0% when $s = 100$ to about -0.5% when $s = 1000$. The QL estimator performs significantly worse than all the other estimators, particularly for large values of s. The ratio $\mathrm{ASE(QL)}/\mathrm{ASE}(\mathrm{QL}_m)$ ranges from about 4 when $s = 100$ to nearly 16 when $s = 1000$.

Tables 38, 39, and 40 show that the ASE's of QL_m, LES, and NI are consistent with the analytical formulas for the expected MSE's given in (22), (33), and (25), respectively. These formulas are accurate for large values of s, but less so for smaller values of s: e.g., in Table 40, RE $\approx -8\%$ when $s = 100$ and RE $\approx -0.033\%$ when $s = 1000$. Tables 43-48 and 49-54 treat the $H_2/M/s+M$ model with $\alpha = 5.0$ and $\alpha = 0.2$, respectively.

3. Simulation Results for the $M/GI/s+M$ Model

In this section, we present simulation results quantifying the performance of the alternative delay estimators with non-exponential service-time distributions; i.e., we consider the $M/GI/s+M$ model. In this model, QL_{ap} coincides with QL_m so we do not include separate results for it. For the service-time distribution, we consider H_2, D, E_{10}, E_4 and E_2 (Erlang, sum of 10, 4, and 2 exponentials, respectively). The results of this section correspond to Tables 55 - 59 of §5.

3.1. The $M/H_2/s+M$ model

Table 55 shows that, with high variability in the service times, the results we get are similar to those we get in the more general cases. The QL_m estimator is the most efficient estimator for this model. The RRASE of QL_m ranges from about 17% when $s = 100$ to about 5% when $s = 1000$. The QL_r estimator is only very slightly outperformed by QL_m (the ratio $\mathrm{ASE}(\mathrm{QL}_m)/\mathrm{ASE}(\mathrm{QL}_r)$ is very close to 1 for all values of s). The LES estimator is relatively accurate as well: The ratio $\mathrm{ASE(LES)}/\mathrm{ASE}(\mathrm{QL}_m)$ is close to 1.8 for all values of s, suggesting possible extensions for our analytical results for the $GI/M/s+M$ model to the $M/GI/s+M$ model. The NI estimator is outperformed by QL_m, QL_r, and LES: The ratio $\mathrm{ASE(NI)}/\mathrm{ASE}(\mathrm{QL}_m)$ is close to 4 for all values of s considered.

3.2. The $M/D/s+M$ model

Table 56 shows that all delay estimators do not perform well in this model. The NI estimator, which uses no information at all beyond the model, is the most effective delay estimator, when $s \geq 300$. (For $s = 100$, QL_m slightly outperforms NI.) But even the NI estimator is not very accurate: The RRASE for NI is roughly equal to 25% for all values of s considered. This suggests that our procedures for estimating delays perform relatively poorly when the service times are deterministic. The ASE's for QL_m, QL_r, QL, and LES do not vary much in this model; e.g., $\mathrm{ASE}(\mathrm{QL}_m)$ varies little about 0.01, for all values of s considered. Alternative delay estimation procedures, appropriate for deterministic service times, remain to be investigated.

3.3. Erlang service times

Tables 57, 58, and 59 show that the proposed delay estimators remain effective, even with very low variability in the service times. The QL_m estimator is the most effective delay estimator for the $M/GI/s+M$ model with E_{10}, E_4, or E_2 service times. The QL_r estimator is nearly identical to QL_m, particularly when s is large enough ($s \geq 300$). Once more, the relative accuracy of the delay estimators improves as s increases. The LES estimator is relatively accurate as well. The NI estimator does not perform as well as LES, nor as bad as QL. The QL estimator is the least efficient estimator. Consistent with §4 of the main paper, all estimators, except QL, have an ASE which is inversely proportional to the number of servers, but mathematical support for the estimators has yet to be provided with non-exponential service-time distributions.

4. Simulation Results for the $M/GI/s + GI$ Model

In this section, we present simulation results quantifying the performance of the alternative estimators in the $M/GI/s+GI$ model, i.e., we consider different combinations of service-time and abandon-time distributions. We do not consider D abandonment times because our QL_{ap} estimator requires a density. Constant service times cause a problem in all cases, but otherwise the estimators perform well. The results of this section correspond to Tables 60 and 61, and Figures 1 - 4.

4.1. The $M/D/s + E_{10}$ model

Figures 1 and 2, and Table 60 show that we get slightly better results with deterministic service times and low-variability abandonment times (Erlang with SCV = 0.1), than those we get with the $M/D/s+M$ model. The LES estimator is the most efficient estimator when the number of servers s is large enough ($s \geq 500$). The RRASE for LES ranges from about 13% when $s = 100$ to about 9% when $s = 1000$, so we see a slight improvement in performance as s increases. The NI estimator is competitive as well, and is the second most efficient estimator when $s \geq 500$. The QL_{ap} estimator is the most efficient estimator when $s \leq 300$, but not otherwise. The QL_m estimator performs poorly, but not as bad as QL which is the least efficient estimator. Figure 2 shows that the products $s\times$ ASE increase nearly linearly with s, for all delay estimators.

4.2. The $M/E_{10}/s + E_{10}$ model

Figures 3 and 4 and Table 61 show that the proposed delay estimators remain effective, with very low variability in the service times, even if combined with low-variability abandonment times. The QL_{ap} estimator is the most effective delay estimator for the $M/E_{10}/s+E_{10}$ model. The NI estimator is competitive as well, and is the second most effective estimator in this model. The LES estimator is relatively accurate as well. The NI estimator does not perform as well as LES, nor as bad as QL. The QL estimator is the least efficient estimator. Figure 4 shows that, except for QL_m, the relative accuracy of the delay estimators improves as s increases. Indeed, the products $s\times$ ASE are nearly constant for all estimators, except QL_m, but mathematical support for the estimators has yet to be provided with non-exponential service-time distributions.

5. Tables and Figures

Efficiency of the estimators in the $M/M/s+M$ model with $\rho = 1.4$ and $\alpha = 1.0$

s	ASE[θ_{QL_m}]	ASE[$\theta_{QL_r^m}$]	ASE[θ_{QL_r}]	ASE[θ_{QL}]	ASE[θ_{LES}]	ASE[θ_{NI}]
100	2.867×10^{-3}	2.869×10^{-3}	3.130×10^{-3}	8.693×10^{-3}	5.772×10^{-3}	1.00×10^{-2}
	$\pm 1.76 \times 10^{-5}$	$\pm 1.78 \times 10^{-5}$	$\pm 1.89 \times 10^{-5}$	$\pm 3.20 \times 10^{-5}$	$\pm 2.79 \times 10^{-5}$	$\pm 5.97 \times 10^{-5}$
300	9.587×10^{-4}	9.601×10^{-4}	1.039×10^{-3}	5.602×10^{-3}	1.922×10^{-3}	3.351×10^{-3}
	$\pm 6.86 \times 10^{-6}$	6.92×10^{-6}	$\pm 6.41 \times 10^{-6}$	$\pm 2.64 \times 10^{-5}$	$\pm 1.50 \times 10^{-5}$	$\pm 6.03 \times 10^{-5}$
500	5.761×10^{-4}	5.661×10^{-4}	6.224×10^{-4}	5.017×10^{-3}	1.153×10^{-3}	2.038×10^{-3}
	$\pm 1.94 \times 10^{-6}$	$\pm 3.86 \times 10^{-6}$	$\pm 2.94 \times 10^{-6}$	$\pm 2.41 \times 10^{-5}$	$\pm 9.99 \times 10^{-6}$	$\pm 2.26 \times 10^{-5}$
700	4.104×10^{-4}	4.201×10^{-4}	4.440×10^{-4}	4.682×10^{-3}	8.166×10^{-4}	1.441×10^{-3}
	$\pm 1.82 \times 10^{-6}$	2.839×10^{-4}	$\pm 2.71 \times 10^{-6}$	$\pm 2.40 \times 10^{-5}$	$\pm 5.78 \times 10^{-6}$	$\pm 1.57 \times 10^{-5}$
1000	2.892×10^{-4}	2.839×10^{-4}	3.136×10^{-4}	4.492×10^{-3}	5.752×10^{-4}	1.019×10^{-3}
	$\pm 3.48 \times 10^{-6}$	$\pm 3.86 \times 10^{-6}$	$\pm 3.09 \times 10^{-6}$	$\pm 1.54 \times 10^{-5}$	$\pm 6.91 \times 10^{-6}$	$\pm 3.00 \times 10^{-5}$

Table 1: Point and confidence interval estimates of the ASEs - average square errors - of the estimators

Efficiency of QL$_m$ in the $M/M/s+M$ model with $\rho = 1.4$ and $\alpha = 1.0$

s	ASE[θ_{QL_m}]	Predicted by (22)	RE (%)
100	2.867×10^{-3}	2.857×10^{-3}	0.362
	$\pm 1.76 \times 10^{-5}$		
300	9.587×10^{-4}	9.524×10^{-4}	0.666
	$\pm 6.86 \times 10^{-6}$		
500	5.761×10^{-4}	5.714×10^{-4}	0.819
	$\pm 1.94 \times 10^{-6}$		
700	4.104×10^{-4}	4.082×10^{-4}	0.546
	$\pm 1.82 \times 10^{-6}$		
1000	2.892×10^{-4}	2.857×10^{-4}	1.21
	$\pm 3.48 \times 10^{-6}$		

Table 2:

Efficiency of LES in the $M/M/s+M$ model with $\rho = 1.4$ and $\alpha = 1.0$

s	ASE$[\theta_{LES}]$	Predicted by (33)	RE (%)
100	5.772×10^{-3} $\pm 2.79 \times 10^{-5}$	5.714×10^{-3}	1.03
300	1.922×10^{-3} $\pm 1.50 \times 10^{-5}$	1.905×10^{-3}	0.905
500	1.153×10^{-3} $\pm 9.99 \times 10^{-6}$	1.143×10^{-3}	0.858
700	8.166×10^{-4} $\pm 5.78 \times 10^{-6}$	8.163×10^{-4}	0.0372
1000	5.752×10^{-4} $\pm 6.91 \times 10^{-6}$	5.714×10^{-4}	0.660

Table 3:

Efficiency of NI in the $M/M/s+M$ model with $\rho = 1.4$ and $\alpha = 1.0$

s	ASE$[\theta_{NI}]$	Predicted by (25)	RE (%)
100	1.00×10^{-2} $\pm 5.97 \times 10^{-5}$	0.01	0.0293
300	3.351×10^{-3} $\pm 6.03 \times 10^{-5}$	3.333×10^{-3}	0.523
500	2.038×10^{-3} $\pm 2.26 \times 10^{-5}$	2.0×10^{-3}	1.89
700	1.441×10^{-3} $\pm 1.57 \times 10^{-5}$	1.429×10^{-3}	1.02
1000	1.019×10^{-3} $\pm 3.00 \times 10^{-5}$	1.0×10^{-3}	0.480

Table 4:

Comparison of the efficiency of LES and QL_m in the $M/M/s+M$ model with $\rho = 1.4$ and $\alpha = 1.0$

s	$ASE[\theta_{QL_m}]$	$ASE[\theta_{LES}]$	$ASE[\theta_{LES}]/ASE[\theta_{QL_m}]$	Predicted ratio by (35)	RE (%)
100	2.867×10^{-3} $\pm 1.76 \times 10^{-5}$	5.772×10^{-3} $\pm 2.79 \times 10^{-5}$	2.013	2	0.661
300	9.587×10^{-4} $\pm 6.86 \times 10^{-6}$	1.922×10^{-3} $\pm 1.50 \times 10^{-5}$	2.005	2	0.238
500	5.761×10^{-4} $\pm 3.82 \times 10^{-6}$	1.153×10^{-3} $\pm 9.99 \times 10^{-6}$	2.001	2	0.0382
700	4.104×10^{-4} $\pm 1.82 \times 10^{-6}$	8.166×10^{-4} $\pm 5.78 \times 10^{-6}$	1.990	2	-0.506
1000	2.892×10^{-4} $\pm 3.85 \times 10^{-6}$	5.752×10^{-4} $\pm 6.91 \times 10^{-6}$	1.989	2	-0.543

