**Statistical Analysis with Little's Law**

**Supplementary Material: More on the Call Center Data**

by
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# 1 Introduction

In this supplement to the main paper [1], we provide more details about the call center data we used. As explained in the main paper, the data are from a telephone call center of a medium-sized American bank from the data archive of Mandelbaum [2], collected from March 26, 2001 to October 26, 2003. This banking call center had sites in New York, Pennsylvania, Rhode Island, and Massachusetts, which were integrated to form a single virtual call center. The virtual call center had $900 - 1200$ agent positions on weekdays and $200 - 500$ agent positions on weekends. The center processed about $300,000$ calls per day during weekdays, with about $60,000$ (20%) handled by agents, with the rest being served by integrated voice response (IVR) technology. As in many modern call centers, in this banking call center there were multiple agent types and multiple call types, with a form of skill-based routing (SBR) used to assign calls to agents.

Since we were only concerned with estimation related to the three parameters $L$, $\lambda$ and $W$, we did not get involved with the full complexity of this system. Specifically, we used data for only one class of customers, denoted by **Summit**. Furthermore, among them, only the sub-calls that had agent interactions during weekdays in May 2001 were considered.

The rest of this supplement is organized as follows. In §2 we briefly describe the full database of Mandelbaum [2] and how we extracted the required data in order to produce the results in the main paper. In §3 we describe the statistics collected for the 18 weekdays in May we used in our analysis and give an overview of the statistics.

# 2 The Data Available and Used

The full database of [2] provides nine pre-processed ACCESS tables for each day in the study period, from March 26, 2001 to October 26, 2003. In the pre-processing, issues such as midnight calls, incorrect time stamps and incorrect identifiers (id's) are already taken care of. As shown on the left pane of Figure 1, the nine database tables are titled: `calls`, `customer sub-calls`, `server sub-calls`, `queue records`, `event details`, `agent events`, `agent profile`, `agent records`, and `agent shifts`. The `calls` table includes general information on each call that enters the call center on a particular day. Each call then consists of sub-call(s) that start and end with a particular service such as IVR, agent interaction and announcement. We focus on the sub-calls of **Summit** customers that involve agent interaction, and hence use only the `customer sub-calls` table of each day.

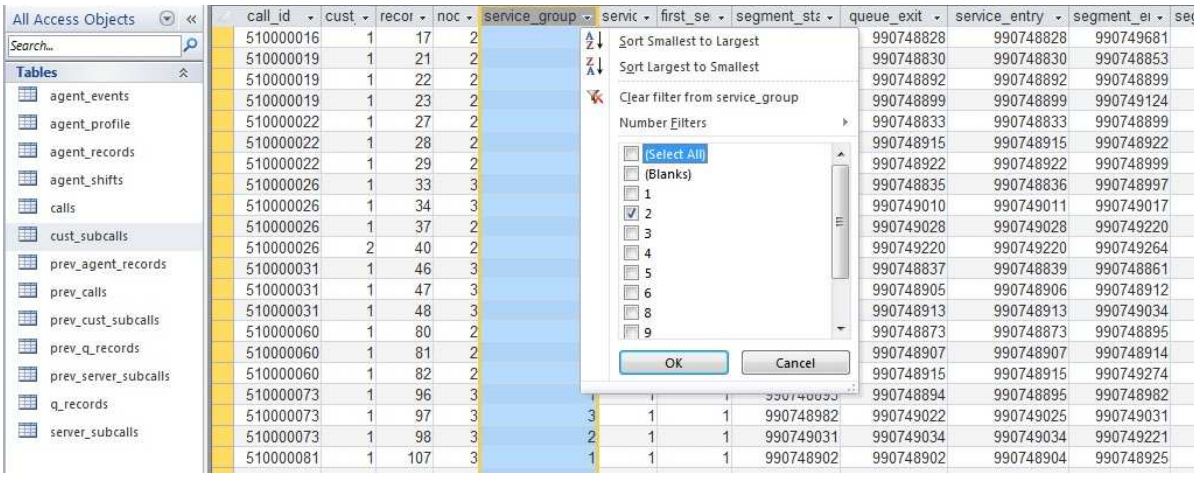


Figure 1: Example of ACCESS tables: call center data from May 25, 2001.