Table 5:

Comparison of the efficiency of NI and QL_m in the $M/M/s+M$ model with $\rho = 1.4$ and $\alpha = 1.0$

s	$ASE[\theta_{QL_m}]$	$ASE[\theta_{NI}]$	$ASE[\theta_{NI}]/ASE[\theta_{QL_m}]$	Predicted ratio by (26)	RE (%)
100	2.867×10^{-3} $\pm 1.76 \times 10^{-5}$	1.00×10^{-2} $\pm 5.97 \times 10^{-5}$	3.488	3.5	-0.332
300	9.587×10^{-4} $\pm 6.86 \times 10^{-6}$	3.351×10^{-3} $\pm 6.03 \times 10^{-5}$	3.495	3.5	-0.142
500	5.761×10^{-4} $\pm 3.82 \times 10^{-6}$	2.038×10^{-3} $\pm 2.26 \times 10^{-5}$	3.537	3.5	1.06
700	4.104×10^{-4} $\pm 1.82 \times 10^{-6}$	1.441×10^{-3} $\pm 1.57 \times 10^{-5}$	3.516	3.5	0.471
1000	2.892×10^{-4} $\pm 3.85 \times 10^{-6}$	1.019×10^{-3} $\pm 3.00 \times 10^{-5}$	3.475	3.5	-0.720

Table 6:

Efficiency of the estimators in the $M/M/s+M$ model with $\rho = 1.4$ and $\alpha = 5.0$

s	ASE[θ_{QL_m}]	ASE[θ_{QL_r}]	ASE[θ_{QL}]	ASE[θ_{LES}]	ASE[θ_{NI}]
100	6.318×10^{-4}	7.172×10^{-4}	1.226×10^{-3}	1.391×10^{-3}	1.809×10^{-3}
	$\pm 1.53 \times 10^{-6}$	$\pm 2.42 \times 10^{-6}$	$\pm 5.06 \times 10^{-6}$	$\pm 3.09 \times 10^{-6}$	$\pm 7.22 \times 10^{-6}$
300	1.935×10^{-4}	2.130×10^{-4}	4.813×10^{-4}	4.035×10^{-4}	6.591×10^{-4}
	$\pm 6.54 \times 10^{-7}$	$\pm 8.69 \times 10^{-7}$	$\pm 1.86 \times 10^{-6}$	$\pm 1.15 \times 10^{-6}$	$\pm 3.06 \times 10^{-6}$
500	1.151×10^{-4}	1.253×10^{-4}	3.467×10^{-4}	2.361×10^{-4}	4.009×10^{-4}
	$\pm 5.41 \times 10^{-7}$	$\pm 4.54 \times 10^{-7}$	$\pm 8.45 \times 10^{-7}$	$\pm 8.67 \times 10^{-7}$	$\pm 3.05 \times 10^{-6}$
700	8.235×10^{-5}	8.965×10^{-5}	2.963×10^{-4}	1.675×10^{-4}	2.872×10^{-4}
	$\pm 4.04 \times 10^{-7}$	$\pm 3.51 \times 10^{-7}$	$\pm 9.01 \times 10^{-7}$	$\pm 8.21 \times 10^{-7}$	$\pm 2.58 \times 10^{-6}$
1000	5.772×10^{-5}	6.261×10^{-5}	2.555×10^{-4}	1.167×10^{-4}	2.022×10^{-4}
	$\pm 2.33 \times 10^{-7}$	$\pm 2.66 \times 10^{-7}$	$\pm 5.44 \times 10^{-7}$	$\pm 6.87 \times 10^{-7}$	$\pm 2.15 \times 10^{-6}$

Table 7: Point and confidence interval estimates of the ASEs - average square errors - of the estimators

Efficiency of $\mathbf{QL}_m$ in the $M/M/s+M$ model with $\rho = 1.4$ and $\alpha = 5.0$

s	ASE[θ_{QL_m}]	Predicted by (22)	RE (%)
100	6.318×10^{-4}	5.714×10^{-4}	10.6
	$\pm 1.53 \times 10^{-6}$		
300	1.935×10^{-4}	1.905×10^{-4}	1.60
	$\pm 6.54 \times 10^{-7}$		
500	1.151×10^{-4}	1.143×10^{-4}	0.730
	$\pm 5.41 \times 10^{-7}$		
700	8.235×10^{-5}	8.163×10^{-5}	0.879
	$\pm 4.04 \times 10^{-7}$		
1000	5.772×10^{-5}	5.714×10^{-5}	1.01
	$\pm 2.33 \times 10^{-7}$		

Table 8:

Efficiency of LES in the $M/M/s+M$ model with $\rho = 1.4$ and $\alpha = 5.0$

s	ASE[θ_{LES}]	Predicted by (33)	RE (%)
100	1.391×10^{-3} $\pm 3.09 \times 10^{-6}$	1.143×10^{-3}	21.7
300	4.035×10^{-4} $\pm 1.15 \times 10^{-6}$	3.810×10^{-4}	5.91
500	2.361×10^{-4} $\pm 8.67 \times 10^{-7}$	2.286×10^{-4}	3.30
700	1.675×10^{-4} $\pm 8.21 \times 10^{-7}$	1.633×10^{-4}	2.61
1000	1.167×10^{-4} $\pm 6.87 \times 10^{-7}$	1.143×10^{-4}	2.10

Table 9:

Efficiency of NI in the $M/M/s+M$ model with $\rho = 1.4$ and $\alpha = 5.0$

s	ASE[θ_{NI}]	Predicted by (25)	RE (%)
100	1.809×10^{-3} $\pm 7.22 \times 10^{-6}$	2.000×10^{-3}	-9.56
300	6.591×10^{-4} $\pm 3.06 \times 10^{-6}$	6.667×10^{-4}	-1.10
500	4.009×10^{-4} $\pm 3.05 \times 10^{-6}$	4.000×10^{-4}	0.235
700	2.872×10^{-4} $\pm 2.58 \times 10^{-6}$	2.857×10^{-4}	0.513
1000	2.022×10^{-4} $\pm 2.15 \times 10^{-6}$	2.000×10^{-4}	1.12

Table 10:

Comparison of the efficiency of LES and QL$_m$ in the $M/M/s+M$ model with $\rho = 1.4$ and $\alpha = 5.0$

s	ASE[θ_{QL_m}]	ASE[θ_{LES}]	ASE[θ_{LES}]/ASE[θ_{QL_m}]	Predicted ratio by (35)	RE (%)
100	6.318×10^{-4}	1.391×10^{-3}	2.202	2.0	10.1
	$\pm 1.53 \times 10^{-6}$	$\pm 3.09 \times 10^{-6}$			
300	1.935×10^{-4}	4.035×10^{-4}	2.085	2.0	4.25
	$\pm 6.54 \times 10^{-7}$	$\pm 1.15 \times 10^{-6}$			
500	1.151×10^{-4}	2.361×10^{-4}	2.051	2.0	2.55
	$\pm 5.41 \times 10^{-7}$	$\pm 8.67 \times 10^{-7}$			
700	8.235×10^{-5}	1.675×10^{-4}	2.034	2.0	1.7
	$\pm 4.04 \times 10^{-7}$	$\pm 8.21 \times 10^{-7}$			
1000	5.772×10^{-5}	1.167×10^{-4}	2.022	2.0	1.1
	$\pm 2.33 \times 10^{-7}$	$\pm 6.87 \times 10^{-7}$			

Table 11:

Comparison of the efficiency of NI and QL$_m$ in the $M/M/s+M$ model with $\rho = 1.4$ and $\alpha = 5.0$

s	ASE[θ_{QL_m}]	ASE[θ_{NI}]	ASE[θ_{NI}]/ASE[θ_{QL_m}]	Predicted ratio by (26)	RE (%)
100	6.318×10^{-4}	1.809×10^{-3}	2.863	3.5	-18.2
	$\pm 1.53 \times 10^{-6}$	$\pm 7.22 \times 10^{-6}$			
300	1.935×10^{-4}	6.591×10^{-4}	3.406	3.5	-2.69
	$\pm 6.54 \times 10^{-7}$	$\pm 3.06 \times 10^{-6}$			
500	1.151×10^{-4}	4.009×10^{-4}	3.483	3.5	-0.494
	$\pm 5.41 \times 10^{-7}$	$\pm 3.05 \times 10^{-6}$			
700	8.235×10^{-5}	2.872×10^{-4}	3.487	3.5	-0.363
	$\pm 4.04 \times 10^{-7}$	$\pm 2.58 \times 10^{-6}$			
1000	5.772×10^{-5}	2.022×10^{-4}	3.504	3.5	0.109
	$\pm 2.33 \times 10^{-7}$	$\pm 2.15 \times 10^{-6}$			

Table 12:

Efficiency of the estimators in the $M/M/s+M$ model with $\rho = 1.4$ and $\alpha = 0.2$

s	ASE[θ_{QL_m}]	ASE[θ_{QL_r}]	ASE[θ_{QL}]	ASE[θ_{LES}]	ASE[θ_{NI}]
100	1.425×10^{-2} $\pm 1.15 \times 10^{-4}$	1.545×10^{-2} $\pm 1.22 \times 10^{-4}$	1.238×10^{-1} $\pm 5.16 \times 10^{-4}$	2.894×10^{-2} $\pm 3.52 \times 10^{-4}$	4.963×10^{-2} $\pm 6.31 \times 10^{-4}$
300	4.705×10^{-3} $\pm 5.33 \times 10^{-5}$	5.099×10^{-3} $\pm 5.95 \times 10^{-5}$	1.094×10^{-1} $\pm 5.04 \times 10^{-4}$	9.573×10^{-3} $\pm 1.20 \times 10^{-4}$	1.657×10^{-2} $\pm 4.98 \times 10^{-4}$
500	2.879×10^{-3} $\pm 4.27 \times 10^{-5}$	3.103×10^{-3} $\pm 3.70 \times 10^{-5}$	1.046×10^{-1} $\pm 4.19 \times 10^{-4}$	5.832×10^{-3} $\pm 7.88 \times 10^{-5}$	9.926×10^{-3} $\pm 5.18 \times 10^{-4}$
700	2.029×10^{-3} $\pm 2.62 \times 10^{-5}$	2.194×10^{-3} $\pm 3.34 \times 10^{-5}$	1.0479×10^{-1} $\pm 5.54 \times 10^{-4}$	4.150×10^{-3} $\pm 1.09 \times 10^{-4}$	7.121×10^{-3} $\pm 2.25 \times 10^{-4}$
1000	1.444×10^{-3} $\pm 4.43 \times 10^{-5}$	1.558×10^{-3} $\pm 4.35 \times 10^{-5}$	1.031×10^{-1} $\pm 3.47 \times 10^{-4}$	2.995×10^{-3} $\pm 6.03 \times 10^{-5}$	4.935×10^{-3} $\pm 3.74 \times 10^{-4}$

Table 13: Point and confidence interval estimates of the ASEs - average square errors - of the estimators