Figure 1 is an example of the `customer sub-calls` table. There are 23 fields in the table, which are: *call_id*, *cust_subcall*, *server_subcall*, *record_id*, *node*, *customer_id*, *customer_type*, *service_group*, *service*, *first_service*, *segment_start*, *queue_exit*, *service_entry*, *segment_end*, *seg_type*, *outcome*, *seg_parties*, *wait_time*, *queue_time*, *preservice_wait*, *service_time*, *hold_time*, and *party_answered*. More information about the different tables, including detailed descriptions of each field, can be found at:

> http://ie.technion.ac.il/Labs/Serveng/files/
> Model_Description_and_Introduction_to_User_Interface.pdf

To create the data set we used in [1], we used the following steps:

- Each sub-call is served by a service group. There are five main service groups, which are IVR, Business line, non-Business line, Announcement and Message. We kept the sub-calls that were handled by the Business line (*service_group* = 2). In the ACCESS `customer sub-calls` table, we filtered out these sub-calls by selecting *service_group* = 2, as illustrated in Figure 1.

- We kept the sub-calls that are from **Summit** customers by keeping records with *service* = 14 (The *service* field indicates the type of service received by the caller. For example, there are *Retail* = 1, *Premier* = 2, *Business* = 3 and *Platinum* = 4).

- We dropped the records with no agent interaction, which involve the caller hanging up (abandoning) while waiting to speak to the next agent. This was done by dropping records with *outcome* = 11, 12, or 13. The *outcome* field indicates the cause of call termination such as whether they were handled, transferred and abandoned. *outcome* = 11 indicates the customer abandoned short (the caller abandons within an abandon threshold time), *outcome* = 12 indicates the customer abandoned (after the abandon threshold time) and *outcome* = 13 indicates the call was not handled with other reason that is not specified in the data).

- To ensure that each sub-call spent positive amount of time with an agent, we omitted records with *service_time* = 0, where *service_time* is defined as the sum of talk time and hold time. It can also be defined as the difference between *segment_end* and *service_entry*. Since we already dropped the records with customer abandonment, there were not many records with *service_time* = 0, less than 5 for each day.

- In order to compute the three parameters $L$, $\lambda$ and $W$, we used the time each sub-call enters the queue, leaves the queue (hence enters the service) and leaves the service. Therefore, we kept only the fields *call_id*, *segment_start* (queue entry time), *queue_exit* (queue exit time), *service_entry* (service entry time) and *segment_end* (service exit time). (The time stamps are records in seconds, using the origin time, 00:00:00 on 01/01/1970.

- Finally, we exported the table to an EXCEL file using the "Export to Excel spreadsheet" function.

The steps above were carried out by the authors for 18 weekdays of May. (In the next section we explain how the 18 days were selected.) The combined data set for all 18 weekdays (*little_weekdays_in_May.xls*) is available from the authors' web sites.

## 3 Statistics from Eighteen Weekdays in May, 2001

There were 23 weekdays in May 2001. (May 1, 2001, was a Tuesday.) Four weekdays were not normal, and so were excluded, for the following reasons:

- May 9 (Wed): shutdown from 4:53:10 AM until 11:28:54 AM

- May 10 (Thurs): shutdown from 2:59:18 PM until 11:31:24 PM

- May 28 (Mon): Memorial Day

- May 31 (Thurs): data missing

In addition, the data from May 3 were excluded because the number of arrivals was extraordinarily high. In particular, the number of arrivals was 8310 on May 3 with about 50% arriving before 9 AM, whereas for the other 18 weekdays, the average number of arrivals was 5410.5 arrivals, with a standard deviation 1080.5. For each day, there were data over a 17-hour period, from 6 AM to 11 PM, referred to as $[6, 23]$. (There were no arrivals before 6 AM and after 11 PM.)

We primarily focused on the number of Summit customers in the system, but we also considered whether they were in service or waiting (in queue). Thus we measured the numbers in the system, in service and in queue. Similarly, we measured the time that each customer spent in the system, in the queue and in service.

Using the data set *little_weekdays_in_May.xls*, we collected the following statistics:

- $L_{sys}$: the number in system

- $L_{ser}$: the number in service

- $L_q$: the number in queue

- $A_{sys} = A_q$: number of arrivals into the system/queue
- $A_{ser}$: number of arrivals into service
- $W_{sys}$: time spent in the system
- $W_{ser}$: time spent in service
- $W_q$: time spent in queue

These are understood to be functions of the measurement interval. For example, $A_{sys} \equiv A_{sys}([9,10])$ is the number of arrivals into the system during the interval $[9,10]$. For the interval $[9,10]$, the average arrival rate per minute is

$$\bar{\lambda}(t) \equiv \frac{A_{sys}([9,10])}{m([9,10])} = \frac{A_{sys}([9,10])}{60}, \tag{1}$$

where $m([9,10]) = 60$ is the number of minutes in the interval $[9,10]$. Thus the statistics are consistent with the definitions in equation (1) of [1].

### 3.1 The Hourly Arrival Rates

Figure 2 shows the overall average (over the full 17-hour day) and the hourly averages of the arrival rates per minute, as defined in (1), together with estimates of the 95% confidence interval (treating the daily values as i.i.d. Gaussian variables) for the 18 weekdays in May. Figure 2 shows that the arrival rate is nonstationary over the day. Figure 2 also shows that the arrival rate is highly variable from day to day, because of the wide confidence intervals for the hourly averages. Part of this day-to-day variation can be explained by day-of-week effect. Figure 3 shows that the average call volume on Mondays is the largest, followed by that of Tuesdays, and then the others. Figures 4-8 further illustrate day-to-day variation in the same day of week.

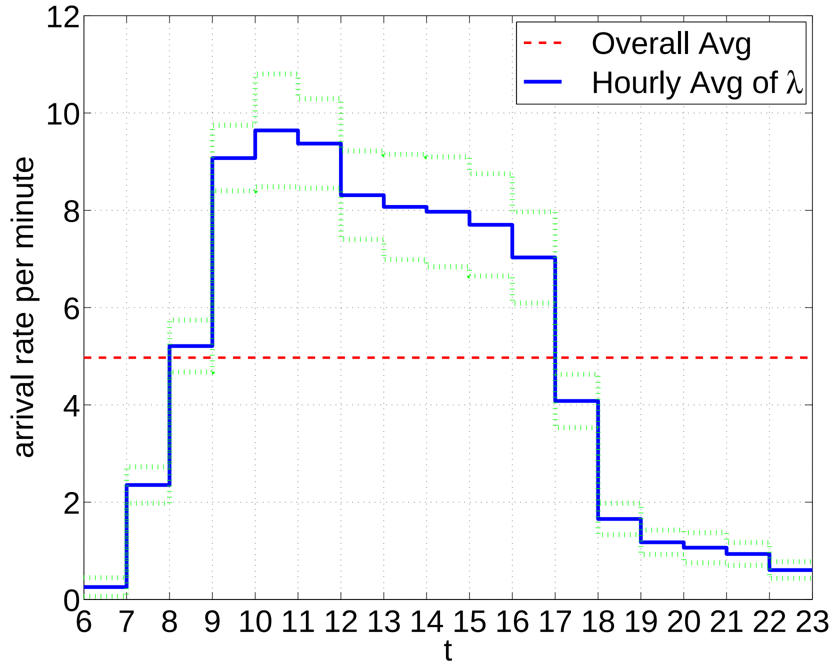


Figure 2: Overall average and hourly average of $\lambda$ and its 95% confidence interval over 18 weekdays in May.