Efficiency of QL$_m$ in the $M/M/s+M$ model with $\rho = 1.4$ and $\alpha = 0.2$

s	ASE[θ_{QL_m}]	Predicted by (22)	RE (%)
100	1.425×10^{-2} $\pm 1.15 \times 10^{-4}$	1.429×10^{-2}	-0.251
300	4.705×10^{-3} $\pm 5.33 \times 10^{-5}$	4.762×10^{-3}	-1.194
500	2.879×10^{-3} $\pm 4.27 \times 10^{-5}$	2.857×10^{-3}	0.762
700	2.029×10^{-3} $\pm 2.62 \times 10^{-5}$	2.041×10^{-3}	-0.582
1000	1.444×10^{-3} $\pm 4.43 \times 10^{-5}$	1.426×10^{-3}	1.05

Table 14:

Efficiency of LES in the $M/M/s+M$ model with $\rho = 1.4$ and $\alpha = 0.2$

s	ASE[θ_{LES}]	Predicted by (33)	RE (%)
100	2.894×10^{-2} $\pm 3.52 \times 10^{-4}$	2.857×10^{-2}	1.29
300	9.573×10^{-3} $\pm 1.20 \times 10^{-4}$	9.524×10^{-3}	0.514
500	5.832×10^{-3} $\pm 7.88 \times 10^{-5}$	5.714×10^{-3}	2.06
700	4.150×10^{-3} $\pm 1.09 \times 10^{-4}$	4.082×10^{-3}	1.69
1000	2.995×10^{-3} $\pm 6.03 \times 10^{-5}$	2.857×10^{-3}	4.81

Table 15:

Efficiency of NI in the $M/M/s+M$ model with $\rho = 1.4$ and $\alpha = 0.2$

s	ASE[θ_{NI}]	Predicted by (25)	RE (%)
100	4.963×10^{-2} $\pm 6.31 \times 10^{-4}$	5.0×10^{-2}	-0.740
300	1.657×10^{-2} $\pm 4.98 \times 10^{-4}$	1.667×10^{-2}	-0.596
500	9.926×10^{-3} $\pm 5.18 \times 10^{-4}$	1.0×10^{-2}	-0.745
700	7.121×10^{-3} $\pm 2.25 \times 10^{-4}$	7.143×10^{-3}	-0.308
1000	4.935×10^{-3} $\pm 3.74 \times 10^{-4}$	5.0×10^{-3}	-1.30

Table 16:

Comparison of the efficiency of LES and QL$_m$ in the $M/M/s+M$ model with $\rho = 1.4$ and $\alpha = 0.2$

s	ASE$[\theta_{QL_m}]$	ASE$[\theta_{LES}]$	ASE$[\theta_{LES}]$/ASE$[\theta_{QL_m}]$	Predicted by (35)	RE (%)
100	1.425×10^{-2} $\pm 1.15 \times 10^{-4}$	2.894×10^{-2} $\pm 3.52 \times 10^{-4}$	2.031	2.0	1.55
300	4.705×10^{-3} $\pm 5.33 \times 10^{-5}$	9.573×10^{-3} $\pm 1.20 \times 10^{-4}$	2.035	2.0	1.72
500	2.879×10^{-3} $\pm 4.27 \times 10^{-5}$	5.832×10^{-3} $\pm 7.88 \times 10^{-5}$	2.026	2.0	1.29
700	2.029×10^{-3} $\pm 2.62 \times 10^{-5}$	4.150×10^{-3} $\pm 1.09 \times 10^{-4}$	2.046	2.0	2.28
1000	1.444×10^{-3} $\pm 4.43 \times 10^{-5}$	2.857×10^{-3} $\pm 6.03 \times 10^{-5}$	2.075	2.0	3.72

Table 17:

Comparison of the efficiency of NI and QL$_m$ in the $M/M/s+M$ model with $\rho = 1.4$ and $\alpha = 0.2$

s	ASE$[\theta_{QL_m}]$	ASE$[\theta_{NI}]$	ASE$[\theta_{NI}]$/ASE$[\theta_{QL_m}]$	Predicted by (26)	RE (%)
100	1.425×10^{-2}	4.963×10^{-2}	3.483	3.5	-0.489
	$\pm 1.15 \times 10^{-4}$	4.963×10^{-2}			
300	4.705×10^{-3}	1.657×10^{-2}	3.521	3.5	0.605
	$\pm 5.33 \times 10^{-5}$	$\pm 4.98 \times 10^{-4}$			
500	2.879×10^{-3}	9.926×10^{-3}	3.448	3.5	-1.50
	$\pm 4.27 \times 10^{-5}$	$\pm 5.18 \times 10^{-4}$			
700	2.029×10^{-3}	7.121×10^{-3}	3.510	3.5	0.275
	$\pm 2.62 \times 10^{-5}$	$\pm 2.25 \times 10^{-4}$			
1000	1.444×10^{-3}	4.935×10^{-3}	3.419	3.5	-2.32
	$\pm 4.43 \times 10^{-5}$	$\pm 3.74 \times 10^{-4}$			

Table 18:

Efficiency of the estimators in the $D/M/s+M$ model with $\rho = 1.4$ and $\alpha = 1.0$

s	ASE$[\theta_{QL_m}]$	ASE$[\theta_{QL_r}]$	ASE$[\theta_{QL}]$	ASE$[\theta_{LES}]$	ASE$[\theta_{NI}]$
100	2.882×10^{-3}	2.994×10^{-3}	7.705×10^{-3}	6.545×10^{-3}	6.496×10^{-3}
	$\pm 7.89 \times 10^{-6}$	$\pm 8.28 \times 10^{-6}$	$\pm 1.22 \times 10^{-5}$	$\pm 1.12 \times 10^{-5}$	$\pm 3.60 \times 10^{-5}$
300	9.520×10^{-4}	9.903×10^{-4}	5.256×10^{-3}	1.243×10^{-3}	2.188×10^{-3}
	$\pm 4.42 \times 10^{-6}$	$\pm 4.73 \times 10^{-6}$	$\pm 8.05 \times 10^{-6}$	$\pm 5.70 \times 10^{-6}$	$\pm 2.50 \times 10^{-5}$
500	5.753×10^{-4}	5.989×10^{-4}	4.774×10^{-3}	7.537×10^{-4}	1.297×10^{-3}
	$\pm 3.51 \times 10^{-6}$	$\pm 3.87 \times 10^{-6}$	$\pm 5.70 \times 10^{-6}$	$\pm 5.44 \times 10^{-6}$	$\pm 1.91 \times 10^{-5}$
700	4.096×10^{-4}	4.260×10^{-4}	4.548×10^{-3}	9.149×10^{-4}	9.537×10^{-4}
	$\pm 3.18 \times 10^{-6}$	$\pm 3.52 \times 10^{-6}$	$\pm 8.71 \times 10^{-6}$	$\pm 4.42 \times 10^{-6}$	$\pm 1.68 \times 10^{-5}$
1000	2.871×10^{-4}	2.979×10^{-4}	4.392×10^{-3}	3.912×10^{-4}	6.697×10^{-4}
	$\pm 3.66 \times 10^{-6}$	$\pm 3.23 \times 10^{-6}$	$\pm 5.34 \times 10^{-6}$	$\pm 5.17 \times 10^{-6}$	$\pm 2.01 \times 10^{-5}$

Table 19: Point and confidence interval estimates of the ASEs - average square errors - of the estimators

Efficiency of QL$_m$ in the $D/M/s+M$ model with $\rho = 1.4$ and $\alpha = 1.0$

s	ASE$[\theta_{QL_m}]$	Predicted by (22)	RE (%)
100	2.882×10^{-3} $\pm 7.89 \times 10^{-6}$	2.857×10^{-3}	0.856
300	9.520×10^{-4} $\pm 4.42 \times 10^{-6}$	9.524×10^{-4}	-0.0369
500	5.753×10^{-4} $\pm 3.51 \times 10^{-6}$	5.714×10^{-4}	0.669
700	4.096×10^{-4} $\pm 3.18 \times 10^{-6}$	4.082×10^{-4}	0.357
1000	2.871×10^{-4} $\pm 3.66 \times 10^{-6}$	2.857×10^{-4}	0.489

Table 20:

Efficiency of LES in the $D/M/s+M$ model with $\rho = 1.4$ and $\alpha = 1.0$

s	ASE$[\theta_{LES}]$	Predicted by (33)	RE (%)
100	3.827×10^{-3} $\pm 1.12 \times 10^{-5}$	3.673×10^{-3}	4.19
300	1.243×10^{-3} $\pm 5.70 \times 10^{-6}$	1.224×10^{-3}	1.52
500	7.537×10^{-4} $\pm 5.44 \times 10^{-6}$	7.347×10^{-4}	2.59
700	5.441×10^{-4} $\pm 4.42 \times 10^{-6}$	5.248×10^{-4}	3.68
1000	3.912×10^{-4} $\pm 5.17 \times 10^{-6}$	3.673×10^{-4}	6.48

Table 21:

Efficiency of NI in the $D/M/s+M$ model with $\rho = 1.4$ and $\alpha = 1.0$

s	ASE[θ_{NI}]	Predicted by (25)	RE (%)
100	6.496×10^{-3} $\pm 3.60 \times 10^{-5}$	6.429×10^{-3}	1.05
300	2.188×10^{-3} $\pm 2.50 \times 10^{-5}$	2.143×10^{-3}	2.09
500	1.297×10^{-3} $\pm 1.91 \times 10^{-5}$	1.286×10^{-3}	0.901
700	9.537×10^{-4} $\pm 1.68 \times 10^{-5}$	9.184×10^{-4}	3.84
1000	6.697×10^{-4} $\pm 2.01 \times 10^{-5}$	6.429×10^{-4}	4.17

Table 22:

Comparison of the efficiency of LES and QL$_m$ in the $D/M/s+M$ model with $\rho = 1.4$ and $\alpha = 1.0$

s	ASE[θ_{QL_m}]	ASE[θ_{LES}]	ASE[θ_{LES}]/ASE[θ_{QL_m}]	Predicted ratio by (36)	RE (%)
100	2.882×10^{-3} $\pm 7.89 \times 10^{-6}$	6.545×10^{-3} $\pm 1.12 \times 10^{-5}$	1.278	1.286	0.622
300	9.520×10^{-4} $\pm 4.42 \times 10^{-6}$	1.243×10^{-3} $\pm 5.70 \times 10^{-6}$	1.255	1.286	2.41
500	5.753×10^{-4} $\pm 3.51 \times 10^{-6}$	7.537×10^{-4} $\pm 5.44 \times 10^{-6}$	1.258	1.286	2.18
700	4.096×10^{-4} $\pm 3.18 \times 10^{-6}$	9.149×10^{-4} $\pm 4.42 \times 10^{-6}$	1.277	1.286	0.700
1000	2.871×10^{-4} $\pm 3.66 \times 10^{-6}$	3.912×10^{-4} $\pm 5.17 \times 10^{-6}$	1.313	1.286	-2.01

Table 23:

Comparison of the efficiency of NI and QL$_m$ in the $D/M/s+M$ model with $\rho = 1.4$ and $\alpha = 1.0$

s	ASE$[\theta_{QL_m}]$	ASE$[\theta_{NI}]$	ASE$[\theta_{NI}]$/ASE$[\theta_{QL_m}]$	Predicted ratio by (26)	RE (%)
100	2.882×10^{-3} $\pm 7.89 \times 10^{-6}$	6.496×10^{-3} $\pm 3.60 \times 10^{-5}$	2.254	2.25	1.89
300	9.520×10^{-4} $\pm 4.42 \times 10^{-6}$	2.188×10^{-3} $\pm 1.91 \times 10^{-5}$	2.298	2.25	2.12
500	5.753×10^{-4} $\pm 3.51 \times 10^{-6}$	1.297×10^{-3} $\pm 1.91 \times 10^{-5}$	2.255	2.25	0.230
700	4.096×10^{-4} $\pm 3.18 \times 10^{-6}$	9.537×10^{-4} $\pm 1.68 \times 10^{-5}$	2.328	2.25	3.47
1000	2.871×10^{-4} $\pm 3.66 \times 10^{-6}$	6.697×10^{-4} $\pm 2.01 \times 10^{-5}$	2.332	2.25	3.67