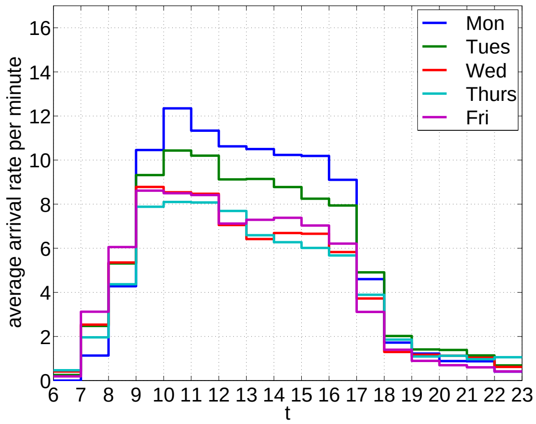


Figure 3: Average arrival rate and the day-of-week effect in May.

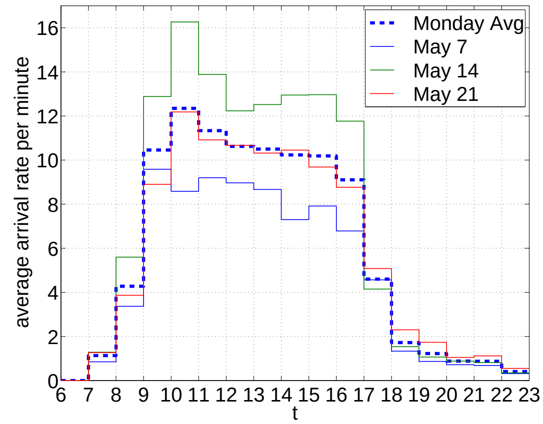


Figure 4: Average arrival rate of Mondays in May.

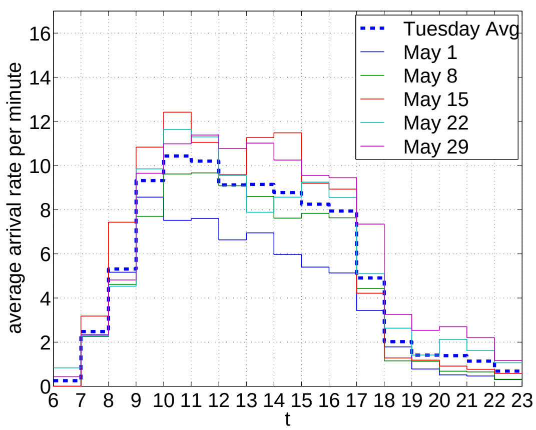


Figure 5: Average arrival rate of Mondays in May.

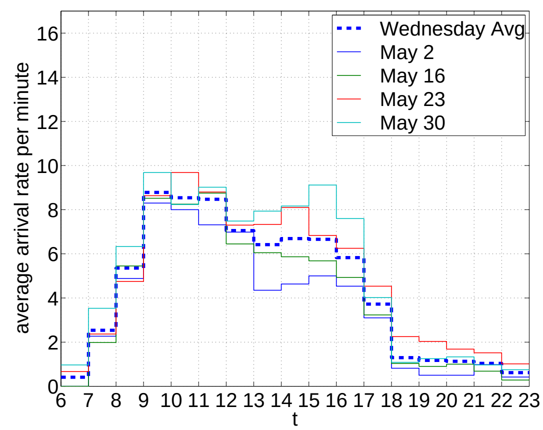


Figure 6: Average arrival rate of Wednesdays in May.

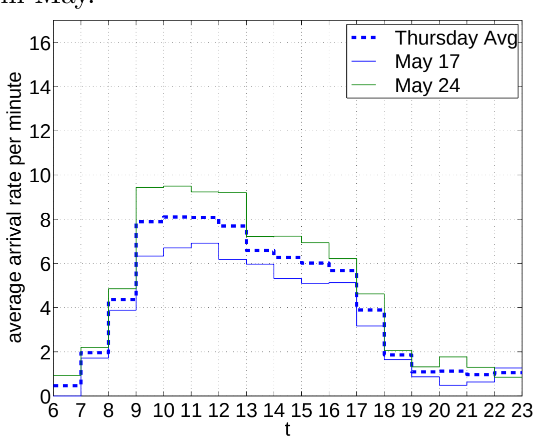


Figure 7: Average arrival rate of Thursdays in May.