Table 24:

Efficiency of the estimators in the $D/M/s+M$ model with $\rho = 1.4$ and $\alpha = 5.0$

s	ASE$[\theta_{QL_m}]$	ASE$[\theta_{QL_r}]$	ASE$[\theta_{QL}]$	ASE$[\theta_{LES}]$	ASE$[\theta_{NI}]$
100	6.0637×10^{-4} $\pm 1.46 \times 10^{-6}$	6.336×10^{-4} $\pm 1.28 \times 10^{-6}$	9.340×10^{-4} $\pm 9.96 \times 10^{-7}$	9.018×10^{-4} $\pm 1.65 \times 10^{-6}$	1.285×10^{-3} $\pm 4.81 \times 10^{-6}$
300	1.929×10^{-4} $\pm 6.27 \times 10^{-7}$	2.011×10^{-4} $\pm 6.51 \times 10^{-7}$	4.081×10^{-4} $\pm 9.46 \times 10^{-7}$	2.625×10^{-4} $\pm 9.16 \times 10^{-7}$	4.329×10^{-4} $\pm 1.84 \times 10^{-6}$
500	1.150×10^{-4} $\pm 3.00 \times 10^{-7}$	1.196×10^{-4} $\pm 3.71 \times 10^{-7}$	3.084×10^{-4} $\pm 7.29 \times 10^{-7}$	1.528×10^{-4} $\pm 4.14 \times 10^{-7}$	2.606×10^{-4} $\pm 1.23 \times 10^{-6}$
700	8.218×10^{-5} $\pm 3.09 \times 10^{-7}$	8.545×10^{-5} $\pm 3.22 \times 10^{-7}$	2.663×10^{-4} $\pm 3.75 \times 10^{-7}$	1.082×10^{-4} $\pm 2.97 \times 10^{-7}$	1.858×10^{-4} $\pm 1.37 \times 10^{-6}$
1000	5.718×10^{-5} $\pm 3.74 \times 10^{-7}$	5.950×10^{-5} $\pm 4.16 \times 10^{-7}$	2.343×10^{-4} $\pm 4.70 \times 10^{-7}$	7.475×10^{-5} $\pm 5.12 \times 10^{-7}$	1.274×10^{-4} $\pm 1.05 \times 10^{-6}$

Table 25: Point and confidence interval estimates of the ASEs - average square errors - of the estimators

Efficiency of QL$_m$ in the $D/M/s+M$ model with $\rho = 1.4$ and $\alpha = 5.0$

s	ASE$[\theta_{QL_m}]$	Predicted by (22)	RE (%)
100	6.064×10^{-4} $\pm 1.46 \times 10^{-6}$	5.714×10^{-4}	6.11
300	1.929×10^{-4} $\pm 6.27 \times 10^{-7}$	1.905×10^{-4}	1.29
500	1.150×10^{-4} $\pm 3.00 \times 10^{-7}$	1.143×10^{-4}	0.660
700	8.218×10^{-5} $\pm 3.09 \times 10^{-7}$	8.163×10^{-5}	0.671
1000	5.718×10^{-5} $\pm 3.74 \times 10^{-7}$	5.714×10^{-5}	0.0650

Table 26:

Efficiency of LES in the $D/M/s+M$ model with $\rho = 1.4$ and $\alpha = 5.0$

s	ASE$[\theta_{LES}]$	Predicted by (33)	RE (%)
100	9.018×10^{-4} $\pm 1.65 \times 10^{-6}$	7.347×10^{-4}	22.7
300	2.625×10^{-4} $\pm 9.16 \times 10^{-7}$	2.449×10^{-4}	7.20
500	1.528×10^{-4} $\pm 4.14 \times 10^{-7}$	1.469×10^{-4}	4.00
700	1.082×10^{-4} $\pm 2.97 \times 10^{-7}$	1.050×10^{-4}	3.07
1000	7.475×10^{-5} $\pm 5.12 \times 10^{-7}$	7.347×10^{-5}	1.74

Table 27:

Efficiency of NI in the $D/M/s+M$ model with $\rho = 1.4$ and $\alpha = 5.0$

s	ASE$[\theta_{NI}]$	Predicted by (25)	RE (%)
100	1.285×10^{-3} $\pm 4.81 \times 10^{-6}$	1.286×10^{-3}	-0.042
300	4.329×10^{-4} $\pm 1.84 \times 10^{-6}$	4.286×10^{-4}	1.01
500	2.606×10^{-4} $\pm 1.23 \times 10^{-6}$	2.571×10^{-4}	1.35
700	1.858×10^{-4} $\pm 1.37 \times 10^{-6}$	1.837×10^{-4}	1.15
1000	1.274×10^{-4} $\pm 1.05 \times 10^{-6}$	1.286×10^{-4}	-0.919

Table 28:

Comparison of the efficiency of LES and QL$_m$ in the $D/M/s+M$ model with $\rho = 1.4$ and $\alpha = 5.0$

s	ASE$[\theta_{QL_m}]$	ASE$[\theta_{LES}]$	ASE$[\theta_{LES}]$/ASE$[\theta_{QL_m}]$	Predicted ratio by (36)	RE (%)
100	6.064×10^{-4} $\pm 1.46 \times 10^{-6}$	9.018×10^{-2} $\pm 1.65 \times 10^{-6}$	1.487	1.286	15.7
300	1.929×10^{-4} $\pm 6.27 \times 10^{-7}$	2.625×10^{-4} $\pm 9.16 \times 10^{-7}$	1.361	1.286	5.84
500	1.150×10^{-4} $\pm 3.00 \times 10^{-7}$	1.528×10^{-4} $\pm 4.14 \times 10^{-7}$	1.328	1.286	3.31
700	8.218×10^{-5} $\pm 3.08 \times 10^{-7}$	1.082×10^{-4} $\pm 2.97 \times 10^{-7}$	1.316	1.286	2.39
1000	5.718×10^{-5} $\pm 3.74 \times 10^{-7}$	7.475×10^{-5} $\pm 5.12 \times 10^{-7}$	1.307	1.286	1.68

Table 29:

Comparison of the efficiency of NI and QL$_m$ in the $D/M/s+M$ model with $\rho = 1.4$ and $\alpha = 5.0$

s	ASE[θ_{QL_m}]	ASE[θ_{NI}]	ASE[θ_{NI}]/ASE[θ_{QL_m}]	Predicted ratio by (26)	RE (%)
100	6.064×10^{-4} $\pm 1.46 \times 10^{-6}$	1.285×10^{-3} $\pm 4.81 \times 10^{-6}$	2.119	2.25	-5.80
300	1.929×10^{-4} $\pm 6.27 \times 10^{-7}$	4.329×10^{-4} $\pm 1.84 \times 10^{-6}$	2.244	2.25	-0.277
500	1.150×10^{-4} $\pm 3.00 \times 10^{-7}$	2.606×10^{-4} $\pm 1.23 \times 10^{-6}$	2.265	2.25	0.684
700	8.218×10^{-5} $\pm 3.08 \times 10^{-7}$	1.858×10^{-4} $\pm 1.37 \times 10^{-6}$	2.261	2.25	0.473
1000	5.718×10^{-5} $\pm 3.74 \times 10^{-7}$	1.274×10^{-4} $\pm 1.05 \times 10^{-6}$	2.228	2.25	-0.983

Table 30:

Efficiency of the estimators in the $D/M/s+M$ model with $\rho = 1.4$ and $\alpha = 0.2$

s	ASE[θ_{QL_m}]	ASE[θ_{QL_r}]	ASE[θ_{QL}]	ASE[θ_{LES}]	ASE[θ_{NI}]
100	1.436×10^{-2} $\pm 9.78 \times 10^{-5}$	1.492×10^{-2} $\pm 9.40 \times 10^{-5}$	1.192×10^{-1} $\pm 1.57 \times 10^{-4}$	1.863×10^{-2} $\pm 1.64 \times 10^{-4}$	3.266×10^{-2} $\pm 5.33 \times 10^{-4}$
300	4.798×10^{-3} $\pm 5.99 \times 10^{-5}$	5.005×10^{-3} $\pm 6.08 \times 10^{-5}$	1.071×10^{-1} $\pm 1.41 \times 10^{-4}$	6.172×10^{-3} $\pm 7.45 \times 10^{-5}$	1.056×10^{-2} $\pm 1.92 \times 10^{-4}$
500	2.865×10^{-3} $\pm 5.43 \times 10^{-5}$	2.966×10^{-3} $\pm 5.24 \times 10^{-5}$	1.044×10^{-1} $\pm 1.071 \times 10^{-4}$	3.672×10^{-3} $\pm 6.67 \times 10^{-5}$	6.641×10^{-3} $\pm 2.933 \times 10^{-4}$
700	2.091×10^{-3} $\pm 2.39 \times 10^{-5}$	2.170×10^{-3} $\pm 1.90 \times 10^{-5}$	1.033×10^{-1} $\pm 1.53803 \times 10^{-4}$	2.691×10^{-3} $\pm 3.23 \times 10^{-5}$	4.802×10^{-3} $\pm 2.26 \times 10^{-4}$
1000	1.435×10^{-3} $\pm 1.15 \times 10^{-5}$	1.507×10^{-3} $\pm 1.52 \times 10^{-5}$	1.026×10^{-1} $\pm 1.20 \times 10^{-4}$	1.859×10^{-3} $\pm 2.06 \times 10^{-5}$	3.030×10^{-3} $\pm 1.05 \times 10^{-4}$

Table 31: Point and confidence interval estimates of the ASEs - average square errors - of the estimators

Efficiency of $\mathbf{QL}_m$ in the $D/M/s+M$ model with $\rho = 1.4$ and $\alpha = 0.2$

s	ASE[θ_{QL_m}]	Predicted by (22)	RE (%)
100	1.436×10^{-2} $\pm 9.78 \times 10^{-5}$	1.429×10^{-2}	0.519
300	4.798×10^{-3} $\pm 5.99 \times 10^{-5}$	4.762×10^{-3}	0.763
500	2.865×10^{-3} $\pm 5.43 \times 10^{-5}$	2.857×10^{-3}	0.283
700	2.091×10^{-3} $\pm 2.39 \times 10^{-5}$	2.041×10^{-3}	2.45
1000	1.435×10^{-3} $\pm 1.15 \times 10^{-5}$	1.429×10^{-3}	0.458

Table 32:

Efficiency of LES in the $D/M/s+M$ model with $\rho = 1.4$ and $\alpha = 0.2$

s	ASE[θ_{LES}]	Predicted by (33)	RE (%)
100	1.863×10^{-2} $\pm 1.64 \times 10^{-4}$	1.837×10^{-2}	1.41
300	6.172×10^{-3} $\pm 7.45 \times 10^{-5}$	6.122×10^{-3}	0.805
500	3.672×10^{-3} $\pm 6.67 \times 10^{-5}$	3.673×10^{-3}	-0.0465
700	2.691×10^{-3} $\pm 3.23 \times 10^{-5}$	2.624×10^{-3}	2.56
1000	1.859×10^{-3} $\pm 2.06 \times 10^{-5}$	1.837×10^{-3}	1.23

Table 33:

Efficiency of NI in the $D/M/s+M$ model with $\rho = 1.4$ and $\alpha = 0.2$

s	ASE[θ_{NI}]	Predicted by (25)	RE (%)
100	3.266×10^{-2} $\pm 5.33 \times 10^{-4}$	3.214×10^{-2}	1.62
300	1.056×10^{-2} $\pm 1.92 \times 10^{-4}$	1.071×10^{-2}	-1.43
500	6.641×10^{-3} $\pm 2.933 \times 10^{-4}$	6.429×10^{-3}	3.31
700	4.802×10^{-3} $\pm 2.26 \times 10^{-4}$	4.592×10^{-3}	4.59
1000	3.030×10^{-3} $\pm 1.05 \times 10^{-4}$	3.214×10^{-3}	-5.75

Table 34:

Comparison of the efficiency of LES and QL$_m$ in the $D/M/s+M$ model with $\rho = 1.4$ and $\alpha = 0.2$

s	ASE[θ_{QL_m}]	ASE[θ_{LES}]	ASE[θ_{LES}]/ASE[θ_{QL_m}]	Predicted ratio by (36)	RE (%)
100	1.436×10^{-2} $\pm 9.78 \times 10^{-5}$	1.863×10^{-2} $\pm 1.642 \times 10^{-4}$	1.297	1.286	0.885
300	4.798×10^{-3} $\pm 5.99 \times 10^{-5}$	6.172×10^{-3} $\pm 7.45 \times 10^{-5}$	1.286	1.286	0.0421
500	2.865×10^{-3} $\pm 5.43 \times 10^{-5}$	3.672×10^{-3} $\pm 6.67 \times 10^{-5}$	1.281	1.286	-0.329
700	2.091×10^{-3} $\pm 2.39 \times 10^{-5}$	2.691×10^{-3} $\pm 3.23 \times 10^{-5}$	1.287	1.286	0.107
1000	1.435×10^{-3} $\pm 1.15 \times 10^{-5}$	1.859×10^{-3} $\pm 2.05 \times 10^{-5}$	1.296	1.286	0.765

Table 35:

Comparison of the efficiency of NI and QL$_m$ in the $D/M/s+M$ model with $\rho = 1.4$ and $\alpha = 0.2$

s	ASE$[\theta_{QL_m}]$	ASE$[\theta_{NI}]$	ASE$[\theta_{NI}]$/ASE$[\theta_{QL_m}]$	Predicted ratio by (26)	RE (%)
100	1.436×10^{-2} $\pm 9.78 \times 10^{-5}$	3.266×10^{-2} $\pm 5.33 \times 10^{-4}$	2.275	2.25	1.09
300	4.798×10^{-3} $\pm 5.99 \times 10^{-5}$	1.056×10^{-2} $\pm 1.92 \times 10^{-4}$	2.201	2.25	-2.18
500	2.865×10^{-3} $\pm 5.43 \times 10^{-5}$	6.641×10^{-3} $\pm 2.933 \times 10^{-4}$	2.318	2.25	3.01
700	2.091×10^{-3} $\pm 2.39 \times 10^{-5}$	4.802×10^{-3} $\pm 2.26 \times 10^{-4}$	2.297	2.25	2.08
1000	1.435×10^{-3} $\pm 1.15 \times 10^{-5}$	3.130×10^{-3} $\pm 1.05 \times 10^{-4}$	2.111	2.25	-3.08

Table 36:

Efficiency of the estimators in the $H_2/M/s+M$ model with $\rho = 1.4$ and $\alpha = 1.0$

s	ASE[θ_{QL_m}]	ASE[θ_{QL_r}]	ASE[θ_{QL}]	ASE[θ_{LES}]	ASE[θ_{NI}]
100	2.898×10^{-3}	3.712×10^{-3}	1.190×10^{-2}	1.129×10^{-2}	1.900×10^{-2}
	$\pm 1.17 \times 10^{-5}$	$\pm 1.65 \times 10^{-5}$	$\pm 5.26 \times 10^{-5}$	$\pm 5.73 \times 10^{-5}$	$\pm 1.31 \times 10^{-4}$
300	9.531×10^{-4}	1.173×10^{-3}	6.652×10^{-3}	3.863×10^{-3}	6.829×10^{-3}
	$\pm 4.79 \times 10^{-6}$	$\pm 9.07 \times 10^{-6}$	$\pm 5.38 \times 10^{-5}$	$\pm 2.15 \times 10^{-5}$	$\pm 1.09 \times 10^{-4}$
500	5.701×10^{-4}	6.903×10^{-4}	5.502×10^{-3}	2.346×10^{-3}	4.118×10^{-3}
	$\pm 3.01 \times 10^{-6}$	$\pm 2.97 \times 10^{-6}$	$\pm 3.12 \times 10^{-5}$	$\pm 1.78 \times 10^{-5}$	$\pm 4.94 \times 10^{-5}$
700	4.120×10^{-4}	4.9888×10^{-4}	5.143×10^{-3}	1.694×10^{-3}	2.939×10^{-3}
	$\pm 2.38 \times 10^{-6}$	$\pm 3.76 \times 10^{-6}$	$\pm 2.83 \times 10^{-5}$	$\pm 1.28 \times 10^{-5}$	$\pm 5.86 \times 10^{-5}$
1000	2.870×10^{-4}	3.477×10^{-4}	4.780×10^{-3}	1.211×10^{-3}	2.117×10^{-3}
	$\pm 3.33 \times 10^{-6}$	$\pm 2.69 \times 10^{-6}$	$\pm 3.30 \times 10^{-5}$	$\pm 1.70 \times 10^{-5}$	$\pm 5.85 \times 10^{-5}$

Table 37: Point and confidence interval estimates of the ASEs - average square errors - of the estimators

Efficiency of QL$_m$ in the $H_2/M/s+M$ model with $\rho = 1.4$ and $\alpha = 1.0$

s	ASE[θ_{QL_m}]	Predicted by (22)	RE (%)
100	2.898×10^{-3}	2.857×10^{-3}	1.44
	$\pm 1.17 \times 10^{-5}$		
300	9.531×10^{-4}	9.524×10^{-4}	0.0797
	$\pm 4.79 \times 10^{-6}$		
500	5.701×10^{-4}	5.714×10^{-4}	-0.233
	$\pm 3.01 \times 10^{-6}$		
700	4.120×10^{-4}	4.082×10^{-4}	0.947
	$\pm 2.38 \times 10^{-6}$		
1000	2.870×10^{-4}	2.857×10^{-4}	0.443
	$\pm 3.33 \times 10^{-6}$		

Table 38:

Efficiency of LES in the $H_2/M/s+M$ model with $\rho = 1.4$ and $\alpha = 1.0$

s	ASE[θ_{LES}]	Predicted by (33)	RE (%)
100	1.129×10^{-2} $\pm 5.73 \times 10^{-5}$	1.184×10^{-2}	-4.64
300	3.863×10^{-3} $\pm 2.15 \times 10^{-5}$	3.946×10^{-3}	-2.10
500	2.346×10^{-3} $\pm 1.78 \times 10^{-5}$	2.367×10^{-3}	-0.891
700	1.694×10^{-3} $\pm 1.28 \times 10^{-5}$	1.691×10^{-3}	0.151
1000	1.211×10^{-3} $\pm 1.70 \times 10^{-5}$	1.184×10^{-3}	2.29

Table 39:

Efficiency of NI in the $H_2/M/s+M$ model with $\rho = 1.4$ and $\alpha = 1.0$

s	ASE[θ_{NI}]	Predicted by (25)	RE (%)
100	1.900×10^{-2} $\pm 1.31 \times 10^{-4}$	2.071×10^{-2}	-7.69
300	6.829×10^{-3} $\pm 1.09 \times 10^{-4}$	6.905×10^{-3}	-0.717
500	4.118×10^{-3} $\pm 4.94 \times 10^{-5}$	4.143×10^{-3}	0.161
700	2.939×10^{-3} $\pm 5.86 \times 10^{-5}$	2.960×10^{-3}	2.52
1000	2.117×10^{-3} $\pm 5.85 \times 10^{-5}$	2.071×10^{-3}	-0.0333

Table 40:

Comparison of the efficiency of LES and $\mathbf{QL}_m$ in the $H_2/M/s+M$ model with $\rho = 1.4$ and $\alpha = 1.0$

s	ASE$[\theta_{QL_m}]$	ASE$[\theta_{LES}]$	ASE$[\theta_{LES}]$/ASE$[\theta_{QL_m}]$	Predicted ratio by (37)	RE (%)
100	2.898×10^{-3} $\pm 1.17 \times 10^{-5}$	1.129×10^{-2} $\pm 5.73 \times 10^{-5}$	3.894	4.143	6.00
300	9.531×10^{-4} $\pm 4.79 \times 10^{-6}$	3.863×10^{-3} $\pm 2.15 \times 10^{-5}$	4.053	4.143	2.18
500	5.701×10^{-4} $\pm 3.01 \times 10^{-6}$	2.346×10^{-3} $\pm 1.78 \times 10^{-5}$	4.115	4.143	0.660
700	4.120×10^{-4} $\pm 2.38 \times 10^{-6}$	1.694×10^{-3} $\pm 1.28 \times 10^{-5}$	4.110	4.143	0.789
1000	2.870×10^{-4} $\pm 3.33 \times 10^{-6}$	1.211×10^{-3} $\pm 1.70 \times 10^{-5}$	4.219	4.143	1.837

Table 41:

Comparison of the efficiency of NI and $\mathbf{QL}_m$ in the $H_2/M/s+M$ model with $\rho = 1.4$ and $\alpha = 1.0$

s	ASE$[\theta_{QL_m}]$	ASE$[\theta_{NI}]$	ASE$[\theta_{NI}]$/ASE$[\theta_{QL_m}]$	Predicted ratio by (26)	RE (%)
100	2.898×10^{-3} $\pm 1.17 \times 10^{-5}$	1.900×10^{-2} $\pm 1.31 \times 10^{-4}$	6.598	7.25	-9.00
300	9.531×10^{-4} $\pm 4.79 \times 10^{-6}$	6.829×10^{-3} $\pm 1.09 \times 10^{-4}$	7.192	7.25	-0.796
500	5.701×10^{-4} $\pm 3.01 \times 10^{-6}$	4.118×10^{-3} $\pm 4.94 \times 10^{-5}$	7.279	7.25	0.394
700	4.120×10^{-4} $\pm 2.38 \times 10^{-6}$	2.939×10^{-3} $\pm 5.86 \times 10^{-5}$	7.363	7.25	1.55
1000	2.870×10^{-4} $\pm 3.33 \times 10^{-6}$	2.117×10^{-3} $\pm 5.85 \times 10^{-5}$	7.216	7.25	-0.475

Table 42:

Efficiency of the estimators in the $H_2/M/s+M$ model with $\rho = 1.4$ and $\alpha = 5.0$					
s	ASE[θ_{QL_m}]	ASE[θ_{QL_r}]	ASE[θ_{QL}]	ASE[θ_{LES}]	ASE[θ_{NI}]
100	7.193×10^{-4}	1.059×10^{-3}	2.217×10^{-3}	2.393×10^{-3}	3.101×10^{-3}
	$\pm 2.63 \times 10^{-6}$	$\pm 4.47 \times 10^{-6}$	$\pm 1.01 \times 10^{-5}$	$\pm 6.72 \times 10^{-6}$	$\pm 1.42 \times 10^{-5}$
300	2.008×10^{-4}	2.675×10^{-4}	7.240×10^{-4}	7.569×10^{-4}	1.169×10^{-3}
	$\pm 7.85 \times 10^{-7}$	$\pm 1.28 \times 10^{-6}$	$\pm 2.63 \times 10^{-6}$	$\pm 2.70 \times 10^{-6}$	$\pm 5.82 \times 10^{-6}$
500	1.167×10^{-4}	1.495×10^{-4}	4.792×10^{-4}	4.540×10^{-4}	7.624×10^{-4}
	$\pm 7.05 \times 10^{-7}$	$\pm 8.78 \times 10^{-7}$	$\pm 2.68 \times 10^{-6}$	$\pm 1.71 \times 10^{-6}$	$\pm 6.07 \times 10^{-6}$
700	8.277×10^{-5}	1.042×10^{-4}	3.856×10^{-4}	3.280×10^{-4}	5.714×10^{-4}
	$\pm 4.12 \times 10^{-7}$	$\pm 6.52 \times 10^{-7}$	$\pm 2.50 \times 10^{-6}$	$\pm 1.27 \times 10^{-6}$	$\pm 4.72 \times 10^{-6}$
1000	5.733×10^{-5}	7.141×10^{-5}	3.184×10^{-4}	2.302×10^{-4}	4.0951×10^{-4}
	$\pm 2.48 \times 10^{-7}$	$\pm 2.44 \times 10^{-7}$	$\pm 1.34 \times 10^{-6}$	$\pm 1.19 \times 10^{-6}$	$\pm 4.15 \times 10^{-6}$

Table 43: Point and confidence interval estimates of the ASEs - average square errors - of the estimators

Efficiency of $\mathbf{QL}_m$ in the $H_2/M/s+M$ model with $\rho = 1.4$ and $\alpha = 5.0$			
s	ASE[θ_{QL_m}]	Predicted by (22)	RE (%)
100	7.193×10^{-4}	5.714×10^{-4}	25.9
	$\pm 2.63 \times 10^{-6}$		
300	2.008×10^{-4}	1.905×10^{-4}	5.44
	$\pm 7.85 \times 10^{-7}$		
500	1.167×10^{-4}	1.143×10^{-4}	2.11
	$\pm 7.05 \times 10^{-7}$		
700	8.277×10^{-5}	8.163×10^{-5}	1.39
	$\pm 4.12 \times 10^{-7}$		
1000	5.733×10^{-5}	5.714×10^{-5}	0.328
	$\pm 2.48 \times 10^{-7}$		

Table 44:

Efficiency of LES in the $H_2/M/s+M$ model with $\rho = 1.4$ and $\alpha = 5.0$

s	ASE[θ_{LES}]	Predicted by (33)	RE (%)
100	2.393×10^{-3} $\pm 6.72 \times 10^{-6}$	2.367×10^{-3}	1.07
300	7.569×10^{-4} $\pm 2.70 \times 10^{-6}$	7.891×10^{-4}	-4.09
500	4.540×10^{-4} $\pm 1.71 \times 10^{-6}$	4.735×10^{-4}	-4.11
700	3.280×10^{-4} $\pm 1.27 \times 10^{-6}$	3.382×10^{-4}	-3.03
1000	2.302×10^{-4} $\pm 1.19 \times 10^{-6}$	2.367×10^{-4}	-2.78

Table 45:

Efficiency of NI in the $H_2/M/s+M$ model with $\rho = 1.4$ and $\alpha = 5.0$

s	ASE[θ_{NI}]	Predicted by (25)	RE (%)
100	3.101×10^{-3} $\pm 1.42 \times 10^{-5}$	4.143×10^{-3}	-25.2
300	1.169×10^{-3} $\pm 5.82 \times 10^{-6}$	1.381×10^{-3}	-15.4
500	7.624×10^{-4} $\pm 6.07 \times 10^{-6}$	8.286×10^{-4}	-7.98
700	5.714×10^{-4} $\pm 4.72 \times 10^{-6}$	5.918×10^{-4}	-3.45
1000	4.0951×10^{-4} $\pm 4.15 \times 10^{-6}$	4.143×10^{-4}	-1.15

Table 46:

Comparison of the efficiency of LES and $\mathbf{QL}_m$ in the $H_2/M/s+M$ model with $\rho = 1.4$ and $\alpha = 5.0$

s	ASE[θ_{QL_m}]	ASE[θ_{LES}]	ASE[θ_{LES}]/ASE[θ_{QL_m}]	Predicted by (37)	RE (%)
100	7.193×10^{-4} $\pm 2.63 \times 10^{-6}$	2.393×10^{-3} $\pm 6.72 \times 10^{-6}$	3.326	4.143	-19.7
300	2.008×10^{-4} $\pm 7.85 \times 10^{-7}$	7.569×10^{-4} $\pm 2.70 \times 10^{-6}$	3.769	4.143	-9.03
500	1.167×10^{-4} $\pm 7.05 \times 10^{-7}$	4.540×10^{-4} $\pm 1.71 \times 10^{-6}$	3.891	4.143	-6.09
700	8.277×10^{-5} $\pm 4.12 \times 10^{-7}$	3.280×10^{-4} $\pm 1.27 \times 10^{-6}$	3.962	4.143	-4.36
1000	5.733×10^{-5} $\pm 2.48 \times 10^{-7}$	2.302×10^{-4} $\pm 1.19 \times 10^{-6}$	4.014	4.143	-3.10

Table 47:

Comparison of the efficiency of NI and $\mathbf{QL}_m$ in the $H_2/M/s+M$ model with $\rho = 1.4$ and $\alpha = 5.0$

s	ASE[θ_{QL_m}]	ASE[θ_{NI}]	ASE[θ_{NI}]/ASE[θ_{QL_m}]	Predicted ratio by (26)	RE (%)
100	7.193×10^{-4} $\pm 2.63 \times 10^{-6}$	3.101×10^{-3} $\pm 1.42 \times 10^{-5}$	4.310	7.25	-40.5
300	2.008×10^{-4} $\pm 7.85 \times 10^{-7}$	1.169×10^{-3} $\pm 5.82 \times 10^{-6}$	5.821	7.25	-19.7
500	1.167×10^{-4} $\pm 7.05 \times 10^{-7}$	7.624×10^{-4} $\pm 6.07 \times 10^{-6}$	6.533	7.25	-9.89
700	8.277×10^{-5} $\pm 4.12 \times 10^{-7}$	5.714×10^{-4} $\pm 4.72 \times 10^{-6}$	6.904	7.25	-4.78
1000	5.733×10^{-5} $\pm 2.48 \times 10^{-7}$	4.0951×10^{-4} $\pm 4.15 \times 10^{-6}$	7.143	7.25	-1.48

Table 48:

Efficiency of the estimators in the $H_2/M/s+M$ model with $\rho = 1.4$ and $\alpha = 0.2$

s	ASE[θ_{QL_m}]	ASE[θ_{QL_r}]	ASE[θ_{QL}]	ASE[θ_{LES}]	ASE[θ_{NI}]
100	1.429×10^{-2}	1.731×10^{-2}	1.370×10^{-1}	5.866×10^{-2}	1.018×10^{-1}
	$\pm 8.51 \times 10^{-5}$	1.50×10^{-4}	9.49×10^{-4}	$\pm 4.87 \times 10^{-4}$	$\pm 1.91 \times 10^{-3}$
300	4.805×10^{-3}	5.747×10^{-3}	1.141×10^{-1}	2.044×10^{-2}	3.426×10^{-2}
	$\pm 1.12 \times 10^{-4}$	$\pm 1.10 \times 10^{-4}$	$\pm 1.01 \times 10^{-3}$	$\pm 3.03 \times 10^{-4}$	$\pm 9.58 \times 10^{-4}$
500	2.865×10^{-3}	3.419×10^{-3}	1.080×10^{-1}	1.189×10^{-2}	2.043×10^{-2}
	$\pm 5.39 \times 10^{-5}$	$\pm 6.99 \times 10^{-5}$	$\pm 1.33 \times 10^{-3}$	$\pm 1.768 \times 10^{-4}$	$\pm 9.72 \times 10^{-4}$
700	2.046×10^{-3}	2.456×10^{-3}	1.070×10^{-1}	8.471×10^{-3}	1.50×10^{-2}
	$\pm 5.45 \times 10^{-5}$	$\pm 6.68 \times 10^{-5}$	$\pm 9.09 \times 10^{-4}$	$\pm 1.72 \times 10^{-4}$	$\pm 8.12 \times 10^{-4}$
1000	1.422×10^{-3}	1.686×10^{-3}	1.037×10^{-1}	5.962×10^{-3}	9.843×10^{-3}
	$\pm 3.61 \times 10^{-5}$	$\pm 3.68 \times 10^{-5}$	$\pm 1.06 \times 10^{-3}$	$\pm 1.24 \times 10^{-4}$	$\pm 4.99 \times 10^{-4}$

Table 49: Point and confidence interval estimates of the ASEs - average square errors - of the estimators

Efficiency of QL$_m$ in the $H_2/M/s+M$ model with $\rho = 1.4$ and $\alpha = 0.2$

s	ASE[θ_{QL_m}]	Predicted by (22)	RE (%)
100	1.429×10^{-2}	1.429×10^{-2}	0.0531
	$\pm 8.51 \times 10^{-5}$		
300	4.805×10^{-3}	4.762×10^{-3}	0.913
	$\pm 1.12 \times 10^{-4}$		
500	2.865×10^{-3}	2.857×10^{-3}	0.263
	$\pm 5.39 \times 10^{-5}$		
700	2.046×10^{-3}	2.041×10^{-3}	0.235
	$\pm 5.45 \times 10^{-5}$		
1000	1.422×10^{-3}	1.429×10^{-3}	-0.440
	$\pm 3.61 \times 10^{-5}$		

Table 50:

Efficiency of LES in the $H_2/M/s+M$ model with $\rho = 1.4$ and $\alpha = 0.2$

s	ASE[θ_{LES}]	Predicted by (33)	RE (%)
100	5.866×10^{-2} $\pm 4.87 \times 10^{-4}$	5.918×10^{-2}	-0.893
300	2.044×10^{-2} $\pm 3.03 \times 10^{-4}$	1.973×10^{-2}	3.63
500	1.189×10^{-2} $\pm 1.768 \times 10^{-4}$	1.187×10^{-2}	0.481
700	8.471×10^{-3} $\pm 1.72 \times 10^{-4}$	8.455×10^{-3}	0.193
1000	5.962×10^{-3} $\pm 1.24 \times 10^{-4}$	5.918×10^{-3}	0.744

Table 51:

Efficiency of NI in the $H_2/M/s+M$ model with $\rho = 1.4$ and $\alpha = 0.2$

s	ASE[θ_{NI}]	Predicted by (25)	RE (%)
100	1.018×10^{-1} $\pm 1.91 \times 10^{-3}$	1.036×10^{-1}	-1.71
300	3.426×10^{-2} 9.58×10^{-4}	3.452×10^{-2}	-0.777
500	2.043×10^{-2} $\pm 9.72 \times 10^{-4}$	2.071×10^{-2}	-1.36
700	1.50×10^{-2} $\pm 8.12 \times 10^{-4}$	1.480×10^{-2}	1.33
1000	9.843×10^{-3} 4.99×10^{-4}	1.036×10^{-2}	-4.97

Table 52:

Comparison of the efficiency of LES and $\mathbf{QL}_m$ in the $H_2/M/s+M$ model with $\rho = 1.4$ and $\alpha = 0.2$

s	ASE$[\theta_{QL_m}]$	ASE$[\theta_{LES}]$	ASE$[\theta_{LES}]$/ASE$[\theta_{QL_m}]$	Predicted by (37)	RE (%)
100	1.429×10^{-2} $\pm 8.51 \times 10^{-5}$	5.866×10^{-2} $\pm 4.87 \times 10^{-4}$	4.104	4.143	-0.946
300	4.805×10^{-3} $\pm 1.12 \times 10^{-4}$	2.044×10^{-2} $\pm 3.03 \times 10^{-4}$	4.255	4.143	2.70
500	2.865×10^{-3} $\pm 5.39 \times 10^{-5}$	1.189×10^{-2} 1.77×10^{-4}	4.152	4.143	0.218
700	2.046×10^{-3} $\pm 5.45 \times 10^{-5}$	8.471×10^{-3} $\pm 1.715 \times 10^{-4}$	4.141	4.143	-0.0422
1000	1.422×10^{-3} $\pm 3.61 \times 10^{-5}$	5.962×10^{-3} $\pm 1.24 \times 10^{-5}$	4.192	4.143	1.19