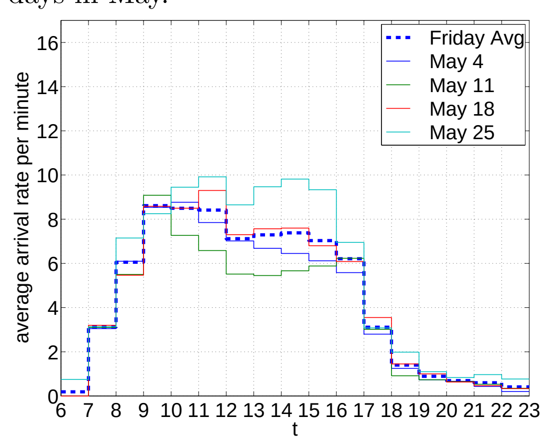


Figure 8: Average arrival rate of Fridays in May.

## 3.2 The Hourly Average Waiting Times

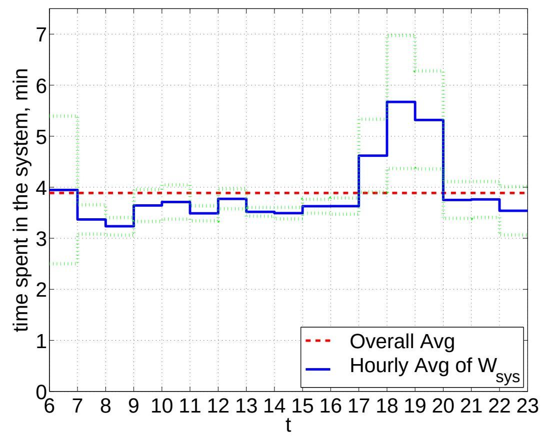


Figure 9: Overall average and hourly average of $W_{sys}$ and its 95% confidence interval over 18 weekdays in May.

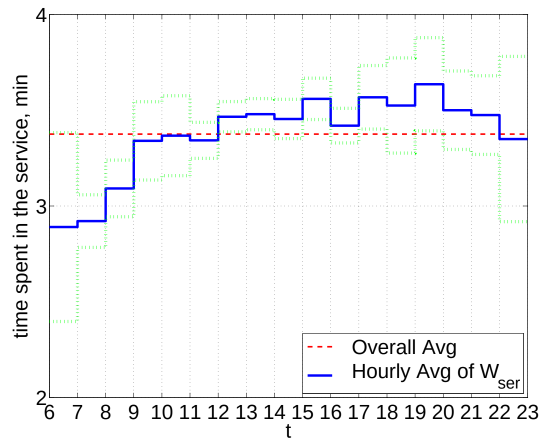


Figure 10: Overall average and hourly average of $W_{ser}$ and its 95% confidence interval over 18 weekdays in May.

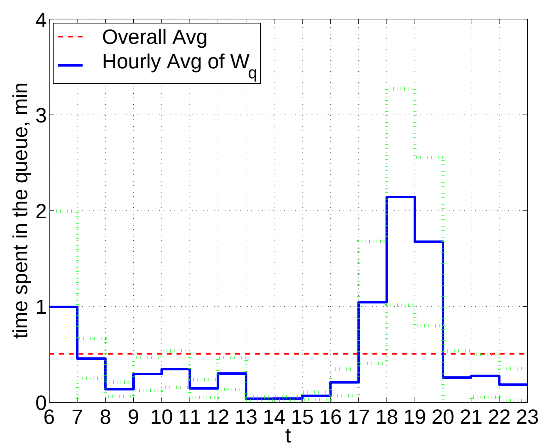


Figure 11: Overall average and hourly average of $W_q$ and its 95% confidence interval over 18 weekdays in May.

Figures 9-11 show the overall average (average of the 18 daily averages) and hourly average of the waiting time in the system, service and queue and their 95% confidence interval of 18 weekdays in May. Figure 10 suggests that the service times are approximately stationary over time. However, by comparing Figure 10 to Figure 9, we can conclude that it is hard to say that the times in system is approximately equal to the service times because the time in queue is too long in the interval [17,20], which might be due to inadequate staffing during this interval. Furthermore, Figure 11 suggests the waiting times in system is not approximately stationary over time, again possibly due to inadequate staffing.

Next, Figure 12 shows the histogram of all waiting times in the interval $[10, 16]$ of Friday, May 25, 2001 in our call center example. In addition, Figure 13 and Figure 14 illustrate the histograms of all waiting times and service times in the interval $[9, 17]$ over 18 weekdays in May in our call center example. As usual for call centers, the distribution is approximately lognormal, but the SCV very close to 1 indicates that an exponential approximation is reasonable.

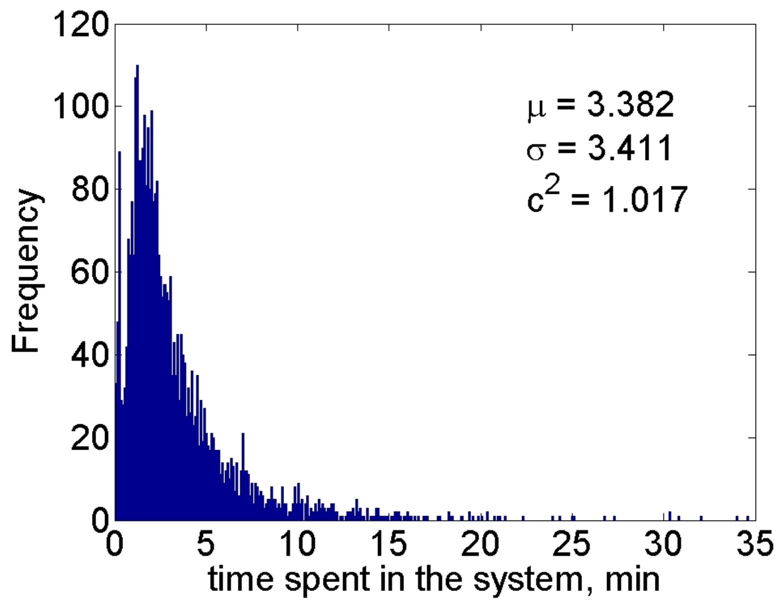


Figure 12: The histogram (empirical distribution) of the times spent in the system of all arrivals during the interval $[10, 16]$.

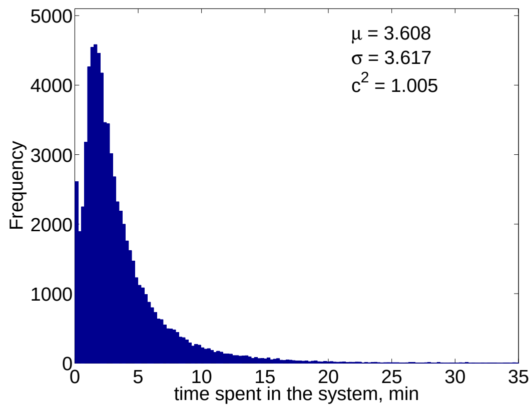


Figure 13: The histogram (empirical distribution) of the times spent in the system of all arrivals during the interval $[9, 17]$ over 18 weekdays in May ($n = 72,535$ and 41 observations that had $W_{sys} > 35$ are not represented).

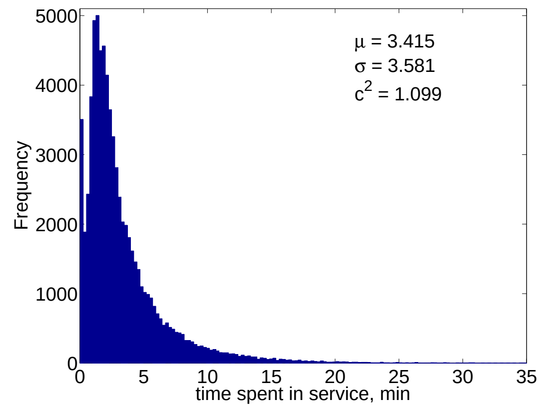


Figure 14: The histogram (empirical distribution) of the times spent in service of all arrivals to the service during the interval $[9, 17]$ over 18 weekdays in May ($n = 72,494$ and 39 observations that had $W_{ser} > 35$ are not represented).

### 3.3 The Hourly Average Number in System)

Figures 15-17 show the overall average (average of the 18 daily averages) and hourly average number in the system, service and queue and their 95% confidence interval of 18 weekdays in May. We observe that the number in system is approximately equal to the number in service, except at the times when there are slightly longer queues, in the intervals [9,13] and [17,20].

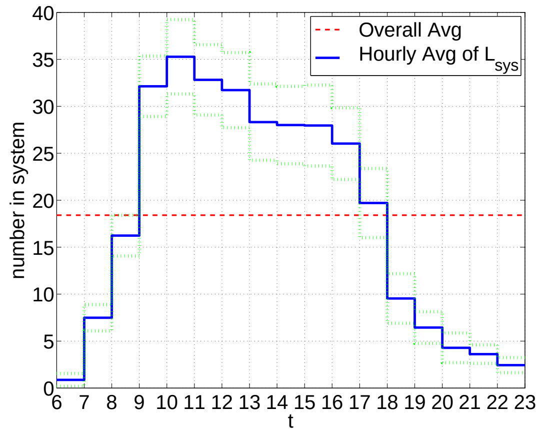


Figure 15: Overall average and hourly average of $L_{sys}$ and its 95% confidence interval over 18 weekdays in May.

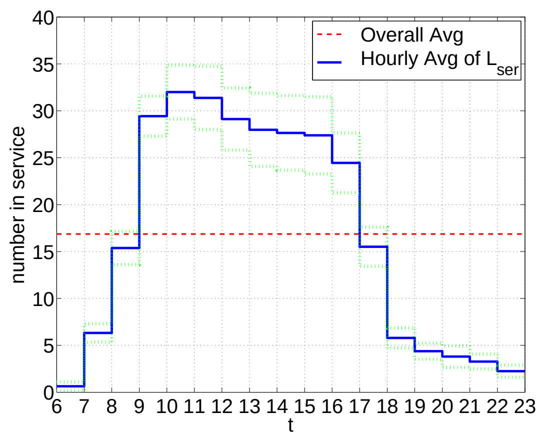


Figure 16: Overall average and hourly average of $L_{ser}$ and its 95% confidence interval over 18 weekdays in May.

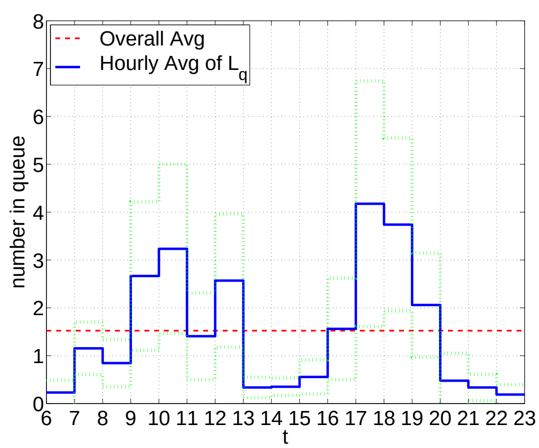


Figure 17: Overall average and hourly average of $L_q$ and its 95% confidence interval over 18 weekdays in May.

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## References

[1] Kim, S., W. Whitt. 2012. Statistical analysis with Little's law, Available from: http://www.columbia.edu/∼ww2040/allpapers.htm

[2] Mandelbaum, A. 2012. Service Engineering of Stochastic Networks web page: http://iew3.technion.ac.il/serveng/