Table 53:

Comparison of the efficiency of NI and $\mathbf{QL}_m$ in the $H_2/M/s+M$ model with $\rho = 1.4$ and $\alpha = 0.2$

s	ASE$[\theta_{QL_m}]$	ASE$[\theta_{NI}]$	ASE$[\theta_{NI}]$/ASE$[\theta_{QL_m}]$	Predicted ratio by (26)	RE (%)
100	1.429×10^{-2} $\pm 8.51 \times 10^{-5}$	1.018×10^{-1} $\pm 1.91 \times 10^{-3}$	7.122	7.25	-1.76
300	4.805×10^{-3} $\pm 1.12 \times 10^{-4}$	3.426×10^{-2} $\pm 9.58 \times 10^{-4}$	7.129	7.25	-1.67
500	2.865×10^{-3} $\pm 5.39 \times 10^{-5}$	2.043×10^{-2} $\pm 9.72 \times 10^{-4}$	7.132	7.25	-1.62
700	2.046×10^{-3} $\pm 5.45 \times 10^{-5}$	1.50×10^{-2} $\pm 8.12 \times 10^{-4}$	7.329	7.25	1.09
1000	1.422×10^{-3} $\pm 3.61 \times 10^{-5}$	9.843×10^{-3} $\pm 4.99 \times 10^{-4}$	6.920	7.25	-4.55

Table 54:

Efficiency of the estimators in the $M/H_2/s+M$ model with $\rho=1.4$ and $\alpha=1.0$					
s	ASE$[\theta_{QL_m}]$	ASE$[\theta_{QL_r}]$	ASE$[\theta_{QL}]$	ASE$[\theta_{LES}]$	ASE$[\theta_{NI}]$
100	3.491×10^{-3}	3.487×10^{-3}	8.720×10^{-3}	6.227×10^{-3}	1.435×10^{-2}
	$\pm2.89\times10^{-5}$	$\pm1.75\times10^{-5}$	$\pm2.66\times10^{-5}$	$\pm3.61\times10^{-5}$	$\pm5.30\times10^{-5}$
300	1.114×10^{-3}	1.117×10^{-3}	5.530×10^{-3}	1.996×10^{-3}	4.893×10^{-3}
	$\pm1.31\times10^{-5}$	$\pm9.83\times10^{-6}$	$\pm2.34\times10^{-5}$	$\pm1.73\times10^{-5}$	$\pm8.37\times10^{-5}$
500	6.660×10^{-4}	6.696×10^{-4}	4.953×10^{-3}	1.190×10^{-3}	2.931×10^{-3}
	$\pm5.68\times10^{-6}$	$\pm5.68\times10^{-6}$	$\pm2.26\times10^{-5}$	$\pm1.14\times10^{-5}$	$\pm5.60\times10^{-5}$
700	4.807×10^{-4}	4.797×10^{-4}	4.672×10^{-3}	8.612×10^{-4}	2.083×10^{-3}
	$\pm6.73\times10^{-6}$	$\pm5.59\times10^{-6}$	$\pm2.46\times10^{-5}$	$\pm1.23\times10^{-5}$	$\pm4.91\times10^{-5}$
1000	3.362×10^{-4}	3.346×10^{-4}	4.489×10^{-3}	6.136×10^{-4}	1.494×10^{-3}
	$\pm3.16\times10^{-6}$	$\pm1.76\times10^{-6}$	$\pm1.62\times10^{-5}$	$\pm9.34\times10^{-6}$	$\pm5.50\times10^{-5}$

Table 55: Point and confidence interval estimates of the ASEs - average square errors - of the estimators

Efficiency of the estimators in the $M/D/s+M$ model with $\rho=1.4$ and $\alpha=1.0$					
s	ASE$[\theta_{QL_m}]$	ASE$[\theta_{QL_r}]$	ASE$[\theta_{QL}]$	ASE$[\theta_{LES}]$	ASE$[\theta_{NI}]$
100	9.171×10^{-3}	1.066×10^{-2}	1.772×10^{-2}	1.525×10^{-2}	9.316×10^{-3}
	$\pm3.08\times10^{-3}$	$\pm3.56\times10^{-3}$	$\pm4.08\times10^{-3}$	$\pm4.12\times10^{-3}$	$\pm1.59\times10^{-3}$
300	1.492×10^{-2}	1.698×10^{-2}	2.400×10^{-2}	2.511×10^{-2}	8.553×10^{-3}
	$\pm1.91\times10^{-3}$	$\pm2.15\times10^{-3}$	$\pm2.48\times10^{-3}$	$\pm3.48\times10^{-3}$	$\pm1.084\times10^{-3}$
500	1.560×10^{-2}	1.771×10^{-2}	2.469×10^{-2}	2.585×10^{-2}	7.806×10^{-3}
	$\pm2.85\times10^{-3}$	$\pm3.23\times10^{-3}$	$\pm3.72\times10^{-3}$	$\pm4.64\times10^{-3}$	$\pm6.00\times10^{-4}$
700	1.259×10^{-2}	1.433×10^{-2}	2.071×10^{-2}	2.015×10^{-2}	8.232×10^{-3}
	1.590×10^{-3}	1.797×10^{-3}	2.053×10^{-3}	2.566×10^{-3}	$\pm9.059\times10^{-4}$
1000	1.417×10^{-2}	1.611×10^{-2}	2.267×10^{-2}	2.246×10^{-2}	7.566×10^{-3}
	$\pm1.515\times10^{-3}$	$\pm1.706\times10^{-3}$	$\pm1.964\times10^{-3}$	$\pm2.64\times10^{-3}$	$\pm4.711\times10^{-4}$

Table 56: Point and confidence interval estimates of the ASEs - average square errors - of the estimators

Efficiency of the estimators in the $M/E_{10}/s+M$ model with $\rho = 1.4$ and $\alpha = 1.0$

s	ASE[θ_{QL_m}]	ASE[θ_{QL_r}]	ASE[θ_{QL}]	ASE[θ_{LES}]	ASE[θ_{NI}]
100	2.024×10^{-3}	2.405×10^{-3}	8.249×10^{-3}	5.052×10^{-3}	6.284×10^{-3}
	$\pm 6.51 \times 10^{-6}$	$\pm 9.91 \times 10^{-6}$	$\pm 4.27 \times 10^{-5}$	$\pm 2.12 \times 10^{-5}$	$\pm 2.63 \times 10^{-5}$
300	6.790×10^{-4}	7.972×10^{-4}	5.439×10^{-3}	1.687×10^{-3}	2.111×10^{-3}
	$\pm 2.48 \times 10^{-6}$	$\pm 2.71 \times 10^{-6}$	$\pm 2.39 \times 10^{-5}$	$\pm 8.44 \times 10^{-6}$	$\pm 2.51 \times 10^{-5}$
500	4.072×10^{-4}	4.775×10^{-4}	4.857×10^{-3}	1.001×10^{-3}	1.266×10^{-3}
	$\pm 2.81 \times 10^{-6}$	$\pm 3.48 \times 10^{-6}$	$\pm 2.04 \times 10^{-5}$	$\pm 7.67 \times 10^{-6}$	$\pm 1.81 \times 10^{-5}$
700	2.946×10^{-4}	3.449×10^{-4}	4.632×10^{-3}	7.147×10^{-4}	9.006×10^{-4}
	$\pm 1.41 \times 10^{-6}$	$\pm 1.84 \times 10^{-6}$	$\pm 2.20 \times 10^{-5}$	$\pm 7.31 \times 10^{-6}$	$\pm 1.64 \times 10^{-5}$
1000	2.063×10^{-4}	2.408×10^{-4}	4.440×10^{-3}	5.073×10^{-4}	6.480×10^{-4}
	$\pm 2.37 \times 10^{-6}$	$\pm 2.89 \times 10^{-6}$	$\pm 2.653 \times 10^{-5}$	$\pm 3.95 \times 10^{-6}$	$\pm 1.55 \times 10^{-5}$

Table 57: Point and confidence interval estimates of the ASEs - average square errors - of the estimators

Efficiency of the estimators in the $M/E_4/s+M$ model with $\rho = 1.4$ and $\alpha = 1.0$

s	ASE[θ_{QL_m}]	ASE[θ_{QL_r}]	ASE[θ_{QL}]	ASE[θ_{LES}]	ASE[θ_{NI}]
100	2.083×10^{-3}	2.421×10^{-3}	8.153×10^{-3}	5.074×10^{-3}	6.962×10^{-3}
	$\pm 5.74 \times 10^{-6}$	$\pm 7.09 \times 10^{-6}$	$\pm 3.57 \times 10^{-5}$	$\pm 1.44 \times 10^{-5}$	$\pm 3.39 \times 10^{-5}$
300	6.973×10^{-4}	8.037×10^{-4}	5.456×10^{-3}	1.672×10^{-3}	2.275×10^{-3}
	$\pm 4.33 \times 10^{-6}$	$\pm 5.26 \times 10^{-6}$	$\pm 2.44 \times 10^{-5}$	$\pm 5.67 \times 10^{-6}$	$\pm 2.56 \times 10^{-5}$
500	4.165×10^{-4}	4.794×10^{-4}	4.861×10^{-3}	1.001×10^{-3}	1.394×10^{-3}
	$\pm 3.63 \times 10^{-6}$	$\pm 4.34 \times 10^{-6}$	$\pm 1.87 \times 10^{-5}$	$\pm 7.29 \times 10^{-6}$	$\pm 2.60 \times 10^{-5}$
700	2.992×10^{-4}	3.447×10^{-4}	4.645×10^{-3}	7.153×10^{-4}	9.989×10^{-4}
	$\pm 1.40 \times 10^{-6}$	$\pm 1.45 \times 10^{-6}$	$\pm 2.20 \times 10^{-5}$	$\pm 3.79 \times 10^{-6}$	$\pm 1.36 \times 10^{-5}$
1000	2.110×10^{-4}	2.417×10^{-4}	4.425×10^{-3}	5.043×10^{-4}	7.197×10^{-4}
	$\pm 1.09 \times 10^{-6}$	$\pm 1.52 \times 10^{-6}$	$\pm 1.57 \times 10^{-5}$	$\pm 5.78 \times 10^{-6}$	$\pm 1.61 \times 10^{-5}$

Table 58: Point and confidence interval estimates of the ASEs - average square errors - of the estimators

Efficiency of the estimators in the $M/E_2/s+M$ model with $\rho = 1.4$ and $\alpha = 1.0$

s	ASE[θ_{QL_m}]	ASE[θ_{QL_r}]	ASE[θ_{QL}]	ASE[θ_{LES}]	ASE[θ_{NI}]
100	2.314×10^{-3}	2.621×10^{-3}	8.301×10^{-3}	5.285×10^{-3}	8.053×10^{-3}
	$\pm 8.02 \times 10^{-6}$	$\pm 8.95 \times 10^{-6}$	$\pm 2.05 \times 10^{-5}$	$\pm 2.54 \times 10^{-5}$	$\pm 3.98 \times 10^{-5}$
300	7.710×10^{-4}	8.655×10^{-4}	5.470×10^{-3}	1.732×10^{-3}	2.643×10^{-3}
	$\pm 3.77 \times 10^{-6}$	$\pm 5.02 \times 10^{-6}$	$\pm 2.98 \times 10^{-5}$	$\pm 8.66 \times 10^{-6}$	$\pm 4.30 \times 10^{-5}$
500	4.593×10^{-4}	5.160×10^{-4}	4.924×10^{-3}	1.032×10^{-3}	1.572×10^{-3}
	$\pm 2.81 \times 10^{-6}$	$\pm 3.03 \times 10^{-6}$	$\pm 2.98 \times 10^{-5}$	$\pm 6.10 \times 10^{-6}$	$\pm 2.58 \times 10^{-5}$
700	3.278×10^{-4}	3.682×10^{-4}	4.621×10^{-3}	7.417×10^{-4}	1.158×10^{-3}
	$\pm 3.08 \times 10^{-6}$	$\pm 3.06 \times 10^{-6}$	$\pm 2.17 \times 10^{-5}$	$\pm 4.64 \times 10^{-6}$	$\pm 1.61 \times 10^{-5}$
1000	2.331×10^{-4}	2.599×10^{-4}	4.460×10^{-3}	5.257×10^{-4}	8.210×10^{-4}
	$\pm 2.72 \times 10^{-6}$	$\pm 2.47 \times 10^{-6}$	$\pm 3.42 \times 10^{-5}$	$\pm 4.68 \times 10^{-6}$	$\pm 2.15 \times 10^{-5}$

Table 59: Point and confidence interval estimates of the ASEs - average square errors - of the estimators

Efficiency of the estimators in the $M/D/s+E_{10}$ model with $\rho = 1.4$ and $\alpha = 1.0$

s	ASE[$\theta_{QL_{ap}}$]	ASE[θ_{QL_m}]	ASE[θ_{QL_r}]	ASE[θ_{QL}]	ASE[θ_{LES}]	ASE[θ_{NI}]
100	8.678×10^{-3}	1.286×10^{-2}	1.038×10^{-2}	8.464×10^{-2}	1.174×10^{-2}	9.016×10^{-3}
	$\pm 3.04 \times 10^{-3}$	$\pm 3.05 \times 10^{-3}$	$\pm 3.49 \times 10^{-3}$	$\pm 3.17 \times 10^{-3}$	$\pm 3.97 \times 10^{-3}$	$\pm 2.92 \times 10^{-3}$
300	7.377×10^{-3}	1.384×10^{-2}	9.827×10^{-3}	7.902×10^{-2}	7.809×10^{-3}	8.230×10^{-3}
	$\pm 9.69 \times 10^{-4}$	$\pm 1.60 \times 10^{-3}$	$\pm 1.81 \times 10^{-3}$	$\pm 1.56 \times 10^{-3}$	$\pm 1.13 \times 10^{-3}$	$\pm 1.63 \times 10^{-3}$
500	7.344×10^{-3}	1.318×10^{-2}	8.821×10^{-3}	7.714×10^{-2}	6.763×10^{-3}	7.088×10^{-3}
	$\pm 1.18 \times 10^{-3}$	$\pm 9.99 \times 10^{-4}$	$\pm 1.10 \times 10^{-3}$	$\pm 9.55 \times 10^{-4}$	$\pm 5.09 \times 10^{-4}$	$\pm 1.11 \times 10^{-3}$
700	7.336×10^{-3}	1.296×10^{-2}	8.412×10^{-3}	7.628×10^{-2}	5.718×10^{-3}	6.805×10^{-3}
	$\pm 9.29 \times 10^{-4}$	$\pm 1.06 \times 10^{-3}$	$\pm 1.18 \times 10^{-3}$	$\pm 8.65 \times 10^{-4}$	$\pm 4.15 \times 10^{-4}$	$\pm 1.11 \times 10^{-3}$
1000	7.269×10^{-3}	1.303×10^{-2}	8.327×10^{-3}	7.575×10^{-2}	5.316×10^{-3}	6.828×10^{-3}
	$\pm 6.57 \times 10^{-4}$	$\pm 8.15 \times 10^{-4}$	$\pm 9.03 \times 10^{-4}$	$\pm 8.63 \times 10^{-4}$	$\pm 3.16 \times 10^{-4}$	$\pm 8.64 \times 10^{-4}$

Table 60: Point and confidence interval estimates of the ASEs - average square errors - of the estimators

Efficiency of the estimators in the $M/E_{10}/s+E_{10}$ model with $\rho = 1.4$ and $\alpha = 1.0$

s	ASE[$\theta_{QL_{ap}}$]	ASE[θ_{QL_m}]	ASE[θ_{QL_r}]	ASE[θ_{QL}]	ASE[θ_{LES}]	ASE[θ_{NI}]
100	3.539×10^{-3}	7.632×10^{-3}	4.457×10^{-3}	7.940×10^{-2}	6.348×10^{-3}	4.011×10^{-3}
	$\pm 1.91 \times 10^{-5}$	$\pm 1.44 \times 10^{-5}$	$\pm 2.25 \times 10^{-5}$	$\pm 2.75 \times 10^{-4}$	$\pm 2.35 \times 10^{-5}$	$\pm 1.78 \times 10^{-5}$
300	1.295×10^{-3}	6.603×10^{-3}	1.502×10^{-3}	7.181×10^{-2}	2.102×10^{-3}	1.364×10^{-3}
	$\pm 7.50 \times 10^{-6}$	$\pm 1.53 \times 10^{-5}$	$\pm 1.45 \times 10^{-5}$	$\pm 2.90 \times 10^{-4}$	$\pm 1.75 \times 10^{-5}$	$\pm 1.52 \times 10^{-5}$
500	8.642×10^{-4}	6.440×10^{-3}	8.984×10^{-4}	7.001×10^{-2}	1.260×10^{-3}	8.660×10^{-4}
	$\pm 1.16 \times 10^{-5}$	$\pm 1.88 \times 10^{-5}$	$\pm 6.61 \times 10^{-6}$	$\pm 2.56 \times 10^{-4}$	$\pm 1.17 \times 10^{-5}$	$\pm 1.33 \times 10^{-5}$
700	6.752×10^{-4}	6.326×10^{-3}	6.440×10^{-4}	6.923×10^{-2}	9.068×10^{-4}	6.771×10^{-4}
	$\pm 9.87 \times 10^{-6}$	$\pm 9.13 \times 10^{-6}$	$\pm 9.15 \times 10^{-6}$	$\pm 1.84 \times 10^{-4}$	$\pm 1.27 \times 10^{-5}$	$\pm 1.15 \times 10^{-5}$
1000	5.413×10^{-4}	6.230×10^{-3}	4.592×10^{-4}	6.890×10^{-2}	6.406×10^{-4}	5.547×10^{-4}
	$\pm 8.62 \times 10^{-6}$	$\pm 2.03 \times 10^{-5}$	$\pm 4.29 \times 10^{-6}$	$\pm 2.70 \times 10^{-4}$	$\pm 6.66 \times 10^{-6}$	$\pm 1.37 \times 10^{-5}$

Table 61: Point and confidence interval estimates of the ASEs - average square errors - of the estimators

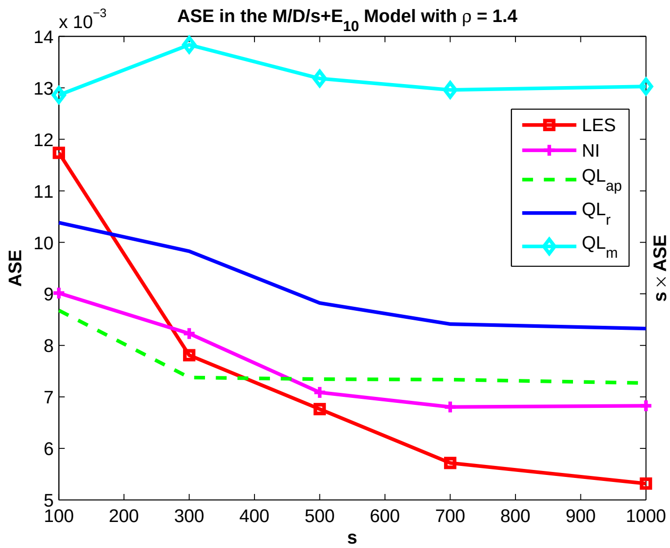

Figure 1:

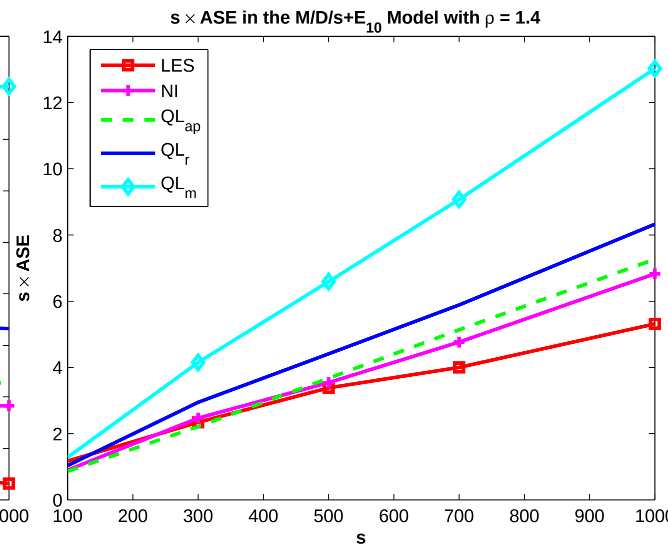

Figure 2:

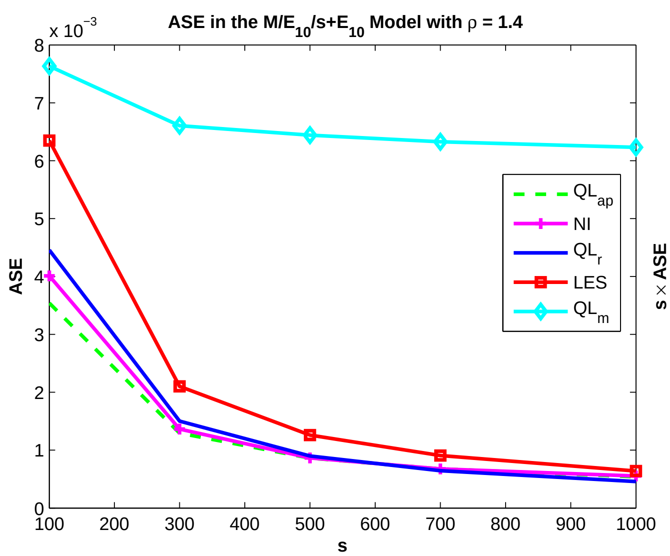

Figure 3:

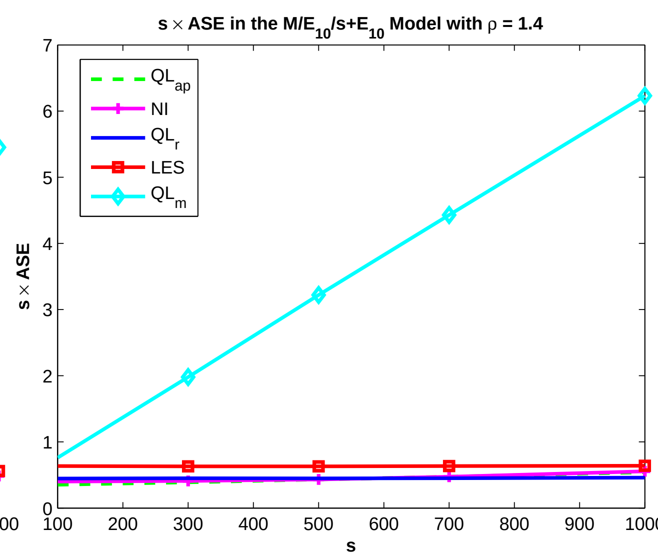

Figure 4: