Online Supplement to "A Rare-Event Simulation Algorithm for Periodic Single-Server Queues"

Ni Ma
Industrial Engineering and Operations Research, Columbia University,nm2692@columbia.edu,

Ward Whitt
Industrial Engineering and Operations Research, Columbia University, ww2040@columbia.edu,
http://www.columbia.edu/ ww2040

(from the main paper) An efficient algorithm is developed to calculate the approximate steady-state distribution of the remaining workload W_y at time yc that within a cycle of length c in a general $G_t/G/1$ single-server queue with a periodic arrival-rate function. A recent heavy-traffic functional functional central limit theorem (FCLT) shows that, if the arrival-rate function is appropriately scaled, then there is a reflected periodic Brownian motion limit that depends on the underlying stochastic processes only through a joint FCLT for the arrival and service processes. Thus, as a first step, we approximate the general $G_t/G/1$ model by an associated $GI_t/GI/1$ model, where a periodic arrival process with the same arrival-rate function is constructed as a deterministic time transformation of a renewal process, with the squared coefficient of variation of the time between renewals set equal to the asymptotic variability parameter in the G_t arrival process FCLT. That definition plus a corresponding construction for the service times guarantees that both models have the same heavy traffic limit. We then compute the exact tail probabilities $P(W_y > b)$ for the approximating $GI_t/GI/1$ model by exploiting a modification of the classic rare-event simulation exploiting importance sampling using an exponential change of measure. We show that the relative error is uniformly bounded in b. That algorithm is then applied to compute related performance measures, such as the mean $E[W_y]$ and variance $Var(W_y)$. Simulation examples demonstrate the accuracy and efficiency of the algorithm, illustrate the heavy-traffic limit and reveal the performance impact of the periodicity.

Key words: periodic queues, ruin probabilities, non-Poisson nonstationary arrival processes, rare-event simulation, exponential change of measure, heavy traffic, reflected periodic Brownian motion
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1. Introduction

This is an online supplement to the main paper Ma and Whitt (2016). In §2 we elaborate on §3 of the main paper. First, we further discuss the tail asymptotics and the asymptotic decay rate needed in the simulation. At the end, we present a couple of additional bounds and approximations. In §3 we report results of additional simulation experiments applying the algorithm developed in the main paper.

2. More Results on the Asymptotics

In this section we continue the discussion of the tail asymptotics in §3.2 of the main paper. We start in §2.1 by conjecturing the asymptotic form of the periodic steady-state distribution of RPBM. Then in §2.2 we review an asymptotic expansion for the stationary $GI/GI/1$ model from Abate and Whitt (1994), which yields an asymptotic expansion for the periodic $GI_t/GI/1$ model, which yields an approximation for the asymptotic decay rate needed in the simulation. In §2.3 we compare the approximate values of the asymptotic decay rate to exact values. Then, in §2.4 we derive the exact form of the asymptotic decay rate for hyperexponential models. Finally, in short §2.5 and 2.6 we briefly discuss other bounds and heuristic approximations.

2.1. Tail Asymptotics for RPBM

It remains to establish tail asymptotics for the periodic steady-state distribution of RPBM. However, we can see the form that tail asymptotics should take from the heavy-traffic scaling and the tail asymptotics established for the $G_t/G/1$ model in §3.2 of the main paper.

Let $Z_y(\infty; c_x)$ be the periodic steady-state distribution of RPBM with variability parameter c_x as in Theorem 2 of the main paper. From Corollary 3, we are led to conjecture that

$$e^{\theta^* b} P(Z_y(\infty; c_x) > b) \to A_y \quad \text{as} \quad b \to \infty, \tag{1}$$

for some constant A_y and

$$\theta^* = \lim_{\rho \uparrow 1} \theta^*_\rho/(1-\rho), \tag{2}$$

where θ^*_ρ is the associated asymptotic decay rate for a family of $G_t/G/1$ models converging to RPBM. We remark that there is a limit-interchange problem for the tail probability asymptotics, closely paralleling the limit-interchange problem associated with the heavy-traffic limit discussed in §6 of the main paper.

Moreover, the asymptotic decay rate of the steady-state distribution of RPBM should coincide with that of RBM, which directly has an exponential steady-state distribution, i.e.,

$P(Z(\infty;c_x) > b) = e^{-2b/c_x^2}$. In the next section we provide support for (2). Our numerical results show how to compute the tail probability $P(Z_y(\infty;c_x) > b)$ assuming that these limits are valid.

2.2. Asymptotic Expansions for the Asymptotic Decay Rate

We can develop useful approximations for the asymptotic decay rate needed in the simulation and we can provide support for (2) making the connection to RPBM in §2.1 by applying asymptotic expansions established for the $GI/GI/1$ model (and more general multichannel queueing models) in Abate and Whitt (1994); corresponding asymptotic expansions for $MAP/GI/1$ queues were established in Choudhury and Whitt (1994). From (4) and (18) of Abate and Whitt (1994), we get the following result. As in the main paper, we fix the service process and introduce the traffic intensity ρ by scaling time in a rate-1 arrival process. That produces a well-defined model as a function of the traffic intensity, where we only change the arrival rate, which we denote by the subscript ρ, as in the main paper.

THEOREM 1. (*asymptotic expansion from Abate and Whitt (1994)*) *For the $GI/GI/1$ model, and thus also the periodic $GI_t/GI/1$ model,*

$$\theta_\rho^* = \frac{2(1-\rho)}{c_a^2 + c_s^2} + C(1-\rho)^2 + O((1-\rho^3)) \quad as \quad \rho \uparrow 1, \tag{3}$$

where C depends on the first three moments of the mean-1 interarrival time U_k and service time V_k, but not ρ, via

$$C \equiv C(c_a^2, d_a; c_s^2, d_s) \equiv \left(\frac{8(d_s - d_a)}{(c_a^2 + c_s^2)^3} - \frac{2(c_a^2 - c_s^2)}{(c_a^2 + c_s^2)^2} \right), \tag{4}$$

with $d_s \equiv (E[V_k^3] - 3c_s^2(c_s^2+1) - 1)/6$. and similarly for d_a using the interarrival time.

In §2.1, we have suggested that we can calculate the RPBM periodic steady-state tail probabilities $P(Z_y(\infty;c_x) > b)$ by calculating associated tail probabilities $P(W_y > b)$ for $GI_t/GI/1$ queues. Now we show that we may be able to choose two different $GI_t/GI/1$ queues that will bound the desired RPBM tail probabilities above and below, and thus bound the error. The following result only applies to the rates, but it explains what we have seen in numerical examples; see Table 1 below and the ratios $P(W_y > b/P(W > b)$ in Tables 5 and 6 in the main paper.

Corollary 1. (*switching interarrival-time and service-time distributions*) *If we switch the interarrival-time and service-time distributions without altering their mean values, and thus switch the pairs* (c_a^2, d_a) *and* (c_s^2, d_s)*, then* C *in* (4) *is unchanged except for its sign, which is reversed. Thus, the one-term asymptotic approximation for* $\theta^*(\rho)$ *is bounded above and below by these special two-term approximations.*

2.3. Approximations for the Asymptotic Decay Rate

In §2.4 we discuss the exact values for the asymptotic decay rates in the special parametric cases in §6.4 of the main paper. For $M_t/M/1$, $\theta^* \equiv \theta_\rho^* = 1-\rho$. For both $M_t/H_2/1$ and $(H_2)_t/M/1$, θ^* is obtained as the solution of quadratic equations. Taylor series approximations produce asymptotic expansions that are consistent with (3).

Table 1 compares the 1-term and 2-term approximations for the asymptotic decay rate θ_ρ^* from the asymptotic expansion in (3) with the exact values for the $M_t/H_2/1$ and $(H_2)_t/M/1$ models, where the H_2 distribution has $c^2 = 2.0$ and balanced means. The actual values θ_ρ^* and the scaled values $\theta_\rho^*/(1-\rho)$ are shown for 6 values of $1-\rho$. (The actual values appear in the top two sections, while the scaled values appear in the bottom two sections.) The asymptotic decay rate for RBM and RPBM are obtained directly from the first term. Table 1 shows that the 2-term approximation can serve as an explicit formula for θ_ρ^* provided that ρ is not too small.

Assuming appropriate limit interchanges are valid, the asymptotic decay rate for RPBM is the same as for RBM, and that common value can be obtained directly from the first term in (3). Assuming that limits for the steady-state quantities follow from the process limits in the HT FCLT in Theorem 2 of the main paper, $(1-\rho)W_{\rho,y} \Rightarrow Z_y(\infty; c^2)$, where $Z_y(\infty; c^2)$ has the steady-state distribution of RPBM. Assuming that the decay rates converge, we should have

$$\theta^* = \lim_{\rho\uparrow 1} \theta_\rho^*/(1-\rho) = 2/(c_a^2 + c_s^2) \qquad (5)$$

from (3). For ordinary RBM, this is immediate because RBM has an exponential steady-state distribution. Since the asymptotic decay rate of $(1-\rho)W_{\rho,y}$ and $(1-\rho)W_\rho$ agrees for all ρ, the same will be true for the limits, provided that the limit interchange is valid.

2.4. Exact Values for the Asymptotic Decay Rate

We now give the exact values for the asymptotic decay rates in the special parametric cases considered in in §3.3 and §4.5 of the main paper. First, for $M_t/M/1$, $\theta^* \equiv \theta_\rho^* = 1-\rho$. For

Table 1 **A comparison of the 1-term and 2-term approximations for the asymptotic decay rate θ^*_ρ from the asymptotic expansion in (3) with the exact values for the $M_t/H_2/1$ and $(H_2)_t/M/1$ models, where the H_2 distribution has $c^2 = 2.0$ and balanced means: The actual values θ^*_ρ and the scaled values $\theta^*_\rho/(1-\rho)$ are shown for 6 values of $1-\rho$.**

	$1-\rho=0.16$	$1-\rho=0.08$	$1-\rho=0.04$	$1-\rho=0.02$	$1-\rho=0.01$	$1-\rho=0.005$
$M_t/H_2/1$ queue						
θ^*						
exact	0.10069	0.05187	0.02631	0.01324	0.006644	0.003328
first term	0.10667	0.05333	0.02667	0.01333	0.006667	0.003333
first two terms	0.10098	0.05191	0.02631	0.01324	0.006644	0.003328
$(H_2)_t/M/1$ queue						
θ^*						
exact	0.11299	0.05483	0.02703	0.01342	0.006689	0.003339
first term	0.10667	0.05333	0.02667	0.01333	0.006667	0.003333
first two terms	0.11236	0.05476	0.02702	0.01342	0.006689	0.003339
$M_t/H_2/1$ queue						
$\theta^*/(1-\rho)$						
exact	0.62934	0.64843	0.65766	0.66219	0.66444	0.66555
first term	0.66667	0.66667	0.66667	0.66667	0.66667	0.66667
first two terms	0.63111	0.64889	0.65778	0.66222	0.66444	0.66556
$(H_2)_t/M/1$ queue						
$\theta^*/(1-\rho)$						
exact	0.70619	0.68542	0.67580	0.67117	0.66890	0.66778
first term	0.66667	0.66667	0.66667	0.66667	0.66667	0.66667
first two terms	0.70222	0.68444	0.67556	0.67111	0.66889	0.66778

both $M_t/H_2/1$ and $(H_2)_t/M/1$, θ^* is obtained as the solution of quadratic equations. The other cases are: $(M+D)_t/M/1$, $M_t/M+D/1$ and $(M+D)_t/(M+D)/1$. The final one is important to treat cases with $c_a^2 + c_s^2 < 1$. The first two cover $1 < c_a^2 + c_s^2 < 2$. We may also want others such as $(H_2)_t/H_2/1$.

We now discuss the exact values for asymptotic decay rates in the special parametric cases in §3.3 of the main paper. In the $GI_t/GI/1$ model, let λ be the average arrival rate and μ be the service rate, then the optimal θ^* is the same as for the $GI/GI/1$ model with rate-λ i.i.d. inter-arrival times U_k and rate-μ i.i.d. service times V_k. First, for $M_t/M/1$, $\theta^* = \mu - \lambda$ and $\theta^* \equiv \theta^*_\rho = 1-\rho$ as a function of ρ if we let $\mu = 1$.

For both the $M_t/H_2/1$ and $(H_2)_t/M/1$ models, θ^* is obtained as the solution of quadratic equations. In the $M_t/H_2/1$ model, let V_k has density $h(x) = p_1\mu_1 e^{-\mu_1 x} + p_2\mu_2 e^{-\mu_2 x}$, where $(p_1/\mu_1) + (p_2/\mu_2) = 1/\mu$. We solve

$$E[e^{\theta^* V}]E[e^{-\theta^* U}] = 1$$

for θ^*, so that

$$E[e^{\theta^* V}]E[e^{-\theta^* U}] = (p_1\mu_1/(\mu_1 - \theta^*) + p_2\mu_2/(\mu_2 - \theta^*))(\lambda/(\lambda + \theta^*)) = 1.$$

This reduces to the quadratic equation

$$(\theta^*)^2 + (\lambda - \mu_1 - \mu_2)\theta^* + (\mu_1\mu_2 - p_2\mu_1\lambda - p_1\mu_2\lambda) = 0$$

or

$$(\theta^*)^2 + (\lambda - \mu_1 - \mu_2)\theta^* + (1-\rho)\mu_1\mu_2 = 0.$$

Hence,

$$\theta^* = [(\mu_1 + \mu_2 - \lambda) \pm \sqrt{\lambda^2 - 2(\mu_1+\mu_2)\lambda + \mu_1^2 + \mu_2^2 + (4\rho - 2)\mu_1\mu_2}]/2,$$

where we choose the value that is appropriate, i.e., satisfying $\mu_1 - \theta^* > 0$, $\mu_2 - \theta^* > 0$, $\lambda + \theta^* > 0$.

Similarly for the $(H_2)_t/M/1$ model, let U_k has density $g(x) = p_1\lambda_1 e^{-\lambda_1 x} + p_2\lambda_2 e^{-\lambda_2 x}$, with $(p_1/\lambda_1) + (p_2/\lambda_2) = 1/\lambda$. Thus, we solve

$$E[e^{\theta^* V}]E[e^{-\theta^* U}] = (\mu/(\mu - \theta^*))(p_1\lambda_1/(\lambda_1 + \theta^*) + p_2\lambda_2/(\lambda_2 + \theta^*)) = 1,$$

which reduces to

$$(\theta^*)^2 + (\lambda_1 + \lambda_2 - \mu)\theta^* + (\lambda_1\lambda_2 - p_2\lambda_1\mu - p_1\lambda_2\mu) = 0$$

or

$$(\theta^*)^2 + (\lambda_1 + \lambda_2 - \mu)\theta^* + \lambda_1\lambda_2(1 - 1/\rho) = 0$$

which has solution

$$\theta^* = [-(\lambda_1 + \lambda_2 - \mu) \pm \sqrt{\mu^2 - 2(\lambda_1+\lambda_2)\mu + \lambda_1^2 + \lambda_2^2 + (4/\rho - 2)\lambda_1\lambda_2}]/2,$$

where we choose the value that is appropriate.

We now briefly discuss other cases, namely, $(M+D)_t/M/1$, $M_t/M+D/1$ and $(M+D)_t/(M+D)/1$. The final one is important to treat cases with $c_a^2 + c_s^2 < 1$. The first two cover $1 < c_a^2 + c_s^2 < 2$. In all of these cases, we need to solve transcendental equations to get θ^*, which is done numerically using Newton's or bisection method. For example, in $(M+D)_t/M/1$ queue, let U_k have parameter pair (d, λ') such that

$$\frac{e^{\lambda' d}}{\lambda'} = \frac{1}{\lambda}$$

. We solve

$$E[e^{\theta^* V}]E[e^{-\theta^* U}] = \frac{\mu}{\mu - \theta^*} e^{-\theta^* d} \frac{\lambda'}{\lambda' + \theta^*} = 1,$$

or

$$(\theta^*)^2 - (\mu - \lambda')\theta^* + \mu\lambda'(e^{-\theta^* d} - 1) = 0.$$

We obtain the following proposition when we compare the exact values of θ^* with the aymptotic expansion in (3).

Proposition 1. *The exact values of θ^* for $M_t/H_2/1$ and $(H_2)_t/M/1$ models are consistent with the two-term asymptotic expansion in* (3).

Proof. We only do this proof for $M_t/H_2/1$ model here; the proof for $(H_2)_t/M/1$ model is similar. For the $M_t/H_2/1$ model, the interarrival time is exponential with $c_a^2 = 1$ and $E[U_k^3] = 6$, then the first term in (3) becomes $2(1-\rho)/(1+c_2^2)$. From (4), the coefficient of the second term is

$$C = \frac{-4(E[V_k^3] - 3c_s^2(c_s^2+1) - E[U_k^3] + 3c_a^2(c_a^2+1)) - 6((c_s^2)^2 - (c_a^2)^2)}{3(c_a^2+c_s^2)^3}$$
$$= \frac{-4E[V_k^3] + 6(c_s^2)^2 + 12c_s^2 + 6}{3(1+c_s^2)^3} \qquad (6)$$

Without loss of generality, we let $\mu = 1$ and thus $\lambda = \rho$. For $M_t/H_2/1$ queue,

$$\theta^* = [(\mu_1 + \mu_2 - \rho) - \sqrt{\rho^2 - 2(\mu_1+\mu_2)\rho + \mu_1^2 + \mu_2^2 + (4\rho - 2)\mu_1\mu_2}]/2 \qquad (7)$$

We use a change of variable with $x = 1-\rho$ and substitute ρ with $1-x$ in (7):

$$\begin{aligned}\theta^* &= \frac{1}{2}(\mu_1+\mu_2-1+x) - \frac{1}{2}\sqrt{x^2 + (2(\mu_1+\mu_2) - 2 - 4\mu_1\mu_2)x + (\mu_1+\mu_2-1)^2} \\ &\equiv \frac{1}{2}(\mu_1+\mu_2-1+x) - \frac{1}{2}f(x) \\ &= \frac{1}{2}(\mu_1+\mu_2-1+x) - \frac{1}{2}(f(0) + f'(0)x + \frac{1}{2}f''(0)x^2 + O(x^3)), \qquad (8)\end{aligned}$$

where we define the function $f(x)$ and do taylor series expansion to get the first two terms of $f(x)$.

First, we look at the constant term of θ^* in (8), it equals

$$\frac{1}{2}(\mu_1+\mu_2-1-f(0)) = \frac{1}{2}(\mu_1+\mu_2-1-|\mu_1+\mu_2-1|) = 0.$$

Because $(p_1/\mu_1) + (p_2/\mu_2) = 1$, we have $(p_1/\mu_1) < 1$ and $(p_2/\mu_2) < 1$. Hence, $\mu_1 + \mu_2 > p_1 + p_2 = 1$ and $|\mu_1+\mu_2-1| = \mu_1+\mu_2-1$. This is consistent with (3) which has no constant term.

Second, we consider the first-order term in the Taylor expansion of θ^* in (8). It equals

$$\begin{aligned}\frac{1}{2}(1-f'(0)) &= \frac{1}{2}(1 - \frac{1}{2}f(x)^{-\frac{1}{2}}(2x + 2(\mu_1+\mu_2) - 2 - 4\mu_1\mu_2)|_{x=0}) \\ &= \frac{1}{2}(1 - \frac{(\mu_1+\mu_2) - 1 - 2\mu_1\mu_2}{\mu_1+\mu_2-1})\end{aligned}$$

$$
\begin{aligned}
&= \frac{\mu_1\mu_2}{\mu_1+\mu_2-1} \\
&= \frac{\mu_1^2\mu_2^2}{(\mu_1+\mu_2-1)\mu_1\mu_2} \\
&= \frac{\mu_1^2\mu_2^2}{(\mu_1+\mu_2-1)(p_1\mu_2+p_2\mu_1)} \\
&= \frac{\mu_1^2\mu_2^2}{p_1\mu_2^2+p_2\mu_1^2} \\
&= \frac{2}{1+c_s^2},
\end{aligned}
$$

where $p_1\mu_2 + p_2\mu_1 = \mu_1\mu_2$ and $p_1\mu_2^2 + p_2\mu_1^2 = ((c_s^2+1)/2)\mu_1^2\mu_2^2$ follow from the first two moments of V_k. Hence, we see that this first-order coefficient is consistent with the first term in (3).

Finally, we examine the second-order term in the expansion of θ^*, which equals

$$
\begin{aligned}
-\frac{1}{4}f''(0) &= -\frac{1}{4}(-\frac{1}{4}f(x)^{-\frac{3}{2}}(2x+2(\mu_1+\mu_2)-2-4\mu_1\mu_2)^2 + \frac{1}{2}f(x)^{-\frac{1}{2}}2)|_{x=0} \\
&= \frac{((\mu_1+\mu_2)-1-2\mu_1\mu_2)^2}{4(\mu_1+\mu_2-1)^3} - \frac{1}{4(\mu_1+\mu_2-1)} \\
&= \frac{\mu_1^2\mu_2^2-\mu_1\mu_2(\mu_1+\mu_2-1)}{(\mu_1+\mu_2-1)^3} \\
&= \frac{(p_1\mu_2+p_2\mu_1)^2-(p_1\mu_2^2+p_2\mu_1^2)}{(\mu_1+\mu_2-1)^3},
\end{aligned}
$$

where we used $\mu_1\mu_2(\mu_1+\mu_2-1) = p_1\mu_2^2+p_2\mu_1^2$ that is derived in the last paragraph. We have also derived previously that

$$
\frac{\mu_1\mu_2}{\mu_1+\mu_2-1} = \frac{2}{1+c_s^2}
$$

. Hence, by substituting c_s^2 in (6), we can write C as

$$
\begin{aligned}
C &= \frac{-24p_1/\mu_1^3 - 24p_2/\mu_2^3 + 6(\frac{2(\mu_1+\mu_2-1)}{\mu_1\mu_2}-1)^2 + 12(\frac{2(\mu_1+\mu_2-1)}{\mu_1\mu_2}-1)+6}{24\frac{(\mu_1+\mu_2-1)^3}{\mu_1^3\mu_2^3}} \\
&= \frac{-p_1\mu_2^3-p_2\mu_1^3+(\mu_1+\mu_2-1)^2\mu_1\mu_2}{(\mu_1+\mu_2-1)^3} \\
&= \frac{-p_1\mu_2^3-p_2\mu_1^3+(p_1\mu_2^2+p_2\mu_1^2)(\mu_1+\mu_2-1)}{(\mu_1+\mu_2-1)^3} \\
&= \frac{(p_1\mu_1\mu_2^2+p_2\mu_1^2\mu_2)-(p_1\mu_2^2+p_2\mu_1^2)}{(\mu_1+\mu_2-1)^3} \\
&= \frac{(p_1\mu_2+p_2\mu_1)^2-(p_1\mu_2^2+p_2\mu_1^2)}{(\mu_1+\mu_2-1)^3}.
\end{aligned}
$$

Therefore, we conclude that this second-order term coefficient in the exact θ^* is consistent with that in (3). ∎

As noted in Corollary 1, the two-term approximations for θ^* in the $M_t/H_2/1$ and $(H_2)_t/M/1$ models approach the one-term approximation in (3) from opposite sides.

Table 1 compares the 1-term and 2-term approximations for the asymptotic decay rate θ^*_ρ from the asymptotic expansion in (3) with the exact values for the $M_t/H_2/1$ and $(H_2)_t/M/1$ models, where the H_2 distribution has $c^2 = 2.0$ and balanced means. The scaled value $\theta^*_\rho/(1-\rho)$ is shown for 6 values of $1-\rho$. the asymptotic decay rate for RBM and RPBM are obtained directly from the first term. Table 1 shows that the 2-term approximation can serve as an explicit formula for θ^*_ρ provided that ρ is not too small.

In this specific case, the asymptotic expansion (3) for θ^* have the following expressions for $M_t/H_2/1$ and $(H_2)_t/M/1$ models respectively:

$$M_t/H_2/1: \quad \theta^*_\rho = \frac{2}{3}(1-\rho) - \frac{2}{9}(1-\rho)^2 + O(1-\rho)^3;$$
$$(H_2)_t/M/1: \quad \theta^*_\rho = \frac{2}{3}(1-\rho) + \frac{2}{9}(1-\rho)^2 + O(1-\rho)^3.$$

2.5. More Bounds

To obtain further bounds, consider the common case in which $\lambda(t) \ge \bar{\lambda}$, $0 \le t \le pc$ while $\lambda(t) \le \bar{\lambda}$, $pc \le t \le c$, for some p, $0 < p < 1$. Then $\tilde{\Lambda}_c(t) = \Lambda(c) - \Lambda(c-t) \le \bar{\lambda}t$, $0 \le t \le c$, while $\tilde{\Lambda}_{pc}(t) = \Lambda(pc) - \Lambda(pc-t) \ge \bar{\lambda}t$, $0 \le t \le c$. As a consequence, $\tilde{\Lambda}_c^{-1}(t) \ge \bar{\lambda}t$, $0 \le t \le c$, while $\tilde{\Lambda}_{pc}^{-1}(t) \le \bar{\lambda}t$, $0 \le t \le c$. Thus,

$$W_0 = W_c \le W \le W_{pc}. \tag{9}$$

It is natural to seek conditions under which $P(W_y > b)$ is increasing in y from a minimum at $y = 0$ to a maximum at $y = pc$ and then is decreasing back to the minimum at $y = c$.

2.6. Heuristic Approximations

Given Lemmas 1 and 2 and Corollary 4 of the main paper, we propose the approximation

$$W_y \approx W - \omega_y, \tag{10}$$

where

$$\omega_y \equiv \frac{-1}{\rho c} \int_0^c (\tilde{\Lambda}_y(s) - \rho s)\, ds. \tag{11}$$

For the sinusoidal case, from Corollary 2 of the main paper, we obtain

$$\omega_y = \frac{\beta\cos(\gamma y)}{\gamma} = \frac{\zeta_y^+ + \zeta_y^-}{2}. \tag{12}$$

Unfortunately, we find that this approximation is not consistently accurate, but it does help us understand roughly how W_y depends on the parameters. In our examples, this approximation consistently underestimates the exact values. Intuitively, that makes sense because we expect the extrema to be larger than the time average.

3. More Simulation Results

For all experiments we use the sinusoidal arrival-rate function

$$\lambda(t) \equiv \bar{\lambda}(1+\beta\sin(\gamma t)), \quad t \geq 0, \tag{13}$$

where β, $0<\beta<1$, is the relative amplitude and the cycle length is $c = 2\pi/\gamma$.

In §3.1 (Tables 2-15) and §3.2 (Tables 16-27) we report results on experiments to estimate the tail probabilities $P(W_y > b)$ in the Markovian $M_t/M/1$ model. In §3.3 (Tables 28-37) and §3.4 (Tables 38-48), respectively, we report results on experiments to estimate the tail probabilities $P(W_y > b)$ in the $(H_2)_t/M/1$ and $M_t/H_2/1$ models. For non-exponential distributions, we use the H_2 distribution (hyperexponential, mixture of two exponential distributions), with with probability density function (pdf) $f(x) = p_1\mu_1 e^{-\mu_1 x} + p_2\mu_2 e^{-\mu_2 x}$, with $p_1 + p_2 = 1$, having parameter triple (p_1, μ_1, μ_2). To reduce the parameters to two (the mean and scv), we assume balanced means, i.e., $p_1/\mu_1 = p_2/\mu_2$, as in (3.7) of Whitt (1982). In all examples, we let the squared coefficient of variation (scv, variance divided by the mean) be $c^2 = 2.0$.

In §3.5 we report additional results on experiments to estimate the mean $E[W_y]$ and standard deviation $SD(W_y)$ using §5.4 of the main paper. Tables 49-51 report results for the $M_t/M/1$ model, while Tables 52 and 53 report results for the $(H_2)_t/M/1$ and $M_t/H_2/1$ models, respectively.

In §3.6 we display analogs of Tables 7 and 11 in the main paper reporting estimates of tail probabilities for the $(H_2)_t/M/1$ model, which requires the adjustment involving $m_{X_1}(\theta^*)$ in (48) of the main paper. That adjustment is required because the first interarrival time has the equilibrium lifetime distribution associated with the H_2 interarrival-time distribution (which is a different H_2 distribution). Tables 54 and 55 show the closely related values when that factor is omitted. These tables are closely related because the steady-state workload and waiting time coincide in the heavy-traffic limit. Table 1 in the main paper shows that the steady-state workload and waiting time in the stationary $H_2/M/1$ model are quite different for the low traffic intensity of $\rho = 0.1$.

3.1. Tail Probability Estimates for the $M_t/M/1$ Periodic Queue, scaled by $\bar{\lambda}=1$

Tables 2-15 display simulation estimates of $P(W_y > b)$ for the $M_t/M/1$ model, scaled to have $\bar{\lambda}=1$ and $\mu=1/\rho$. in subsequent tables, the scaling was changed to have $\bar{\lambda}=\rho$ and $\mu=1$, as in the main paper. An approximation for A_y is shown; it is discussed in §**??**.

Tables 2-6 show estimates for 12 values of b ranging from 5 to 90 to show that the simulation accuracy tends to be independent of b, as intended for rare-event simulation. To check the simulation algorithm and for a basis of comparison, Table 2 shows simulation results for the $M/M/1$ queue, where the exact results are known. Then Tables 3-6 show the estimates for 3 different values of γ in (13) ($\gamma=10$, $\gamma=1.0$, and $\gamma=0.1$) and 4 different cases of y. Here the cycle length is chosen to be $c=2\pi/\gamma$, so the four values of y are $0c$, $0.25c=\pi/2\gamma$, $0.50c=\pi/\gamma$ and $0.75c=3\pi/2\gamma$. All these examples have $\rho=0.8$ and $\beta=0.2$. We regard this as our base model, and regard $\gamma=1.0$ and 0.1 as our base examples illustrating shorter and longer cycles, respectively.

There are 8 columns. The first column gives n, the number of replications. The second column gives the tail probability estimate $\hat{p}\equiv P(W_y > b)\equiv A_y e{-}\theta^* b$ and then the third and fourth columns give the components $e{-}\theta^* b$ and $A\equiv A_y$. The fifth column gives the standard error (s.e.), while the sixth and seventh columns give the lower bound (lb) and upper bound (ub) of the associated 95% confidence interval (CI). The final eight column gives the relative error (r.e.), which is the estimated s.e divided by the estimated value itself.

Tables 7-15 show the estimates as a function of y for 40 values of y within the cycle in 9 different cases. As noted above, in all these cases $\bar{\lambda}=1$ and $\mu=1/\rho$. Tables 7-9 consider three values of the pair (γ,b) for fixed $(\rho,\beta)=(0.8,0.2)$, in particular, $(\gamma,b)=(10,20)$, $(\gamma,b)=(0.1,50)$ and $(\gamma,b)=(0.01,300)$. Tables 10 and 11 consider three values of the pair (γ,b) for fixed $(\rho,\beta)=(0.9,0.2)$, in particular, $(\gamma,b)=(1,20)$ and $(\gamma,b)=(0.1,50)$. Tables 12-14 consider three values of the pair (γ,b) for fixed $(\rho,\beta)=(0.8,0.5)$, in particular, $(\gamma,b)=(10,20)$, $(\gamma,b)=(1.0,20)$ and $(\gamma,b)=(0.1,100)$. Finally, Table 15 shows estimates as a function of y for 40 values of y within a small subinterval in the center of the cycle, in an attempt to verify that the maximum occurs in the middle of the cycle, i.e, at $y=0.5$. Table 15 has the parameter 4-tuple $(\gamma,\beta\rho,b)=(0.1,0.2,0.8,20)$.

Tables 2-15 display simulation estimates of $P(W_y > b)$ for the $M_t/M/1$ model, scaled to have $\bar{\lambda}=1$ and $\mu=1/\rho$. in subsequent tables, the scaling was changed to have $\bar{\lambda}=\rho$ and $\mu=1$, as in the main paper.

Table 2 **Estimates of $\hat{p} \equiv P(W > b) \equiv Ae^{-\theta^* b}$ in the $M/M/1$ model with $\rho = 0.8, \bar{\lambda} = 1, \mu = 1.25$ based on $n = 5000$ replications.**

$\beta = 0$	b	n	$\hat{p}$	$exp(-\theta^* b)$	A	s.e.	95% CI (lb)	(ub)	r.e.
	5	5000	0.229	0.287	0.799	6.60E-04	0.228	0.230	0.00289
	10	5000	0.0656	0.0821	0.799	1.90E-04	0.0652	0.0660	0.00289
	15	5000	0.0187	0.0235	0.797	5.48E-05	0.0186	0.0188	0.00293
	20	5000	0.00541	0.00674	0.803	1.52E-05	0.00538	0.00544	0.00280
	25	5000	1.54E-03	1.93E-03	0.797	4.51E-06	0.00153	0.00155	0.00293
	30	5000	4.43E-04	5.53E-04	0.800	1.29E-06	0.000440	0.000445	0.00290
	40	5000	3.64E-05	4.54E-05	0.802	1.05E-07	3.62E-05	3.66E-05	0.00288
	50	5000	2.98E-06	3.73E-06	0.800	8.51E-09	2.97E-06	3.00E-06	0.00285
	60	5000	2.45E-07	3.06E-07	0.800	6.97E-10	2.43E-07	2.46E-07	0.00285
	70	5000	2.01E-08	2.51E-08	0.802	5.75E-11	2.00E-08	2.02E-08	0.00286
	80	5000	1.65E-09	2.06E-09	0.798	4.73E-12	1.64E-09	1.65E-09	0.00287
	90	5000	1.35E-10	1.69E-10	0.795	4.04E-13	1.34E-10	1.35E-10	0.00300

Table 3 **Simulation estimates of $\hat{p} \equiv P(W_y > b) \equiv A_y e^{-\theta^* b}$ in the $M_t/M/1$ model for $y = 0.0$ as a function of γ and b based on $n = 5,000$ replications: $\rho = 0.8, \bar{\lambda} = 1, \mu = 1.25, \beta = 0.2$**

	b	n	$\hat{p}$	$exp(-\theta^* b)$	A	s.e.	95% CI (lb)	(ub)	r.e.
$\gamma = 10$	5	5000	0.228	0.287	0.797	6.55E-04	0.227	0.230	0.00287
	10	5000	0.0654	0.0821	0.797	1.87E-04	0.0651	0.0658	0.00286
	15	5000	0.0188	0.0235	0.799	5.32E-05	0.0187	0.0189	0.00283
	20	5000	0.00537	0.00674	0.797	1.55E-05	0.00534	0.00540	0.00289
	25	5000	1.53E-03	1.93E-03	0.795	4.37E-06	0.00153	0.00154	0.00285
	30	5000	4.40E-04	5.53E-04	0.795	1.28E-06	4.37E-04	4.42E-04	0.00290
	40	5000	3.61E-05	4.54E-05	0.795	1.05E-07	3.59E-05	3.63E-05	0.00290
	50	5000	2.97E-06	3.73E-06	0.796	8.59E-09	2.95E-06	2.99E-06	0.00289
	60	5000	2.44E-07	3.06E-07	0.798	7.02E-10	2.43E-07	2.45E-07	0.00288
	70	5000	2.01E-08	2.51E-08	0.799	5.67E-11	1.99E-08	2.02E-08	0.00283
	80	5000	1.64E-09	2.06E-09	0.796	4.82E-12	1.63E-09	1.65E-09	0.00294
	90	5000	1.35E-10	1.69E-10	0.797	3.88E-13	1.34E-10	1.36E-10	0.00288
$\gamma = 1$	5	5000	0.219	0.287	0.764	6.38E-04	0.218	0.220	0.00292
	10	5000	0.0628	0.0821	0.765	1.87E-04	0.0624	0.0632	0.00298
	15	5000	0.0179	0.0235	0.762	5.19E-05	0.0178	0.0180	0.00290
	20	5000	0.00516	0.00674	0.766	1.51E-05	0.00513	0.00519	0.00292
	25	5000	1.48E-03	1.93E-03	0.764	4.29E-06	0.00147	0.00148	0.00291
	30	5000	4.25E-04	5.53E-04	0.769	1.20E-06	4.23E-04	4.27E-04	0.00283
	40	5000	3.49E-05	4.54E-05	0.769	1.00E-07	3.47E-05	3.51E-05	0.00287
	50	5000	2.85E-06	3.73E-06	0.764	8.40E-09	2.83E-06	2.86E-06	0.00295
	60	5000	2.34E-07	3.06E-07	0.766	6.85E-10	2.33E-07	2.36E-07	0.00292
	70	5000	1.92E-08	2.51E-08	0.763	5.61E-11	1.90E-08	1.93E-08	0.00293
	80	5000	1.58E-09	2.06E-09	0.767	4.65E-12	1.57E-09	1.59E-09	0.00294
	90	5000	1.29E-10	1.69E-10	0.764	3.86E-13	1.28E-10	1.30E-10	0.00299
$\gamma = 0.1$	5	5000	0.161	0.287	0.563	8.88E-04	0.160	0.163	0.00550
	10	5000	0.0413	0.0821	0.503	2.33E-04	0.0409	0.0418	0.00565
	15	5000	0.0122	0.0235	0.520	6.77E-05	0.0121	0.0124	0.00554
	20	5000	0.00360	0.00674	0.535	1.98E-05	0.00356	0.00364	0.00550
	25	5000	1.06E-03	1.93E-03	0.551	5.72E-06	0.00105	0.00107	0.00538
	30	5000	3.04E-04	5.53E-04	0.550	1.66E-06	3.01E-04	3.08E-04	0.00546
	40	5000	2.50E-05	4.54E-05	0.551	1.37E-07	2.47E-05	2.53E-05	0.00548
	50	5000	2.04E-06	3.73E-06	0.547	1.10E-08	2.02E-06	2.06E-06	0.00538
	60	5000	1.67E-07	3.06E-07	0.546	9.25E-10	1.65E-07	1.69E-07	0.00553
	70	5000	1.37E-08	2.51E-08	0.544	7.59E-11	1.35E-08	1.38E-08	0.00556
	80	5000	1.12E-09	2.06E-09	0.545	6.20E-12	1.11E-09	1.14E-09	0.00552
	90	5000	9.21E-11	1.69E-10	0.544	5.01E-13	9.11E-11	9.31E-11	0.00544

Table 4 **Simulation estimates of $\hat{p} \equiv P(W_y > b) \equiv A_y e^{-\theta^* b}$ in the $M_t/M/1$ model for $y = \pi/2\gamma$ as a function of γ and b based on $n = 5,000$ replications: $\rho = 0.8, \bar{\lambda} = 1, \mu = 1.25, \beta = 0.2$**

	b	n	$\hat{p}$	$exp(-\theta^* b)$	A	s.e.	95% CI (lb)	(ub)	r.e.
$\gamma = 10$	5	5000	0.229	0.287	0.801	6.61E-04	0.228	0.231	0.00288
	10	5000	0.0659	0.0821	0.803	1.88E-04	0.0655	0.0663	0.00285
	15	5000	0.0187	0.0235	0.797	5.57E-05	0.0186	0.0188	0.00297
	20	5000	0.00538	0.00674	0.799	1.59E-05	0.00535	0.00541	0.00296
	25	5000	1.55E-03	1.93E-03	0.801	4.45E-06	0.00154	0.00156	0.00288
	30	5000	4.42E-04	5.53E-04	0.800	1.28E-06	4.40E-04	4.45E-04	0.00288
	40	5000	3.64E-05	4.54E-05	0.802	1.05E-07	3.62E-05	3.66E-05	0.00288
	50	5000	3.00E-06	3.73E-06	0.806	8.51E-09	2.99E-06	3.02E-06	0.00283
	60	5000	2.44E-07	3.06E-07	0.797	7.13E-10	2.43E-07	2.45E-07	0.00292
	70	5000	2.02E-08	2.51E-08	0.805	5.76E-11	2.01E-08	2.03E-08	0.00285
	80	5000	1.64E-09	2.06E-09	0.798	4.81E-12	1.64E-09	1.65E-09	0.00293
	90	5000	1.35E-10	1.69E-10	0.799	3.84E-13	1.34E-10	1.36E-10	0.00284
$\gamma = 1$	5	5000	0.230	0.287	0.804	6.81E-04	0.229	0.232	0.00295
	10	5000	0.0659	0.0821	0.803	1.92E-04	0.0655	0.0663	0.00292
	15	5000	0.0188	0.0235	0.801	5.67E-05	0.0187	0.0189	0.00301
	20	5000	0.00540	0.00674	0.801	1.58E-05	0.00536	0.00543	0.00294
	25	5000	1.54E-03	1.93E-03	0.799	4.59E-06	0.00153	0.00155	0.00298
	30	5000	4.45E-04	5.53E-04	0.805	1.28E-06	4.43E-04	4.48E-04	0.00287
	40	5000	3.63E-05	4.54E-05	0.800	1.07E-07	3.61E-05	3.65E-05	0.00294
	50	5000	2.97E-06	3.73E-06	0.798	8.98E-09	2.96E-06	2.99E-06	0.00302
	60	5000	2.46E-07	3.06E-07	0.803	7.18E-10	2.44E-07	2.47E-07	0.00292
	70	5000	2.02E-08	2.51E-08	0.804	5.90E-11	2.01E-08	2.03E-08	0.00293
	80	5000	1.66E-09	2.06E-09	0.806	4.74E-12	1.65E-09	1.67E-09	0.00285
	90	5000	1.36E-10	1.69E-10	0.804	4.00E-13	1.35E-10	1.37E-10	0.00294
$\gamma = 0.1$	5	5000	0.293	0.287	1.024	1.24E-03	0.291	0.296	0.00421
	10	5000	0.0828	0.0821	1.008	4.06E-04	0.0820	0.0836	0.00491
	15	5000	0.0217	0.0235	0.924	1.20E-04	0.0215	0.0220	0.00553
	20	5000	0.00600	0.00674	0.891	3.37E-05	0.00594	0.00607	0.00561
	25	5000	1.71E-03	1.93E-03	0.887	9.53E-06	0.00169	0.00173	0.00556
	30	5000	4.95E-04	5.53E-04	0.895	2.76E-06	4.90E-04	5.00E-04	0.00558
	40	5000	4.13E-05	4.54E-05	0.910	2.23E-07	4.09E-05	4.18E-05	0.00539
	50	5000	3.37E-06	3.73E-06	0.904	1.86E-08	3.33E-06	3.40E-06	0.00551
	60	5000	2.73E-07	3.06E-07	0.893	1.51E-09	2.70E-07	2.76E-07	0.00554
	70	5000	2.27E-08	2.51E-08	0.902	1.25E-10	2.24E-08	2.29E-08	0.00551
	80	5000	1.85E-09	2.06E-09	0.896	1.01E-11	1.83E-09	1.87E-09	0.00547
	90	5000	1.52E-10	1.69E-10	0.900	8.36E-13	1.51E-10	1.54E-10	0.00549

Table 5 **Simulation estimates of $\hat{p} \equiv P(W_y > b) \equiv A_y e^{-\theta^* b}$ in the $M_t/M/1$ model for $y = \pi/\gamma$ as a function of γ and b based on $n = 5,000$ replications: $\rho = 0.8, \bar{\lambda} = 1, \mu = 1.25, \beta = 0.2$**

	b	n	$\hat{p}$	$exp(-\theta^* b)$	A	s.e.	95% CI (lb)	(ub)	r.e.
$\gamma = 10$	5	5000	0.232	0.287	0.808	6.64E-04	0.230	0.233	0.00286
	10	5000	0.0657	0.0821	0.800	1.93E-04	0.0653	0.0661	0.00294
	15	5000	0.0190	0.0235	0.807	5.39E-05	0.0189	0.0191	0.00284
	20	5000	0.00546	0.00674	0.810	1.53E-05	0.00543	0.00549	0.00281
	25	5000	1.55E-03	1.93E-03	0.804	4.49E-06	0.00154	0.00156	0.00289
	30	5000	4.46E-04	5.53E-04	0.807	1.28E-06	4.44E-04	4.49E-04	0.00286
	40	5000	3.64E-05	4.54E-05	0.802	1.06E-07	3.62E-05	3.66E-05	0.00291
	50	5000	3.00E-06	3.73E-06	0.804	8.59E-09	2.98E-06	3.01E-06	0.00286
	60	5000	2.46E-07	3.06E-07	0.803	7.21E-10	2.44E-07	2.47E-07	0.00294
	70	5000	2.02E-08	2.51E-08	0.804	5.76E-11	2.01E-08	2.03E-08	0.00285
	80	5000	1.65E-09	2.06E-09	0.803	4.79E-12	1.65E-09	1.66E-09	0.00289
	90	5000	1.36E-10	1.69E-10	0.805	3.88E-13	1.35E-10	1.37E-10	0.00285
$\gamma = 1$	5	5000	0.242	0.287	0.846	6.96E-04	0.241	0.244	0.00287
	10	5000	0.0691	0.0821	0.842	2.05E-04	0.0687	0.0695	0.00297
	15	5000	0.0198	0.0235	0.841	5.89E-05	0.0197	0.0199	0.00298
	20	5000	0.00570	0.00674	0.846	1.65E-05	0.00567	0.00573	0.00290
	25	5000	1.62E-03	1.93E-03	0.840	4.80E-06	0.00161	0.00163	0.00296
	30	5000	4.68E-04	5.53E-04	0.847	1.36E-06	4.66E-04	4.71E-04	0.00289
	40	5000	3.81E-05	4.54E-05	0.840	1.15E-07	3.79E-05	3.83E-05	0.00303
	50	5000	3.14E-06	3.73E-06	0.843	9.16E-09	3.12E-06	3.16E-06	0.00292
	60	5000	2.59E-07	3.06E-07	0.847	7.62E-10	2.58E-07	2.61E-07	0.00294
	70	5000	2.13E-08	2.51E-08	0.849	6.11E-11	2.12E-08	2.14E-08	0.00287
	80	5000	1.74E-09	2.06E-09	0.842	5.18E-12	1.73E-09	1.75E-09	0.00298
	90	5000	1.42E-10	1.69E-10	0.839	4.25E-13	1.41E-10	1.43E-10	0.00300
$\gamma = 0.1$	5	5000	0.342	0.287	1.195	1.99E-03	0.338	0.346	0.00581
	10	5000	0.1181	0.0821	1.438	6.32E-04	0.1168	0.1193	0.00535
	15	5000	0.0366	0.0235	1.554	1.86E-04	0.0362	0.0369	0.00508
	20	5000	0.01038	0.00674	1.541	5.42E-05	0.01027	0.01049	0.00522
	25	5000	2.93E-03	1.93E-03	1.516	1.57E-05	0.00289	0.00296	0.00536
	30	5000	8.22E-04	5.53E-04	1.485	4.51E-06	8.13E-04	8.30E-04	0.00549
	40	5000	6.67E-05	4.54E-05	1.470	3.68E-07	6.60E-05	6.75E-05	0.00552
	50	5000	5.49E-06	3.73E-06	1.473	3.01E-08	5.43E-06	5.55E-06	0.00548
	60	5000	4.58E-07	3.06E-07	1.499	2.52E-09	4.54E-07	4.63E-07	0.00549
	70	5000	3.75E-08	2.51E-08	1.495	2.06E-10	3.71E-08	3.79E-08	0.00549
	80	5000	3.07E-09	2.06E-09	1.490	1.69E-11	3.04E-09	3.10E-09	0.00552
	90	5000	2.49E-10	1.69E-10	1.474	1.38E-12	2.47E-10	2.52E-10	0.00554

Table 6 **Simulation estimates of $\hat{p} \equiv P(W_y > b) \equiv A_y e^{-\theta^* b}$ in the $M_t/M/1$ model for $y = 3\pi/2\gamma$ as a function of γ and b based on $n = 5,000$ replications: $\rho = 0.8, \bar{\lambda} = 1, \mu = 1.25, \beta = 0.2$**

	b	n	$\hat{p}$	$exp(-\theta^* b)$	A	s.e.	95% CI (lb)	(ub)	r.e.
$\gamma = 10$	5	5000	0.229	0.287	0.798	6.66E-04	0.227	0.230	0.00291
	10	5000	0.0657	0.0821	0.801	1.89E-04	0.0654	0.0661	0.00287
	15	5000	0.0187	0.0235	0.794	5.49E-05	0.0186	0.0188	0.00294
	20	5000	0.00541	0.00674	0.803	1.54E-05	0.00538	0.00544	0.00284
	25	5000	1.55E-03	1.93E-03	0.801	4.43E-06	0.00154	0.00155	0.00286
	30	5000	4.43E-04	5.53E-04	0.801	1.28E-06	4.40E-04	4.45E-04	0.00290
	40	5000	3.63E-05	4.54E-05	0.800	1.05E-07	3.61E-05	3.65E-05	0.00289
	50	5000	2.98E-06	3.73E-06	0.798	8.62E-09	2.96E-06	2.99E-06	0.00290
	60	5000	2.46E-07	3.06E-07	0.803	6.95E-10	2.44E-07	2.47E-07	0.00283
	70	5000	2.01E-08	2.51E-08	0.799	5.81E-11	2.00E-08	2.02E-08	0.00289
	80	5000	1.66E-09	2.06E-09	0.803	4.74E-12	1.65E-09	1.67E-09	0.00286
	90	5000	1.36E-10	1.69E-10	0.802	3.93E-13	1.35E-10	1.37E-10	0.00290
$\gamma = 1$	5	5000	0.231	0.287	0.807	6.63E-04	0.230	0.232	0.00287
	10	5000	0.0659	0.0821	0.803	1.92E-04	0.0655	0.0663	0.00291
	15	5000	0.0189	0.0235	0.803	5.53E-05	0.0188	0.0190	0.00293
	20	5000	0.00539	0.00674	0.800	1.58E-05	0.00536	0.00542	0.00294
	25	5000	1.55E-03	1.93E-03	0.801	4.60E-06	0.00154	0.00155	0.00298
	30	5000	4.44E-04	5.53E-04	0.803	1.29E-06	4.42E-04	4.47E-04	0.00290
	40	5000	3.66E-05	4.54E-05	0.807	1.06E-07	3.64E-05	3.68E-05	0.00290
	50	5000	2.98E-06	3.73E-06	0.798	8.82E-09	2.96E-06	2.99E-06	0.00296
	60	5000	2.45E-07	3.06E-07	0.800	7.21E-10	2.43E-07	2.46E-07	0.00294
	70	5000	2.01E-08	2.51E-08	0.802	5.91E-11	2.00E-08	2.03E-08	0.00293
	80	5000	1.66E-09	2.06E-09	0.803	4.90E-12	1.65E-09	1.67E-09	0.00296
	90	5000	1.36E-10	1.69E-10	0.805	4.00E-13	1.35E-10	1.37E-10	0.00293
$\gamma = 0.1$	5	5000	0.201	0.287	0.701	1.14E-03	0.199	0.203	0.00568
	10	5000	0.0658	0.0821	0.801	3.83E-04	0.0650	0.0665	0.00581
	15	5000	0.0205	0.0235	0.872	1.15E-04	0.0203	0.0207	0.00562
	20	5000	0.00612	0.00674	0.908	3.30E-05	0.00605	0.00618	0.00540
	25	5000	1.77E-03	1.93E-03	0.918	9.62E-06	0.00175	0.00179	0.00543
	30	5000	5.01E-04	5.53E-04	0.906	2.72E-06	4.96E-04	5.06E-04	0.00543
	40	5000	4.10E-05	4.54E-05	0.903	2.27E-07	4.06E-05	4.14E-05	0.00555
	50	5000	3.33E-06	3.73E-06	0.893	1.84E-08	3.29E-06	3.37E-06	0.00552
	60	5000	2.76E-07	3.06E-07	0.901	1.51E-09	2.73E-07	2.79E-07	0.00549
	70	5000	2.28E-08	2.51E-08	0.908	1.24E-10	2.26E-08	2.30E-08	0.00544
	80	5000	1.87E-09	2.06E-09	0.905	1.02E-11	1.84E-09	1.89E-09	0.00549
	90	5000	1.52E-10	1.69E-10	0.898	8.37E-13	1.50E-10	1.54E-10	0.00551

Table 7 Simulation estimates of $\hat{p} \equiv P(W_y > b) \equiv A_y e^{-\theta^* b}$ in the $M_t/M/1$ model as a function of y based on $n = 5,000$ replications: $\gamma = 10, b = 20, \rho = 0.8, \bar{\lambda} = 1, \mu = 1.25, \beta = 0.2$

$\gamma = 10$	position	n	$\hat{p}$	$exp(-\theta^* b)$	A_y	A_y approx	s.e.	95% CI (lb)	(ub)	r.e.
	0.000	10000	0.0053699	0.00674	0.797	0.796	1.08E-05	0.0053487	0.0053911	0.00202
	0.025	10000	0.0053537	0.00674	0.795	0.796	1.09E-05	0.0053323	0.0053751	0.00204
	0.050	10000	0.0053577	0.00674	0.795	0.796	1.11E-05	0.0053359	0.0053795	0.00208
	0.075	10000	0.0053619	0.00674	0.796	0.796	1.10E-05	0.0053403	0.0053835	0.00206
	0.100	10000	0.0053614	0.00674	0.796	0.797	1.09E-05	0.0053400	0.0053829	0.00204
	0.125	10000	0.0053859	0.00674	0.799	0.797	1.09E-05	0.0053646	0.0054073	0.00202
	0.150	10000	0.0053805	0.00674	0.799	0.798	1.09E-05	0.0053590	0.0054019	0.00203
	0.175	10000	0.0053653	0.00674	0.796	0.798	1.09E-05	0.0053439	0.0053867	0.00204
	0.200	10000	0.0053969	0.00674	0.801	0.799	1.09E-05	0.0053755	0.0054183	0.00202
	0.225	10000	0.0053956	0.00674	0.801	0.799	1.10E-05	0.0053740	0.0054172	0.00204
	0.250	10000	0.0053814	0.00674	0.799	0.800	1.10E-05	0.0053598	0.0054029	0.00204
	0.275	10000	0.0053804	0.00674	0.799	0.801	1.10E-05	0.0053588	0.0054020	0.00205
	0.300	10000	0.0053728	0.00674	0.797	0.801	1.11E-05	0.0053510	0.0053945	0.00207
	0.325	10000	0.0053793	0.00674	0.798	0.802	1.12E-05	0.0053574	0.0054012	0.00208
	0.350	10000	0.0054018	0.00674	0.802	0.802	1.12E-05	0.0053799	0.0054238	0.00207
	0.375	10000	0.0053946	0.00674	0.801	0.803	1.12E-05	0.0053727	0.0054165	0.00207
	0.400	10000	0.0054297	0.00674	0.806	0.803	1.10E-05	0.0054081	0.0054514	0.00203
	0.425	10000	0.0054067	0.00674	0.802	0.804	1.10E-05	0.0053851	0.0054283	0.00204
	0.450	10000	0.0054257	0.00674	0.805	0.804	1.11E-05	0.0054040	0.0054474	0.00204
	0.475	10000	0.0054453	0.00674	0.808	0.804	1.09E-05	0.0054238	0.0054667	0.00201
	0.500	10000	0.0054138	0.00674	0.803	0.804	1.11E-05	0.0053920	0.0054356	0.00206
	0.525	10000	0.0054315	0.00674	0.806	0.804	1.10E-05	0.0054099	0.0054532	0.00203
	0.550	10000	0.0054065	0.00674	0.802	0.804	1.12E-05	0.0053846	0.0054284	0.00206
	0.575	10000	0.0054207	0.00674	0.805	0.804	1.11E-05	0.0053990	0.0054425	0.00205
	0.600	10000	0.0054270	0.00674	0.805	0.803	1.09E-05	0.0054057	0.0054484	0.00201
	0.625	10000	0.0054153	0.00674	0.804	0.803	1.09E-05	0.0053938	0.0054367	0.00202
	0.650	10000	0.0054065	0.00674	0.802	0.802	1.10E-05	0.0053849	0.0054281	0.00204
	0.675	10000	0.0054121	0.00674	0.803	0.802	1.09E-05	0.0053908	0.0054334	0.00201
	0.700	10000	0.0054175	0.00674	0.804	0.801	1.10E-05	0.0053960	0.0054390	0.00202
	0.725	10000	0.0053797	0.00674	0.798	0.801	1.11E-05	0.0053580	0.0054014	0.00206
	0.750	10000	0.0053901	0.00674	0.800	0.800	1.10E-05	0.0053686	0.0054116	0.00203
	0.775	10000	0.0053580	0.00674	0.795	0.799	1.11E-05	0.0053361	0.0053798	0.00208
	0.800	10000	0.0053783	0.00674	0.798	0.799	1.10E-05	0.0053568	0.0053998	0.00204
	0.825	10000	0.0053843	0.00674	0.799	0.798	1.08E-05	0.0053630	0.0054056	0.00201
	0.850	10000	0.0053946	0.00674	0.801	0.798	1.09E-05	0.0053733	0.0054160	0.00202
	0.875	10000	0.0053783	0.00674	0.798	0.797	1.09E-05	0.0053569	0.0053997	0.00203
	0.900	10000	0.0053758	0.00674	0.798	0.797	1.10E-05	0.0053543	0.0053974	0.00205
	0.925	10000	0.0053714	0.00674	0.797	0.796	1.08E-05	0.0053502	0.0053926	0.00201
	0.950	10000	0.0053435	0.00674	0.793	0.796	1.10E-05	0.0053220	0.0053651	0.00206
	0.975	10000	0.0053681	0.00674	0.797	0.796	1.09E-05	0.0053468	0.0053895	0.00203

Table 8 Simulation estimates of $\hat{p} \equiv P(W_y > b) \equiv A_y e^{-\theta^* b}$ in the $M_t/M/1$ model as a function of y based on $n = 5,000$ replications: $\gamma = 0.1, b = 50, \rho = 0.8, \bar{\lambda} = 1, \mu = 1.25, \beta = 0.2$

$\gamma = 0.1$	position	n	$\hat{p}$	$exp(-\theta^* b)$	A_y	A_y approx	A_y LB	A_y UB	s.e.	95% CI (lb)	(ub)	r.e.
	0.000	10000	2.04E-06	3.73E-06	0.548	0.485	0.294	0.800	7.95E-09	2.03E-06	2.06E-06	0.00389
	0.025	10000	2.05E-06	3.73E-06	0.551	0.488	0.296	0.805	7.94E-09	2.04E-06	2.07E-06	0.00387
	0.050	10000	2.09E-06	3.73E-06	0.560	0.497	0.302	0.820	8.08E-09	2.07E-06	2.10E-06	0.00387
	0.075	10000	2.15E-06	3.73E-06	0.577	0.512	0.311	0.845	8.34E-09	2.13E-06	2.17E-06	0.00388
	0.100	10000	2.24E-06	3.73E-06	0.602	0.534	0.324	0.880	8.68E-09	2.22E-06	2.26E-06	0.00387
	0.125	10000	2.37E-06	3.73E-06	0.635	0.562	0.341	0.926	9.21E-09	2.35E-06	2.38E-06	0.00389
	0.150	10000	2.50E-06	3.73E-06	0.671	0.596	0.362	0.983	9.70E-09	2.48E-06	2.52E-06	0.00388
	0.175	10000	2.68E-06	3.73E-06	0.719	0.638	0.387	1.051	1.05E-08	2.66E-06	2.70E-06	0.00392
	0.200	10000	2.87E-06	3.73E-06	0.770	0.685	0.416	1.130	1.12E-08	2.85E-06	2.89E-06	0.00392
	0.225	10000	3.10E-06	3.73E-06	0.832	0.740	0.449	1.220	1.20E-08	3.08E-06	3.12E-06	0.00387
	0.250	10000	3.36E-06	3.73E-06	0.902	0.800	0.485	1.319	1.31E-08	3.33E-06	3.39E-06	0.00390
	0.275	10000	3.63E-06	3.73E-06	0.974	0.865	0.525	1.426	1.42E-08	3.60E-06	3.66E-06	0.00390
	0.300	10000	3.90E-06	3.73E-06	1.045	0.934	0.566	1.539	1.51E-08	3.87E-06	3.93E-06	0.00389
	0.325	10000	4.19E-06	3.73E-06	1.126	1.004	0.609	1.655	1.63E-08	4.16E-06	4.23E-06	0.00389
	0.350	10000	4.50E-06	3.73E-06	1.208	1.073	0.651	1.770	1.76E-08	4.47E-06	4.54E-06	0.00391
	0.375	10000	4.80E-06	3.73E-06	1.289	1.139	0.691	1.878	1.84E-08	4.77E-06	4.84E-06	0.00383
	0.400	10000	5.04E-06	3.73E-06	1.352	1.199	0.727	1.977	1.96E-08	5.00E-06	5.08E-06	0.00389
	0.425	10000	5.27E-06	3.73E-06	1.413	1.249	0.758	2.059	2.03E-08	5.23E-06	5.31E-06	0.00386
	0.450	10000	5.39E-06	3.73E-06	1.446	1.287	0.781	2.122	2.09E-08	5.35E-06	5.43E-06	0.00388
	0.475	10000	5.54E-06	3.73E-06	1.487	1.311	0.795	2.161	2.14E-08	5.50E-06	5.58E-06	0.00387
	0.500	10000	5.54E-06	3.73E-06	1.488	1.319	0.800	2.175	2.15E-08	5.50E-06	5.59E-06	0.00388
	0.525	10000	5.51E-06	3.73E-06	1.479	1.311	0.795	2.161	2.14E-08	5.47E-06	5.55E-06	0.00388
	0.550	10000	5.46E-06	3.73E-06	1.466	1.287	0.781	2.122	2.11E-08	5.42E-06	5.50E-06	0.00386
	0.575	10000	5.22E-06	3.73E-06	1.401	1.249	0.758	2.059	2.04E-08	5.18E-06	5.26E-06	0.00390
	0.600	10000	5.04E-06	3.73E-06	1.353	1.199	0.727	1.977	1.95E-08	5.00E-06	5.08E-06	0.00387
	0.625	10000	4.77E-06	3.73E-06	1.281	1.139	0.691	1.878	1.87E-08	4.74E-06	4.81E-06	0.00391
	0.650	10000	4.55E-06	3.73E-06	1.221	1.073	0.651	1.770	1.75E-08	4.52E-06	4.58E-06	0.00386
	0.675	10000	4.23E-06	3.73E-06	1.134	1.004	0.609	1.655	1.64E-08	4.19E-06	4.26E-06	0.00388
	0.700	10000	3.91E-06	3.73E-06	1.049	0.934	0.566	1.539	1.52E-08	3.88E-06	3.94E-06	0.00389
	0.725	10000	3.63E-06	3.73E-06	0.975	0.865	0.525	1.426	1.40E-08	3.61E-06	3.66E-06	0.00385
	0.750	10000	3.35E-06	3.73E-06	0.899	0.800	0.485	1.319	1.30E-08	3.33E-06	3.38E-06	0.00387
	0.775	10000	3.09E-06	3.73E-06	0.829	0.740	0.449	1.220	1.21E-08	3.07E-06	3.11E-06	0.00390
	0.800	10000	2.86E-06	3.73E-06	0.768	0.685	0.416	1.130	1.12E-08	2.84E-06	2.88E-06	0.00392
	0.825	10000	2.68E-06	3.73E-06	0.719	0.638	0.387	1.051	1.04E-08	2.66E-06	2.70E-06	0.00388
	0.850	10000	2.49E-06	3.73E-06	0.669	0.596	0.362	0.983	9.65E-09	2.47E-06	2.51E-06	0.00387
	0.875	10000	2.35E-06	3.73E-06	0.630	0.562	0.341	0.926	9.06E-09	2.33E-06	2.37E-06	0.00386
	0.900	10000	2.24E-06	3.73E-06	0.602	0.534	0.324	0.880	8.72E-09	2.23E-06	2.26E-06	0.00389
	0.925	10000	2.16E-06	3.73E-06	0.578	0.512	0.311	0.845	8.38E-09	2.14E-06	2.17E-06	0.00389
	0.950	10000	2.08E-06	3.73E-06	0.557	0.497	0.302	0.820	8.08E-09	2.06E-06	2.09E-06	0.00389
	0.975	10000	2.04E-06	3.73E-06	0.548	0.488	0.296	0.805	7.99E-09	2.03E-06	2.06E-06	0.00391

Table 9 Simulation estimates of $\hat{p} \equiv P(W_y > b) \equiv A_y e^{-\theta^* b}$ in the $M_t/M/1$ model as a function of y based on $n = 5{,}000$ replications: $\gamma = 0.01, b = 300, \rho = 0.8, \bar{\lambda} = 1, \mu = 1.25, \beta = 0.2$

$\gamma = 0.01$	position	n	$\hat{p}$	$exp(-\theta^* b)$	A_y	A_y approx	A_y LB	A_y UB	s.e.	95% CI (lb)	(ub)	r.e.
	0.000	10000	4.71E-34	2.68E-33	0.176	0.005	0.00004	0.800	7.10E-36	4.57E-34	4.85E-34	0.0151
	0.025	10000	5.02E-34	2.68E-33	0.187	0.006	0.00004	0.851	7.56E-36	4.87E-34	5.17E-34	0.0151
	0.050	10000	5.78E-34	2.68E-33	0.216	0.007	0.00005	1.022	8.95E-36	5.60E-34	5.95E-34	0.0155
	0.075	10000	7.64E-34	2.68E-33	0.285	0.009	0.00006	1.380	1.19E-35	7.41E-34	7.88E-34	0.0155
	0.100	10000	1.07E-33	2.68E-33	0.401	0.014	0.00009	2.079	1.75E-35	1.04E-33	1.11E-33	0.0163
	0.125	10000	1.72E-33	2.68E-33	0.642	0.023	0.00016	3.460	2.88E-35	1.66E-33	1.78E-33	0.0167
	0.150	10000	2.86E-33	2.68E-33	1.066	0.042	0.00029	6.284	4.98E-35	2.76E-33	2.95E-33	0.0175
	0.175	10000	5.36E-33	2.68E-33	2.003	0.083	0.00056	12.267	9.71E-35	5.17E-33	5.56E-33	0.0181
	0.200	10000	1.07E-32	2.68E-33	3.994	0.171	0.00115	25.324	1.98E-34	1.03E-32	1.11E-32	0.0185
	0.225	10000	2.30E-32	2.68E-33	8.587	0.366	0.00247	54.309	4.24E-34	2.22E-32	2.38E-32	0.0184
	0.250	10000	4.95E-32	2.68E-33	18.490	0.800	0.00539	118.731	9.29E-34	4.77E-32	5.13E-32	0.0188
	0.275	10000	1.12E-31	2.68E-33	41.970	1.749	0.01178	259.571	2.08E-33	1.08E-31	1.16E-31	0.0185
	0.300	10000	2.60E-31	2.68E-33	96.956	3.751	0.02527	556.653	4.65E-33	2.51E-31	2.69E-31	0.0179
	0.325	10000	5.72E-31	2.68E-33	213.720	7.743	0.05217	1149.186	9.85E-33	5.53E-31	5.92E-31	0.0172
	0.350	10000	1.21E-30	2.68E-33	453.072	15.116	0.10185	2243.478	1.98E-32	1.17E-30	1.25E-30	0.0163
	0.375	10000	2.35E-30	2.68E-33	875.663	27.451	0.18496	4074.040	3.72E-32	2.27E-30	2.42E-30	0.0158
	0.400	10000	4.22E-30	2.68E-33	1574.049	45.693	0.30788	6781.417	6.33E-32	4.09E-30	4.34E-30	0.0150
	0.425	10000	6.54E-30	2.68E-33	2440.946	68.847	0.46389	10217.825	9.60E-32	6.35E-30	6.73E-30	0.0147
	0.450	10000	9.11E-30	2.68E-33	3399.220	92.957	0.62634	13796.070	1.30E-31	8.85E-30	9.36E-30	0.0143
	0.475	10000	1.13E-29	2.68E-33	4227.878	111.642	0.75224	16569.156	1.58E-31	1.10E-29	1.16E-29	0.0139
	0.500	10000	1.21E-29	2.68E-33	4505.760	118.731	0.80000	17621.173	1.67E-31	1.17E-29	1.24E-29	0.0138
	0.525	10000	1.13E-29	2.68E-33	4211.232	111.642	0.75224	16569.156	1.56E-31	1.10E-29	1.16E-29	0.0138
	0.550	10000	9.29E-30	2.68E-33	3469.383	92.957	0.62634	13796.070	1.29E-31	9.04E-30	9.55E-30	0.0138
	0.575	10000	6.88E-30	2.68E-33	2567.108	68.847	0.46389	10217.825	9.59E-32	6.69E-30	7.06E-30	0.0139
	0.600	10000	4.67E-30	2.68E-33	1744.227	45.693	0.30788	6781.417	6.36E-32	4.55E-30	4.80E-30	0.0136
	0.625	10000	2.74E-30	2.68E-33	1023.988	27.451	0.18496	4074.040	3.78E-32	2.67E-30	2.82E-30	0.0138
	0.650	10000	1.48E-30	2.68E-33	553.534	15.116	0.10185	2243.478	2.07E-32	1.44E-30	1.52E-30	0.0140
	0.675	10000	7.60E-31	2.68E-33	283.764	7.743	0.05217	1149.186	1.07E-32	7.39E-31	7.81E-31	0.0140
	0.700	10000	3.72E-31	2.68E-33	138.823	3.751	0.02527	556.653	5.19E-33	3.62E-31	3.82E-31	0.0140
	0.725	10000	1.78E-31	2.68E-33	66.314	1.749	0.01178	259.571	2.44E-33	1.73E-31	1.82E-31	0.0137
	0.750	10000	7.88E-32	2.68E-33	29.402	0.800	0.00539	118.731	1.10E-33	7.66E-32	8.09E-32	0.0140
	0.775	10000	3.58E-32	2.68E-33	13.368	0.366	0.00247	54.309	4.98E-34	3.48E-32	3.68E-32	0.0139
	0.800	10000	1.61E-32	2.68E-33	6.025	0.171	0.00115	25.324	2.32E-34	1.57E-32	1.66E-32	0.0144
	0.825	10000	7.97E-33	2.68E-33	2.977	0.083	0.00056	12.267	1.13E-34	7.75E-33	8.19E-33	0.0142
	0.850	10000	3.99E-33	2.68E-33	1.488	0.042	0.00029	6.284	5.76E-35	3.87E-33	4.10E-33	0.0144
	0.875	10000	2.20E-33	2.68E-33	0.820	0.023	0.00016	3.460	3.14E-35	2.14E-33	2.26E-33	0.0143
	0.900	10000	1.30E-33	2.68E-33	0.487	0.014	0.00009	2.079	1.89E-35	1.27E-33	1.34E-33	0.0145
	0.925	10000	8.79E-34	2.68E-33	0.328	0.009	0.00006	1.380	1.26E-35	8.54E-34	9.04E-34	0.0143
	0.950	10000	6.41E-34	2.68E-33	0.239	0.007	0.00005	1.022	9.26E-36	6.23E-34	6.59E-34	0.0145
	0.975	10000	5.19E-34	2.68E-33	0.194	0.006	0.00004	0.851	7.67E-36	5.04E-34	5.34E-34	0.0148

Table 10 Simulation estimates of $\hat{p} \equiv P(W_y > b) \equiv A_y e^{-\theta^* b}$ in the $M_t/M/1$ model as a function of y based on $n = 5,000$ replications: $\gamma = 1, b = 20, \rho = 0.9, \bar{\lambda} = 1, \mu = 1.11, \beta = 0.2$

$\gamma = 1$	position	n	$\hat{p}$	$exp(-\theta^* b)$	A_y	A_y approx	A_yUB	A_yLB	s.e.	95% CI (lb)	(ub)	r.e.
	0.000	10000	0.0954	0.108	0.881	0.880	0.861	0.900	9.60E-05	0.0953	0.0956	0.00101
	0.025	10000	0.0956	0.108	0.883	0.880	0.861	0.900	9.63E-05	0.0954	0.0958	0.00101
	0.050	10000	0.0957	0.108	0.883	0.881	0.862	0.901	9.73E-05	0.0955	0.0959	0.00102
	0.075	10000	0.0956	0.108	0.883	0.882	0.863	0.902	9.72E-05	0.0955	0.0958	0.00102
	0.100	10000	0.0961	0.108	0.887	0.884	0.865	0.904	9.68E-05	0.0959	0.0963	0.00101
	0.125	10000	0.0961	0.108	0.886	0.886	0.866	0.906	9.82E-05	0.0959	0.0963	0.00102
	0.150	10000	0.0963	0.108	0.889	0.888	0.869	0.908	9.80E-05	0.0961	0.0965	0.00102
	0.175	10000	0.0968	0.108	0.893	0.891	0.871	0.911	9.79E-05	0.0966	0.0970	0.00101
	0.200	10000	0.0970	0.108	0.895	0.894	0.874	0.914	9.86E-05	0.0968	0.0972	0.00102
	0.225	10000	0.0973	0.108	0.898	0.897	0.877	0.917	9.84E-05	0.0972	0.0975	0.00101
	0.250	10000	0.0976	0.108	0.901	0.900	0.880	0.920	9.81E-05	0.0974	0.0978	0.00100
	0.275	10000	0.0980	0.108	0.904	0.903	0.883	0.923	1.00E-04	0.0978	0.0982	0.00102
	0.300	10000	0.0984	0.108	0.908	0.906	0.886	0.927	9.93E-05	0.0982	0.0986	0.00101
	0.325	10000	0.0985	0.108	0.909	0.909	0.889	0.930	1.01E-04	0.0983	0.0987	0.00103
	0.350	10000	0.0992	0.108	0.915	0.912	0.892	0.932	1.00E-04	0.0990	0.0994	0.00101
	0.375	10000	0.0993	0.108	0.916	0.914	0.894	0.935	9.93E-05	0.0991	0.0994	0.00100
	0.400	10000	0.0994	0.108	0.917	0.916	0.896	0.937	1.02E-04	0.0992	0.0996	0.00102
	0.425	10000	0.0997	0.108	0.920	0.918	0.898	0.939	1.02E-04	0.0995	0.0999	0.00102
	0.450	10000	0.0997	0.108	0.920	0.919	0.899	0.940	1.02E-04	0.0995	0.0999	0.00102
	0.475	10000	0.0998	0.108	0.921	0.920	0.900	0.941	1.01E-04	0.0996	0.1000	0.00102
	0.500	10000	0.0998	0.108	0.921	0.920	0.900	0.941	1.02E-04	0.0996	0.1000	0.00102
	0.525	10000	0.0997	0.108	0.920	0.920	0.900	0.941	1.02E-04	0.0995	0.0999	0.00102
	0.550	10000	0.0998	0.108	0.921	0.919	0.899	0.940	1.02E-04	0.0996	0.1000	0.00102
	0.575	10000	0.0997	0.108	0.920	0.918	0.898	0.939	1.01E-04	0.0995	0.0998	0.00101
	0.600	10000	0.0993	0.108	0.917	0.916	0.896	0.937	1.01E-04	0.0991	0.0995	0.00101
	0.625	10000	0.0993	0.108	0.916	0.914	0.894	0.935	1.02E-04	0.0991	0.0995	0.00103
	0.650	10000	0.0987	0.108	0.911	0.912	0.892	0.932	1.03E-04	0.0985	0.0989	0.00104
	0.675	10000	0.0988	0.108	0.912	0.909	0.889	0.930	9.90E-05	0.0986	0.0990	0.00100
	0.700	10000	0.0984	0.108	0.908	0.906	0.886	0.927	9.98E-05	0.0982	0.0986	0.00101
	0.725	10000	0.0979	0.108	0.904	0.903	0.883	0.923	1.00E-04	0.0978	0.0981	0.00102
	0.750	10000	0.0978	0.108	0.902	0.900	0.880	0.920	9.81E-05	0.0976	0.0980	0.00100
	0.775	10000	0.0974	0.108	0.899	0.897	0.877	0.917	9.85E-05	0.0972	0.0976	0.00101
	0.800	10000	0.0972	0.108	0.897	0.894	0.874	0.914	9.73E-05	0.0970	0.0973	0.00100
	0.825	10000	0.0964	0.108	0.890	0.891	0.871	0.911	1.02E-04	0.0962	0.0966	0.00106
	0.850	10000	0.0963	0.108	0.889	0.888	0.869	0.908	9.84E-05	0.0961	0.0965	0.00102
	0.875	10000	0.0961	0.108	0.887	0.886	0.866	0.906	9.93E-05	0.0960	0.0963	0.00103
	0.900	10000	0.0963	0.108	0.888	0.884	0.865	0.904	9.43E-05	0.0961	0.0964	0.00098
	0.925	10000	0.0958	0.108	0.884	0.882	0.863	0.902	9.82E-05	0.0956	0.0960	0.00103
	0.950	10000	0.0956	0.108	0.882	0.881	0.862	0.901	9.70E-05	0.0954	0.0958	0.00101
	0.975	10000	0.0957	0.108	0.883	0.880	0.861	0.900	9.63E-05	0.0955	0.0959	0.00101

Table 11 Simulation estimates of $\hat{p} \equiv P(W_y > b) \equiv A_y e^{-\theta^* b}$ in the $M_t/M/1$ model as a function of y based on $n = 5,000$ replications: $\gamma = 0.1, b = 50, \rho = 0.9, \bar{\lambda} = 1, \mu = 1.11, \beta = 0.2$

$\gamma = 0.1$	position	n	$\hat{p}$	$exp(-\theta^* b)$	A_y	A_y approx	A_yUB	A_yLB	s.e.	95% CI (lb)	(ub)	r.e.
	0.000	10000	0.00292	0.00387	0.757	0.721	0.577	0.900	5.22E-06	0.00291	0.00293	0.00178
	0.025	10000	0.00293	0.00387	0.759	0.723	0.579	0.902	5.23E-06	0.00292	0.00294	0.00178
	0.050	10000	0.00296	0.00387	0.766	0.729	0.583	0.910	5.32E-06	0.00295	0.00297	0.00180
	0.075	10000	0.00300	0.00387	0.776	0.738	0.591	0.922	5.32E-06	0.00299	0.00301	0.00177
	0.100	10000	0.00306	0.00387	0.792	0.752	0.602	0.939	5.42E-06	0.00305	0.00307	0.00177
	0.125	10000	0.00312	0.00387	0.808	0.769	0.616	0.961	5.56E-06	0.00311	0.00314	0.00178
	0.150	10000	0.00321	0.00387	0.829	0.790	0.632	0.986	5.75E-06	0.00319	0.00322	0.00179
	0.175	10000	0.00331	0.00387	0.857	0.814	0.652	1.016	5.86E-06	0.00330	0.00332	0.00177
	0.200	10000	0.00342	0.00387	0.884	0.840	0.673	1.049	6.07E-06	0.00340	0.00343	0.00178
	0.225	10000	0.00353	0.00387	0.914	0.869	0.696	1.086	6.30E-06	0.00352	0.00355	0.00178
	0.250	10000	0.00365	0.00387	0.944	0.900	0.721	1.124	6.55E-06	0.00364	0.00366	0.00180
	0.275	10000	0.00378	0.00387	0.978	0.932	0.746	1.164	6.74E-06	0.00377	0.00379	0.00178
	0.300	10000	0.00392	0.00387	1.014	0.964	0.772	1.204	7.02E-06	0.00391	0.00394	0.00179
	0.325	10000	0.00404	0.00387	1.045	0.996	0.797	1.243	7.20E-06	0.00403	0.00405	0.00178
	0.350	10000	0.00417	0.00387	1.078	1.026	0.821	1.281	7.45E-06	0.00415	0.00418	0.00179
	0.375	10000	0.00428	0.00387	1.108	1.053	0.843	1.315	7.71E-06	0.00427	0.00430	0.00180
	0.400	10000	0.00438	0.00387	1.132	1.077	0.863	1.345	7.86E-06	0.00436	0.00439	0.00180
	0.425	10000	0.00445	0.00387	1.152	1.097	0.878	1.370	7.89E-06	0.00444	0.00447	0.00177
	0.450	10000	0.00452	0.00387	1.168	1.112	0.890	1.388	8.10E-06	0.00450	0.00453	0.00179
	0.475	10000	0.00454	0.00387	1.174	1.121	0.898	1.400	8.11E-06	0.00452	0.00455	0.00179
	0.500	10000	0.00456	0.00387	1.179	1.124	0.900	1.404	8.24E-06	0.00454	0.00457	0.00181
	0.525	10000	0.00455	0.00387	1.177	1.121	0.898	1.400	8.09E-06	0.00453	0.00457	0.00178
	0.550	10000	0.00452	0.00387	1.170	1.112	0.890	1.388	8.01E-06	0.00451	0.00454	0.00177
	0.575	10000	0.00446	0.00387	1.153	1.097	0.878	1.370	7.94E-06	0.00444	0.00447	0.00178
	0.600	10000	0.00437	0.00387	1.131	1.077	0.863	1.345	7.79E-06	0.00436	0.00439	0.00178
	0.625	10000	0.00427	0.00387	1.106	1.053	0.843	1.315	7.65E-06	0.00426	0.00429	0.00179
	0.650	10000	0.00416	0.00387	1.077	1.026	0.821	1.281	7.43E-06	0.00415	0.00418	0.00179
	0.675	10000	0.00405	0.00387	1.047	0.996	0.797	1.243	7.16E-06	0.00403	0.00406	0.00177
	0.700	10000	0.00391	0.00387	1.013	0.964	0.772	1.204	7.05E-06	0.00390	0.00393	0.00180
	0.725	10000	0.00378	0.00387	0.977	0.932	0.746	1.164	6.78E-06	0.00376	0.00379	0.00179
	0.750	10000	0.00366	0.00387	0.946	0.900	0.721	1.124	6.51E-06	0.00365	0.00367	0.00178
	0.775	10000	0.00353	0.00387	0.914	0.869	0.696	1.086	6.27E-06	0.00352	0.00355	0.00177
	0.800	10000	0.00341	0.00387	0.882	0.840	0.673	1.049	6.16E-06	0.00340	0.00342	0.00181
	0.825	10000	0.00329	0.00387	0.850	0.814	0.652	1.016	5.94E-06	0.00328	0.00330	0.00181
	0.850	10000	0.00320	0.00387	0.829	0.790	0.632	0.986	5.75E-06	0.00319	0.00321	0.00180
	0.875	10000	0.00312	0.00387	0.806	0.769	0.616	0.961	5.52E-06	0.00311	0.00313	0.00177
	0.900	10000	0.00305	0.00387	0.789	0.752	0.602	0.939	5.43E-06	0.00304	0.00306	0.00178
	0.925	10000	0.00300	0.00387	0.776	0.738	0.591	0.922	5.33E-06	0.00299	0.00301	0.00178
	0.950	10000	0.00295	0.00387	0.764	0.729	0.583	0.910	5.28E-06	0.00294	0.00296	0.00179
	0.975	10000	0.00294	0.00387	0.760	0.723	0.579	0.902	5.23E-06	0.00293	0.00295	0.00178

Table 12 **Simulation estimates of $\hat{p} \equiv P(W_y > b) \equiv A_y e^{-\theta^* b}$ in the $M_t/M/1$ model as a function of y based on $n = 5,000$ replications: $\gamma = 10, b = 20, \rho = 0.8, \bar{\lambda} = 1, \mu = 1.25, \beta = 0.5$**

$\gamma = 10$	position	n	$\hat{p}$	$exp(-\theta^* b)$	A	A_y approx	s.e.	95% CI (lb)	(ub)	r.e.
	0.000	10000	0.005312	0.00674	0.788	0.790	1.10E-05	0.005290	0.005333	0.00206
	0.025	10000	0.005315	0.00674	0.789	0.790	1.09E-05	0.005294	0.005337	0.00204
	0.050	10000	0.005323	0.00674	0.790	0.791	1.08E-05	0.005302	0.005345	0.00203
	0.075	10000	0.005347	0.00674	0.794	0.791	1.08E-05	0.005326	0.005369	0.00202
	0.100	10000	0.005318	0.00674	0.789	0.792	1.10E-05	0.005297	0.005340	0.00208
	0.125	10000	0.005334	0.00674	0.792	0.793	1.10E-05	0.005312	0.005355	0.00206
	0.150	10000	0.005356	0.00674	0.795	0.794	1.10E-05	0.005334	0.005377	0.00205
	0.175	10000	0.005373	0.00674	0.797	0.795	1.09E-05	0.005351	0.005394	0.00203
	0.200	10000	0.005383	0.00674	0.799	0.797	1.09E-05	0.005362	0.005405	0.00203
	0.225	10000	0.005387	0.00674	0.799	0.798	1.10E-05	0.005365	0.005408	0.00205
	0.250	10000	0.005409	0.00674	0.803	0.800	1.09E-05	0.005388	0.005430	0.00201
	0.275	10000	0.005417	0.00674	0.804	0.802	1.11E-05	0.005396	0.005439	0.00204
	0.300	10000	0.005408	0.00674	0.803	0.803	1.10E-05	0.005386	0.005429	0.00204
	0.325	10000	0.005427	0.00674	0.805	0.805	1.09E-05	0.005405	0.005448	0.00200
	0.350	10000	0.005432	0.00674	0.806	0.806	1.10E-05	0.005410	0.005453	0.00202
	0.375	10000	0.005449	0.00674	0.809	0.807	1.12E-05	0.005427	0.005471	0.00205
	0.400	10000	0.005437	0.00674	0.807	0.808	1.12E-05	0.005415	0.005459	0.00206
	0.425	10000	0.005467	0.00674	0.811	0.809	1.10E-05	0.005445	0.005489	0.00202
	0.450	10000	0.005453	0.00674	0.809	0.810	1.12E-05	0.005431	0.005475	0.00206
	0.475	10000	0.005462	0.00674	0.811	0.810	1.11E-05	0.005440	0.005484	0.00204
	0.500	10000	0.005451	0.00674	0.809	0.810	1.11E-05	0.005429	0.005472	0.00203
	0.525	10000	0.005440	0.00674	0.807	0.810	1.11E-05	0.005418	0.005462	0.00205
	0.550	10000	0.005443	0.00674	0.808	0.810	1.13E-05	0.005421	0.005465	0.00208
	0.575	10000	0.005475	0.00674	0.813	0.809	1.09E-05	0.005454	0.005497	0.00200
	0.600	10000	0.005445	0.00674	0.808	0.808	1.12E-05	0.005423	0.005467	0.00205
	0.625	10000	0.005434	0.00674	0.806	0.807	1.12E-05	0.005412	0.005456	0.00206
	0.650	10000	0.005440	0.00674	0.807	0.806	1.10E-05	0.005418	0.005462	0.00203
	0.675	10000	0.005424	0.00674	0.805	0.805	1.12E-05	0.005402	0.005446	0.00206
	0.700	10000	0.005400	0.00674	0.801	0.803	1.12E-05	0.005378	0.005422	0.00207
	0.725	10000	0.005408	0.00674	0.803	0.802	1.11E-05	0.005386	0.005430	0.00205
	0.750	10000	0.005375	0.00674	0.798	0.800	1.11E-05	0.005353	0.005397	0.00207
	0.775	10000	0.005374	0.00674	0.798	0.798	1.10E-05	0.005352	0.005396	0.00205
	0.800	10000	0.005404	0.00674	0.802	0.797	1.08E-05	0.005383	0.005426	0.00200
	0.825	10000	0.005361	0.00674	0.796	0.795	1.09E-05	0.005340	0.005383	0.00203
	0.850	10000	0.005351	0.00674	0.794	0.794	1.10E-05	0.005329	0.005372	0.00205
	0.875	10000	0.005358	0.00674	0.795	0.793	1.08E-05	0.005337	0.005380	0.00201
	0.900	10000	0.005349	0.00674	0.794	0.792	1.08E-05	0.005328	0.005371	0.00202
	0.925	10000	0.005346	0.00674	0.793	0.791	1.08E-05	0.005325	0.005367	0.00203
	0.950	10000	0.005323	0.00674	0.790	0.791	1.08E-05	0.005302	0.005344	0.00203
	0.975	10000	0.005319	0.00674	0.789	0.790	1.10E-05	0.005297	0.005340	0.00206

Table 13 Simulation estimates of $\hat{p} \equiv P(W_y > b) \equiv A_y e^{-\theta^* b}$ in the $M_t/M/1$ model as a function of y based on $n = 5,000$ replications: $\gamma = 1, b = 20, \rho = 0.8, \bar{\lambda} = 1, \mu = 1.25, \beta = 0.5$

$\gamma = 1$	position	n	$\hat{p}$	$exp(-\theta^* b)$	A	A_y approx	A_yUB	A_yLB	s.e.	95% CI (lb)	(ub)	r.e.
	0.000	10000	0.00486	0.00674	0.721	0.706	0.623	0.800	1.07E-05	0.00484	0.00488	0.00219
	0.025	10000	0.00485	0.00674	0.720	0.707	0.624	0.801	1.09E-05	0.00483	0.00487	0.00224
	0.050	10000	0.00490	0.00674	0.727	0.710	0.627	0.805	1.08E-05	0.00488	0.00492	0.00220
	0.075	10000	0.00492	0.00674	0.731	0.716	0.632	0.811	1.09E-05	0.00490	0.00495	0.00221
	0.100	10000	0.00497	0.00674	0.738	0.723	0.638	0.819	1.11E-05	0.00495	0.00500	0.00222
	0.125	10000	0.00505	0.00674	0.750	0.732	0.646	0.830	1.11E-05	0.00503	0.00507	0.00220
	0.150	10000	0.00512	0.00674	0.759	0.743	0.656	0.842	1.15E-05	0.00509	0.00514	0.00224
	0.175	10000	0.00521	0.00674	0.773	0.756	0.667	0.857	1.16E-05	0.00518	0.00523	0.00223
	0.200	10000	0.00528	0.00674	0.784	0.770	0.679	0.872	1.17E-05	0.00526	0.00531	0.00222
	0.225	10000	0.00540	0.00674	0.801	0.785	0.692	0.889	1.19E-05	0.00537	0.00542	0.00221
	0.250	10000	0.00550	0.00674	0.816	0.800	0.706	0.907	1.22E-05	0.00547	0.00552	0.00221
	0.275	10000	0.00563	0.00674	0.835	0.816	0.720	0.924	1.25E-05	0.00560	0.00565	0.00222
	0.300	10000	0.00576	0.00674	0.854	0.832	0.734	0.942	1.25E-05	0.00573	0.00578	0.00218
	0.325	10000	0.00583	0.00674	0.865	0.847	0.747	0.959	1.28E-05	0.00580	0.00585	0.00220
	0.350	10000	0.00593	0.00674	0.880	0.861	0.760	0.976	1.29E-05	0.00590	0.00595	0.00218
	0.375	10000	0.00601	0.00674	0.892	0.874	0.771	0.990	1.34E-05	0.00598	0.00603	0.00222
	0.400	10000	0.00605	0.00674	0.898	0.885	0.781	1.003	1.36E-05	0.00602	0.00608	0.00225
	0.425	10000	0.00617	0.00674	0.916	0.894	0.789	1.013	1.35E-05	0.00615	0.00620	0.00219
	0.450	10000	0.00618	0.00674	0.917	0.901	0.795	1.021	1.40E-05	0.00615	0.00621	0.00226
	0.475	10000	0.00624	0.00674	0.926	0.905	0.799	1.026	1.37E-05	0.00621	0.00626	0.00220
	0.500	10000	0.00621	0.00674	0.922	0.907	0.800	1.027	1.38E-05	0.00619	0.00624	0.00222
	0.525	10000	0.00624	0.00674	0.927	0.905	0.799	1.026	1.37E-05	0.00622	0.00627	0.00219
	0.550	10000	0.00620	0.00674	0.920	0.901	0.795	1.021	1.38E-05	0.00617	0.00623	0.00222
	0.575	10000	0.00615	0.00674	0.912	0.894	0.789	1.013	1.39E-05	0.00612	0.00618	0.00225
	0.600	10000	0.00609	0.00674	0.904	0.885	0.781	1.003	1.35E-05	0.00606	0.00611	0.00222
	0.625	10000	0.00600	0.00674	0.890	0.874	0.771	0.990	1.35E-05	0.00597	0.00602	0.00224
	0.650	10000	0.00593	0.00674	0.881	0.861	0.760	0.976	1.31E-05	0.00591	0.00596	0.00220
	0.675	10000	0.00582	0.00674	0.864	0.847	0.747	0.959	1.30E-05	0.00579	0.00585	0.00223
	0.700	10000	0.00571	0.00674	0.847	0.832	0.734	0.942	1.27E-05	0.00568	0.00573	0.00223
	0.725	10000	0.00562	0.00674	0.834	0.816	0.720	0.924	1.25E-05	0.00559	0.00564	0.00222
	0.750	10000	0.00551	0.00674	0.818	0.800	0.706	0.907	1.23E-05	0.00548	0.00553	0.00223
	0.775	10000	0.00541	0.00674	0.804	0.785	0.692	0.889	1.19E-05	0.00539	0.00544	0.00220
	0.800	10000	0.00527	0.00674	0.782	0.770	0.679	0.872	1.18E-05	0.00525	0.00529	0.00224
	0.825	10000	0.00520	0.00674	0.772	0.756	0.667	0.857	1.16E-05	0.00518	0.00523	0.00223
	0.850	10000	0.00510	0.00674	0.757	0.743	0.656	0.842	1.14E-05	0.00508	0.00513	0.00223
	0.875	10000	0.00505	0.00674	0.749	0.732	0.646	0.830	1.12E-05	0.00503	0.00507	0.00222
	0.900	10000	0.00497	0.00674	0.738	0.723	0.638	0.819	1.09E-05	0.00495	0.00500	0.00219
	0.925	10000	0.00493	0.00674	0.732	0.716	0.632	0.811	1.10E-05	0.00491	0.00495	0.00223
	0.950	10000	0.00487	0.00674	0.723	0.710	0.627	0.805	1.09E-05	0.00485	0.00489	0.00224
	0.975	10000	0.00488	0.00674	0.724	0.707	0.624	0.801	1.08E-05	0.00485749	0.004899702	0.00221

Table 14 Summary of simulation results for a fixed b and differing y in a cycle:

$\gamma = 0.1, b = 100, \rho = 0.8, \bar{\lambda} = 1, \mu = 1.25, \beta = 0.5$

$\gamma = 0.1$	position	n	$\hat{p}$	$exp(-\theta^* b)$	A	A_y approx	A_yUB	A_yLB	s.e.	95% CI (lb)	(ub)	r.e.
	0.000	10000	5.89E-12	1.39E-11	0.424	0.229	0.066	0.800	3.74E-14	5.82E-12	5.96E-12	0.00635
	0.025	10000	5.99E-12	1.39E-11	0.431	0.233	0.067	0.812	3.75E-14	5.92E-12	6.06E-12	0.00627
	0.050	10000	6.26E-12	1.39E-11	0.451	0.244	0.070	0.850	3.93E-14	6.18E-12	6.34E-12	0.00628
	0.075	10000	6.71E-12	1.39E-11	0.483	0.263	0.075	0.917	4.24E-14	6.63E-12	6.80E-12	0.00632
	0.100	10000	7.44E-12	1.39E-11	0.536	0.291	0.083	1.016	4.72E-14	7.35E-12	7.53E-12	0.00634
	0.125	10000	8.50E-12	1.39E-11	0.612	0.331	0.095	1.154	5.34E-14	8.39E-12	8.60E-12	0.00629
	0.150	10000	9.87E-12	1.39E-11	0.711	0.384	0.110	1.339	6.20E-14	9.75E-12	9.99E-12	0.00628
	0.175	10000	1.16E-11	1.39E-11	0.832	0.454	0.130	1.583	7.29E-14	1.14E-11	1.17E-11	0.00631
	0.200	10000	1.39E-11	1.39E-11	1.004	0.544	0.156	1.898	8.76E-14	1.38E-11	1.41E-11	0.00628
	0.225	10000	1.69E-11	1.39E-11	1.217	0.658	0.188	2.296	1.06E-13	1.67E-11	1.71E-11	0.00628
	0.250	10000	2.05E-11	1.39E-11	1.474	0.800	0.229	2.792	1.29E-13	2.02E-11	2.07E-11	0.00628
	0.275	10000	2.49E-11	1.39E-11	1.796	0.973	0.279	3.395	1.59E-13	2.46E-11	2.52E-11	0.00636
	0.300	10000	3.04E-11	1.39E-11	2.185	1.177	0.337	4.109	1.91E-13	3.00E-11	3.07E-11	0.00630
	0.325	10000	3.61E-11	1.39E-11	2.599	1.411	0.404	4.925	2.27E-13	3.56E-11	3.65E-11	0.00630
	0.350	10000	4.33E-11	1.39E-11	3.117	1.668	0.478	5.822	2.72E-13	4.28E-11	4.38E-11	0.00628
	0.375	10000	4.96E-11	1.39E-11	3.568	1.936	0.555	6.758	3.12E-13	4.89E-11	5.02E-11	0.00629
	0.400	10000	5.61E-11	1.39E-11	4.040	2.199	0.630	7.676	3.56E-13	5.54E-11	5.68E-11	0.00634
	0.425	10000	6.24E-11	1.39E-11	4.492	2.437	0.698	8.505	3.94E-13	6.16E-11	6.32E-11	0.00631
	0.450	10000	6.78E-11	1.39E-11	4.879	2.627	0.753	9.168	4.27E-13	6.69E-11	6.86E-11	0.00630
	0.475	10000	7.05E-11	1.39E-11	5.076	2.750	0.788	9.597	4.43E-13	6.96E-11	7.14E-11	0.00629
	0.500	10000	7.11E-11	1.39E-11	5.122	2.792	0.800	9.746	4.49E-13	7.02E-11	7.20E-11	0.00631
	0.525	10000	7.05E-11	1.39E-11	5.076	2.750	0.788	9.597	4.46E-13	6.96E-11	7.14E-11	0.00633
	0.550	10000	6.80E-11	1.39E-11	4.895	2.627	0.753	9.168	4.26E-13	6.72E-11	6.88E-11	0.00627
	0.575	10000	6.24E-11	1.39E-11	4.492	2.437	0.698	8.505	3.96E-13	6.16E-11	6.32E-11	0.00635
	0.600	10000	5.59E-11	1.39E-11	4.022	2.199	0.630	7.676	3.54E-13	5.52E-11	5.66E-11	0.00633
	0.625	10000	4.98E-11	1.39E-11	3.588	1.936	0.555	6.758	3.15E-13	4.92E-11	5.04E-11	0.00631
	0.650	10000	4.27E-11	1.39E-11	3.078	1.668	0.478	5.822	2.70E-13	4.22E-11	4.33E-11	0.00631
	0.675	10000	3.64E-11	1.39E-11	2.624	1.411	0.404	4.925	2.29E-13	3.60E-11	3.69E-11	0.00629
	0.700	10000	3.03E-11	1.39E-11	2.180	1.177	0.337	4.109	1.91E-13	2.99E-11	3.06E-11	0.00630
	0.725	10000	2.47E-11	1.39E-11	1.782	0.973	0.279	3.395	1.58E-13	2.44E-11	2.51E-11	0.00638
	0.750	10000	2.05E-11	1.39E-11	1.476	0.800	0.229	2.792	1.30E-13	2.02E-11	2.08E-11	0.00635
	0.775	10000	1.69E-11	1.39E-11	1.218	0.658	0.188	2.296	1.05E-13	1.67E-11	1.71E-11	0.00623
	0.800	10000	1.39E-11	1.39E-11	1.000	0.544	0.156	1.898	8.80E-14	1.37E-11	1.41E-11	0.00634
	0.825	10000	1.16E-11	1.39E-11	0.835	0.454	0.130	1.583	7.35E-14	1.15E-11	1.17E-11	0.00633
	0.850	10000	9.80E-12	1.39E-11	0.706	0.384	0.110	1.339	6.25E-14	9.68E-12	9.93E-12	0.00638
	0.875	10000	8.56E-12	1.39E-11	0.617	0.331	0.095	1.154	5.39E-14	8.46E-12	8.67E-12	0.00630
	0.900	10000	7.43E-12	1.39E-11	0.535	0.291	0.083	1.016	4.68E-14	7.34E-12	7.52E-12	0.00630
	0.925	10000	6.77E-12	1.39E-11	0.487	0.263	0.075	0.917	4.24E-14	6.68E-12	6.85E-12	0.00626
	0.950	10000	6.18E-12	1.39E-11	0.445	0.244	0.070	0.850	3.96E-14	6.10E-12	6.25E-12	0.00641
	0.975	10000	6.03E-12	1.39E-11	0.434	0.233	0.067	0.812	3.80E-14	5.96E-12	6.11E-12	0.00629

Table 15 Simulation estimates of $\hat{p} \equiv P(W_y > b) \equiv A_y e^{-\theta^* b}$ in the $M_t/M/1$ model as a function of y, for y in the small interval $[0.45, 0.55]$ based on $n = 5,000$ replications: $\gamma = 0.1, b = 20, \rho = 0.8, \bar{\lambda} = 1, \mu = 1.25, \beta = 0.2$

$\gamma = 0.1$	position	n	$\hat{p}$	$exp(-\theta^* b)$	A_y	s.e.	95% CI (lb)	(ub)	r.e.
	0.4500	10000	0.0102042	0.00674	1.514	3.81E-05	0.0101296	0.0102789	0.00373
	0.4525	10000	0.0102480	0.00674	1.521	3.85E-05	0.0101726	0.0103234	0.00375
	0.4550	10000	0.0102694	0.00674	1.524	3.83E-05	0.0101944	0.0103445	0.00373
	0.4575	10000	0.0102476	0.00674	1.521	3.84E-05	0.0101723	0.0103229	0.00375
	0.4600	10000	0.0103224	0.00674	1.532	3.86E-05	0.0102467	0.0103981	0.00374
	0.4625	10000	0.0101856	0.00674	1.512	3.85E-05	0.0101101	0.0102611	0.00378
	0.4650	10000	0.0103051	0.00674	1.529	3.87E-05	0.0102292	0.0103810	0.00376
	0.4675	10000	0.0102580	0.00674	1.522	3.84E-05	0.0101826	0.0103333	0.00375
	0.4700	10000	0.0103188	0.00674	1.531	3.83E-05	0.0102438	0.0103938	0.00371
	0.4725	10000	0.0103469	0.00674	1.536	3.87E-05	0.0102711	0.0104227	0.00374
	0.4750	10000	0.0102930	0.00674	1.528	3.83E-05	0.0102179	0.0103681	0.00372
	0.4775	10000	0.0103730	0.00674	1.539	3.90E-05	0.0102966	0.0104495	0.00376
	0.4800	10000	0.0103778	0.00674	1.540	3.82E-05	0.0103029	0.0104528	0.00369
	0.4825	10000	0.0103410	0.00674	1.535	3.88E-05	0.0102649	0.0104172	0.00376
	0.4850	10000	0.0103687	0.00674	1.539	3.88E-05	0.0102926	0.0104448	0.00374
	0.4875	10000	0.0104335	0.00674	1.548	3.88E-05	0.0103574	0.0105097	0.00372
	0.4900	10000	0.0103590	0.00674	1.537	3.86E-05	0.0102833	0.0104346	0.00373
	0.4925	10000	0.0103960	0.00674	1.543	3.89E-05	0.0103197	0.0104723	0.00374
	0.4950	10000	0.0103041	0.00674	1.529	3.87E-05	0.0102282	0.0103800	0.00376
	0.4975	10000	0.0104239	0.00674	1.547	3.92E-05	0.0103472	0.0105007	0.00376
	0.5000	10000	0.0104064	0.00674	1.544	3.89E-05	0.0103300	0.0104827	0.00374
	0.5025	10000	0.0103887	0.00674	1.542	3.88E-05	0.0103125	0.0104648	0.00374
	0.5050	10000	0.0104046	0.00674	1.544	3.90E-05	0.0103281	0.0104811	0.00375
	0.5075	10000	0.0103907	0.00674	1.542	3.89E-05	0.0103144	0.0104670	0.00375
	0.5100	10000	0.0103596	0.00674	1.538	3.88E-05	0.0102835	0.0104357	0.00375
	0.5125	10000	0.0103260	0.00674	1.533	3.83E-05	0.0102509	0.0104010	0.00371
	0.5150	10000	0.0104469	0.00674	1.550	3.87E-05	0.0103711	0.0105226	0.00370
	0.5175	10000	0.0103561	0.00674	1.537	3.85E-05	0.0102806	0.0104316	0.00372
	0.5200	10000	0.0104290	0.00674	1.548	3.86E-05	0.0103534	0.0105047	0.00370
	0.5225	10000	0.0103480	0.00674	1.536	3.84E-05	0.0102727	0.0104232	0.00371
	0.5250	10000	0.0103970	0.00674	1.543	3.84E-05	0.0103218	0.0104723	0.00369
	0.5275	10000	0.0102753	0.00674	1.525	3.83E-05	0.0102004	0.0103503	0.00372
	0.5300	10000	0.0102461	0.00674	1.521	3.86E-05	0.0101706	0.0103217	0.00376
	0.5325	10000	0.0102789	0.00674	1.526	3.82E-05	0.0102039	0.0103538	0.00372
	0.5350	10000	0.0102817	0.00674	1.526	3.82E-05	0.0102067	0.0103566	0.00372
	0.5375	10000	0.0102432	0.00674	1.520	3.80E-05	0.0101688	0.0103176	0.00371
	0.5400	10000	0.0102212	0.00674	1.517	3.81E-05	0.0101465	0.0102958	0.00373
	0.5425	10000	0.0102356	0.00674	1.519	3.78E-05	0.0101615	0.0103097	0.00369
	0.5450	10000	0.0102132	0.00674	1.516	3.81E-05	0.0101386	0.0102878	0.00373
	0.5475	10000	0.0101162	0.00674	1.501	3.77E-05	0.0100424	0.0101901	0.00372
	0.5500	10000	0.0101235	0.00674	1.502	3.78E-05	0.0100495	0.0101976	0.00373

3.2. Tail Probability Estimates for the $M_t/M/1$ Periodic Queue, scaled by $\mu = 1$

Table 16 displays simulation results for what we regard as our base case, having parameter 4-tuple $\rho, \beta, \gamma, b) = (0.8, 0.2, 0.1, 20)$, which corresponds to the general framework in (23) of the main paper, i.e.,

$$(\bar{\lambda}_\rho, \beta_\rho, \gamma_\rho, b_\rho) = (\rho, (1-\rho)\beta, (1-\rho)^2\gamma, (1-\rho)^{-1}b), \tag{14}$$

where (β, γ, b) is a feasible base triple of positive constants with $\beta < 1$ when the base triple is $(\beta, \gamma, b) = (1, 2.5, 4.0)$ as in (24) of the main paper. The bounds for A_y are discussed in Corollary 4 of the main paper, while the approximation is discussed at the end here in §**??**.

Tables 17-24 give results for the framework in (14) for the base triple $(\beta, \gamma, b) = (1, 25, 4.0)$. The results for different ρ ranging from $\rho = 0.84$ to $\rho = 0.99$ are summarized in Tables 25, 26 and 27. These summaries strongly supports the heavy-traffic scaling in (14).

Table 16 Simulation estimates of $\hat{p} \equiv P(W_y > b) \equiv A_y e^{-\theta^* b}$ in the $M_t/M/1$ model as a function of y based on $5,000$ replications: $\gamma = 0.1, b = 20, \rho = 0.8, \bar{\lambda} = 0.8, \mu = 1, \beta = 0.2$

$\gamma = 1$	position	n	$\hat{p}$	$exp(-\theta^* b)$	A_y	A_y approx	A_y LB	A_y UB	s.e.	95% CI (lb)	(ub)	r.e.
	0.000	5000	1.05E-02	1.83E-02	0.571	0.536	0.359	0.800	5.00E-05	1.04E-02	1.06E-02	4.78E-03
	0.025	5000	1.05E-02	1.83E-02	0.572	0.539	0.361	0.804	5.06E-05	1.04E-02	1.06E-02	4.83E-03
	0.050	5000	1.06E-02	1.83E-02	0.580	0.547	0.367	0.816	5.08E-05	1.05E-02	1.07E-02	4.79E-03
	0.075	5000	1.09E-02	1.83E-02	0.593	0.560	0.375	0.836	5.27E-05	1.08E-02	1.10E-02	4.85E-03
	0.100	5000	1.13E-02	1.83E-02	0.616	0.579	0.388	0.864	5.47E-05	1.12E-02	1.14E-02	4.85E-03
	0.125	5000	1.17E-02	1.83E-02	0.639	0.603	0.404	0.899	5.62E-05	1.16E-02	1.18E-02	4.80E-03
	0.150	5000	1.24E-02	1.83E-02	0.678	0.632	0.424	0.943	5.99E-05	1.23E-02	1.25E-02	4.82E-03
	0.175	5000	1.30E-02	1.83E-02	0.711	0.667	0.447	0.995	6.29E-05	1.29E-02	1.31E-02	4.83E-03
	0.200	5000	1.40E-02	1.83E-02	0.762	0.707	0.474	1.055	6.79E-05	1.38E-02	1.41E-02	4.87E-03
	0.225	5000	1.48E-02	1.83E-02	0.806	0.751	0.504	1.121	7.21E-05	1.46E-02	1.49E-02	4.88E-03
	0.250	5000	1.60E-02	1.83E-02	0.873	0.800	0.536	1.193	7.67E-05	1.58E-02	1.61E-02	4.79E-03
	0.275	5000	1.72E-02	1.83E-02	0.938	0.852	0.571	1.271	8.32E-05	1.70E-02	1.73E-02	4.84E-03
	0.300	5000	1.83E-02	1.83E-02	1.001	0.905	0.607	1.350	8.74E-05	1.82E-02	1.85E-02	4.77E-03
	0.325	5000	1.95E-02	1.83E-02	1.067	0.959	0.643	1.431	9.28E-05	1.94E-02	1.97E-02	4.75E-03
	0.350	5000	2.10E-02	1.83E-02	1.146	1.012	0.678	1.510	9.69E-05	2.08E-02	2.12E-02	4.62E-03
	0.375	5000	2.17E-02	1.83E-02	1.184	1.062	0.712	1.584	1.01E-04	2.15E-02	2.19E-02	4.68E-03
	0.400	5000	2.27E-02	1.83E-02	1.238	1.106	0.741	1.649	1.05E-04	2.25E-02	2.29E-02	4.65E-03
	0.425	5000	2.35E-02	1.83E-02	1.285	1.143	0.766	1.704	1.10E-04	2.33E-02	2.38E-02	4.66E-03
	0.450	5000	2.42E-02	1.83E-02	1.323	1.170	0.784	1.746	1.12E-04	2.40E-02	2.45E-02	4.61E-03
	0.475	5000	2.45E-02	1.83E-02	1.337	1.188	0.796	1.772	1.13E-04	2.43E-02	2.47E-02	4.61E-03
	0.500	5000	2.47E-02	1.83E-02	1.350	1.193	0.800	1.780	1.13E-04	2.45E-02	2.49E-02	4.56E-03
	0.525	5000	2.43E-02	1.83E-02	1.326	1.188	0.796	1.772	1.12E-04	2.41E-02	2.45E-02	4.62E-03
	0.550	5000	2.40E-02	1.83E-02	1.309	1.170	0.784	1.746	1.10E-04	2.38E-02	2.42E-02	4.58E-03
	0.575	5000	2.34E-02	1.83E-02	1.278	1.143	0.766	1.704	1.08E-04	2.32E-02	2.36E-02	4.63E-03
	0.600	5000	2.26E-02	1.83E-02	1.234	1.106	0.741	1.649	1.04E-04	2.24E-02	2.28E-02	4.61E-03
	0.625	5000	2.15E-02	1.83E-02	1.174	1.062	0.712	1.584	1.01E-04	2.13E-02	2.17E-02	4.68E-03
	0.650	5000	2.04E-02	1.83E-02	1.116	1.012	0.678	1.510	9.51E-05	2.02E-02	2.06E-02	4.66E-03
	0.675	5000	1.94E-02	1.83E-02	1.061	0.959	0.643	1.431	8.93E-05	1.93E-02	1.96E-02	4.60E-03
	0.700	5000	1.81E-02	1.83E-02	0.988	0.905	0.607	1.350	8.47E-05	1.79E-02	1.83E-02	4.68E-03
	0.725	5000	1.71E-02	1.83E-02	0.934	0.852	0.571	1.271	8.01E-05	1.69E-02	1.73E-02	4.68E-03
	0.750	5000	1.60E-02	1.83E-02	0.873	0.800	0.536	1.193	7.54E-05	1.58E-02	1.61E-02	4.72E-03
	0.775	5000	1.50E-02	1.83E-02	0.817	0.751	0.504	1.121	7.14E-05	1.48E-02	1.51E-02	4.77E-03
	0.800	5000	1.40E-02	1.83E-02	0.764	0.707	0.474	1.055	6.71E-05	1.39E-02	1.41E-02	4.79E-03
	0.825	5000	1.31E-02	1.83E-02	0.718	0.667	0.447	0.995	6.22E-05	1.30E-02	1.33E-02	4.73E-03
	0.850	5000	1.25E-02	1.83E-02	0.683	0.632	0.424	0.943	6.00E-05	1.24E-02	1.26E-02	4.80E-03
	0.875	5000	1.19E-02	1.83E-02	0.652	0.603	0.404	0.899	5.69E-05	1.18E-02	1.21E-02	4.77E-03
	0.900	5000	1.15E-02	1.83E-02	0.625	0.579	0.388	0.864	5.48E-05	1.13E-02	1.16E-02	4.79E-03
	0.925	5000	1.10E-02	1.83E-02	0.601	0.560	0.375	0.836	5.31E-05	1.09E-02	1.11E-02	4.82E-03
	0.950	5000	1.07E-02	1.83E-02	0.586	0.547	0.367	0.816	5.17E-05	1.06E-02	1.08E-02	4.81E-03
	0.975	5000	1.05E-02	1.83E-02	0.575	0.539	0.361	0.804	5.11E-05	1.04E-02	1.06E-02	4.86E-03

Table 17 Simulation estimates of $\hat{p} \equiv P(W_y > b) \equiv A_y e^{-\theta^* b}$ in the $M_t/M/1$ model as a function of y based on $5,000$ replications: $\gamma = 1, b = 20, \rho = 0.8, \bar{\lambda} = 0.8, \mu = 1, \beta = 0.2$

position	n	$\hat{p}$	$exp(-\theta^* b)$	A_y	A_y approx	A_y LB	A_y UB	s.e.	95% CI (lb)	(ub)	r.e.
0.000	5000	0.014162	0.0183	0.773	0.769	0.738	0.800	4.07E-05	0.01408	0.01424	0.00288
0.025	5000	0.014104	0.0183	0.770	0.769	0.739	0.800	4.12E-05	0.01402	0.01419	0.00292
0.050	5000	0.014038	0.0183	0.766	0.770	0.740	0.802	4.22E-05	0.01396	0.01412	0.00301
0.075	5000	0.014227	0.0183	0.777	0.772	0.742	0.803	4.04E-05	0.01415	0.01431	0.00284
0.100	5000	0.014197	0.0183	0.775	0.775	0.744	0.806	4.11E-05	0.01412	0.01428	0.00289
0.125	5000	0.014289	0.0183	0.780	0.778	0.747	0.809	4.11E-05	0.01421	0.01437	0.00287
0.150	5000	0.014311	0.0183	0.781	0.781	0.751	0.813	4.21E-05	0.01423	0.01439	0.00294
0.175	5000	0.014465	0.0183	0.790	0.786	0.755	0.818	4.18E-05	0.01438	0.01455	0.00289
0.200	5000	0.014520	0.0183	0.793	0.790	0.759	0.822	4.21E-05	0.01444	0.01460	0.00290
0.225	5000	0.014620	0.0183	0.798	0.795	0.764	0.827	4.24E-05	0.01454	0.01470	0.00290
0.250	5000	0.014725	0.0183	0.804	0.800	0.769	0.833	4.22E-05	0.01464	0.01481	0.00286
0.275	5000	0.014810	0.0183	0.809	0.805	0.773	0.838	4.28E-05	0.01473	0.01489	0.00289
0.300	5000	0.014879	0.0183	0.812	0.810	0.778	0.843	4.36E-05	0.01479	0.01496	0.00293
0.325	5000	0.014961	0.0183	0.817	0.815	0.783	0.848	4.35E-05	0.01488	0.01505	0.00291
0.350	5000	0.015099	0.0183	0.824	0.819	0.787	0.852	4.36E-05	0.01501	0.01518	0.00289
0.375	5000	0.015093	0.0183	0.824	0.823	0.791	0.857	4.43E-05	0.01501	0.01518	0.00293
0.400	5000	0.015156	0.0183	0.827	0.826	0.794	0.860	4.44E-05	0.01507	0.01524	0.00293
0.425	5000	0.015162	0.0183	0.828	0.829	0.797	0.863	4.45E-05	0.01508	0.01525	0.00293
0.450	5000	0.015274	0.0183	0.834	0.831	0.798	0.865	4.46E-05	0.01519	0.01536	0.00292
0.475	5000	0.015280	0.0183	0.834	0.832	0.800	0.866	4.39E-05	0.01519	0.01537	0.00287
0.500	5000	0.015332	0.0183	0.837	0.833	0.800	0.867	4.43E-05	0.01524	0.01542	0.00289
0.525	5000	0.015291	0.0183	0.835	0.832	0.800	0.866	4.48E-05	0.01520	0.01538	0.00293
0.550	5000	0.015307	0.0183	0.836	0.831	0.798	0.865	4.42E-05	0.01522	0.01539	0.00289
0.575	5000	0.015218	0.0183	0.831	0.829	0.797	0.863	4.41E-05	0.01513	0.01530	0.00290
0.600	5000	0.015178	0.0183	0.829	0.826	0.794	0.860	4.32E-05	0.01509	0.01526	0.00284
0.625	5000	0.015175	0.0183	0.829	0.823	0.791	0.857	4.30E-05	0.01509	0.01526	0.00283
0.650	5000	0.015123	0.0183	0.826	0.819	0.787	0.852	4.30E-05	0.01504	0.01521	0.00284
0.675	5000	0.015090	0.0183	0.824	0.815	0.783	0.848	4.29E-05	0.01501	0.01517	0.00284
0.700	5000	0.014905	0.0183	0.814	0.810	0.778	0.843	4.28E-05	0.01482	0.01499	0.00287
0.725	5000	0.014770	0.0183	0.806	0.805	0.773	0.838	4.31E-05	0.01469	0.01485	0.00292
0.750	5000	0.014647	0.0183	0.800	0.800	0.769	0.833	4.30E-05	0.01456	0.01473	0.00294
0.775	5000	0.014614	0.0183	0.798	0.795	0.764	0.827	4.26E-05	0.01453	0.01470	0.00291
0.800	5000	0.014500	0.0183	0.792	0.790	0.759	0.822	4.29E-05	0.01442	0.01458	0.00296
0.825	5000	0.014415	0.0183	0.787	0.786	0.755	0.818	4.23E-05	0.01433	0.01450	0.00294
0.850	5000	0.014291	0.0183	0.780	0.781	0.751	0.813	4.29E-05	0.01421	0.01437	0.00300
0.875	5000	0.014214	0.0183	0.776	0.778	0.747	0.809	4.17E-05	0.01413	0.01430	0.00294
0.900	5000	0.014238	0.0183	0.777	0.775	0.744	0.806	4.12E-05	0.01416	0.01432	0.00289
0.925	5000	0.014138	0.0183	0.772	0.772	0.742	0.803	4.16E-05	0.01406	0.01422	0.00294
0.950	5000	0.014165	0.0183	0.773	0.770	0.740	0.802	4.06E-05	0.01409	0.01424	0.00287
0.975	5000	0.014140	0.0183	0.772	0.769	0.739	0.800	4.11E-05	0.01406	0.01422	0.00291

Table 18 Simulation estimates of $\hat{p} \equiv P(W_y > b) \equiv A_y e^{-\theta^* b}$ in the $M_t/M/1$ model as a function of y based on $5,000$ replications: $\gamma = 0.25, b = 40, \rho = 0.9, \bar{\lambda} = 0.9, \mu = 1, \beta = 0.1$

position	n	$\hat{p}$	$exp(-\theta^* b)$	A_y	A_y approx	A_y LB	A_y UB	s.e.	95% CI (lb)	(ub)	r.e.
0.000	5000	0.015877	0.0183	0.867	0.865	0.831	0.900	2.36E-05	0.01583	0.01592	0.00148
0.025	5000	0.015880	0.0183	0.867	0.865	0.831	0.900	2.33E-05	0.01583	0.01593	0.00147
0.050	5000	0.015923	0.0183	0.869	0.866	0.832	0.902	2.32E-05	0.01588	0.01597	0.00146
0.075	5000	0.015942	0.0183	0.870	0.868	0.834	0.904	2.36E-05	0.01590	0.01599	0.00148
0.100	5000	0.015996	0.0183	0.873	0.871	0.837	0.907	2.41E-05	0.01595	0.01604	0.00150
0.125	5000	0.016072	0.0183	0.878	0.875	0.841	0.911	2.30E-05	0.01603	0.01612	0.00143
0.150	5000	0.016184	0.0183	0.884	0.879	0.845	0.915	2.35E-05	0.01614	0.01623	0.00145
0.175	5000	0.016232	0.0183	0.886	0.884	0.849	0.920	2.38E-05	0.01618	0.01628	0.00147
0.200	5000	0.016293	0.0183	0.890	0.889	0.854	0.925	2.45E-05	0.01625	0.01634	0.00150
0.225	5000	0.016422	0.0183	0.897	0.894	0.859	0.931	2.43E-05	0.01637	0.01647	0.00148
0.250	5000	0.016556	0.0183	0.904	0.900	0.865	0.937	2.36E-05	0.01651	0.01660	0.00142
0.275	5000	0.016641	0.0183	0.909	0.906	0.870	0.943	2.43E-05	0.01659	0.01669	0.00146
0.300	5000	0.016743	0.0183	0.914	0.911	0.875	0.948	2.45E-05	0.01669	0.01679	0.00147
0.325	5000	0.016778	0.0183	0.916	0.916	0.881	0.954	2.49E-05	0.01673	0.01683	0.00149
0.350	5000	0.016989	0.0183	0.928	0.921	0.885	0.959	2.48E-05	0.01694	0.01704	0.00146
0.375	5000	0.017009	0.0183	0.929	0.926	0.890	0.964	2.50E-05	0.01696	0.01706	0.00147
0.400	5000	0.017058	0.0183	0.931	0.930	0.893	0.968	2.52E-05	0.01701	0.01711	0.00148
0.425	5000	0.017128	0.0183	0.935	0.933	0.896	0.971	2.55E-05	0.01708	0.01718	0.00149
0.450	5000	0.017153	0.0183	0.937	0.935	0.898	0.973	2.50E-05	0.01710	0.01720	0.00146
0.475	5000	0.017128	0.0183	0.935	0.936	0.900	0.974	2.60E-05	0.01708	0.01718	0.00152
0.500	5000	0.017247	0.0183	0.942	0.937	0.900	0.975	2.46E-05	0.01720	0.01730	0.00143
0.525	5000	0.017189	0.0183	0.938	0.936	0.900	0.974	2.52E-05	0.01714	0.01724	0.00147
0.550	5000	0.017176	0.0183	0.938	0.935	0.898	0.973	2.53E-05	0.01713	0.01723	0.00147
0.575	5000	0.017126	0.0183	0.935	0.933	0.896	0.971	2.56E-05	0.01708	0.01718	0.00150
0.600	5000	0.017047	0.0183	0.931	0.930	0.893	0.968	2.51E-05	0.01700	0.01710	0.00147
0.625	5000	0.016999	0.0183	0.928	0.926	0.890	0.964	2.56E-05	0.01695	0.01705	0.00150
0.650	5000	0.016917	0.0183	0.924	0.921	0.885	0.959	2.54E-05	0.01687	0.01697	0.00150
0.675	5000	0.016858	0.0183	0.920	0.916	0.881	0.954	2.44E-05	0.01681	0.01691	0.00144
0.700	5000	0.016752	0.0183	0.915	0.911	0.875	0.948	2.46E-05	0.01670	0.01680	0.00147
0.725	5000	0.016642	0.0183	0.909	0.906	0.870	0.943	2.45E-05	0.01659	0.01669	0.00147
0.750	5000	0.016563	0.0183	0.904	0.900	0.865	0.937	2.39E-05	0.01652	0.01661	0.00145
0.775	5000	0.016440	0.0183	0.898	0.894	0.859	0.931	2.37E-05	0.01639	0.01649	0.00144
0.800	5000	0.016280	0.0183	0.889	0.889	0.854	0.925	2.50E-05	0.01623	0.01633	0.00154
0.825	5000	0.016219	0.0183	0.886	0.884	0.849	0.920	2.40E-05	0.01617	0.01627	0.00148
0.850	5000	0.016130	0.0183	0.881	0.879	0.845	0.915	2.42E-05	0.01608	0.01618	0.00150
0.875	5000	0.016051	0.0183	0.876	0.875	0.841	0.911	2.43E-05	0.01600	0.01610	0.00151
0.900	5000	0.015954	0.0183	0.871	0.871	0.837	0.907	2.44E-05	0.01591	0.01600	0.00153
0.925	5000	0.015943	0.0183	0.870	0.868	0.834	0.904	2.40E-05	0.01590	0.01599	0.00150
0.950	5000	0.015877	0.0183	0.867	0.866	0.832	0.902	2.38E-05	0.01583	0.01592	0.00150
0.975	5000	0.015857	0.0183	0.866	0.865	0.831	0.900	2.37E-05	0.01581	0.01590	0.00149

Table 19 Simulation estimates of $\hat{p} \equiv P(W_y > b) \equiv A_y e^{-\theta^* b}$ in the $M_t/M/1$ model as a function of y based on $5,000$ replications: $\gamma = \frac{1}{16}, b = 80, \rho = 0.95, \bar{\lambda} = 0.95, \mu = 1, \beta = 0.05$

position	n	$\hat{p}$	$exp(-\theta^* b)$	A_y	A_y approx	A_y LB	A_y UB	s.e.	95% CI (lb)	(ub)	r.e.
0.000	5000	0.01676	0.0183	0.915	0.913	0.877	0.950	1.35E-05	0.01674	0.01679	8.03E-04
0.025	5000	0.01678	0.0183	0.916	0.913	0.877	0.950	1.36E-05	0.01675	0.01680	8.13E-04
0.050	5000	0.01681	0.0183	0.918	0.915	0.879	0.952	1.36E-05	0.01678	0.01684	8.06E-04
0.075	5000	0.01685	0.0183	0.920	0.917	0.881	0.954	1.37E-05	0.01682	0.01687	8.12E-04
0.100	5000	0.01689	0.0183	0.922	0.920	0.884	0.957	1.40E-05	0.01686	0.01692	8.31E-04
0.125	5000	0.01697	0.0183	0.926	0.924	0.887	0.961	1.37E-05	0.01694	0.01700	8.07E-04
0.150	5000	0.01701	0.0183	0.929	0.928	0.892	0.966	1.42E-05	0.01698	0.01704	8.32E-04
0.175	5000	0.01713	0.0183	0.935	0.933	0.896	0.971	1.41E-05	0.01710	0.01716	8.22E-04
0.200	5000	0.01725	0.0183	0.942	0.938	0.902	0.977	1.40E-05	0.01722	0.01727	8.09E-04
0.225	5000	0.01732	0.0183	0.946	0.944	0.907	0.983	1.40E-05	0.01730	0.01735	8.09E-04
0.250	5000	0.01744	0.0183	0.952	0.950	0.913	0.989	1.42E-05	0.01741	0.01747	8.14E-04
0.275	5000	0.01754	0.0183	0.958	0.956	0.918	0.995	1.44E-05	0.01752	0.01757	8.19E-04
0.300	5000	0.01769	0.0183	0.966	0.962	0.924	1.001	1.42E-05	0.01766	0.01771	8.03E-04
0.325	5000	0.01778	0.0183	0.971	0.967	0.929	1.007	1.44E-05	0.01775	0.01781	8.12E-04
0.350	5000	0.01787	0.0183	0.976	0.973	0.934	1.012	1.46E-05	0.01784	0.01790	8.18E-04
0.375	5000	0.01794	0.0183	0.980	0.977	0.939	1.017	1.47E-05	0.01791	0.01797	8.20E-04
0.400	5000	0.01801	0.0183	0.983	0.981	0.943	1.021	1.46E-05	0.01798	0.01804	8.11E-04
0.425	5000	0.01809	0.0183	0.988	0.984	0.946	1.025	1.48E-05	0.01806	0.01812	8.20E-04
0.450	5000	0.01813	0.0183	0.990	0.987	0.948	1.027	1.48E-05	0.01810	0.01816	8.16E-04
0.475	5000	0.01817	0.0183	0.992	0.988	0.950	1.029	1.46E-05	0.01814	0.01820	8.06E-04
0.500	5000	0.01815	0.0183	0.991	0.989	0.950	1.029	1.50E-05	0.01812	0.01817	8.26E-04
0.525	5000	0.01817	0.0183	0.992	0.988	0.950	1.029	1.46E-05	0.01814	0.01819	8.05E-04
0.550	5000	0.01813	0.0183	0.990	0.987	0.948	1.027	1.48E-05	0.01810	0.01816	8.14E-04
0.575	5000	0.01811	0.0183	0.989	0.984	0.946	1.025	1.44E-05	0.01808	0.01814	7.94E-04
0.600	5000	0.01804	0.0183	0.985	0.981	0.943	1.021	1.44E-05	0.01801	0.01806	8.00E-04
0.625	5000	0.01796	0.0183	0.980	0.977	0.939	1.017	1.49E-05	0.01793	0.01798	8.33E-04
0.650	5000	0.01786	0.0183	0.975	0.973	0.934	1.012	1.44E-05	0.01783	0.01788	8.08E-04
0.675	5000	0.01775	0.0183	0.969	0.967	0.929	1.007	1.47E-05	0.01772	0.01778	8.26E-04
0.700	5000	0.01767	0.0183	0.965	0.962	0.924	1.001	1.43E-05	0.01764	0.01769	8.09E-04
0.725	5000	0.01757	0.0183	0.959	0.956	0.918	0.995	1.40E-05	0.01754	0.01759	8.00E-04
0.750	5000	0.01743	0.0183	0.952	0.950	0.913	0.989	1.45E-05	0.01740	0.01746	8.34E-04
0.775	5000	0.01734	0.0183	0.947	0.944	0.907	0.983	1.42E-05	0.01731	0.01737	8.20E-04
0.800	5000	0.01724	0.0183	0.941	0.938	0.902	0.977	1.41E-05	0.01721	0.01726	8.20E-04
0.825	5000	0.01713	0.0183	0.935	0.933	0.896	0.971	1.37E-05	0.01710	0.01715	8.00E-04
0.850	5000	0.01705	0.0183	0.931	0.928	0.892	0.966	1.37E-05	0.01702	0.01708	8.06E-04
0.875	5000	0.01697	0.0183	0.926	0.924	0.887	0.961	1.37E-05	0.01694	0.01699	8.08E-04
0.900	5000	0.01689	0.0183	0.922	0.920	0.884	0.957	1.39E-05	0.01686	0.01692	8.21E-04
0.925	5000	0.01685	0.0183	0.920	0.917	0.881	0.954	1.40E-05	0.01682	0.01688	8.32E-04
0.950	5000	0.01678	0.0183	0.916	0.915	0.879	0.952	1.36E-05	0.01675	0.01681	8.11E-04
0.975	5000	0.01676	0.0183	0.915	0.913	0.877	0.950	1.37E-05	0.01674	0.01679	8.18E-04

Table 20 Simulation estimates of $\hat{p} \equiv P(W_y > b) \equiv A_y e^{-\theta^* b}$ in the $M_t/M/1$ model as a function of y based on $5,000$ replications: $\gamma = 0.64, b = 25, \rho = 0.84, \bar{\lambda} = 0.84, \mu = 1, \beta = 0.16$

position	n	$\hat{p}$	$exp(-\theta^* b)$	A_y	A_y approx	A_y LB	A_y UB	s.e.	95% CI (lb)	(ub)	r.e.
0.000	5000	0.014826	0.0183	0.809	0.807	0.775	0.840	3.46E-05	0.01476	0.01489	0.00233
0.025	5000	0.014831	0.0183	0.810	0.807	0.776	0.840	3.47E-05	0.01476	0.01490	0.00234
0.050	5000	0.014861	0.0183	0.811	0.809	0.777	0.842	3.46E-05	0.01479	0.01493	0.00233
0.075	5000	0.014898	0.0183	0.813	0.811	0.779	0.844	3.48E-05	0.01483	0.01497	0.00233
0.100	5000	0.014968	0.0183	0.817	0.813	0.781	0.846	3.48E-05	0.01490	0.01504	0.00233
0.125	5000	0.015019	0.0183	0.820	0.817	0.785	0.850	3.50E-05	0.01495	0.01509	0.00233
0.150	5000	0.015084	0.0183	0.824	0.820	0.788	0.854	3.52E-05	0.01502	0.01515	0.00234
0.175	5000	0.015164	0.0183	0.828	0.825	0.793	0.859	3.54E-05	0.01509	0.01523	0.00234
0.200	5000	0.015243	0.0183	0.832	0.830	0.797	0.864	3.55E-05	0.01517	0.01531	0.00233
0.225	5000	0.015338	0.0183	0.837	0.835	0.802	0.869	3.57E-05	0.01527	0.01541	0.00233
0.250	5000	0.015455	0.0183	0.844	0.840	0.807	0.874	3.59E-05	0.01538	0.01553	0.00232
0.275	5000	0.015555	0.0183	0.849	0.845	0.812	0.880	3.62E-05	0.01548	0.01563	0.00232
0.300	5000	0.015650	0.0183	0.854	0.850	0.817	0.885	3.63E-05	0.01558	0.01572	0.00232
0.325	5000	0.015733	0.0183	0.859	0.855	0.822	0.890	3.65E-05	0.01566	0.01580	0.00232
0.350	5000	0.015816	0.0183	0.863	0.860	0.826	0.895	3.68E-05	0.01574	0.01589	0.00232
0.375	5000	0.015889	0.0183	0.868	0.864	0.830	0.899	3.70E-05	0.01582	0.01596	0.00233
0.400	5000	0.015947	0.0183	0.871	0.868	0.834	0.903	3.71E-05	0.01587	0.01602	0.00233
0.425	5000	0.016011	0.0183	0.874	0.870	0.836	0.906	3.70E-05	0.01594	0.01608	0.00231
0.450	5000	0.016053	0.0183	0.876	0.873	0.838	0.908	3.71E-05	0.01598	0.01613	0.00231
0.475	5000	0.016065	0.0183	0.877	0.874	0.840	0.910	3.73E-05	0.01599	0.01614	0.00232
0.500	5000	0.016088	0.0183	0.878	0.874	0.840	0.910	3.73E-05	0.01601	0.01616	0.00232
0.525	5000	0.016074	0.0183	0.878	0.874	0.840	0.910	3.73E-05	0.01600	0.01615	0.00232
0.550	5000	0.016037	0.0183	0.876	0.873	0.838	0.908	3.73E-05	0.01596	0.01611	0.00233
0.575	5000	0.016004	0.0183	0.874	0.870	0.836	0.906	3.71E-05	0.01593	0.01608	0.00232
0.600	5000	0.015944	0.0183	0.871	0.868	0.834	0.903	3.70E-05	0.01587	0.01602	0.00232
0.625	5000	0.015879	0.0183	0.867	0.864	0.830	0.899	3.68E-05	0.01581	0.01595	0.00232
0.650	5000	0.015802	0.0183	0.863	0.860	0.826	0.895	3.65E-05	0.01573	0.01587	0.00231
0.675	5000	0.015720	0.0183	0.858	0.855	0.822	0.890	3.63E-05	0.01565	0.01579	0.00231
0.700	5000	0.015624	0.0183	0.853	0.850	0.817	0.885	3.60E-05	0.01555	0.01569	0.00230
0.725	5000	0.015530	0.0183	0.848	0.845	0.812	0.880	3.57E-05	0.01546	0.01560	0.00230
0.750	5000	0.015420	0.0183	0.842	0.840	0.807	0.874	3.57E-05	0.01535	0.01549	0.00232
0.775	5000	0.015316	0.0183	0.836	0.835	0.802	0.869	3.56E-05	0.01525	0.01539	0.00233
0.800	5000	0.015214	0.0183	0.831	0.830	0.797	0.864	3.56E-05	0.01514	0.01528	0.00234
0.825	5000	0.015126	0.0183	0.826	0.825	0.793	0.859	3.53E-05	0.01506	0.01520	0.00234
0.850	5000	0.015050	0.0183	0.822	0.820	0.788	0.854	3.51E-05	0.01498	0.01512	0.00233
0.875	5000	0.014981	0.0183	0.818	0.817	0.785	0.850	3.49E-05	0.01491	0.01505	0.00233
0.900	5000	0.014927	0.0183	0.815	0.813	0.781	0.846	3.46E-05	0.01486	0.01499	0.00232
0.925	5000	0.014869	0.0183	0.812	0.811	0.779	0.844	3.46E-05	0.01480	0.01494	0.00233
0.950	5000	0.014835	0.0183	0.810	0.809	0.777	0.842	3.46E-05	0.01477	0.01490	0.00233
0.975	5000	0.014820	0.0183	0.809	0.807	0.776	0.840	3.46E-05	0.01475	0.01489	0.00233

Table 21 Simulation estimates of $\hat{p} \equiv P(W_y > b) \equiv A_y e^{-\theta^* b}$ in the $M_t/M/1$ model as a function of y based on $5,000$ replications: $\gamma = 0.16, b = 50, \rho = 0.92, \bar{\lambda} = 0.92, \mu = 1, \beta = 0.08$

position	n	$\hat{p}$	$exp(-\theta^* b)$	A_y	A_y approx	A_y LB	A_y UB	s.e.	95% CI (lb)	(ub)	r.e.
0.000	5000	0.016264	0.0183	0.888	0.884	0.849	0.920	1.95E-05	0.01623	0.01630	0.00120
0.025	5000	0.016268	0.0183	0.888	0.884	0.850	0.920	1.96E-05	0.01623	0.01631	0.00120
0.050	5000	0.016297	0.0183	0.890	0.886	0.851	0.922	1.97E-05	0.01626	0.01634	0.00121
0.075	5000	0.016327	0.0183	0.891	0.888	0.853	0.924	1.97E-05	0.01629	0.01637	0.00121
0.100	5000	0.016389	0.0183	0.895	0.891	0.856	0.927	1.97E-05	0.01635	0.01643	0.00120
0.125	5000	0.016460	0.0183	0.899	0.894	0.859	0.931	1.98E-05	0.01642	0.01650	0.00120
0.150	5000	0.016532	0.0183	0.903	0.899	0.863	0.935	1.98E-05	0.01649	0.01657	0.00120
0.175	5000	0.016615	0.0183	0.907	0.903	0.868	0.940	1.98E-05	0.01658	0.01665	0.00119
0.200	5000	0.016718	0.0183	0.913	0.909	0.873	0.946	2.00E-05	0.01668	0.01676	0.00119
0.225	5000	0.016825	0.0183	0.919	0.914	0.878	0.952	2.01E-05	0.01679	0.01686	0.00119
0.250	5000	0.016926	0.0183	0.924	0.920	0.884	0.958	2.02E-05	0.01689	0.01697	0.00119
0.275	5000	0.017021	0.0183	0.929	0.926	0.889	0.964	2.03E-05	0.01698	0.01706	0.00119
0.300	5000	0.017132	0.0183	0.935	0.931	0.895	0.969	2.03E-05	0.01709	0.01717	0.00118
0.325	5000	0.017229	0.0183	0.941	0.937	0.900	0.975	2.04E-05	0.01719	0.01727	0.00118
0.350	5000	0.017318	0.0183	0.946	0.942	0.905	0.980	2.06E-05	0.01728	0.01736	0.00119
0.375	5000	0.017400	0.0183	0.950	0.946	0.909	0.985	2.09E-05	0.01736	0.01744	0.00120
0.400	5000	0.017465	0.0183	0.954	0.950	0.913	0.989	2.09E-05	0.01742	0.01751	0.00120
0.425	5000	0.017529	0.0183	0.957	0.953	0.916	0.992	2.09E-05	0.01749	0.01757	0.00119
0.450	5000	0.017573	0.0183	0.959	0.956	0.918	0.995	2.10E-05	0.01753	0.01761	0.00120
0.475	5000	0.017604	0.0183	0.961	0.957	0.920	0.996	2.11E-05	0.01756	0.01765	0.00120
0.500	5000	0.017604	0.0183	0.961	0.958	0.920	0.997	2.10E-05	0.01756	0.01764	0.00119
0.525	5000	0.017595	0.0183	0.961	0.957	0.920	0.996	2.10E-05	0.01755	0.01764	0.00119
0.550	5000	0.017559	0.0183	0.959	0.956	0.918	0.995	2.10E-05	0.01752	0.01760	0.00119
0.575	5000	0.017512	0.0183	0.956	0.953	0.916	0.992	2.09E-05	0.01747	0.01755	0.00119
0.600	5000	0.017445	0.0183	0.952	0.950	0.913	0.989	2.07E-05	0.01740	0.01749	0.00119
0.625	5000	0.017372	0.0183	0.948	0.946	0.909	0.985	2.06E-05	0.01733	0.01741	0.00119
0.650	5000	0.017298	0.0183	0.944	0.942	0.905	0.980	2.05E-05	0.01726	0.01734	0.00119
0.675	5000	0.017212	0.0183	0.940	0.937	0.900	0.975	2.05E-05	0.01717	0.01725	0.00119
0.700	5000	0.017114	0.0183	0.934	0.931	0.895	0.969	2.04E-05	0.01707	0.01715	0.00119
0.725	5000	0.017014	0.0183	0.929	0.926	0.889	0.964	2.03E-05	0.01697	0.01705	0.00119
0.750	5000	0.016918	0.0183	0.924	0.920	0.884	0.958	2.01E-05	0.01688	0.01696	0.00119
0.775	5000	0.016822	0.0183	0.918	0.914	0.878	0.952	1.98E-05	0.01678	0.01686	0.00118
0.800	5000	0.016727	0.0183	0.913	0.909	0.873	0.946	1.97E-05	0.01669	0.01677	0.00118
0.825	5000	0.016626	0.0183	0.908	0.903	0.868	0.940	1.97E-05	0.01659	0.01666	0.00118
0.850	5000	0.016535	0.0183	0.903	0.899	0.863	0.935	1.95E-05	0.01650	0.01657	0.00118
0.875	5000	0.016462	0.0183	0.899	0.894	0.859	0.931	1.95E-05	0.01642	0.01650	0.00119
0.900	5000	0.016390	0.0183	0.895	0.891	0.856	0.927	1.96E-05	0.01635	0.01643	0.00119
0.925	5000	0.016332	0.0183	0.892	0.888	0.853	0.924	1.95E-05	0.01629	0.01637	0.00120
0.950	5000	0.016291	0.0183	0.889	0.886	0.851	0.922	1.95E-05	0.01625	0.01633	0.00120
0.975	5000	0.016271	0.0183	0.888	0.884	0.850	0.920	1.95E-05	0.01623	0.01631	0.00120

Table 22 Simulation estimates of $\hat{p} \equiv P(W_y > b) \equiv A_y e^{-\theta^* b}$ in the $M_t/M/1$ model as a function of y based on $5,000$ replications: $\gamma = 0.04, b = 100, \rho = 0.96, \bar{\lambda} = 0.96, \mu = 1, \beta = 0.04$

position	n	$\hat{p}$	$exp(-\theta^* b)$	A_y	A_y approx	A_y LB	A_y UB	s.e.	95% CI (lb)	(ub)	r.e.
0.000	5000	0.01695	0.0183	0.926	0.922	0.886	0.960	1.18E-05	0.01693	0.01698	6.96E-04
0.025	5000	0.01696	0.0183	0.926	0.923	0.887	0.960	1.18E-05	0.01693	0.01698	6.98E-04
0.050	5000	0.01698	0.0183	0.927	0.924	0.888	0.962	1.19E-05	0.01695	0.01700	7.02E-04
0.075	5000	0.01702	0.0183	0.929	0.926	0.890	0.964	1.19E-05	0.01699	0.01704	6.97E-04
0.100	5000	0.01707	0.0183	0.932	0.929	0.893	0.967	1.20E-05	0.01705	0.01710	7.00E-04
0.125	5000	0.01714	0.0183	0.936	0.933	0.897	0.971	1.19E-05	0.01712	0.01717	6.97E-04
0.150	5000	0.01722	0.0183	0.940	0.938	0.901	0.976	1.21E-05	0.01719	0.01724	7.01E-04
0.175	5000	0.01731	0.0183	0.945	0.943	0.906	0.981	1.21E-05	0.01729	0.01733	7.00E-04
0.200	5000	0.01741	0.0183	0.951	0.948	0.911	0.987	1.22E-05	0.01739	0.01743	6.99E-04
0.225	5000	0.01752	0.0183	0.956	0.954	0.917	0.993	1.23E-05	0.01749	0.01754	7.04E-04
0.250	5000	0.01763	0.0183	0.962	0.960	0.922	0.999	1.22E-05	0.01760	0.01765	6.94E-04
0.275	5000	0.01774	0.0183	0.968	0.966	0.928	1.005	1.24E-05	0.01771	0.01776	6.98E-04
0.300	5000	0.01784	0.0183	0.974	0.972	0.934	1.012	1.26E-05	0.01782	0.01787	7.05E-04
0.325	5000	0.01794	0.0183	0.979	0.978	0.939	1.017	1.27E-05	0.01791	0.01796	7.07E-04
0.350	5000	0.01804	0.0183	0.985	0.983	0.944	1.023	1.28E-05	0.01801	0.01806	7.08E-04
0.375	5000	0.01812	0.0183	0.989	0.988	0.949	1.028	1.27E-05	0.01809	0.01814	7.02E-04
0.400	5000	0.01820	0.0183	0.993	0.992	0.953	1.032	1.27E-05	0.01817	0.01822	6.99E-04
0.425	5000	0.01826	0.0183	0.997	0.995	0.956	1.035	1.27E-05	0.01824	0.01829	6.95E-04
0.450	5000	0.01830	0.0183	0.999	0.997	0.958	1.038	1.29E-05	0.01827	0.01833	7.03E-04
0.475	5000	0.01833	0.0183	1.001	0.999	0.960	1.039	1.30E-05	0.01830	0.01835	7.07E-04
0.500	5000	0.01834	0.0183	1.001	0.999	0.960	1.040	1.30E-05	0.01831	0.01836	7.09E-04
0.525	5000	0.01834	0.0183	1.001	0.999	0.960	1.039	1.29E-05	0.01831	0.01836	7.06E-04
0.550	5000	0.01832	0.0183	1.000	0.997	0.958	1.038	1.28E-05	0.01829	0.01834	7.00E-04
0.575	5000	0.01828	0.0183	0.998	0.995	0.956	1.035	1.27E-05	0.01825	0.01830	6.93E-04
0.600	5000	0.01822	0.0183	0.995	0.992	0.953	1.032	1.27E-05	0.01819	0.01824	6.95E-04
0.625	5000	0.01815	0.0183	0.991	0.988	0.949	1.028	1.26E-05	0.01813	0.01818	6.95E-04
0.650	5000	0.01807	0.0183	0.986	0.983	0.944	1.023	1.25E-05	0.01804	0.01809	6.92E-04
0.675	5000	0.01797	0.0183	0.981	0.978	0.939	1.017	1.24E-05	0.01795	0.01799	6.89E-04
0.700	5000	0.01788	0.0183	0.976	0.972	0.934	1.012	1.23E-05	0.01785	0.01790	6.87E-04
0.725	5000	0.01777	0.0183	0.970	0.966	0.928	1.005	1.22E-05	0.01774	0.01779	6.89E-04
0.750	5000	0.01766	0.0183	0.964	0.960	0.922	0.999	1.22E-05	0.01763	0.01768	6.94E-04
0.775	5000	0.01755	0.0183	0.958	0.954	0.917	0.993	1.22E-05	0.01752	0.01757	6.93E-04
0.800	5000	0.01744	0.0183	0.952	0.948	0.911	0.987	1.22E-05	0.01741	0.01746	6.97E-04
0.825	5000	0.01734	0.0183	0.947	0.943	0.906	0.981	1.20E-05	0.01731	0.01736	6.92E-04
0.850	5000	0.01724	0.0183	0.941	0.938	0.901	0.976	1.20E-05	0.01722	0.01727	6.98E-04
0.875	5000	0.01717	0.0183	0.937	0.933	0.897	0.971	1.19E-05	0.01714	0.01719	6.93E-04
0.900	5000	0.01710	0.0183	0.933	0.929	0.893	0.967	1.18E-05	0.01707	0.01712	6.90E-04
0.925	5000	0.01704	0.0183	0.930	0.926	0.890	0.964	1.18E-05	0.01701	0.01706	6.93E-04
0.950	5000	0.01699	0.0183	0.928	0.924	0.888	0.962	1.18E-05	0.01697	0.01701	6.96E-04
0.975	5000	0.01696	0.0183	0.926	0.923	0.887	0.960	1.18E-05	0.01694	0.01699	6.98E-04

Table 23 Simulation estimates of $\hat{p} \equiv P(W_y > b) \equiv A_y e^{-\theta^* b}$ in the $M_t/M/1$ model as a function of y based on $5,000$ replications: $\gamma = 0.01, b = 200, \rho = 0.98, \bar{\lambda} = 0.98, \mu = 1, \beta = 0.02$

	position	n	$\hat{p}$	$exp(-\theta^* b)$	A_y	A_y approx	A_y LB	A_y UB	s.e.	95% CI (lb)	(ub)	r.e.
0.000	5000	0.017295	0.0183	0.944	0.942	0.905	0.980	8.47E-06	0.01728	0.01731	4.90E-04	
0.025	5000	0.017303	0.0183	0.945	0.942	0.905	0.980	8.47E-06	0.01729	0.01732	4.89E-04	
0.050	5000	0.017327	0.0183	0.946	0.943	0.906	0.982	8.51E-06	0.01731	0.01734	4.91E-04	
0.075	5000	0.017370	0.0183	0.948	0.946	0.909	0.984	8.51E-06	0.01735	0.01739	4.90E-04	
0.100	5000	0.017426	0.0183	0.951	0.949	0.912	0.988	8.52E-06	0.01741	0.01744	4.89E-04	
0.125	5000	0.017499	0.0183	0.955	0.953	0.915	0.992	8.50E-06	0.01748	0.01752	4.86E-04	
0.150	5000	0.017588	0.0183	0.960	0.957	0.920	0.996	8.52E-06	0.01757	0.01761	4.85E-04	
0.175	5000	0.017676	0.0183	0.965	0.962	0.925	1.002	8.64E-06	0.01766	0.01769	4.89E-04	
0.200	5000	0.017778	0.0183	0.971	0.968	0.930	1.007	8.72E-06	0.01776	0.01779	4.90E-04	
0.225	5000	0.017885	0.0183	0.976	0.974	0.936	1.014	8.79E-06	0.01787	0.01790	4.91E-04	
0.250	5000	0.017994	0.0183	0.982	0.980	0.942	1.020	8.85E-06	0.01798	0.01801	4.92E-04	
0.275	5000	0.018114	0.0183	0.989	0.986	0.947	1.026	8.85E-06	0.01810	0.01813	4.89E-04	
0.300	5000	0.018221	0.0183	0.995	0.992	0.953	1.033	8.95E-06	0.01820	0.01824	4.91E-04	
0.325	5000	0.018322	0.0183	1.000	0.998	0.959	1.039	8.99E-06	0.01830	0.01834	4.91E-04	
0.350	5000	0.018420	0.0183	1.006	1.003	0.964	1.044	9.05E-06	0.01840	0.01844	4.91E-04	
0.375	5000	0.018515	0.0183	1.011	1.008	0.969	1.049	9.09E-06	0.01850	0.01853	4.91E-04	
0.400	5000	0.018592	0.0183	1.015	1.012	0.973	1.054	9.05E-06	0.01857	0.01861	4.87E-04	
0.425	5000	0.018647	0.0183	1.018	1.016	0.976	1.057	9.06E-06	0.01863	0.01866	4.86E-04	
0.450	5000	0.018691	0.0183	1.021	1.018	0.978	1.060	9.09E-06	0.01867	0.01871	4.86E-04	
0.475	5000	0.018717	0.0183	1.022	1.019	0.980	1.061	9.20E-06	0.01870	0.01873	4.92E-04	
0.500	5000	0.018729	0.0183	1.023	1.020	0.980	1.062	9.20E-06	0.01871	0.01875	4.91E-04	
0.525	5000	0.018720	0.0183	1.022	1.019	0.980	1.061	9.15E-06	0.01870	0.01874	4.89E-04	
0.550	5000	0.018688	0.0183	1.020	1.018	0.978	1.060	9.13E-06	0.01867	0.01871	4.89E-04	
0.575	5000	0.018647	0.0183	1.018	1.016	0.976	1.057	9.14E-06	0.01863	0.01866	4.90E-04	
0.600	5000	0.018589	0.0183	1.015	1.012	0.973	1.054	9.17E-06	0.01857	0.01861	4.93E-04	
0.625	5000	0.018515	0.0183	1.011	1.008	0.969	1.049	9.13E-06	0.01850	0.01853	4.93E-04	
0.650	5000	0.018432	0.0183	1.006	1.003	0.964	1.044	9.05E-06	0.01841	0.01845	4.91E-04	
0.675	5000	0.018331	0.0183	1.001	0.998	0.959	1.039	8.94E-06	0.01831	0.01835	4.88E-04	
0.700	5000	0.018222	0.0183	0.995	0.992	0.953	1.033	8.81E-06	0.01821	0.01824	4.83E-04	
0.725	5000	0.018117	0.0183	0.989	0.986	0.947	1.026	8.71E-06	0.01810	0.01813	4.81E-04	
0.750	5000	0.017999	0.0183	0.983	0.980	0.942	1.020	8.61E-06	0.01798	0.01802	4.78E-04	
0.775	5000	0.017891	0.0183	0.977	0.974	0.936	1.014	8.65E-06	0.01787	0.01791	4.84E-04	
0.800	5000	0.017786	0.0183	0.971	0.968	0.930	1.007	8.62E-06	0.01777	0.01780	4.84E-04	
0.825	5000	0.017674	0.0183	0.965	0.962	0.925	1.002	8.62E-06	0.01766	0.01769	4.88E-04	
0.850	5000	0.017581	0.0183	0.960	0.957	0.920	0.996	8.59E-06	0.01756	0.01760	4.89E-04	
0.875	5000	0.017493	0.0183	0.955	0.953	0.915	0.992	8.62E-06	0.01748	0.01751	4.93E-04	
0.900	5000	0.017422	0.0183	0.951	0.949	0.912	0.988	8.60E-06	0.01741	0.01744	4.94E-04	
0.925	5000	0.017366	0.0183	0.948	0.946	0.909	0.984	8.56E-06	0.01735	0.01738	4.93E-04	
0.950	5000	0.017327	0.0183	0.946	0.943	0.906	0.982	8.48E-06	0.01731	0.01734	4.89E-04	
0.975	5000	0.017299	0.0183	0.944	0.942	0.905	0.980	8.50E-06	0.01728	0.01732	4.91E-04	

Table 24 Simulation estimates of $\hat{p} \equiv P(W_y > b) \equiv A_y e^{-\theta^* b}$ in the $M_t/M/1$ model as a function of y based on $5,000$ replications: $\gamma = 0.0025, b = 400, \rho = 0.99, \bar{\lambda} = 0.99, \mu = 1, \beta = 0.01$

position	n	$\hat{p}$	$exp(-\theta^* b)$	A_y	A_y approx	A_y LB	A_y UB	s.e.	95% CI (lb)	(ub)	r.e.
0.000	5000	0.017470	0.0183	0.954	0.951	0.914	0.990	7.39E-06	0.01746	0.01748	4.23E-04
0.025	5000	0.017482	0.0183	0.954	0.952	0.914	0.990	7.30E-06	0.01747	0.01750	4.18E-04
0.050	5000	0.017506	0.0183	0.956	0.953	0.916	0.992	7.47E-06	0.01749	0.01752	4.27E-04
0.075	5000	0.017559	0.0183	0.959	0.955	0.918	0.994	7.48E-06	0.01754	0.01757	4.26E-04
0.100	5000	0.017617	0.0183	0.962	0.958	0.921	0.998	7.31E-06	0.01760	0.01763	4.15E-04
0.125	5000	0.017676	0.0183	0.965	0.962	0.925	1.002	7.45E-06	0.01766	0.01769	4.21E-04
0.150	5000	0.017760	0.0183	0.970	0.967	0.929	1.006	7.53E-06	0.01775	0.01777	4.24E-04
0.175	5000	0.017868	0.0183	0.976	0.972	0.934	1.012	7.56E-06	0.01785	0.01788	4.23E-04
0.200	5000	0.017955	0.0183	0.980	0.978	0.939	1.018	7.59E-06	0.01794	0.01797	4.23E-04
0.225	5000	0.018087	0.0183	0.987	0.984	0.945	1.024	7.68E-06	0.01807	0.01810	4.25E-04
0.250	5000	0.018188	0.0183	0.993	0.990	0.951	1.030	7.68E-06	0.01817	0.01820	4.22E-04
0.275	5000	0.018299	0.0183	0.999	0.996	0.957	1.037	7.64E-06	0.01828	0.01831	4.18E-04
0.300	5000	0.018407	0.0183	1.005	1.002	0.963	1.043	7.83E-06	0.01839	0.01842	4.25E-04
0.325	5000	0.018526	0.0183	1.011	1.008	0.969	1.049	7.78E-06	0.01851	0.01854	4.20E-04
0.350	5000	0.018632	0.0183	1.017	1.014	0.974	1.055	7.86E-06	0.01862	0.01865	4.22E-04
0.375	5000	0.018704	0.0183	1.021	1.018	0.978	1.060	7.89E-06	0.01869	0.01872	4.22E-04
0.400	5000	0.018775	0.0183	1.025	1.023	0.982	1.064	7.90E-06	0.01876	0.01879	4.21E-04
0.425	5000	0.018842	0.0183	1.029	1.026	0.986	1.068	7.97E-06	0.01883	0.01886	4.23E-04
0.450	5000	0.018875	0.0183	1.031	1.028	0.988	1.070	7.90E-06	0.01886	0.01889	4.19E-04
0.475	5000	0.018911	0.0183	1.033	1.030	0.990	1.072	8.03E-06	0.01890	0.01893	4.25E-04
0.500	5000	0.018931	0.0183	1.034	1.030	0.990	1.072	7.97E-06	0.01892	0.01895	4.21E-04
0.525	5000	0.018921	0.0183	1.033	1.030	0.990	1.072	8.04E-06	0.01891	0.01894	4.25E-04
0.550	5000	0.018884	0.0183	1.031	1.028	0.988	1.070	7.91E-06	0.01887	0.01890	4.19E-04
0.575	5000	0.018839	0.0183	1.029	1.026	0.986	1.068	7.99E-06	0.01882	0.01885	4.24E-04
0.600	5000	0.018780	0.0183	1.025	1.023	0.982	1.064	7.93E-06	0.01876	0.01880	4.22E-04
0.625	5000	0.018713	0.0183	1.022	1.018	0.978	1.060	7.80E-06	0.01870	0.01873	4.17E-04
0.650	5000	0.018621	0.0183	1.017	1.014	0.974	1.055	7.79E-06	0.01861	0.01864	4.19E-04
0.675	5000	0.018513	0.0183	1.011	1.008	0.969	1.049	7.85E-06	0.01850	0.01853	4.24E-04
0.700	5000	0.018408	0.0183	1.005	1.002	0.963	1.043	7.71E-06	0.01839	0.01842	4.19E-04
0.725	5000	0.018294	0.0183	0.999	0.996	0.957	1.037	7.73E-06	0.01828	0.01831	4.23E-04
0.750	5000	0.018176	0.0183	0.992	0.990	0.951	1.030	7.71E-06	0.01816	0.01819	4.24E-04
0.775	5000	0.018061	0.0183	0.986	0.984	0.945	1.024	7.60E-06	0.01805	0.01808	4.21E-04
0.800	5000	0.017962	0.0183	0.981	0.978	0.939	1.018	7.55E-06	0.01795	0.01798	4.20E-04
0.825	5000	0.017862	0.0183	0.975	0.972	0.934	1.012	7.60E-06	0.01785	0.01788	4.25E-04
0.850	5000	0.017768	0.0183	0.970	0.967	0.929	1.006	7.52E-06	0.01775	0.01778	4.23E-04
0.875	5000	0.017687	0.0183	0.966	0.962	0.925	1.002	7.45E-06	0.01767	0.01770	4.21E-04
0.900	5000	0.017608	0.0183	0.961	0.958	0.921	0.998	7.38E-06	0.01759	0.01762	4.19E-04
0.925	5000	0.017547	0.0183	0.958	0.955	0.918	0.994	7.42E-06	0.01753	0.01756	4.23E-04
0.950	5000	0.017499	0.0183	0.955	0.953	0.916	0.992	7.43E-06	0.01748	0.01751	4.25E-04
0.975	5000	0.017482	0.0183	0.954	0.952	0.914	0.990	7.40E-06	0.01747	0.01750	4.23E-04

Table 25 **Comparison of ratio $P(W_y > b)/P(W > b) = A_y/\rho$ for different ρ's with base parameter $(\beta, \gamma, b) = (1, 25, 4)$ using the scaling in (23) of the main paper**

position	$\rho = 0.84$	$\rho = 0.92$	$\rho = 0.96$	$\rho = 0.98$	$\rho = 0.99$
0.000	0.96364	0.96523	0.96424	0.96357	0.96344
0.025	0.96397	0.96543	0.96436	0.96398	0.96412
0.050	0.96596	0.96718	0.96545	0.96531	0.96545
0.075	0.96832	0.96896	0.96778	0.96771	0.96839
0.100	0.97289	0.97264	0.97102	0.97086	0.97156
0.125	0.97619	0.97686	0.97504	0.97493	0.97482
0.150	0.98044	0.98109	0.97919	0.97989	0.97945
0.175	0.98562	0.98605	0.98442	0.98475	0.98539
0.200	0.99074	0.99215	0.99018	0.99043	0.99019
0.225	0.99693	0.99851	0.99614	0.99642	0.99747
0.250	1.00456	1.00450	1.00255	1.00251	1.00305
0.275	1.01102	1.01015	1.00875	1.00918	1.00918
0.300	1.01721	1.01669	1.01482	1.01513	1.01514
0.325	1.02258	1.02247	1.02006	1.02079	1.02171
0.350	1.02797	1.02776	1.02572	1.02622	1.02757
0.375	1.03278	1.03264	1.03035	1.03152	1.03152
0.400	1.03650	1.03649	1.03484	1.03582	1.03546
0.425	1.04065	1.04030	1.03871	1.03886	1.03911
0.450	1.04342	1.04286	1.04079	1.04134	1.04094
0.475	1.04420	1.04475	1.04237	1.04276	1.04294
0.500	1.04565	1.04470	1.04278	1.04346	1.04405
0.525	1.04475	1.04420	1.04296	1.04291	1.04351
0.550	1.04236	1.04204	1.04183	1.04118	1.04142
0.575	1.04021	1.03925	1.03955	1.03886	1.03895
0.600	1.03634	1.03532	1.03597	1.03564	1.03569
0.625	1.03213	1.03096	1.03230	1.03150	1.03204
0.650	1.02712	1.02655	1.02745	1.02690	1.02695
0.675	1.02178	1.02146	1.02203	1.02124	1.02099
0.700	1.01554	1.01565	1.01682	1.01521	1.01517
0.725	1.00944	1.00971	1.01054	1.00935	1.00888
0.750	1.00225	1.00404	1.00425	1.00277	1.00241
0.775	0.99551	0.99831	0.99785	0.99674	0.99604
0.800	0.98888	0.99266	0.99178	0.99088	0.99059
0.825	0.98318	0.98671	0.98604	0.98464	0.98509
0.850	0.97825	0.98128	0.98059	0.97950	0.97991
0.875	0.97371	0.97696	0.97629	0.97457	0.97545
0.900	0.97022	0.97270	0.97228	0.97063	0.97108
0.925	0.96647	0.96926	0.96887	0.96748	0.96772
0.950	0.96422	0.96681	0.96626	0.96531	0.96507
0.975	0.96328	0.96564	0.96472	0.96377	0.96414
avg diff w.r.t. last column	0.00037	0.00112	0.00015	-0.00019	0.00000
avg. abs. diff w.r.t. last column	0.00099	0.00121	0.00081	0.00039	0.00000
rmse w.r.t. last column	0.00116	0.00134	0.00096	0.00049	0.00000

Table 26 **Summary of simulation results for $M_t/M/1$ queue at $y = 0$ as a function of $1 - \rho$ with base parameter $(\beta, \gamma, b) = (1, 25, 4)$ using the scaling in (23) of the main paper:**

	$1-\rho = 0.16$	$1-\rho = 0.08$	$1-\rho = 0.04$	$1-\rho = 0.02$	$1-\rho = 0.01$	$1-\rho = 0.005$	$1-\rho = 0.0025$
n	5000	5000	5000	5000	5000	5000	5000
$\hat{p}$	0.014834	0.016239	0.016941	0.017298	0.017462	0.017566	0.017596
$e^{-\theta^* b}$	0.0183	0.0183	0.0183	0.0183	0.0183	0.0183	0.0183
A_y	0.810	0.887	0.925	0.944	0.953	0.959	0.961
A_y approxi	0.807	0.884	0.922	0.942	0.951	0.956	0.958
A_y LB	0.775	0.849	0.886	0.905	0.914	0.919	0.921
A_y UB	0.840	0.920	0.960	0.980	0.990	0.995	0.998
s.e.	3.42E-05	1.99E-05	1.16E-05	8.35E-06	7.38E-06	7.09E-06	7.02E-06
95% CI (lb)	0.01477	0.01620	0.01692	0.01728	0.01745	0.01755	0.01758
(ub)	0.01490	0.01628	0.01696	0.01731	0.01748	0.01758	0.01761
r.e.	0.002303	0.001222	0.000685	0.000483	0.000422	0.000403	0.000399
$P(W_y > b)/P(W > b)$							
ratio	0.96419	0.96375	0.96349	0.96370	0.96301	0.96386	0.96312
diff	-0.00107	-0.00062	-0.00037	-0.00058	0.00011	-0.00074	0.00000
abs diff	0.00107	0.00062	0.00037	0.00058	0.00011	0.00074	0.00000

Table 27 **Summary of simulation results for $M_t/M/1$ queue at $y=0$ and at $y=0.5$ as a function of $1-\rho$ with base parameter $(\beta,\gamma,b)=(1,2.5,4)$ using the scaling in (23) of the main paper**

	$1-\rho=0.16$	$1-\rho=0.08$	$1-\rho=0.04$	$1-\rho=0.02$	$1-\rho=0.01$
n	40000	40000	40000	40000	40000
$y=0$					
$\hat{p}$	0.011053	0.012192	0.012814	0.013122	0.013263
$e^{-\theta^* b}$	0.0183	0.0183	0.0183	0.0183	0.0183
A_y	0.604	0.666	0.700	0.716	0.724
A_y approxi	0.563	0.617	0.644	0.657	0.664
A_y LB	0.377	0.413	0.431	0.440	0.445
A_y UB	0.840	0.920	0.960	0.980	0.990
s.e.	1.75E-05	1.69E-05	1.71E-05	1.73E-05	1.74E-05
95% CI (lb)	0.01102	0.01216	0.01278	0.01309	0.01323
(ub)	0.01109	0.01223	0.01285	0.01316	0.01330
r.e.	0.001582	0.001387	0.001333	0.001319	0.001313
$P(W_y>b)/P(W>b)$					
ratio	0.71845	0.72356	0.72879	0.73103	0.73144
diff w.r.t. last column	0.01298	0.00788	0.00264	0.00041	0.00000
abs diff	0.01298	0.00788	0.00264	0.00041	0.00000
$y=0.5$					
$\hat{p}$	0.025888	0.028396	0.029551	0.030110	0.030430
$e^{-\theta^* b}$	0.0183	0.0183	0.0183	0.0183	0.0183
A_y	1.413	1.550	1.613	1.644	1.661
A_y approxi	1.253	1.372	1.432	1.462	1.477
A_y LB	0.840	0.920	0.960	0.980	0.990
A_y UB	1.869	2.047	2.137	2.181	2.203
s.e.	3.87E-05	3.74E-05	3.80E-05	3.86E-05	3.89E-05
95% CI (lb)	0.02581	0.02832	0.02948	0.03003	0.03035
(ub)	0.02596	0.02847	0.02963	0.03019	0.03051
r.e.	0.001496	0.001318	0.001286	0.001281	0.001279
$P(W_y>b)/P(W>b)$					
ratio	1.68266	1.68517	1.68068	1.67751	1.67821
diff w.r.t. last column	-0.00445	-0.00696	-0.00247	0.00071	0.00000
abs diff	0.00445	0.00696	0.00247	0.00071	0.00000

3.3. Tail Probability Estimates for the $(H_2)_t/M/1$ Periodic Queue

Tables 28-37 present results for the $(H_2)_t/M/1$ model paralleling the results for the $M_t/M/1$ model in Tables 17-27.

Table 28 **Simulation estimates of $\hat{p} \equiv P(W_y > b) \equiv A_y e^{-\theta^* b}$ in the $(H_2)_t/M/1$ model as a function of y based on $5,000$ replications: $\gamma = 1, b = 20, \rho = 0.8, \bar{\lambda} = 0.8, \mu = 1, \beta = 0.2, \theta^* = 0.173$**

position	n	$\hat{p}$	$exp(-\theta^* b)$	A_y	A_y/A approx	A_y/A LB	A_y/A UB	s.e.	95% CI (lb)	(ub)	r.e.
0.000	5000	0.025326	0.0317	0.799	0.966	0.933	1.000	6.42E-05	0.02520	0.02545	0.00254
0.025	5000	0.025266	0.0317	0.797	0.966	0.934	1.000	6.35E-05	0.02514	0.02539	0.00251
0.050	5000	0.025358	0.0317	0.800	0.968	0.935	1.002	6.40E-05	0.02523	0.02548	0.00252
0.075	5000	0.025503	0.0317	0.805	0.970	0.937	1.004	6.39E-05	0.02538	0.02563	0.00251
0.100	5000	0.025516	0.0317	0.805	0.972	0.939	1.007	6.45E-05	0.02539	0.02564	0.00253
0.125	5000	0.025714	0.0317	0.811	0.976	0.943	1.010	6.39E-05	0.02559	0.02584	0.00248
0.150	5000	0.025790	0.0317	0.814	0.980	0.947	1.014	6.43E-05	0.02566	0.02592	0.00249
0.175	5000	0.025819	0.0317	0.815	0.984	0.951	1.019	6.43E-05	0.02569	0.02595	0.00249
0.200	5000	0.026039	0.0317	0.822	0.989	0.956	1.024	6.38E-05	0.02591	0.02616	0.00245
0.225	5000	0.026254	0.0317	0.828	0.995	0.961	1.030	6.54E-05	0.02613	0.02638	0.00249
0.250	5000	0.026343	0.0317	0.831	1.000	0.966	1.035	6.56E-05	0.02621	0.02647	0.00249
0.275	5000	0.026580	0.0317	0.839	1.005	0.971	1.041	6.42E-05	0.02645	0.02671	0.00241
0.300	5000	0.026623	0.0317	0.840	1.011	0.976	1.046	6.57E-05	0.02649	0.02675	0.00247
0.325	5000	0.026778	0.0317	0.845	1.016	0.981	1.051	6.71E-05	0.02665	0.02691	0.00251
0.350	5000	0.026745	0.0317	0.844	1.020	0.986	1.056	6.79E-05	0.02661	0.02688	0.00254
0.375	5000	0.026965	0.0317	0.851	1.025	0.990	1.061	6.68E-05	0.02683	0.02710	0.00248
0.400	5000	0.027035	0.0317	0.853	1.028	0.993	1.064	6.72E-05	0.02690	0.02717	0.00248
0.425	5000	0.027097	0.0317	0.855	1.031	0.996	1.067	6.84E-05	0.02696	0.02723	0.00253
0.450	5000	0.027116	0.0317	0.856	1.033	0.998	1.070	6.71E-05	0.02698	0.02725	0.00247
0.475	5000	0.027069	0.0317	0.854	1.035	1.000	1.071	6.82E-05	0.02694	0.02720	0.00252
0.500	5000	0.027280	0.0317	0.861	1.035	1.000	1.071	6.79E-05	0.02715	0.02741	0.00249
0.525	5000	0.027020	0.0317	0.853	1.035	1.000	1.071	6.96E-05	0.02688	0.02716	0.00257
0.550	5000	0.027095	0.0317	0.855	1.033	0.998	1.070	6.87E-05	0.02696	0.02723	0.00253
0.575	5000	0.026990	0.0317	0.852	1.031	0.996	1.067	6.80E-05	0.02686	0.02712	0.00252
0.600	5000	0.027078	0.0317	0.854	1.028	0.993	1.064	6.78E-05	0.02695	0.02721	0.00250
0.625	5000	0.026855	0.0317	0.847	1.025	0.990	1.061	6.82E-05	0.02672	0.02699	0.00254
0.650	5000	0.026811	0.0317	0.846	1.020	0.986	1.056	6.78E-05	0.02668	0.02694	0.00253
0.675	5000	0.026697	0.0317	0.842	1.016	0.981	1.051	6.72E-05	0.02657	0.02683	0.00252
0.700	5000	0.026616	0.0317	0.840	1.011	0.976	1.046	6.48E-05	0.02649	0.02674	0.00243
0.725	5000	0.026456	0.0317	0.835	1.005	0.971	1.041	6.62E-05	0.02633	0.02659	0.00250
0.750	5000	0.026376	0.0317	0.832	1.000	0.966	1.035	6.46E-05	0.02625	0.02650	0.00245
0.775	5000	0.026222	0.0317	0.827	0.995	0.961	1.030	6.41E-05	0.02610	0.02635	0.00245
0.800	5000	0.025962	0.0317	0.819	0.989	0.956	1.024	6.52E-05	0.02583	0.02609	0.00251
0.825	5000	0.025856	0.0317	0.816	0.984	0.951	1.019	6.53E-05	0.02573	0.02598	0.00252
0.850	5000	0.025724	0.0317	0.812	0.980	0.947	1.014	6.53E-05	0.02560	0.02585	0.00254
0.875	5000	0.025635	0.0317	0.809	0.976	0.943	1.010	6.60E-05	0.02551	0.02576	0.00257
0.900	5000	0.025539	0.0317	0.806	0.972	0.939	1.007	6.43E-05	0.02541	0.02566	0.00252
0.925	5000	0.025462	0.0317	0.803	0.970	0.937	1.004	6.30E-05	0.02534	0.02559	0.00247
0.950	5000	0.025424	0.0317	0.802	0.968	0.935	1.002	6.36E-05	0.02530	0.02555	0.00250
0.975	5000	0.025442	0.0317	0.803	0.966	0.934	1.000	6.29E-05	0.02532	0.02557	0.00247

Table 29 Simulation estimates of $\hat{p} \equiv P(W_y > b) \equiv A_y e^{-\theta^* b}$ in the $(H_2)_t/M/1$ model as a function of y based on $5,000$ replications: $\gamma = 0.25, b = 40, \rho = 0.9, \bar{\lambda} = 0.9, \mu = 1, \beta = 0.1, \theta^* = 0.0761$

position	n	$\hat{p}$	$exp(-\theta^* b)$	A_y	A_y/A approx	A_y/A LB	A_y/A UB	s.e.	95% CI (lb)	(ub)	r.e.
0.000	5000	0.042734	0.0477	0.897	0.970	0.941	1.000	4.96E-05	0.04264	0.04283	0.00116
0.025	5000	0.042861	0.0477	0.899	0.970	0.941	1.000	4.79E-05	0.04277	0.04295	0.00112
0.050	5000	0.042816	0.0477	0.898	0.971	0.942	1.001	4.75E-05	0.04272	0.04291	0.00111
0.075	5000	0.042915	0.0477	0.901	0.973	0.944	1.003	4.85E-05	0.04282	0.04301	0.00113
0.100	5000	0.043023	0.0477	0.903	0.976	0.946	1.006	4.79E-05	0.04293	0.04312	0.00111
0.125	5000	0.043116	0.0477	0.905	0.979	0.949	1.009	4.93E-05	0.04302	0.04321	0.00114
0.150	5000	0.043219	0.0477	0.907	0.982	0.953	1.013	4.94E-05	0.04312	0.04332	0.00114
0.175	5000	0.043533	0.0477	0.914	0.986	0.957	1.017	4.85E-05	0.04344	0.04363	0.00111
0.200	5000	0.043765	0.0477	0.918	0.991	0.961	1.021	4.93E-05	0.04367	0.04386	0.00113
0.225	5000	0.043885	0.0477	0.921	0.995	0.965	1.026	4.90E-05	0.04379	0.04398	0.00112
0.250	5000	0.044134	0.0477	0.926	1.000	0.970	1.031	4.91E-05	0.04404	0.04423	0.00111
0.275	5000	0.044318	0.0477	0.930	1.005	0.975	1.036	4.99E-05	0.04422	0.04442	0.00113
0.300	5000	0.044483	0.0477	0.933	1.009	0.979	1.041	4.99E-05	0.04439	0.04458	0.00112
0.325	5000	0.044729	0.0477	0.939	1.014	0.984	1.045	4.97E-05	0.04463	0.04483	0.00111
0.350	5000	0.044932	0.0477	0.943	1.018	0.988	1.050	5.00E-05	0.04483	0.04503	0.00111
0.375	5000	0.045040	0.0477	0.945	1.022	0.991	1.053	4.98E-05	0.04494	0.04514	0.00110
0.400	5000	0.045175	0.0477	0.948	1.025	0.994	1.057	5.12E-05	0.04507	0.04528	0.00113
0.425	5000	0.045244	0.0477	0.949	1.027	0.997	1.059	5.07E-05	0.04514	0.04534	0.00112
0.450	5000	0.045360	0.0477	0.952	1.029	0.999	1.061	5.19E-05	0.04526	0.04546	0.00114
0.475	5000	0.045519	0.0477	0.955	1.031	1.000	1.062	5.07E-05	0.04542	0.04562	0.00111
0.500	5000	0.045536	0.0477	0.956	1.031	1.000	1.063	5.00E-05	0.04544	0.04563	0.00110
0.525	5000	0.045435	0.0477	0.953	1.031	1.000	1.062	5.13E-05	0.04533	0.04554	0.00113
0.550	5000	0.045563	0.0477	0.956	1.029	0.999	1.061	4.95E-05	0.04547	0.04566	0.00109
0.575	5000	0.045329	0.0477	0.951	1.027	0.997	1.059	5.08E-05	0.04523	0.04543	0.00112
0.600	5000	0.045185	0.0477	0.948	1.025	0.994	1.057	5.07E-05	0.04509	0.04528	0.00112
0.625	5000	0.045032	0.0477	0.945	1.022	0.991	1.053	5.11E-05	0.04493	0.04513	0.00113
0.650	5000	0.044887	0.0477	0.942	1.018	0.988	1.050	5.12E-05	0.04479	0.04499	0.00114
0.675	5000	0.044731	0.0477	0.939	1.014	0.984	1.045	4.90E-05	0.04463	0.04483	0.00110
0.700	5000	0.044457	0.0477	0.933	1.009	0.979	1.041	5.14E-05	0.04436	0.04456	0.00116
0.725	5000	0.044321	0.0477	0.930	1.005	0.975	1.036	4.92E-05	0.04422	0.04442	0.00111
0.750	5000	0.044170	0.0477	0.927	1.000	0.970	1.031	4.93E-05	0.04407	0.04427	0.00112
0.775	5000	0.043813	0.0477	0.919	0.995	0.965	1.026	5.05E-05	0.04371	0.04391	0.00115
0.800	5000	0.043666	0.0477	0.916	0.991	0.961	1.021	4.94E-05	0.04357	0.04376	0.00113
0.825	5000	0.043504	0.0477	0.913	0.986	0.957	1.017	4.80E-05	0.04341	0.04360	0.00110
0.850	5000	0.043330	0.0477	0.909	0.982	0.953	1.013	4.91E-05	0.04323	0.04343	0.00113
0.875	5000	0.043244	0.0477	0.907	0.979	0.949	1.009	4.73E-05	0.04315	0.04334	0.00109
0.900	5000	0.043098	0.0477	0.904	0.976	0.946	1.006	4.82E-05	0.04300	0.04319	0.00112
0.925	5000	0.042836	0.0477	0.899	0.973	0.944	1.003	4.91E-05	0.04274	0.04293	0.00115
0.950	5000	0.042714	0.0477	0.896	0.971	0.942	1.001	4.87E-05	0.04262	0.04281	0.00114
0.975	5000	0.042777	0.0477	0.898	0.970	0.941	1.000	4.81E-05	0.04268	0.04287	0.00112

Table 30 Simulation estimates of $\hat{p} \equiv P(W_y > b) \equiv A_y e^{-\theta^* b}$ in the $(H_2)_t/M/1$ model as a function of y based on $5,000$ replications: $\gamma = \frac{1}{16}, b = 80, \rho = 0.95, \bar{\lambda} = 0.95, \mu = 1, \beta = 0.05, \theta^* = 0.0356$

position	n	$\hat{p}$	$exp(-\theta^* b)$	A_y	A_y/A approx	A_y/A LB	A_y/A UB	s.e.	95% CI (lb)	(ub)	r.e.
0.000	5000	0.054303	0.0578	0.939	0.972	0.945	1.000	3.13E-05	0.05424	0.05436	5.77E-04
0.025	5000	0.054300	0.0578	0.939	0.972	0.945	1.000	3.15E-05	0.05424	0.05436	5.81E-04
0.050	5000	0.054360	0.0578	0.940	0.973	0.946	1.001	3.12E-05	0.05430	0.05442	5.74E-04
0.075	5000	0.054519	0.0578	0.943	0.975	0.948	1.003	3.09E-05	0.05446	0.05458	5.66E-04
0.100	5000	0.054550	0.0578	0.943	0.977	0.950	1.005	3.18E-05	0.05449	0.05461	5.83E-04
0.125	5000	0.054786	0.0578	0.947	0.980	0.953	1.008	3.16E-05	0.05472	0.05485	5.76E-04
0.150	5000	0.054931	0.0578	0.950	0.983	0.956	1.012	3.16E-05	0.05487	0.05499	5.76E-04
0.175	5000	0.055150	0.0578	0.954	0.987	0.959	1.016	3.15E-05	0.05509	0.05521	5.71E-04
0.200	5000	0.055383	0.0578	0.958	0.991	0.963	1.020	3.21E-05	0.05532	0.05545	5.79E-04
0.225	5000	0.055608	0.0578	0.961	0.996	0.968	1.024	3.18E-05	0.05555	0.05567	5.72E-04
0.250	5000	0.055865	0.0578	0.966	1.000	0.972	1.029	3.24E-05	0.05580	0.05593	5.80E-04
0.275	5000	0.056159	0.0578	0.971	1.004	0.976	1.034	3.27E-05	0.05610	0.05622	5.82E-04
0.300	5000	0.056380	0.0578	0.975	1.009	0.980	1.038	3.23E-05	0.05632	0.05644	5.73E-04
0.325	5000	0.056566	0.0578	0.978	1.013	0.985	1.042	3.30E-05	0.05650	0.05663	5.83E-04
0.350	5000	0.056841	0.0578	0.983	1.017	0.988	1.046	3.26E-05	0.05678	0.05691	5.74E-04
0.375	5000	0.056992	0.0578	0.985	1.020	0.992	1.050	3.39E-05	0.05693	0.05706	5.95E-04
0.400	5000	0.057139	0.0578	0.988	1.023	0.995	1.053	3.27E-05	0.05708	0.05720	5.72E-04
0.425	5000	0.057324	0.0578	0.991	1.026	0.997	1.055	3.36E-05	0.05726	0.05739	5.86E-04
0.450	5000	0.057391	0.0578	0.992	1.027	0.999	1.057	3.35E-05	0.05733	0.05746	5.83E-04
0.475	5000	0.057464	0.0578	0.994	1.029	1.000	1.058	3.32E-05	0.05740	0.05753	5.77E-04
0.500	5000	0.057466	0.0578	0.994	1.029	1.000	1.059	3.29E-05	0.05740	0.05753	5.73E-04
0.525	5000	0.057460	0.0578	0.993	1.029	1.000	1.058	3.34E-05	0.05739	0.05753	5.82E-04
0.550	5000	0.057412	0.0578	0.993	1.027	0.999	1.057	3.32E-05	0.05735	0.05748	5.79E-04
0.575	5000	0.057343	0.0578	0.991	1.026	0.997	1.055	3.30E-05	0.05728	0.05741	5.75E-04
0.600	5000	0.057165	0.0578	0.988	1.023	0.995	1.053	3.33E-05	0.05710	0.05723	5.83E-04
0.625	5000	0.057041	0.0578	0.986	1.020	0.992	1.050	3.35E-05	0.05697	0.05711	5.87E-04
0.650	5000	0.056788	0.0578	0.982	1.017	0.988	1.046	3.20E-05	0.05673	0.05685	5.63E-04
0.675	5000	0.056650	0.0578	0.979	1.013	0.985	1.042	3.20E-05	0.05659	0.05671	5.65E-04
0.700	5000	0.056391	0.0578	0.975	1.009	0.980	1.038	3.29E-05	0.05633	0.05646	5.84E-04
0.725	5000	0.056128	0.0578	0.970	1.004	0.976	1.034	3.23E-05	0.05606	0.05619	5.76E-04
0.750	5000	0.055929	0.0578	0.967	1.000	0.972	1.029	3.15E-05	0.05587	0.05599	5.62E-04
0.775	5000	0.055577	0.0578	0.961	0.996	0.968	1.024	3.30E-05	0.05551	0.05564	5.94E-04
0.800	5000	0.055409	0.0578	0.958	0.991	0.963	1.020	3.15E-05	0.05535	0.05547	5.68E-04
0.825	5000	0.055163	0.0578	0.954	0.987	0.959	1.016	3.18E-05	0.05510	0.05523	5.76E-04
0.850	5000	0.054896	0.0578	0.949	0.983	0.956	1.012	3.20E-05	0.05483	0.05496	5.84E-04
0.875	5000	0.054714	0.0578	0.946	0.980	0.953	1.008	3.15E-05	0.05465	0.05478	5.76E-04
0.900	5000	0.054613	0.0578	0.944	0.977	0.950	1.005	3.16E-05	0.05455	0.05467	5.79E-04
0.925	5000	0.054457	0.0578	0.942	0.975	0.948	1.003	3.25E-05	0.05439	0.05452	5.96E-04
0.950	5000	0.054428	0.0578	0.941	0.973	0.946	1.001	3.22E-05	0.05437	0.05449	5.91E-04
0.975	5000	0.054358	0.0578	0.940	0.972	0.945	1.000	3.12E-05	0.05430	0.05442	5.75E-04

Table 31 Simulation estimates of $\hat{p} \equiv P(W_y > b) \equiv A_y e^{-\theta^* b}$ in the $(H_2)_t/M/1$ model as a function of y based on $5,000$ replications: $\gamma = 0.64, b = 25, \rho = 0.84, \bar{\lambda} = 0.84, \mu = 1, \beta = 0.16, \theta^* = 0.131$

position	n	$\hat{p}$	$exp(-\theta^* b)$	A_y	A_y approx	A_y LB	A_y UB	s.e.	95% CI (lb)	(ub)	r.e.
0.000	5000	0.031500	0.0374	0.842	0.842	0.815	0.870	5.96E-05	0.03138	0.03162	0.00189
0.025	5000	0.031531	0.0374	0.843	0.843	0.815	0.871	6.05E-05	0.03141	0.03165	0.00192
0.050	5000	0.031537	0.0374	0.843	0.844	0.816	0.872	6.04E-05	0.03142	0.03166	0.00192
0.075	5000	0.031558	0.0374	0.844	0.845	0.818	0.873	5.96E-05	0.03144	0.03167	0.00189
0.100	5000	0.031603	0.0374	0.845	0.848	0.820	0.876	5.99E-05	0.03149	0.03172	0.00190
0.125	5000	0.031834	0.0374	0.851	0.850	0.823	0.879	6.01E-05	0.03172	0.03195	0.00189
0.150	5000	0.031906	0.0374	0.853	0.854	0.826	0.882	6.11E-05	0.03179	0.03203	0.00191
0.175	5000	0.032097	0.0374	0.858	0.857	0.830	0.886	6.08E-05	0.03198	0.03222	0.00189
0.200	5000	0.032215	0.0374	0.862	0.862	0.834	0.890	6.17E-05	0.03209	0.03234	0.00192
0.225	5000	0.032294	0.0374	0.864	0.866	0.838	0.895	6.19E-05	0.03217	0.03242	0.00192
0.250	5000	0.032581	0.0374	0.871	0.870	0.842	0.899	6.10E-05	0.03246	0.03270	0.00187
0.275	5000	0.032785	0.0374	0.877	0.875	0.847	0.904	6.30E-05	0.03266	0.03291	0.00192
0.300	5000	0.032967	0.0374	0.882	0.879	0.851	0.909	6.17E-05	0.03285	0.03309	0.00187
0.325	5000	0.033044	0.0374	0.884	0.883	0.855	0.913	6.23E-05	0.03292	0.03317	0.00189
0.350	5000	0.033178	0.0374	0.887	0.887	0.859	0.917	6.31E-05	0.03305	0.03330	0.00190
0.375	5000	0.033343	0.0374	0.892	0.891	0.862	0.921	6.36E-05	0.03322	0.03347	0.00191
0.400	5000	0.033383	0.0374	0.893	0.894	0.865	0.924	6.31E-05	0.03326	0.03351	0.00189
0.425	5000	0.033410	0.0374	0.894	0.896	0.867	0.926	6.42E-05	0.03328	0.03354	0.00192
0.450	5000	0.033533	0.0374	0.897	0.898	0.869	0.928	6.36E-05	0.03341	0.03366	0.00190
0.475	5000	0.033599	0.0374	0.899	0.899	0.870	0.929	6.42E-05	0.03347	0.03372	0.00191
0.500	5000	0.033561	0.0374	0.898	0.899	0.870	0.929	6.35E-05	0.03344	0.03369	0.00189
0.525	5000	0.033613	0.0374	0.899	0.899	0.870	0.929	6.43E-05	0.03349	0.03374	0.00191
0.550	5000	0.033546	0.0374	0.897	0.898	0.869	0.928	6.49E-05	0.03342	0.03367	0.00193
0.575	5000	0.033541	0.0374	0.897	0.896	0.867	0.926	6.26E-05	0.03342	0.03366	0.00187
0.600	5000	0.033365	0.0374	0.892	0.894	0.865	0.924	6.31E-05	0.03324	0.03349	0.00189
0.625	5000	0.033284	0.0374	0.890	0.891	0.862	0.921	6.47E-05	0.03316	0.03341	0.00194
0.650	5000	0.033196	0.0374	0.888	0.887	0.859	0.917	6.38E-05	0.03307	0.03332	0.00192
0.675	5000	0.033028	0.0374	0.883	0.883	0.855	0.913	6.36E-05	0.03290	0.03315	0.00193
0.700	5000	0.032913	0.0374	0.880	0.879	0.851	0.909	6.09E-05	0.03279	0.03303	0.00185
0.725	5000	0.032711	0.0374	0.875	0.875	0.847	0.904	6.35E-05	0.03259	0.03284	0.00194
0.750	5000	0.032600	0.0374	0.872	0.870	0.842	0.899	6.05E-05	0.03248	0.03272	0.00186
0.775	5000	0.032468	0.0374	0.868	0.866	0.838	0.895	6.07E-05	0.03235	0.03259	0.00187
0.800	5000	0.032151	0.0374	0.860	0.862	0.834	0.890	6.25E-05	0.03203	0.03227	0.00194
0.825	5000	0.032071	0.0374	0.858	0.857	0.830	0.886	6.10E-05	0.03195	0.03219	0.00190
0.850	5000	0.031876	0.0374	0.852	0.854	0.826	0.882	6.17E-05	0.03175	0.03200	0.00193
0.875	5000	0.031610	0.0374	0.845	0.850	0.823	0.879	6.17E-05	0.03149	0.03173	0.00195
0.900	5000	0.031681	0.0374	0.847	0.848	0.820	0.876	5.95E-05	0.03156	0.03180	0.00188
0.925	5000	0.031634	0.0374	0.846	0.845	0.818	0.873	5.98E-05	0.03152	0.03175	0.00189
0.950	5000	0.031516	0.0374	0.843	0.844	0.816	0.872	6.01E-05	0.03140	0.03163	0.00191
0.975	5000	0.031469	0.0374	0.842	0.843	0.815	0.871	5.99E-05	0.03135	0.03159	0.00190

Table 32 Simulation estimates of $\hat{p} \equiv P(W_y > b) \equiv A_y e^{-\theta^* b}$ in the $(H_2)_t/M/1$ model as a function of y based on $5,000$ replications: $\gamma = 0.16, b = 50, \rho = 0.92, \bar{\lambda} = 0.92, \mu = 1, \beta = 0.08, \theta^* = 0.0593$

position	n	$\hat{p}$	$exp(-\theta^* b)$	A_y	A_y approx	A_y LB	A_y UB	s.e.	95% CI (lb)	(ub)	r.e.
0.000	5000	0.047148	0.0515	0.915	0.913	0.887	0.941	4.20E-05	0.04707	0.04723	8.92E-04
0.025	5000	0.047196	0.0515	0.916	0.914	0.887	0.941	4.21E-05	0.04711	0.04728	8.91E-04
0.050	5000	0.047178	0.0515	0.915	0.915	0.888	0.942	4.21E-05	0.04710	0.04726	8.92E-04
0.075	5000	0.047238	0.0515	0.916	0.916	0.890	0.944	4.29E-05	0.04715	0.04732	9.07E-04
0.100	5000	0.047472	0.0515	0.921	0.919	0.892	0.946	9.77E-05	0.04728	0.04766	2.06E-03
0.125	5000	0.047550	0.0515	0.922	0.921	0.894	0.949	4.30E-05	0.04747	0.04763	9.04E-04
0.150	5000	0.047720	0.0515	0.926	0.925	0.898	0.952	4.24E-05	0.04764	0.04780	8.89E-04
0.175	5000	0.048040	0.0515	0.932	0.928	0.901	0.956	8.37E-05	0.04788	0.04820	1.74E-03
0.200	5000	0.048026	0.0515	0.932	0.932	0.905	0.960	4.39E-05	0.04794	0.04811	9.15E-04
0.225	5000	0.048282	0.0515	0.937	0.936	0.909	0.965	4.40E-05	0.04820	0.04837	9.11E-04
0.250	5000	0.048515	0.0515	0.941	0.941	0.913	0.969	4.47E-05	0.04843	0.04860	9.21E-04
0.275	5000	0.048691	0.0515	0.945	0.945	0.918	0.974	4.48E-05	0.04860	0.04878	9.20E-04
0.300	5000	0.048982	0.0515	0.950	0.950	0.922	0.978	4.54E-05	0.04889	0.04907	9.28E-04
0.325	5000	0.049243	0.0515	0.955	0.954	0.926	0.982	4.34E-05	0.04916	0.04933	8.81E-04
0.350	5000	0.049461	0.0515	0.960	0.957	0.929	0.986	4.57E-05	0.04937	0.04955	9.24E-04
0.375	5000	0.049662	0.0515	0.963	0.961	0.933	0.990	4.31E-05	0.04958	0.04975	8.68E-04
0.400	5000	0.049779	0.0515	0.966	0.964	0.936	0.993	4.39E-05	0.04969	0.04987	8.83E-04
0.425	5000	0.050002	0.0515	0.970	0.966	0.938	0.995	7.26E-05	0.04986	0.05014	1.45E-03
0.450	5000	0.049994	0.0515	0.970	0.968	0.939	0.997	4.41E-05	0.04991	0.05008	8.82E-04
0.475	5000	0.049949	0.0515	0.969	0.969	0.941	0.998	4.50E-05	0.04986	0.05004	9.01E-04
0.500	5000	0.050020	0.0515	0.970	0.969	0.941	0.998	4.44E-05	0.04993	0.05011	8.88E-04
0.525	5000	0.050090	0.0515	0.972	0.969	0.941	0.998	4.49E-05	0.05000	0.05018	8.97E-04
0.550	5000	0.050077	0.0515	0.971	0.968	0.939	0.997	4.31E-05	0.04999	0.05016	8.60E-04
0.575	5000	0.049931	0.0515	0.969	0.966	0.938	0.995	4.36E-05	0.04985	0.05002	8.72E-04
0.600	5000	0.049756	0.0515	0.965	0.964	0.936	0.993	4.45E-05	0.04967	0.04984	8.95E-04
0.625	5000	0.049611	0.0515	0.962	0.961	0.933	0.990	4.33E-05	0.04953	0.04970	8.72E-04
0.650	5000	0.049456	0.0515	0.959	0.957	0.929	0.986	4.43E-05	0.04937	0.04954	8.96E-04
0.675	5000	0.049202	0.0515	0.954	0.954	0.926	0.982	4.41E-05	0.04912	0.04929	8.95E-04
0.700	5000	0.048966	0.0515	0.950	0.950	0.922	0.978	4.47E-05	0.04888	0.04905	9.12E-04
0.725	5000	0.048780	0.0515	0.946	0.945	0.918	0.974	4.40E-05	0.04869	0.04887	9.02E-04
0.750	5000	0.048635	0.0515	0.944	0.941	0.913	0.969	4.28E-05	0.04855	0.04872	8.80E-04
0.775	5000	0.048339	0.0515	0.938	0.936	0.909	0.965	4.29E-05	0.04826	0.04842	8.88E-04
0.800	5000	0.048207	0.0515	0.935	0.932	0.905	0.960	4.21E-05	0.04812	0.04829	8.72E-04
0.825	5000	0.047963	0.0515	0.930	0.928	0.901	0.956	4.17E-05	0.04788	0.04804	8.69E-04
0.850	5000	0.047699	0.0515	0.925	0.925	0.898	0.952	4.32E-05	0.04761	0.04778	9.05E-04
0.875	5000	0.047584	0.0515	0.923	0.921	0.894	0.949	4.17E-05	0.04750	0.04767	8.77E-04
0.900	5000	0.047438	0.0515	0.920	0.919	0.892	0.946	4.14E-05	0.04736	0.04752	8.73E-04
0.925	5000	0.047343	0.0515	0.918	0.916	0.890	0.944	4.15E-05	0.04726	0.04742	8.76E-04
0.950	5000	0.047215	0.0515	0.916	0.915	0.888	0.942	4.18E-05	0.04713	0.04730	8.86E-04
0.975	5000	0.047194	0.0515	0.916	0.914	0.887	0.941	4.18E-05	0.04711	0.04728	8.87E-04

Table 33 Simulation estimates of $\hat{p} \equiv P(W_y > b) \equiv A_y e^{-\theta^* b}$ in the $(H_2)_t/M/1$ model as a function of y based on $5,000$ replications: $\gamma = 0.01, b = 200, \rho = 0.98, \bar{\lambda} = 0.98, \mu = 1, \beta = 0.02, \theta^* = 0.0137$

position	n	$\hat{p}$	$exp(-\theta^* b)$	A_y	A_y/A approx	A_y/A LB	A_y/A UB	s.e.	95% CI (lb)	(ub)	r.e.
0.000	5000	0.062152	0.0647	0.961	0.973	0.947	1.000	2.09E-05	0.06211	0.06219	3.36E-04
0.025	5000	0.062156	0.0647	0.961	0.973	0.947	1.000	2.05E-05	0.06212	0.06220	3.30E-04
0.050	5000	0.062226	0.0647	0.962	0.974	0.948	1.001	2.09E-05	0.06218	0.06227	3.37E-04
0.075	5000	0.062304	0.0647	0.964	0.976	0.950	1.003	2.14E-05	0.06226	0.06235	3.43E-04
0.100	5000	0.062430	0.0647	0.966	0.978	0.952	1.005	2.10E-05	0.06239	0.06247	3.36E-04
0.125	5000	0.062626	0.0647	0.969	0.981	0.954	1.008	2.08E-05	0.06258	0.06267	3.32E-04
0.150	5000	0.062851	0.0647	0.972	0.984	0.957	1.011	2.08E-05	0.06281	0.06289	3.30E-04
0.175	5000	0.063082	0.0647	0.976	0.988	0.961	1.015	2.11E-05	0.06304	0.06312	3.34E-04
0.200	5000	0.063343	0.0647	0.980	0.992	0.965	1.019	2.11E-05	0.06330	0.06338	3.32E-04
0.225	5000	0.063591	0.0647	0.984	0.996	0.969	1.023	2.11E-05	0.06355	0.06363	3.32E-04
0.250	5000	0.063883	0.0647	0.988	1.000	0.973	1.028	2.14E-05	0.06384	0.06393	3.35E-04
0.275	5000	0.064143	0.0647	0.992	1.004	0.977	1.032	2.14E-05	0.06410	0.06418	3.34E-04
0.300	5000	0.064370	0.0647	0.996	1.008	0.981	1.037	2.15E-05	0.06433	0.06441	3.35E-04
0.325	5000	0.064690	0.0647	1.001	1.013	0.985	1.041	2.17E-05	0.06465	0.06473	3.35E-04
0.350	5000	0.064920	0.0647	1.004	1.016	0.989	1.044	2.18E-05	0.06488	0.06496	3.36E-04
0.375	5000	0.065129	0.0647	1.007	1.020	0.992	1.048	2.16E-05	0.06509	0.06517	3.32E-04
0.400	5000	0.065284	0.0647	1.010	1.022	0.995	1.051	2.21E-05	0.06524	0.06533	3.38E-04
0.425	5000	0.065469	0.0647	1.013	1.025	0.997	1.053	2.17E-05	0.06543	0.06551	3.31E-04
0.450	5000	0.065561	0.0647	1.014	1.026	0.999	1.055	2.19E-05	0.06552	0.06560	3.34E-04
0.475	5000	0.065605	0.0647	1.015	1.027	1.000	1.056	2.17E-05	0.06556	0.06565	3.31E-04
0.500	5000	0.065625	0.0647	1.015	1.028	1.000	1.056	2.18E-05	0.06558	0.06567	3.32E-04
0.525	5000	0.065597	0.0647	1.015	1.027	1.000	1.056	2.25E-05	0.06555	0.06564	3.42E-04
0.550	5000	0.065522	0.0647	1.013	1.026	0.999	1.055	2.17E-05	0.06548	0.06556	3.31E-04
0.575	5000	0.065497	0.0647	1.013	1.025	0.997	1.053	2.19E-05	0.06545	0.06554	3.35E-04
0.600	5000	0.065314	0.0647	1.010	1.022	0.995	1.051	2.23E-05	0.06527	0.06536	3.41E-04
0.625	5000	0.065144	0.0647	1.008	1.020	0.992	1.048	2.18E-05	0.06510	0.06519	3.35E-04
0.650	5000	0.064897	0.0647	1.004	1.016	0.989	1.044	2.20E-05	0.06485	0.06494	3.39E-04
0.675	5000	0.064678	0.0647	1.000	1.013	0.985	1.041	2.17E-05	0.06464	0.06472	3.36E-04
0.700	5000	0.064436	0.0647	0.997	1.008	0.981	1.037	2.13E-05	0.06439	0.06448	3.30E-04
0.725	5000	0.064149	0.0647	0.992	1.004	0.977	1.032	2.15E-05	0.06411	0.06419	3.36E-04
0.750	5000	0.063882	0.0647	0.988	1.000	0.973	1.028	2.12E-05	0.06384	0.06392	3.32E-04
0.775	5000	0.063605	0.0647	0.984	0.996	0.969	1.023	2.16E-05	0.06356	0.06365	3.39E-04
0.800	5000	0.063313	0.0647	0.979	0.992	0.965	1.019	2.10E-05	0.06327	0.06335	3.32E-04
0.825	5000	0.063053	0.0647	0.975	0.988	0.961	1.015	2.08E-05	0.06301	0.06309	3.30E-04
0.850	5000	0.062886	0.0647	0.973	0.984	0.957	1.011	2.09E-05	0.06285	0.06293	3.32E-04
0.875	5000	0.062639	0.0647	0.969	0.981	0.954	1.008	2.05E-05	0.06260	0.06268	3.27E-04
0.900	5000	0.062504	0.0647	0.967	0.978	0.952	1.005	2.07E-05	0.06246	0.06254	3.30E-04
0.925	5000	0.062342	0.0647	0.964	0.976	0.950	1.003	2.10E-05	0.06230	0.06238	3.36E-04
0.950	5000	0.062243	0.0647	0.963	0.974	0.948	1.001	2.05E-05	0.06220	0.06228	3.29E-04
0.975	5000	0.062175	0.0647	0.962	0.973	0.947	1.000	2.02E-05	0.06214	0.06221	3.25E-04

Table 34 Simulation estimates of $\hat{p} \equiv P(W_y > b) \equiv A_y e^{-\theta^* b}$ in the $(H_2)_t/M/1$ model as a function of y based on $5,000$ replications: $\gamma = 0.0025, b = 400, \rho = 0.99, \bar{\lambda} = 0.99, \mu = 1, \beta = 0.01, \theta^* = 0.00676$

position	n	$\hat{p}$	$exp(-\theta^* b)$	A_y	A_y/A approx	A_y/A LB	A_y/A UB	s.e.	95% CI (lb)	(ub)	r.e.
0.000	5000	0.064912	0.0670	0.968	0.973	0.947	1.000	1.88E-05	0.06488	0.06495	2.90E-04
0.025	5000	0.064916	0.0670	0.968	0.974	0.948	1.000	1.86E-05	0.06488	0.06495	2.86E-04
0.050	5000	0.064997	0.0670	0.970	0.975	0.949	1.001	1.86E-05	0.06496	0.06503	2.87E-04
0.075	5000	0.065107	0.0670	0.971	0.976	0.950	1.003	1.86E-05	0.06507	0.06514	2.86E-04
0.100	5000	0.065259	0.0670	0.973	0.978	0.952	1.005	1.86E-05	0.06522	0.06529	2.85E-04
0.125	5000	0.065414	0.0670	0.976	0.981	0.955	1.008	1.85E-05	0.06538	0.06545	2.83E-04
0.150	5000	0.065616	0.0670	0.979	0.984	0.958	1.011	1.88E-05	0.06558	0.06565	2.86E-04
0.175	5000	0.065908	0.0670	0.983	0.988	0.961	1.015	1.88E-05	0.06587	0.06594	2.85E-04
0.200	5000	0.066145	0.0670	0.987	0.992	0.965	1.019	1.89E-05	0.06611	0.06618	2.86E-04
0.225	5000	0.066400	0.0670	0.990	0.996	0.969	1.023	1.91E-05	0.06636	0.06644	2.88E-04
0.250	5000	0.066691	0.0670	0.995	1.000	0.973	1.027	1.89E-05	0.06665	0.06673	2.83E-04
0.275	5000	0.067005	0.0670	0.999	1.004	0.977	1.032	1.93E-05	0.06697	0.06704	2.88E-04
0.300	5000	0.067244	0.0670	1.003	1.008	0.981	1.036	1.91E-05	0.06721	0.06728	2.84E-04
0.325	5000	0.067532	0.0670	1.007	1.012	0.985	1.040	1.94E-05	0.06749	0.06757	2.87E-04
0.350	5000	0.067757	0.0670	1.011	1.016	0.989	1.044	1.93E-05	0.06772	0.06780	2.86E-04
0.375	5000	0.067990	0.0670	1.014	1.019	0.992	1.047	1.95E-05	0.06795	0.06803	2.86E-04
0.400	5000	0.068184	0.0670	1.017	1.022	0.995	1.050	1.93E-05	0.06815	0.06822	2.84E-04
0.425	5000	0.068312	0.0670	1.019	1.024	0.997	1.052	1.96E-05	0.06827	0.06835	2.86E-04
0.450	5000	0.068399	0.0670	1.020	1.026	0.999	1.054	1.95E-05	0.06836	0.06844	2.86E-04
0.475	5000	0.068526	0.0670	1.022	1.027	1.000	1.055	1.96E-05	0.06849	0.06856	2.86E-04
0.500	5000	0.068541	0.0670	1.022	1.027	1.000	1.056	1.96E-05	0.06850	0.06858	2.86E-04
0.525	5000	0.068500	0.0670	1.022	1.027	1.000	1.055	1.96E-05	0.06846	0.06854	2.86E-04
0.550	5000	0.068422	0.0670	1.021	1.026	0.999	1.054	1.95E-05	0.06838	0.06846	2.85E-04
0.575	5000	0.068339	0.0670	1.019	1.024	0.997	1.052	1.94E-05	0.06830	0.06838	2.84E-04
0.600	5000	0.068154	0.0670	1.017	1.022	0.995	1.050	1.95E-05	0.06812	0.06819	2.86E-04
0.625	5000	0.068002	0.0670	1.014	1.019	0.992	1.047	1.95E-05	0.06796	0.06804	2.87E-04
0.650	5000	0.067769	0.0670	1.011	1.016	0.989	1.044	1.93E-05	0.06773	0.06781	2.85E-04
0.675	5000	0.067519	0.0670	1.007	1.012	0.985	1.040	1.91E-05	0.06748	0.06756	2.83E-04
0.700	5000	0.067221	0.0670	1.003	1.008	0.981	1.036	1.94E-05	0.06718	0.06726	2.88E-04
0.725	5000	0.066975	0.0670	0.999	1.004	0.977	1.032	1.92E-05	0.06694	0.06701	2.86E-04
0.750	5000	0.066729	0.0670	0.995	1.000	0.973	1.027	1.92E-05	0.06669	0.06677	2.88E-04
0.775	5000	0.066409	0.0670	0.991	0.996	0.969	1.023	1.89E-05	0.06637	0.06645	2.84E-04
0.800	5000	0.066165	0.0670	0.987	0.992	0.965	1.019	1.86E-05	0.06613	0.06620	2.81E-04
0.825	5000	0.065873	0.0670	0.983	0.988	0.961	1.015	1.89E-05	0.06584	0.06591	2.87E-04
0.850	5000	0.065640	0.0670	0.979	0.984	0.958	1.011	1.87E-05	0.06560	0.06568	2.85E-04
0.875	5000	0.065434	0.0670	0.976	0.981	0.955	1.008	1.85E-05	0.06540	0.06547	2.83E-04
0.900	5000	0.065241	0.0670	0.973	0.978	0.952	1.005	1.85E-05	0.06521	0.06528	2.83E-04
0.925	5000	0.065099	0.0670	0.971	0.976	0.950	1.003	1.86E-05	0.06506	0.06514	2.85E-04
0.950	5000	0.064994	0.0670	0.969	0.975	0.949	1.001	1.87E-05	0.06496	0.06503	2.87E-04
0.975	5000	0.064916	0.0670	0.968	0.974	0.948	1.000	1.84E-05	0.06488	0.06495	2.84E-04

Table 35 Comparison of ratio $P(W_y > b)/\rho$ in the $(H_2)_t/M/1$ queue as a function of ρ with base parameter $(\beta, \gamma, b) = (1, 25, 4)$ using the scaling in (23) of the main paper.

position	$\rho = 0.8$	$\rho = 0.9$	$\rho = 0.95$	$\rho = 0.98$	$\rho = 0.99$
0.000	0.99885	0.99642	0.98833	0.98093	0.97804
0.025	0.99648	0.99938	0.98828	0.98099	0.97809
0.050	1.00011	0.99832	0.98936	0.98209	0.97932
0.075	1.00585	1.00064	0.99226	0.98332	0.98096
0.100	1.00635	1.00316	0.99283	0.98532	0.98325
0.125	1.01416	1.00532	0.99712	0.98840	0.98560
0.150	1.01717	1.00772	0.99975	0.99195	0.98864
0.175	1.01831	1.01505	1.00374	0.99561	0.99304
0.200	1.02699	1.02045	1.00798	0.99972	0.99660
0.225	1.03546	1.02326	1.01207	1.00363	1.00044
0.250	1.03895	1.02906	1.01676	1.00825	1.00483
0.275	1.04831	1.03335	1.02211	1.01234	1.00956
0.300	1.05001	1.03720	1.02614	1.01593	1.01317
0.325	1.05613	1.04294	1.02952	1.02099	1.01750
0.350	1.05483	1.04767	1.03452	1.02461	1.02090
0.375	1.06350	1.05019	1.03727	1.02791	1.02441
0.400	1.06624	1.05333	1.03995	1.03035	1.02733
0.425	1.06871	1.05493	1.04330	1.03327	1.02927
0.450	1.06943	1.05766	1.04453	1.03473	1.03056
0.475	1.06759	1.06136	1.04585	1.03542	1.03248
0.500	1.07590	1.06174	1.04589	1.03574	1.03270
0.525	1.06565	1.05938	1.04578	1.03530	1.03209
0.550	1.06860	1.06238	1.04491	1.03412	1.03092
0.575	1.06446	1.05693	1.04366	1.03372	1.02966
0.600	1.06795	1.05356	1.04041	1.03083	1.02688
0.625	1.05916	1.05001	1.03815	1.02815	1.02458
0.650	1.05741	1.04661	1.03355	1.02425	1.02107
0.675	1.05293	1.04298	1.03105	1.02080	1.01731
0.700	1.04973	1.03660	1.02633	1.01698	1.01282
0.725	1.04343	1.03342	1.02154	1.01245	1.00912
0.750	1.04025	1.02989	1.01792	1.00823	1.00540
0.775	1.03420	1.02157	1.01150	1.00387	1.00059
0.800	1.02393	1.01814	1.00845	0.99925	0.99691
0.825	1.01977	1.01437	1.00398	0.99515	0.99251
0.850	1.01453	1.01032	0.99913	0.99251	0.98901
0.875	1.01105	1.00830	0.99581	0.98861	0.98590
0.900	1.00724	1.00491	0.99396	0.98648	0.98299
0.925	1.00420	0.99879	0.99113	0.98393	0.98085
0.950	1.00273	0.99595	0.99061	0.98237	0.97926
0.975	1.00342	0.99743	0.98932	0.98129	0.97809
avg diff w.r.t. last column	0.03168	0.02345	0.01205	0.00318	0.00000
avg. abs. diff w.r.t. last column	0.03168	0.02345	0.01205	0.00318	0.00000
rmse w.r.t. last column	0.03234	0.02369	0.01213	0.00322	0.00000

Table 36 Summary of simulation results for the $(H_2)_t/M/1$ queue at $y=0$ as a function of $1-\rho$ with base parameter $(\beta,\gamma,b)=(1,25,4)$ using the scaling in (23) of the main paper.

	$1-\rho=0.16$	$1-\rho=0.08$	$1-\rho=0.04$	$1-\rho=0.02$	$1-\rho=0.01$	$1-\rho=0.005$
θ^*	0.113	0.0548	0.0270	0.0134	0.00669	0.00334
n	5000	5000	5000	5000	5000	5000
$\hat{p}$	0.051165	0.059299	0.063514	0.065607	0.066689	0.067212
$e^{-\theta^* b}$	0.0593	0.0645	0.0670	0.0682	0.0689	0.0692
A_y	0.862	0.920	0.948	0.961	0.968	0.972
A_y approxi	0.861	0.919	0.947	0.960	0.967	0.970
A_y LB	0.837	0.895	0.922	0.935	0.942	0.945
A_y UB	0.885	0.945	0.973	0.987	0.993	0.997
s.e.	8.57E-05	5.04E-05	2.97E-05	2.15E-05	1.89E-05	1.82E-05
95% CI (lb)	0.05100	0.05920	0.06346	0.06557	0.06665	0.06718
(ub)	0.05133	0.05940	0.06357	0.06565	0.06673	0.06725
r.e.	0.001675	0.000849	0.000467	0.000327	0.000284	0.000271
$P(W_y>b)/P(W>b)$						
ratio	0.97418	0.97338	0.97468	0.97445	0.97493	0.97491
diff w.r.t. last column	0.00074	0.00153	0.00023	0.00046	-0.00002	0.00000
abs diff w.r.t. last column	0.00074	0.00153	0.00023	0.00046	0.00002	0.00000
A_y/ρ						
ratio	1.02676	0.99988	0.98758	0.98100	0.97819	0.97652
diff w.r.t. last column	-0.05024	-0.02336	-0.01106	-0.00448	-0.00167	0.00000
abs diff w.r.t. last column	0.05024	0.02336	0.01106	0.00448	0.00167	0.00000

Table 37 Summary of simulation results for the $(H_2)_t/M/1$ queue at $y=0$ and $y=0.5$ as a function of $1-\rho$ with base parameter $(\beta,\gamma,b)=(1,2.5,4)$ using the scaling in (23) of the main paper.

	$1-\rho=0.16$	$1-\rho=0.08$	$1-\rho=0.04$	$1-\rho=0.02$	$1-\rho=0.01$
*theta**	0.113	0.0548	0.0270	0.0134	0.00669
n	40000	40000	40000	40000	40000
$y=0$					
$\hat{p}$	0.041099	0.047976	0.051467	0.053499	0.054240
$e^{-\theta^* b}$	0.0593	0.0645	0.0670	0.0682	0.0689
A_y	0.693	0.744	0.768	0.784	0.788
A_y approxi	0.669	0.718	0.743	0.754	0.760
A_y LB	0.504	0.546	0.567	0.577	0.582
A_y UB	0.887	0.945	0.973	0.987	0.993
s.e.	4.62E-05	4.68E-05	4.82E-05	1.72E-04	4.96E-05
95% CI (lb)	0.04101	0.04788	0.05137	0.05316	0.05414
(ub)	0.04119	0.04807	0.05156	0.05384	0.05434
r.e.	0.001125	0.000975	0.000936	0.003208	0.000914
$P(W_y>b)/P(W>b)$					
ratio	0.78064	0.78762	0.78945	0.79463	0.79294
diff	0.01230	0.00532	0.00349	-0.00169	0.00000
abs diff	0.01230	0.00532	0.00349	0.00169	0.00000
A_y/ρ					
ratio	0.82476	0.80897	0.80027	0.79995	0.79559
diff	-0.02916	-0.01337	-0.00467	-0.00436	0.00000
abs diff	0.02916	0.01337	0.00467	0.00436	0.00000
$y=0.5$					
$\hat{p}$	0.075260	0.086414	0.092196	0.095157	0.096491
$e^{-\theta^* b}$	0.0593	0.0645	0.0670	0.0682	0.0689
A_y	1.269	1.341	1.376	1.394	1.401
A_y approxi	1.177	1.243	1.275	1.290	1.298
A_y LB	0.887	0.945	0.973	0.987	0.993
A_y UB	1.561	1.635	1.671	1.688	1.696
s.e.	8.03E-05	7.92E-05	8.02E-05	1.83E-04	8.25E-05
95% CI (lb)	0.07510	0.08626	0.09204	0.09480	0.09633
(ub)	0.07542	0.08657	0.09235	0.09552	0.09665
r.e.	0.001067	0.000916	0.000870	0.001921	0.000855
$P(W_y>b)/P(W>b)$					
ratio	1.42950	1.41863	1.41419	1.41339	1.41060
diff	-0.01891	-0.00803	-0.00360	-0.00279	0.00000
abs diff	0.01891	0.00803	0.00360	0.00279	0.00000
A_y/ρ					
ratio	1.51029	1.45708	1.43357	1.42285	1.41532
diff	-0.09497	-0.04176	-0.01825	-0.00753	0.00000
abs diff	0.09497	0.04176	0.01825	0.00753	0.00000

3.4. Tail Probability Estimates for the $M_t/H_2/1$ Periodic Queue

Tables 38-48 present results for the $M_t/H_2/1$ model paralleling the results for the $M_t/M/1$ model in Tables 17-27 and the results for the $(H_2)_t/M/1$ model in Tables 28-37.

Table 38 **Simulation estimates of** $\hat{p} \equiv P(W_y > b) \equiv A_y e^{-\theta^* b}$ **in the** $M_t/H_2/1$ **model as a function of** y **based on** $5,000$ **replications:** $\gamma = 1, b = 20, \rho = 0.8, \bar{\lambda} = 0.8, \mu = 1, \beta = 0.2, \theta^* = 0.124$

position	n	$\hat{p}$	$exp(-\theta^* b)$	A_y	A_y/A approx	A_y/A LB	A_y/A UB	s.e.	95% CI (lb)	(ub)	r.e.
0.000	5000	0.061043	0.0839	0.728	0.976	0.952	1.000	2.46E-04	0.06056	0.06153	0.00403
0.025	5000	0.060935	0.0839	0.726	0.976	0.952	1.000	2.50E-04	0.06045	0.06142	0.00410
0.050	5000	0.060934	0.0839	0.726	0.977	0.953	1.001	2.47E-04	0.06045	0.06142	0.00406
0.075	5000	0.060531	0.0839	0.721	0.978	0.954	1.003	2.53E-04	0.06004	0.06103	0.00417
0.100	5000	0.061014	0.0839	0.727	0.980	0.956	1.005	2.50E-04	0.06052	0.06150	0.00410
0.125	5000	0.061186	0.0839	0.729	0.983	0.959	1.007	2.50E-04	0.06070	0.06168	0.00409
0.150	5000	0.061205	0.0839	0.729	0.986	0.961	1.010	2.56E-04	0.06070	0.06171	0.00418
0.175	5000	0.061299	0.0839	0.731	0.989	0.965	1.014	2.59E-04	0.06079	0.06181	0.00422
0.200	5000	0.062072	0.0839	0.740	0.992	0.968	1.017	2.50E-04	0.06158	0.06256	0.00402
0.225	5000	0.062331	0.0839	0.743	0.996	0.972	1.021	2.52E-04	0.06184	0.06283	0.00404
0.250	5000	0.062644	0.0839	0.747	1.000	0.976	1.025	2.50E-04	0.06215	0.06313	0.00399
0.275	5000	0.062369	0.0839	0.743	1.004	0.979	1.029	2.59E-04	0.06186	0.06288	0.00415
0.300	5000	0.063145	0.0839	0.753	1.008	0.983	1.033	2.57E-04	0.06264	0.06365	0.00407
0.325	5000	0.063175	0.0839	0.753	1.011	0.987	1.037	2.61E-04	0.06266	0.06369	0.00413
0.350	5000	0.063013	0.0839	0.751	1.015	0.990	1.040	2.61E-04	0.06250	0.06352	0.00414
0.375	5000	0.063367	0.0839	0.755	1.018	0.993	1.043	2.63E-04	0.06285	0.06388	0.00414
0.400	5000	0.063504	0.0839	0.757	1.020	0.995	1.046	2.64E-04	0.06299	0.06402	0.00415
0.425	5000	0.063472	0.0839	0.756	1.022	0.997	1.048	2.66E-04	0.06295	0.06399	0.00419
0.450	5000	0.063690	0.0839	0.759	1.024	0.999	1.050	2.65E-04	0.06317	0.06421	0.00415
0.475	5000	0.063951	0.0839	0.762	1.025	1.000	1.050	2.59E-04	0.06344	0.06446	0.00404
0.500	5000	0.063853	0.0839	0.761	1.025	1.000	1.051	2.65E-04	0.06333	0.06437	0.00415
0.525	5000	0.064030	0.0839	0.763	1.025	1.000	1.050	2.59E-04	0.06352	0.06454	0.00404
0.550	5000	0.063536	0.0839	0.757	1.024	0.999	1.050	2.63E-04	0.06302	0.06405	0.00415
0.575	5000	0.063183	0.0839	0.753	1.022	0.997	1.048	2.65E-04	0.06266	0.06370	0.00419
0.600	5000	0.063351	0.0839	0.755	1.020	0.995	1.046	2.68E-04	0.06283	0.06388	0.00423
0.625	5000	0.062683	0.0839	0.747	1.018	0.993	1.043	2.64E-04	0.06216	0.06320	0.00422
0.650	5000	0.063185	0.0839	0.753	1.015	0.990	1.040	2.57E-04	0.06268	0.06369	0.00407
0.675	5000	0.063070	0.0839	0.752	1.011	0.987	1.037	2.61E-04	0.06256	0.06358	0.00414
0.700	5000	0.062820	0.0839	0.749	1.008	0.983	1.033	2.59E-04	0.06231	0.06333	0.00412
0.725	5000	0.062393	0.0839	0.744	1.004	0.979	1.029	2.55E-04	0.06189	0.06289	0.00409
0.750	5000	0.062807	0.0839	0.749	1.000	0.976	1.025	2.52E-04	0.06231	0.06330	0.00401
0.775	5000	0.061698	0.0839	0.735	0.996	0.972	1.021	2.58E-04	0.06119	0.06220	0.00418
0.800	5000	0.061308	0.0839	0.731	0.992	0.968	1.017	2.57E-04	0.06080	0.06181	0.00419
0.825	5000	0.061566	0.0839	0.734	0.989	0.965	1.014	2.56E-04	0.06106	0.06207	0.00416
0.850	5000	0.060905	0.0839	0.726	0.986	0.961	1.010	2.57E-04	0.06040	0.06141	0.00423
0.875	5000	0.061046	0.0839	0.728	0.983	0.959	1.007	2.52E-04	0.06055	0.06154	0.00412
0.900	5000	0.060828	0.0839	0.725	0.980	0.956	1.005	2.51E-04	0.06034	0.06132	0.00412
0.925	5000	0.060998	0.0839	0.727	0.978	0.954	1.003	2.48E-04	0.06051	0.06148	0.00407
0.950	5000	0.060592	0.0839	0.722	0.977	0.953	1.001	2.51E-04	0.06010	0.06108	0.00414
0.975	5000	0.061300	0.0839	0.731	0.976	0.952	1.000	2.50E-04	0.06081	0.06179	0.00407

Table 39 **Simulation estimates of $\hat{p} \equiv P(W_y > b) \equiv A_y e^{-\theta^* b}$ in the $M_t/H_2/1$ model as a function of y based on $5,000$ replications: $\gamma = 0.25, b = 40, \rho = 0.9, \bar{\lambda} = 0.9, \mu = 1, \beta = 0.1, \theta^* = 0.0644$**

position	n	$\hat{p}$	$exp(-\theta^* b)$	A_y	A_y/A approx	A_y/A LB	A_y/A UB	s.e.	95% CI (lb)	(ub)	r.e.
0.000	5000	0.064535	0.0762	0.847	0.975	0.950	1.000	1.34E-04	0.06427	0.06480	0.00208
0.025	5000	0.064520	0.0762	0.847	0.975	0.950	1.000	1.32E-04	0.06426	0.06478	0.00204
0.050	5000	0.064459	0.0762	0.846	0.976	0.951	1.001	1.35E-04	0.06419	0.06472	0.00210
0.075	5000	0.064732	0.0762	0.850	0.977	0.952	1.003	1.35E-04	0.06447	0.06500	0.00208
0.100	5000	0.064849	0.0762	0.851	0.979	0.954	1.005	1.35E-04	0.06458	0.06511	0.00209
0.125	5000	0.064980	0.0762	0.853	0.982	0.957	1.008	1.39E-04	0.06471	0.06525	0.00214
0.150	5000	0.065339	0.0762	0.858	0.985	0.960	1.011	1.35E-04	0.06507	0.06560	0.00207
0.175	5000	0.065442	0.0762	0.859	0.988	0.963	1.014	1.37E-04	0.06517	0.06571	0.00210
0.200	5000	0.065886	0.0762	0.865	0.992	0.967	1.018	1.34E-04	0.06562	0.06615	0.00203
0.225	5000	0.065748	0.0762	0.863	0.996	0.971	1.022	1.39E-04	0.06548	0.06602	0.00212
0.250	5000	0.066450	0.0762	0.873	1.000	0.975	1.026	1.38E-04	0.06618	0.06672	0.00207
0.275	5000	0.066500	0.0762	0.873	1.004	0.979	1.030	1.40E-04	0.06622	0.06677	0.00211
0.300	5000	0.066845	0.0762	0.878	1.008	0.982	1.034	1.40E-04	0.06657	0.06712	0.00210
0.325	5000	0.066998	0.0762	0.880	1.012	0.986	1.038	1.37E-04	0.06673	0.06727	0.00205
0.350	5000	0.067298	0.0762	0.884	1.015	0.989	1.042	1.40E-04	0.06702	0.06757	0.00208
0.375	5000	0.067413	0.0762	0.885	1.018	0.992	1.045	1.40E-04	0.06714	0.06769	0.00207
0.400	5000	0.067933	0.0762	0.892	1.021	0.995	1.048	1.37E-04	0.06766	0.06820	0.00201
0.425	5000	0.067857	0.0762	0.891	1.023	0.997	1.050	1.40E-04	0.06758	0.06813	0.00207
0.450	5000	0.067828	0.0762	0.891	1.025	0.999	1.052	1.39E-04	0.06756	0.06810	0.00205
0.475	5000	0.067711	0.0762	0.889	1.026	1.000	1.053	1.43E-04	0.06743	0.06799	0.00211
0.500	5000	0.068304	0.0762	0.897	1.026	1.000	1.053	1.38E-04	0.06803	0.06857	0.00202
0.525	5000	0.068189	0.0762	0.895	1.026	1.000	1.053	1.39E-04	0.06792	0.06846	0.00204
0.550	5000	0.067899	0.0762	0.892	1.025	0.999	1.052	1.39E-04	0.06763	0.06817	0.00205
0.575	5000	0.067971	0.0762	0.892	1.023	0.997	1.050	1.42E-04	0.06769	0.06825	0.00209
0.600	5000	0.067537	0.0762	0.887	1.021	0.995	1.048	1.43E-04	0.06726	0.06782	0.00212
0.625	5000	0.067407	0.0762	0.885	1.018	0.992	1.045	1.39E-04	0.06713	0.06768	0.00207
0.650	5000	0.067242	0.0762	0.883	1.015	0.989	1.042	1.38E-04	0.06697	0.06751	0.00205
0.675	5000	0.067221	0.0762	0.883	1.012	0.986	1.038	1.37E-04	0.06695	0.06749	0.00203
0.700	5000	0.066949	0.0762	0.879	1.008	0.982	1.034	1.38E-04	0.06668	0.06722	0.00206
0.725	5000	0.066561	0.0762	0.874	1.004	0.979	1.030	1.40E-04	0.06629	0.06684	0.00210
0.750	5000	0.066419	0.0762	0.872	1.000	0.975	1.026	1.38E-04	0.06615	0.06669	0.00207
0.775	5000	0.065957	0.0762	0.866	0.996	0.971	1.022	1.39E-04	0.06569	0.06623	0.00210
0.800	5000	0.065570	0.0762	0.861	0.992	0.967	1.018	1.36E-04	0.06530	0.06584	0.00208
0.825	5000	0.065372	0.0762	0.858	0.988	0.963	1.014	1.38E-04	0.06510	0.06564	0.00211
0.850	5000	0.065319	0.0762	0.858	0.985	0.960	1.011	1.35E-04	0.06505	0.06558	0.00207
0.875	5000	0.065107	0.0762	0.855	0.982	0.957	1.008	1.34E-04	0.06484	0.06537	0.00206
0.900	5000	0.064955	0.0762	0.853	0.979	0.954	1.005	1.35E-04	0.06469	0.06522	0.00208
0.925	5000	0.064617	0.0762	0.848	0.977	0.952	1.003	1.36E-04	0.06435	0.06488	0.00211
0.950	5000	0.064682	0.0762	0.849	0.976	0.951	1.001	1.34E-04	0.06442	0.06495	0.00208
0.975	5000	0.064639	0.0762	0.849	0.975	0.950	1.000	1.35E-04	0.06437	0.06490	0.00209

Table 40 **Simulation estimates of $\hat{p} \equiv P(W_y > b) \equiv A_y e^{-\theta^* b}$ in the $M_t/H_2/1$ model as a function of y based on $5,000$ replications:** $\gamma = \frac{1}{16}, b = 80, \rho = 0.95, \bar{\lambda} = 0.95, \mu = 1, \beta = 0.05, \theta^* = 0.0328$

position	n	$\hat{p}$	$exp(-\theta^* b)$	A_y	A_y/A approx	A_y/A LB	A_y/A UB	s.e.	95% CI (lb)	(ub)	r.e.
0.000	5000	0.066280	0.0727	0.912	0.974	0.949	1.000	7.09E-05	0.06614	0.06642	0.00107
0.025	5000	0.066204	0.0727	0.911	0.974	0.949	1.000	7.10E-05	0.06607	0.06634	0.00107
0.050	5000	0.066377	0.0727	0.913	0.975	0.950	1.001	7.10E-05	0.06624	0.06652	0.00107
0.075	5000	0.066435	0.0727	0.914	0.977	0.952	1.003	6.96E-05	0.06630	0.06657	0.00105
0.100	5000	0.066556	0.0727	0.916	0.979	0.954	1.005	7.14E-05	0.06642	0.06670	0.00107
0.125	5000	0.066726	0.0727	0.918	0.982	0.956	1.008	7.11E-05	0.06659	0.06687	0.00106
0.150	5000	0.067002	0.0727	0.922	0.985	0.959	1.011	7.22E-05	0.06686	0.06714	0.00108
0.175	5000	0.067169	0.0727	0.924	0.988	0.963	1.014	7.11E-05	0.06703	0.06731	0.00106
0.200	5000	0.067501	0.0727	0.929	0.992	0.966	1.018	7.24E-05	0.06736	0.06764	0.00107
0.225	5000	0.067670	0.0727	0.931	0.996	0.970	1.022	7.21E-05	0.06753	0.06781	0.00107
0.250	5000	0.068039	0.0727	0.936	1.000	0.974	1.027	7.11E-05	0.06790	0.06818	0.00105
0.275	5000	0.068288	0.0727	0.939	1.004	0.978	1.031	7.34E-05	0.06814	0.06843	0.00107
0.300	5000	0.068467	0.0727	0.942	1.008	0.982	1.035	7.39E-05	0.06832	0.06861	0.00108
0.325	5000	0.068874	0.0727	0.947	1.012	0.986	1.039	7.23E-05	0.06873	0.06902	0.00105
0.350	5000	0.069024	0.0727	0.950	1.016	0.989	1.043	7.61E-05	0.06888	0.06917	0.00110
0.375	5000	0.069298	0.0727	0.953	1.019	0.992	1.046	7.50E-05	0.06915	0.06945	0.00108
0.400	5000	0.069309	0.0727	0.953	1.021	0.995	1.049	7.57E-05	0.06916	0.06946	0.00109
0.425	5000	0.069602	0.0727	0.957	1.024	0.997	1.051	7.45E-05	0.06946	0.06975	0.00107
0.450	5000	0.069535	0.0727	0.957	1.025	0.999	1.052	7.69E-05	0.06938	0.06969	0.00111
0.475	5000	0.069728	0.0727	0.959	1.026	1.000	1.053	7.48E-05	0.06958	0.06987	0.00107
0.500	5000	0.069779	0.0727	0.960	1.027	1.000	1.054	7.44E-05	0.06963	0.06992	0.00107
0.525	5000	0.069683	0.0727	0.959	1.026	1.000	1.053	7.60E-05	0.06953	0.06983	0.00109
0.550	5000	0.069647	0.0727	0.958	1.025	0.999	1.052	7.77E-05	0.06950	0.06980	0.00112
0.575	5000	0.069576	0.0727	0.957	1.024	0.997	1.051	7.56E-05	0.06943	0.06972	0.00109
0.600	5000	0.069369	0.0727	0.954	1.021	0.995	1.049	7.52E-05	0.06922	0.06952	0.00108
0.625	5000	0.069258	0.0727	0.953	1.019	0.992	1.046	7.44E-05	0.06911	0.06940	0.00107
0.650	5000	0.069145	0.0727	0.951	1.016	0.989	1.043	7.24E-05	0.06900	0.06929	0.00105
0.675	5000	0.068683	0.0727	0.945	1.012	0.986	1.039	7.49E-05	0.06854	0.06883	0.00109
0.700	5000	0.068628	0.0727	0.944	1.008	0.982	1.035	7.22E-05	0.06849	0.06877	0.00105
0.725	5000	0.068246	0.0727	0.939	1.004	0.978	1.031	7.47E-05	0.06810	0.06839	0.00109
0.750	5000	0.067919	0.0727	0.934	1.000	0.974	1.027	7.29E-05	0.06778	0.06806	0.00107
0.775	5000	0.067731	0.0727	0.932	0.996	0.970	1.022	7.39E-05	0.06759	0.06788	0.00109
0.800	5000	0.067406	0.0727	0.927	0.992	0.966	1.018	7.36E-05	0.06726	0.06755	0.00109
0.825	5000	0.067147	0.0727	0.924	0.988	0.963	1.014	7.26E-05	0.06700	0.06729	0.00108
0.850	5000	0.066820	0.0727	0.919	0.985	0.959	1.011	7.29E-05	0.06668	0.06696	0.00109
0.875	5000	0.066765	0.0727	0.918	0.982	0.956	1.008	7.14E-05	0.06662	0.06690	0.00107
0.900	5000	0.066668	0.0727	0.917	0.979	0.954	1.005	7.14E-05	0.06653	0.06681	0.00107
0.925	5000	0.066418	0.0727	0.914	0.977	0.952	1.003	7.21E-05	0.06628	0.06656	0.00109
0.950	5000	0.066375	0.0727	0.913	0.975	0.950	1.001	7.11E-05	0.06624	0.06651	0.00107
0.975	5000	0.066309	0.0727	0.912	0.974	0.949	1.000	7.06E-05	0.06617	0.06645	0.00106

Table 41 Simulation estimates of $\hat{p} \equiv P(W_y > b) \equiv A_y e^{-\theta^* b}$ in the $M_t/H_2/1$ model as a function of y based on $5,000$ replications: $\gamma = 0.64, b = 25, \rho = 0.84, \bar{\lambda} = 0.84, \mu = 1, \beta = 0.16, \theta^* = 0.101$

position	n	$\hat{p}$	$exp(-\theta^* b)$	A_y	A_y approx	A_y LB	A_y UB	s.e.	95% CI (lb)	(ub)	r.e.
0.000	5000	0.062304	0.0807	0.772	0.771	0.752	0.791	2.07E-04	0.06190	0.06271	0.00332
0.025	5000	0.062294	0.0807	0.772	0.771	0.752	0.791	2.07E-04	0.06189	0.06270	0.00333
0.050	5000	0.062341	0.0807	0.773	0.772	0.753	0.792	2.08E-04	0.06193	0.06275	0.00333
0.075	5000	0.062355	0.0807	0.773	0.773	0.754	0.793	2.08E-04	0.06195	0.06276	0.00333
0.100	5000	0.062523	0.0807	0.775	0.775	0.756	0.795	2.08E-04	0.06212	0.06293	0.00333
0.125	5000	0.062689	0.0807	0.777	0.777	0.758	0.797	2.08E-04	0.06228	0.06310	0.00332
0.150	5000	0.062893	0.0807	0.780	0.779	0.760	0.799	2.09E-04	0.06248	0.06330	0.00332
0.175	5000	0.063137	0.0807	0.783	0.782	0.762	0.802	2.09E-04	0.06273	0.06355	0.00332
0.200	5000	0.063326	0.0807	0.785	0.785	0.765	0.805	2.10E-04	0.06291	0.06374	0.00332
0.225	5000	0.063531	0.0807	0.788	0.788	0.768	0.808	2.11E-04	0.06312	0.06394	0.00332
0.250	5000	0.063743	0.0807	0.790	0.791	0.771	0.811	2.12E-04	0.06333	0.06416	0.00333
0.275	5000	0.063997	0.0807	0.793	0.794	0.774	0.814	2.13E-04	0.06358	0.06441	0.00333
0.300	5000	0.064233	0.0807	0.796	0.797	0.777	0.817	2.14E-04	0.06381	0.06465	0.00333
0.325	5000	0.064497	0.0807	0.800	0.800	0.780	0.820	2.14E-04	0.06408	0.06492	0.00332
0.350	5000	0.064751	0.0807	0.803	0.803	0.783	0.823	2.14E-04	0.06433	0.06517	0.00331
0.375	5000	0.064921	0.0807	0.805	0.805	0.785	0.826	2.15E-04	0.06450	0.06534	0.00331
0.400	5000	0.065042	0.0807	0.806	0.807	0.787	0.828	2.16E-04	0.06462	0.06546	0.00332
0.425	5000	0.065179	0.0807	0.808	0.809	0.789	0.829	2.16E-04	0.06476	0.06560	0.00331
0.450	5000	0.065249	0.0807	0.809	0.810	0.790	0.831	2.16E-04	0.06482	0.06567	0.00332
0.475	5000	0.065345	0.0807	0.810	0.811	0.791	0.831	2.17E-04	0.06492	0.06577	0.00332
0.500	5000	0.065349	0.0807	0.810	0.811	0.791	0.832	2.17E-04	0.06492	0.06577	0.00332
0.525	5000	0.065313	0.0807	0.810	0.811	0.791	0.831	2.17E-04	0.06489	0.06574	0.00333
0.550	5000	0.065244	0.0807	0.809	0.810	0.790	0.831	2.17E-04	0.06482	0.06567	0.00333
0.575	5000	0.065193	0.0807	0.808	0.809	0.789	0.829	2.17E-04	0.06477	0.06562	0.00332
0.600	5000	0.065069	0.0807	0.807	0.807	0.787	0.828	2.16E-04	0.06465	0.06549	0.00332
0.625	5000	0.064912	0.0807	0.805	0.805	0.785	0.826	2.16E-04	0.06449	0.06534	0.00333
0.650	5000	0.064713	0.0807	0.802	0.803	0.783	0.823	2.16E-04	0.06429	0.06514	0.00333
0.675	5000	0.064523	0.0807	0.800	0.800	0.780	0.820	2.15E-04	0.06410	0.06494	0.00333
0.700	5000	0.064290	0.0807	0.797	0.797	0.777	0.817	2.14E-04	0.06387	0.06471	0.00333
0.725	5000	0.064135	0.0807	0.795	0.794	0.774	0.814	2.13E-04	0.06372	0.06455	0.00332
0.750	5000	0.063932	0.0807	0.792	0.791	0.771	0.811	2.12E-04	0.06352	0.06435	0.00332
0.775	5000	0.063708	0.0807	0.790	0.788	0.768	0.808	2.11E-04	0.06330	0.06412	0.00331
0.800	5000	0.063435	0.0807	0.786	0.785	0.765	0.805	2.10E-04	0.06302	0.06385	0.00331
0.825	5000	0.063174	0.0807	0.783	0.782	0.762	0.802	2.09E-04	0.06276	0.06358	0.00331
0.850	5000	0.062899	0.0807	0.780	0.779	0.760	0.799	2.09E-04	0.06249	0.06331	0.00332
0.875	5000	0.062675	0.0807	0.777	0.777	0.758	0.797	2.08E-04	0.06227	0.06308	0.00332
0.900	5000	0.062508	0.0807	0.775	0.775	0.756	0.795	2.08E-04	0.06210	0.06292	0.00333
0.925	5000	0.062427	0.0807	0.774	0.773	0.754	0.793	2.08E-04	0.06202	0.06283	0.00332
0.950	5000	0.062330	0.0807	0.773	0.772	0.753	0.792	2.07E-04	0.06192	0.06274	0.00333
0.975	5000	0.062298	0.0807	0.772	0.771	0.752	0.791	2.07E-04	0.06189	0.06270	0.00332

Table 42 **Simulation estimates of $\hat{p} \equiv P(W_y > b) \equiv A_y e^{-\theta^* b}$ in the $M_t/H_2/1$ model as a function of y based on $5,000$ replications:** $\gamma = 0.16, b = 50, \rho = 0.92, \bar{\lambda} = 0.92, \mu = 1, \beta = 0.08, \theta^* = 0.0519$

position	n	$\hat{p}$	$exp(-\theta^* b)$	A_y	A_y approx	A_y LB	A_y UB	s.e.	95% CI (lb)	(ub)	r.e.
0.000	5000	0.065304	0.0747	0.874	0.873	0.851	0.896	1.07E-04	0.06509	0.06551	0.00164
0.025	5000	0.065295	0.0747	0.874	0.873	0.851	0.896	1.07E-04	0.06508	0.06551	0.00165
0.050	5000	0.065349	0.0747	0.874	0.874	0.852	0.897	1.08E-04	0.06514	0.06556	0.00165
0.075	5000	0.065448	0.0747	0.876	0.875	0.853	0.898	1.08E-04	0.06524	0.06566	0.00165
0.100	5000	0.065563	0.0747	0.877	0.877	0.855	0.900	1.08E-04	0.06535	0.06577	0.00164
0.125	5000	0.065746	0.0747	0.880	0.880	0.857	0.903	1.08E-04	0.06553	0.06596	0.00164
0.150	5000	0.065973	0.0747	0.883	0.882	0.860	0.905	1.08E-04	0.06576	0.06618	0.00163
0.175	5000	0.066185	0.0747	0.886	0.885	0.863	0.909	1.08E-04	0.06597	0.06640	0.00164
0.200	5000	0.066471	0.0747	0.889	0.889	0.866	0.912	1.09E-04	0.06626	0.06668	0.00164
0.225	5000	0.066755	0.0747	0.893	0.892	0.869	0.916	1.09E-04	0.06654	0.06697	0.00163
0.250	5000	0.067060	0.0747	0.897	0.896	0.873	0.919	1.09E-04	0.06685	0.06727	0.00163
0.275	5000	0.067334	0.0747	0.901	0.899	0.876	0.923	1.10E-04	0.06712	0.06755	0.00163
0.300	5000	0.067595	0.0747	0.904	0.903	0.880	0.927	1.11E-04	0.06738	0.06781	0.00164
0.325	5000	0.067848	0.0747	0.908	0.906	0.883	0.930	1.11E-04	0.06763	0.06807	0.00164
0.350	5000	0.068096	0.0747	0.911	0.910	0.886	0.934	1.11E-04	0.06788	0.06831	0.00163
0.375	5000	0.068321	0.0747	0.914	0.912	0.889	0.936	1.11E-04	0.06810	0.06854	0.00163
0.400	5000	0.068484	0.0747	0.916	0.915	0.891	0.939	1.12E-04	0.06826	0.06870	0.00163
0.425	5000	0.068620	0.0747	0.918	0.917	0.893	0.941	1.12E-04	0.06840	0.06884	0.00163
0.450	5000	0.068664	0.0747	0.919	0.918	0.895	0.942	1.12E-04	0.06844	0.06888	0.00164
0.475	5000	0.068750	0.0747	0.920	0.919	0.896	0.943	1.12E-04	0.06853	0.06897	0.00164
0.500	5000	0.068803	0.0747	0.921	0.919	0.896	0.944	1.12E-04	0.06858	0.06902	0.00163
0.525	5000	0.068795	0.0747	0.920	0.919	0.896	0.943	1.12E-04	0.06858	0.06901	0.00163
0.550	5000	0.068749	0.0747	0.920	0.918	0.895	0.942	1.12E-04	0.06853	0.06897	0.00163
0.575	5000	0.068648	0.0747	0.918	0.917	0.893	0.941	1.12E-04	0.06843	0.06887	0.00163
0.600	5000	0.068517	0.0747	0.917	0.915	0.891	0.939	1.12E-04	0.06830	0.06874	0.00163
0.625	5000	0.068336	0.0747	0.914	0.912	0.889	0.936	1.11E-04	0.06812	0.06855	0.00163
0.650	5000	0.068135	0.0747	0.912	0.910	0.886	0.934	1.11E-04	0.06792	0.06835	0.00163
0.675	5000	0.067861	0.0747	0.908	0.906	0.883	0.930	1.11E-04	0.06764	0.06808	0.00163
0.700	5000	0.067609	0.0747	0.905	0.903	0.880	0.927	1.10E-04	0.06739	0.06783	0.00163
0.725	5000	0.067362	0.0747	0.901	0.899	0.876	0.923	1.10E-04	0.06715	0.06758	0.00163
0.750	5000	0.067105	0.0747	0.898	0.896	0.873	0.919	1.10E-04	0.06689	0.06732	0.00163
0.775	5000	0.066853	0.0747	0.894	0.892	0.869	0.916	1.09E-04	0.06664	0.06707	0.00163
0.800	5000	0.066597	0.0747	0.891	0.889	0.866	0.912	1.09E-04	0.06638	0.06681	0.00163
0.825	5000	0.066355	0.0747	0.888	0.885	0.863	0.909	1.08E-04	0.06614	0.06657	0.00163
0.850	5000	0.066106	0.0747	0.884	0.882	0.860	0.905	1.08E-04	0.06589	0.06632	0.00164
0.875	5000	0.065883	0.0747	0.881	0.880	0.857	0.903	1.08E-04	0.06567	0.06609	0.00164
0.900	5000	0.065680	0.0747	0.879	0.877	0.855	0.900	1.08E-04	0.06547	0.06589	0.00164
0.925	5000	0.065529	0.0747	0.877	0.875	0.853	0.898	1.07E-04	0.06532	0.06574	0.00164
0.950	5000	0.065427	0.0747	0.875	0.874	0.852	0.897	1.07E-04	0.06522	0.06564	0.00164
0.975	5000	0.065332	0.0747	0.874	0.873	0.851	0.896	1.07E-04	0.06512	0.06554	0.00164

Table 43 Simulation estimates of $\hat{p} \equiv P(W_y > b) \equiv A_y e^{-\theta^* b}$ in the $M_t/H_2/1$ model as a function of y based on $5,000$ replications: $\gamma = 0.04, b = 100, \rho = 0.96, \bar{\lambda} = 0.96, \mu = 1, \beta = 0.04, \theta^* = 0.0263$

position	n	$\hat{p}$	$exp(-\theta^* b)$	A_y	A_y approx	A_y LB	A_y UB	s.e.	95% CI (lb)	(ub)	r.e.
0.000	5000	0.066528	0.0720	0.924	0.923	0.899	0.947	5.86E-05	0.06641	0.06664	8.80E-04
0.025	5000	0.066579	0.0720	0.924	0.923	0.899	0.947	5.86E-05	0.06646	0.06669	8.79E-04
0.050	5000	0.066657	0.0720	0.925	0.924	0.900	0.948	5.86E-05	0.06654	0.06677	8.79E-04
0.075	5000	0.066735	0.0720	0.926	0.925	0.901	0.950	5.91E-05	0.06662	0.06685	8.86E-04
0.100	5000	0.066889	0.0720	0.929	0.927	0.903	0.952	5.90E-05	0.06677	0.06700	8.82E-04
0.125	5000	0.067072	0.0720	0.931	0.930	0.906	0.954	5.89E-05	0.06696	0.06719	8.79E-04
0.150	5000	0.067302	0.0720	0.934	0.933	0.908	0.957	5.89E-05	0.06719	0.06742	8.76E-04
0.175	5000	0.067528	0.0720	0.937	0.936	0.912	0.961	5.88E-05	0.06741	0.06764	8.71E-04
0.200	5000	0.067773	0.0720	0.941	0.939	0.915	0.964	5.92E-05	0.06766	0.06789	8.73E-04
0.225	5000	0.068063	0.0720	0.945	0.943	0.919	0.968	5.96E-05	0.06795	0.06818	8.76E-04
0.250	5000	0.068348	0.0720	0.949	0.947	0.923	0.972	5.97E-05	0.06823	0.06846	8.73E-04
0.275	5000	0.068644	0.0720	0.953	0.951	0.926	0.976	5.97E-05	0.06853	0.06876	8.70E-04
0.300	5000	0.068937	0.0720	0.957	0.955	0.930	0.980	6.00E-05	0.06882	0.06906	8.70E-04
0.325	5000	0.069186	0.0720	0.960	0.958	0.934	0.984	6.02E-05	0.06907	0.06930	8.70E-04
0.350	5000	0.069408	0.0720	0.964	0.962	0.937	0.988	6.04E-05	0.06929	0.06953	8.70E-04
0.375	5000	0.069613	0.0720	0.966	0.965	0.940	0.991	6.06E-05	0.06949	0.06973	8.71E-04
0.400	5000	0.069791	0.0720	0.969	0.967	0.942	0.993	6.11E-05	0.06967	0.06991	8.75E-04
0.425	5000	0.069939	0.0720	0.971	0.970	0.944	0.995	6.15E-05	0.06982	0.07006	8.79E-04
0.450	5000	0.070041	0.0720	0.972	0.971	0.946	0.997	6.17E-05	0.06992	0.07016	8.80E-04
0.475	5000	0.070110	0.0720	0.973	0.972	0.947	0.998	6.18E-05	0.06999	0.07023	8.82E-04
0.500	5000	0.070128	0.0720	0.974	0.972	0.947	0.998	6.20E-05	0.07001	0.07025	8.85E-04
0.525	5000	0.070100	0.0720	0.973	0.972	0.947	0.998	6.19E-05	0.06998	0.07022	8.82E-04
0.550	5000	0.070048	0.0720	0.972	0.971	0.946	0.997	6.19E-05	0.06993	0.07017	8.84E-04
0.575	5000	0.069921	0.0720	0.971	0.970	0.944	0.995	6.19E-05	0.06980	0.07004	8.86E-04
0.600	5000	0.069775	0.0720	0.969	0.967	0.942	0.993	6.19E-05	0.06965	0.06990	8.87E-04
0.625	5000	0.069596	0.0720	0.966	0.965	0.940	0.991	6.14E-05	0.06948	0.06972	8.82E-04
0.650	5000	0.069396	0.0720	0.963	0.962	0.937	0.988	6.11E-05	0.06928	0.06952	8.81E-04
0.675	5000	0.069136	0.0720	0.960	0.958	0.934	0.984	6.09E-05	0.06902	0.06926	8.80E-04
0.700	5000	0.068862	0.0720	0.956	0.955	0.930	0.980	6.05E-05	0.06874	0.06898	8.78E-04
0.725	5000	0.068604	0.0720	0.952	0.951	0.926	0.976	5.98E-05	0.06849	0.06872	8.72E-04
0.750	5000	0.068341	0.0720	0.949	0.947	0.923	0.972	5.98E-05	0.06822	0.06846	8.75E-04
0.775	5000	0.068085	0.0720	0.945	0.943	0.919	0.968	5.95E-05	0.06797	0.06820	8.74E-04
0.800	5000	0.067825	0.0720	0.942	0.939	0.915	0.964	5.95E-05	0.06771	0.06794	8.78E-04
0.825	5000	0.067534	0.0720	0.938	0.936	0.912	0.961	5.94E-05	0.06742	0.06765	8.80E-04
0.850	5000	0.067285	0.0720	0.934	0.933	0.908	0.957	5.89E-05	0.06717	0.06740	8.75E-04
0.875	5000	0.067081	0.0720	0.931	0.930	0.906	0.954	5.89E-05	0.06697	0.06720	8.78E-04
0.900	5000	0.066872	0.0720	0.928	0.927	0.903	0.952	5.89E-05	0.06676	0.06699	8.81E-04
0.925	5000	0.066735	0.0720	0.926	0.925	0.901	0.950	5.86E-05	0.06662	0.06685	8.78E-04
0.950	5000	0.066627	0.0720	0.925	0.924	0.900	0.948	5.85E-05	0.06651	0.06674	8.78E-04
0.975	5000	0.066558	0.0720	0.924	0.923	0.899	0.947	5.88E-05	0.06644	0.06667	8.84E-04

Table 44 Simulation estimates of $\hat{p} \equiv P(W_y > b) \equiv A_y e^{-\theta^* b}$ in the $M_t/H_2/1$ model as a function of y based on $5,000$ replications: $\gamma = 0.01, b = 200, \rho = 0.98, \bar{\lambda} = 0.98, \mu = 1, \beta = 0.02, \theta^* = 0.0132$

position	n	$\hat{p}$	$exp(-\theta^* b)$	A_y	A_y/A approx	A_y/A LB	A_y/A UB	s.e.	95% CI (lb)	(ub)	r.e.
0.000	5000	0.067202	0.0707	0.950	0.974	0.948	1.000	3.33E-05	0.06714	0.06727	4.95E-04
0.025	5000	0.067195	0.0707	0.950	0.974	0.949	1.000	3.30E-05	0.06713	0.06726	4.91E-04
0.050	5000	0.067252	0.0707	0.951	0.975	0.950	1.001	3.35E-05	0.06719	0.06732	4.98E-04
0.075	5000	0.067346	0.0707	0.952	0.977	0.951	1.003	3.39E-05	0.06728	0.06741	5.04E-04
0.100	5000	0.067486	0.0707	0.954	0.979	0.953	1.005	3.39E-05	0.06742	0.06755	5.02E-04
0.125	5000	0.067691	0.0707	0.957	0.981	0.956	1.008	3.41E-05	0.06762	0.06776	5.04E-04
0.150	5000	0.067868	0.0707	0.959	0.985	0.959	1.011	3.40E-05	0.06780	0.06794	5.01E-04
0.175	5000	0.068178	0.0707	0.964	0.988	0.962	1.015	3.35E-05	0.06811	0.06824	4.92E-04
0.200	5000	0.068420	0.0707	0.967	0.992	0.966	1.018	3.40E-05	0.06835	0.06849	4.96E-04
0.225	5000	0.068639	0.0707	0.970	0.996	0.970	1.023	3.46E-05	0.06857	0.06871	5.04E-04
0.250	5000	0.068940	0.0707	0.975	1.000	0.974	1.027	3.42E-05	0.06887	0.06901	4.96E-04
0.275	5000	0.069295	0.0707	0.980	1.004	0.978	1.031	3.42E-05	0.06923	0.06936	4.94E-04
0.300	5000	0.069531	0.0707	0.983	1.008	0.982	1.035	3.45E-05	0.06946	0.06960	4.97E-04
0.325	5000	0.069780	0.0707	0.986	1.012	0.986	1.039	3.48E-05	0.06971	0.06985	4.99E-04
0.350	5000	0.069954	0.0707	0.989	1.016	0.989	1.043	3.57E-05	0.06988	0.07002	5.10E-04
0.375	5000	0.070282	0.0707	0.994	1.019	0.992	1.046	3.38E-05	0.07022	0.07035	4.81E-04
0.400	5000	0.070463	0.0707	0.996	1.022	0.995	1.049	3.51E-05	0.07039	0.07053	4.98E-04
0.425	5000	0.070639	0.0707	0.999	1.024	0.997	1.051	3.44E-05	0.07057	0.07071	4.87E-04
0.450	5000	0.070742	0.0707	1.000	1.026	0.999	1.053	3.43E-05	0.07068	0.07081	4.85E-04
0.475	5000	0.070811	0.0707	1.001	1.027	1.000	1.054	3.48E-05	0.07074	0.07088	4.91E-04
0.500	5000	0.070867	0.0707	1.002	1.027	1.000	1.054	3.46E-05	0.07080	0.07093	4.88E-04
0.525	5000	0.070781	0.0707	1.001	1.027	1.000	1.054	3.54E-05	0.07071	0.07085	5.00E-04
0.550	5000	0.070717	0.0707	1.000	1.026	0.999	1.053	3.55E-05	0.07065	0.07079	5.02E-04
0.575	5000	0.070637	0.0707	0.999	1.024	0.997	1.051	3.60E-05	0.07057	0.07071	5.09E-04
0.600	5000	0.070510	0.0707	0.997	1.022	0.995	1.049	3.47E-05	0.07044	0.07058	4.92E-04
0.625	5000	0.070218	0.0707	0.993	1.019	0.992	1.046	3.51E-05	0.07015	0.07029	5.00E-04
0.650	5000	0.070101	0.0707	0.991	1.016	0.989	1.043	3.44E-05	0.07003	0.07017	4.90E-04
0.675	5000	0.069818	0.0707	0.987	1.012	0.986	1.039	3.40E-05	0.06975	0.06988	4.87E-04
0.700	5000	0.069552	0.0707	0.983	1.008	0.982	1.035	3.40E-05	0.06949	0.06962	4.89E-04
0.725	5000	0.069181	0.0707	0.978	1.004	0.978	1.031	3.47E-05	0.06911	0.06925	5.02E-04
0.750	5000	0.068975	0.0707	0.975	1.000	0.974	1.027	3.45E-05	0.06891	0.06904	5.00E-04
0.775	5000	0.068746	0.0707	0.972	0.996	0.970	1.023	3.38E-05	0.06868	0.06881	4.92E-04
0.800	5000	0.068349	0.0707	0.966	0.992	0.966	1.018	3.44E-05	0.06828	0.06842	5.03E-04
0.825	5000	0.068149	0.0707	0.963	0.988	0.962	1.015	3.31E-05	0.06808	0.06821	4.86E-04
0.850	5000	0.067861	0.0707	0.959	0.985	0.959	1.011	3.41E-05	0.06779	0.06793	5.02E-04
0.875	5000	0.067708	0.0707	0.957	0.981	0.956	1.008	3.37E-05	0.06764	0.06777	4.98E-04
0.900	5000	0.067490	0.0707	0.954	0.979	0.953	1.005	3.31E-05	0.06742	0.06755	4.91E-04
0.925	5000	0.067377	0.0707	0.952	0.977	0.951	1.003	3.38E-05	0.06731	0.06744	5.02E-04
0.950	5000	0.067222	0.0707	0.950	0.975	0.950	1.001	3.39E-05	0.06716	0.06729	5.04E-04
0.975	5000	0.067210	0.0707	0.950	0.974	0.949	1.000	3.31E-05	0.06715	0.06727	4.92E-04

Table 45 Simulation estimates of $\hat{p} \equiv P(W_y > b) \equiv A_y e^{-\theta^* b}$ in the $M_t/H_2/1$ model as a function of y based on 5,000 replications: $\gamma = 0.0025, b = 400, \rho = 0.99, \bar{\lambda} = 0.99, \mu = 1, \beta = 0.01, \theta^* = 0.00664$

position	n	$\hat{p}$	$exp(-\theta^* b)$	A_y	A_y/A approx	A_y/A LB	A_y/A UB	s.e.	95% CI (lb)	(ub)	r.e.
0.000	5000	0.067433	0.0701	0.962	0.974	0.948	1.000	2.31E-05	0.06739	0.06748	3.43E-04
0.025	5000	0.067502	0.0701	0.963	0.974	0.949	1.000	2.24E-05	0.06746	0.06755	3.32E-04
0.050	5000	0.067568	0.0701	0.964	0.975	0.949	1.001	2.27E-05	0.06752	0.06761	3.36E-04
0.075	5000	0.067632	0.0701	0.965	0.977	0.951	1.003	2.31E-05	0.06759	0.06768	3.42E-04
0.100	5000	0.067785	0.0701	0.967	0.979	0.953	1.005	2.30E-05	0.06774	0.06783	3.39E-04
0.125	5000	0.068011	0.0701	0.970	0.981	0.956	1.008	2.27E-05	0.06797	0.06806	3.34E-04
0.150	5000	0.068171	0.0701	0.972	0.984	0.959	1.011	2.32E-05	0.06813	0.06822	3.41E-04
0.175	5000	0.068453	0.0701	0.976	0.988	0.962	1.015	2.32E-05	0.06841	0.06850	3.39E-04
0.200	5000	0.068726	0.0701	0.980	0.992	0.966	1.019	2.33E-05	0.06868	0.06877	3.40E-04
0.225	5000	0.069005	0.0701	0.984	0.996	0.970	1.023	2.32E-05	0.06896	0.06905	3.37E-04
0.250	5000	0.069337	0.0701	0.989	1.000	0.974	1.027	2.31E-05	0.06929	0.06938	3.33E-04
0.275	5000	0.069615	0.0701	0.993	1.004	0.978	1.031	2.35E-05	0.06957	0.06966	3.37E-04
0.300	5000	0.069855	0.0701	0.996	1.008	0.982	1.035	2.36E-05	0.06981	0.06990	3.38E-04
0.325	5000	0.070150	0.0701	1.001	1.012	0.986	1.039	2.34E-05	0.07010	0.07020	3.34E-04
0.350	5000	0.070378	0.0701	1.004	1.016	0.989	1.043	2.38E-05	0.07033	0.07043	3.39E-04
0.375	5000	0.070605	0.0701	1.007	1.019	0.992	1.046	2.40E-05	0.07056	0.07065	3.40E-04
0.400	5000	0.070764	0.0701	1.009	1.022	0.995	1.049	2.39E-05	0.07072	0.07081	3.38E-04
0.425	5000	0.070956	0.0701	1.012	1.024	0.997	1.052	2.33E-05	0.07091	0.07100	3.29E-04
0.450	5000	0.071038	0.0701	1.013	1.026	0.999	1.053	2.38E-05	0.07099	0.07108	3.36E-04
0.475	5000	0.071140	0.0701	1.015	1.027	1.000	1.054	2.39E-05	0.07109	0.07119	3.36E-04
0.500	5000	0.071167	0.0701	1.015	1.027	1.000	1.055	2.43E-05	0.07112	0.07121	3.41E-04
0.525	5000	0.071103	0.0701	1.014	1.027	1.000	1.054	2.43E-05	0.07106	0.07115	3.42E-04
0.550	5000	0.071106	0.0701	1.014	1.026	0.999	1.053	2.42E-05	0.07106	0.07115	3.40E-04
0.575	5000	0.070928	0.0701	1.012	1.024	0.997	1.052	2.39E-05	0.07088	0.07097	3.38E-04
0.600	5000	0.070775	0.0701	1.010	1.022	0.995	1.049	2.44E-05	0.07073	0.07082	3.45E-04
0.625	5000	0.070609	0.0701	1.007	1.019	0.992	1.046	2.36E-05	0.07056	0.07066	3.34E-04
0.650	5000	0.070368	0.0701	1.004	1.016	0.989	1.043	2.37E-05	0.07032	0.07041	3.36E-04
0.675	5000	0.070112	0.0701	1.000	1.012	0.986	1.039	2.39E-05	0.07007	0.07016	3.41E-04
0.700	5000	0.069854	0.0701	0.996	1.008	0.982	1.035	2.36E-05	0.06981	0.06990	3.38E-04
0.725	5000	0.069574	0.0701	0.992	1.004	0.978	1.031	2.32E-05	0.06953	0.06962	3.33E-04
0.750	5000	0.069314	0.0701	0.989	1.000	0.974	1.027	2.34E-05	0.06927	0.06936	3.37E-04
0.775	5000	0.069002	0.0701	0.984	0.996	0.970	1.023	2.32E-05	0.06896	0.06905	3.36E-04
0.800	5000	0.068719	0.0701	0.980	0.992	0.966	1.019	2.31E-05	0.06867	0.06876	3.36E-04
0.825	5000	0.068468	0.0701	0.977	0.988	0.962	1.015	2.29E-05	0.06842	0.06851	3.35E-04
0.850	5000	0.068245	0.0701	0.973	0.984	0.959	1.011	2.32E-05	0.06820	0.06829	3.40E-04
0.875	5000	0.067991	0.0701	0.970	0.981	0.956	1.008	2.28E-05	0.06795	0.06804	3.35E-04
0.900	5000	0.067803	0.0701	0.967	0.979	0.953	1.005	2.33E-05	0.06776	0.06785	3.43E-04
0.925	5000	0.067694	0.0701	0.966	0.977	0.951	1.003	2.22E-05	0.06765	0.06774	3.27E-04
0.950	5000	0.067575	0.0701	0.964	0.975	0.949	1.001	2.23E-05	0.06753	0.06762	3.30E-04
0.975	5000	0.067501	0.0701	0.963	0.974	0.949	1.000	2.33E-05	0.06746	0.06755	3.45E-04

Table 46 Comparison of ratio $P(W_y > b)/\rho$ as a function of ρ in $M_t/H_2/1$ queue with base parameter $(\beta,\gamma,b) = (1,25,4)$ using the scaling in (23) of the main paper.

position	$\rho = 0.8$	$\rho = 0.9$	$\rho = 0.95$	$\rho = 0.98$	$\rho = 0.99$
0.000	0.90943	0.94152	0.95975	0.96940	0.97159
0.025	0.90782	0.94131	0.95866	0.96930	0.97258
0.050	0.90781	0.94042	0.96116	0.97012	0.97353
0.075	0.90180	0.94440	0.96200	0.97147	0.97445
0.100	0.90900	0.94611	0.96375	0.97349	0.97667
0.125	0.91156	0.94802	0.96621	0.97645	0.97992
0.150	0.91184	0.95325	0.97021	0.97901	0.98223
0.175	0.91325	0.95476	0.97262	0.98348	0.98629
0.200	0.92476	0.96123	0.97744	0.98696	0.99022
0.225	0.92863	0.95922	0.97988	0.99013	0.99423
0.250	0.93328	0.96945	0.98522	0.99447	0.99903
0.275	0.92918	0.97018	0.98883	0.99960	1.00303
0.300	0.94075	0.97523	0.99142	1.00300	1.00649
0.325	0.94119	0.97746	0.99731	1.00659	1.01073
0.350	0.93878	0.98184	0.99949	1.00910	1.01403
0.375	0.94405	0.98350	1.00346	1.01383	1.01730
0.400	0.94610	0.99109	1.00361	1.01644	1.01958
0.425	0.94562	0.98998	1.00786	1.01898	1.02235
0.450	0.94886	0.98957	1.00688	1.02047	1.02353
0.475	0.95275	0.98786	1.00967	1.02146	1.02500
0.500	0.95129	0.99651	1.01042	1.02227	1.02538
0.525	0.95393	0.99483	1.00902	1.02103	1.02447
0.550	0.94658	0.99060	1.00851	1.02011	1.02451
0.575	0.94131	0.99165	1.00748	1.01895	1.02194
0.600	0.94382	0.98532	1.00448	1.01711	1.01974
0.625	0.93386	0.98342	1.00288	1.01291	1.01735
0.650	0.94133	0.98102	1.00125	1.01122	1.01388
0.675	0.93963	0.98072	0.99455	1.00713	1.01019
0.700	0.93590	0.97674	0.99375	1.00330	1.00647
0.725	0.92954	0.97109	0.98823	0.99795	1.00244
0.750	0.93571	0.96901	0.98348	0.99497	0.99869
0.775	0.91919	0.96227	0.98076	0.99167	0.99420
0.800	0.91338	0.95662	0.97606	0.98594	0.99012
0.825	0.91722	0.95373	0.97230	0.98306	0.98650
0.850	0.90737	0.95296	0.96758	0.97891	0.98328
0.875	0.90948	0.94987	0.96677	0.97670	0.97963
0.900	0.90623	0.94765	0.96537	0.97355	0.97691
0.925	0.90875	0.94272	0.96176	0.97193	0.97535
0.950	0.90272	0.94366	0.96113	0.96969	0.97363
0.975	0.91325	0.94305	0.96017	0.96952	0.97257
avg diff w.r.t. last column	-0.07108	-0.03151	-0.01397	-0.00346	0.00000
avg. abs. diff w.r.t. last column	0.07108	0.03151	0.01397	0.00346	0.00000
rmse w.r.t. last column	0.07126	0.03157	0.01402	0.00351	0.00000

Table 47 Summary of simulation results for $M_t/H_2/1$ queue at $y=0$ as a function of $1-\rho$ with base parameter $(\beta,\gamma,b)=(1,25,4)$ using the scaling in (23) of the main paper

	$1-\rho=0.16$	$1-\rho=0.08$	$1-\rho=0.04$	$1-\rho=0.02$	$1-\rho=0.01$	$1-\rho=0.005$
θ^*	0.101	0.0519	0.0263	0.0132	0.00664	0.00333
n	5000	5000	5000	5000	5000	5000
$\hat{p}$	0.061910	0.065213	0.066492	0.067148	0.067429	0.067641
$e^{-\theta^* b}$	0.0807	0.0747	0.0720	0.0707	0.0701	0.0698
A_y	0.767	0.873	0.923	0.949	0.962	0.969
A_y approxi	0.766	0.873	0.921	0.948	0.961	0.967
A_y LB	0.747	0.851	0.897	0.923	0.936	0.942
A_y UB	0.786	0.896	0.945	0.973	0.987	0.993
s.e.	2.06E-04	1.09E-04	5.88E-05	3.28E-05	2.27E-05	1.92E-05
95% CI (lb)	0.06151	0.06500	0.06638	0.06708	0.06738	0.06760
(ub)	0.06231	0.06543	0.06661	0.06721	0.06747	0.06768
r.e.	0.003327	0.001665	0.000885	0.000489	0.000337	0.000283
$P(W_y > b)/P(W > b)$						
ratio	0.97659	0.97396	0.97632	0.97526	0.97480	0.97562
diff w.r.t. last column	-0.00097	0.00166	-0.00070	0.00036	0.00082	0.00000
abs diff w.r.t. last column	0.00097	0.00166	0.00070	0.00036	0.00082	0.00000
A_y/ρ						
ratio	0.91361	0.94838	0.96155	0.96861	0.97153	0.97403
diff w.r.t. last column	0.06042	0.02564	0.01248	0.00541	0.00250	0.00000
abs diff w.r.t. last column	0.06042	0.02564	0.01248	0.00541	0.00250	0.00000

Table 48 Summary of simulation results for $M_t/H_2/1$ queue at $y=0$ and $y=0.5$ as a function of $1-\rho$ with base parameter $(\beta,\gamma,b)=(1,25,4)$ using the scaling in (23) of the main paper

	$1-\rho=0.16$	$1-\rho=0.08$	$1-\rho=0.04$	$1-\rho=0.02$	$1-\rho=0.01$
θ^*	0.101	0.0519	0.0263	0.0132	0.00664
n	40000	40000	40000	40000	40000
$y=0$					
$\hat{p}$	0.050594	0.052946	0.054024	0.054544	0.054904
$e^{-\theta^* b}$	0.0807	0.0747	0.0720	0.0707	0.0701
A_y	0.627	0.708	0.750	0.771	0.783
A_y approxi	0.613	0.690	0.728	0.747	0.756
A_y LB	0.477	0.532	0.560	0.573	0.580
A_y UB	0.789	0.894	0.947	0.974	0.987
s.e.	7.49E-05	5.64E-05	5.13E-05	5.03E-05	5.01E-05
95% CI (lb)	0.05045	0.05284	0.05392	0.05445	0.05481
(ub)	0.05074	0.05306	0.05412	0.05464	0.05500
r.e.	0.001480	0.001065	0.000950	0.000923	0.000913
$P(W_y>b)/P(W>b)$					
ratio	0.79534	0.79246	0.79200	0.79200	0.79377
diff w.r.t. last column	-0.00158	0.00131	0.00177	0.00177	0.00000
abs diff	0.00158	0.00131	0.00177	0.00177	0.00000
A_y/ρ					
ratio	0.74662	0.76999	0.78125	0.78680	0.79107
diff w.r.t. last column	0.04445	0.02108	0.00982	0.00427	0.00000
abs diff	0.04445	0.02108	0.00982	0.00427	0.00000
$y=0.5$					
$\hat{p}$	0.086646	0.092721	0.095707	0.096711	0.097186
$e^{-\theta^* b}$	0.0807	0.0747	0.0720	0.0707	0.0701
A_y	1.074	1.241	1.329	1.367	1.386
A_y approxi	1.014	1.159	1.232	1.269	1.287
A_y LB	0.789	0.894	0.947	0.974	0.987
A_y UB	1.305	1.502	1.603	1.654	1.679
s.e.	1.25E-04	9.42E-05	8.49E-05	8.28E-05	8.28E-05
95% CI (lb)	0.08640	0.09254	0.09554	0.09655	0.09702
(ub)	0.08689	0.09291	0.09587	0.09687	0.09735
r.e.	0.001442	0.001016	0.000887	0.000856	0.000852
$P(W_y>b)/P(W>b)$					
ratio	1.36208	1.38777	1.40307	1.40428	1.40505
diff w.r.t. last column	0.04297	0.01728	0.00198	0.00077	0.00000
abs diff	0.04297	0.01728	0.00198	0.00077	0.00000
A_y/ρ					
ratio	1.27865	1.34842	1.38403	1.39507	1.40028
diff w.r.t. last column	0.12163	0.05186	0.01625	0.00521	0.00000
abs diff	0.12163	0.05186	0.01625	0.00521	0.00000

3.5. Estimates of the Mean and Standard Deviation

In §3.5 we report additional results on experiments to estimate the mean $E[W_y]$ and standard deviation $SD(W_y)$ using §5.4 of the main paper. Tables 49-51 report results for the $M_t/M/1$ model, while Tables 52 and 53 report results for the $(H_2)_t/M/1$ and $M_t/H_2/1$ models, respectively. The parameters n_s and δ are the parameters for the discrete sum approximations of the integrals; n_s is the number of terms after truncation and δ is the time increment.

Table 49 Estimated mean and standard deviation of the steady-state waiting time in $M/M/1$ queue as a function of $1-\rho$: $\mu=1, \bar{\lambda}=\rho$

$1-\rho$	0.16	0.08	0.04	0.02	0.01
n_s	40,000	40,000	40,000	40,000	40,000
δ	0.001	0.001	0.001	0.001	0.001
b	41	86	173	345	691
$P(W_y>0)$	0.8396	0.9201	0.9601	0.9799	0.9900
s.e. of $P(W_y>0)$	6.86E-04	3.71E-04	1.93E-04	9.73E-05	4.98E-05
%95 CI of $P(W_y>0)$	[0.8383, 0.8410]	[0.9194, 0.9209]	[0.9598, 0.9605]	[0.9797, 0.9801]	[0.9899, 0.9901]
$E[W_y]$	5.249	11.499	23.999	49.000	99.000
s.e. of $E[W_y]$	1.59E-03	1.27E-03	9.51E-04	6.93E-04	4.94E-04
%95 CI of $E[W_y]$	[5.246, 5.252]	[11.497, 11.502]	[23.997, 24.001]	[48.999, 49.001]	[98.999, 99.001]
$E[W_y\|W_y>0]$	6.251	12.497	24.995	50.003	100.005
%95 CI of $E[W_y\|W_y>0]$	[6.238, 6.265]	[12.485, 12.510]	[24.983, 25.007]	[49.992, 50.014]	[99.994, 100.015]
$E[W_y^2]$	65.624	287.494	1199.982	4899.957	19800.030
s.e. of $E[W_y^2]$	1.50E-02	2.33E-02	3.40E-02	4.92E-02	7.04E-02
%95 CI of $E[W_y^2]$	[65.595, 65.654]	[287.449, 287.540]	[1199.916, 1200.049]	[4899.860, 4900.053]	[19799.892, 19800.168]
$SD[W_y]$	6.170	12.460	24.981	49.990	99.995
$P(W_y>0)/\rho$	0.9995	1.0002	1.0001	0.9999	1.0000
$(1-\rho)E[W_y]$	0.8398	0.9200	0.9600	0.9800	0.9900
$(1-\rho)SD[W_y]$	0.9873	0.9968	0.9992	0.9998	0.9999
$(1-\rho)E[W_y]/\rho$	0.9998	0.9999	0.9999	1.0000	1.0000
$(1-\rho)SD[W_y]/\rho$	0.8293	0.9171	0.9593	0.9798	0.9899
$(1-\rho)E[W_y\|W_y>0]$	1.0002	0.9998	0.9998	1.0001	1.0000
$(1-\rho)SD[W_y\|W_y>0]$	1.0002	1.0000	1.0000	1.0000	1.0000

Table 50 Estimated mean $E[W_y]$ and standard deviation $SD(W_y)$ as a function of $1-\rho$ for five cases of the $M_t/M/1$ queue at $y=0.0$ and $y=0.5$: $\mu=1, \bar{\lambda}=\rho$ and base parameter pair $(\beta,\gamma)=(1,2.5)$ using the scaling in (23) of the main paper.

$1-\rho$	0.16	0.08	0.04	0.02	0.01
n_s	40,000	40,000	40,000	40,000	40,000
δ	0.001	0.001	0.001	0.001	0.001
b	41	86	173	345	691
$\mathbf{y=0}$					
$P(W_y>0)$	0.8028	0.9013	0.9507	0.9751	0.9874
s.e. of $P(W_y>0)$	8.22E-04	5.22E-04	3.36E-04	2.23E-04	1.61E-04
%95 CI of $P(W_y>0)$	[0.8012, 0.8044]	[0.9003, 0.9024]	[0.9501, 0.9514]	[0.9747, 0.9755]	[0.9870, 0.9877]
$E[W_y]$	4.249	9.416	19.714	40.309	81.624
std of $E[W_y]$	3.07E-03	5.93E-03	1.19E-02	2.38E-02	4.72E-02
%95 CI of $E[W_y]$	[4.243, 4.255]	[9.404, 9.427]	[19.691, 19.737]	[40.262, 40.355]	[81.531, 81.716]
$E[W_y\|W_y>0]$	5.293	10.446	20.736	41.337	82.669
%95 CI of $E[W_y\|W_y>0]$	[5.275, 5.311]	[10.422, 10.471]	[20.697, 20.775]	[41.271, 41.404]	[82.549, 82.789]
$E[W_y^2]$	48.677	213.860	892.838	3644.475	14740.585
std of $E[W_y^2]$	3.50E-02	1.40E-01	5.66E-01	2.279	9.123
%95 CI of $E[W_y^2]$	[48.608, 48.745]	[213.585, 214.135]	[891.729, 893.948]	[3640.009, 3648.942]	[14722.703, 14758.466]
$SD[W_y]$	5.534	11.190	22.454	44.941	89.878
$P(W_y>0)/\rho$	0.9557	0.9797	0.9903	0.9950	0.9973
$(1-\rho)E[W_y]$	0.6798	0.7532	0.7886	0.8062	0.8162
$(1-\rho)SD[W_y]$	0.8854	0.8952	0.8982	0.8988	0.8988
$(1-\rho)E[W_y]/\rho$	0.8093	0.8187	0.8214	0.8226	0.8245
$(1-\rho)SD[W_y]/\rho$	0.7437	0.8236	0.8622	0.8808	0.8898
$(1-\rho)E[W_y\|W_y>0]$	0.8469	0.8357	0.8294	0.8267	0.8267
$(1-\rho)SD[W_y\|W_y>0]$	0.9138	0.9056	0.9026	0.9008	0.8997
$\mathbf{y=0.5}$					
$P(W_y>0)$	0.8801	0.9411	0.9714	0.9851	0.9930
s.e. of $P(W_y>0)$	9.85E-04	6.54E-04	4.51E-04	2.92E-04	2.19E-04
%95 CI of $P(W_y>0)$	[0.8782, 0.8820]	[0.9399, 0.9424]	[0.9705, 0.9723]	[0.9845, 0.9856]	[0.9926, 0.9934]
$E[W_y]$	6.839	14.927	31.194	63.667	128.411
std of $E[W_y]$	6.42E-03	1.20E-02	2.36E-02	4.69E-02	9.30E-02
%95 CI of $E[W_y]$	[6.827, 6.852]	[14.903, 14.950]	[31.147, 31.240]	[63.575, 63.759]	[128.228, 128.593]
$E[W_y\|W_y>0]$	7.771	15.860	32.113	64.632	129.315
%95 CI of $E[W_y\|W_y>0]$	[7.740, 7.803]	[15.814, 15.907]	[32.036, 32.189]	[64.501, 64.763]	[129.075, 129.554]
$E[W_y^2]$	97.057	427.685	1795.344	7344.665	29673.770
std of $E[W_y^2]$	7.81E-02	0.302	1.207	4.829	19.314
%95 CI of $E[W_y^2]$	[96.904, 97.210]	[427.092, 428.277]	[1792.979, 1797.709]	[7335.201, 7354.129]	[29635.915, 29711.625]
$SD[W_y]$	7.091	14.314	28.676	57.369	114.824
$P(W_y>0)/\rho$	1.0478	1.0230	1.0119	1.0052	1.0030
$(1-\rho)E[W_y]$	1.0943	1.1941	1.2477	1.2733	1.2841
$(1-\rho)SD[W_y]$	1.1345	1.1451	1.1470	1.1474	1.1482
$(1-\rho)E[W_y]/\rho$	1.3028	1.2980	1.2997	1.2993	1.2971
$(1-\rho)SD[W_y]/\rho$	0.9530	1.0535	1.1011	1.1244	1.1368
$(1-\rho)E[W_y\|W_y>0]$	1.2434	1.2688	1.2845	1.2926	1.2931
$(1-\rho)SD[W_y\|W_y>0]$	1.1301	1.1395	1.1433	1.1452	1.1472

Table 51 Estimated mean $E[W_y]$ and standard deviation $SD(W_y)$ as a function of $1-\rho$ for five cases of the $M_t/M/1$ queue at $y=0.0$ and $y=0.5$: $\mu=1, \bar{\lambda}=\rho$ and base parameter pair $(\beta,\gamma)=(4,2.5)$ (with longer cycles than in Table 50) using the scaling in (23) of the main paper.

$1-\rho$	0.16	0.08	0.04	0.02	0.01
n_s	40,000	40,000	40,000	40,000	40,000
δ	0.001	0.001	0.001	0.001	0.001
b	41	86	173	345	691
$\mathbf{y=0}$					
$P(W_y>0)$	0.7346	0.8679	0.9349	0.9665	0.9828
s.e. of $P(W_y>0)$	1.28E-03	9.20E-04	6.45E-04	4.75E-04	3.46E-04
%95 CI of $P(W_y>0)$	[0.7321, 0.7371]	[0.8661, 0.8697]	[0.9336, 0.9361]	[0.9656, 0.9675]	[0.9821, 0.9835]
$E[W_y]$	3.115	7.091	15.097	31.129	63.073
std of $E[W_y]$	5.46E-03	1.10E-02	2.21E-02	4.36E-02	8.71E-02
%95 CI of $E[W_y]$	[3.104, 3.126]	[7.091, 7.134]	[15.054, 15.141]	[31.043, 31.214]	[62.902, 63.243]
$E[W_y\|W_y>0]$	4.240	8.171	16.149	32.206	64.178
%95 CI $E[W_y\|W_y>0]$	[4.211, 4.269]	[8.154, 8.237]	[16.081, 16.218]	[32.087, 32.326]	[63.960, 64.396]
$E[W_y^2]$	33.071	147.266	619.769	2547.465	10295.922
std of $E[W_y^2]$	5.99E-02	2.50E-01	1.028	4.144	0.733
%95 CI of $E[W_y^2]$	[32.954, 33.189]	[146.775, 147.756]	[617.754, 621.784]	[2539, 2555]	[10263, 10328]
$SD[W_y]$	4.834	9.832	19.795	39.730	79.484
$P(W_y>0)/\rho$	0.8745	0.9433	0.9738	0.9863	0.9927
$(1-\rho)E[W_y]$	0.4984	0.5673	0.6039	0.6226	0.6307
$(1-\rho)SD[W_y]$	0.7735	0.7866	0.7918	0.7946	0.7948
$(1-\rho)E[W_y]/\rho$	0.5933	0.6166	0.6291	0.6353	0.6371
$(1-\rho)SD[W_y]/\rho$	0.6497	0.7237	0.7601	0.7787	0.7869
$(1-\rho)E[W_y\|W_y>0]$	0.6784	0.6537	0.6460	0.6441	0.6418
$(1-\rho)SD[W_y\|W_y>0]$	0.8320	0.8116	0.8022	0.7996	0.7973
$\mathbf{y=0.5}$					
$P(W_y>0)$	0.9728	0.9883	0.9967	0.9965	0.9993
s.e. of $P(W_y>0)$	3.61E-03	2.69E-03	2.05E-03	1.16E-03	8.52E-04
%95 CI of $P(W_y>0)$	[0.9657, 0.9799]	[0.9831, 0.9936]	[0.9927, 1.0000]	[0.9943, 0.9988]	[0.9976, 1.0000]
$E[W_y]$	15.148	33.583	70.677	145.183	294.222
std of $E[W_y]$	5.58E-02	1.13E-01	2.27E-01	4.59E-01	9.15E-01
%95 CI $E[W_y]$	[15.039, 15.258]	[33.362, 33.805]	[70.232, 71.121]	[144.284, 146.081]	[292.428, 296.016]
$E[W_y\|W_y>0]$	15.572	33.980	70.909	145.690	294.437
%95 CI of $E[W_y\|W_y>0]$	[15.348, 15.799]	[33.576, 34.387]	[70.232, 71.643]	[144.458, 146.926]	[292.428, 296.728]
$E[W_y^2]$	331.868	1528.127	6547.951	27092.166	110239.942
std of $E[W_y^2]$	1.023	4.263	17.227	69.632	0.785
%95 CI of $E[W_y^2]$	[329.864, 333.873]	[1519.773, 1536.481]	[6514.187, 6581.716]	[26955, 27228]	[109691, 110787]
$SD[W_y]$	10.119	20.007	39.405	77.551	153.861
$P(W_y>0)/\rho$	1.1581	1.0743	1.0383	1.0169	1.0094
$(1-\rho)E[W_y]$	2.4237	2.6867	2.8271	2.9037	2.9422
$(1-\rho)SD[W_y]$	1.6190	1.6006	1.5762	1.5510	1.5386
$(1-\rho)E[W_y]/\rho$	2.8854	2.9203	2.9449	2.9629	2.9719
$(1-\rho)SD[W_y]/\rho$	1.3600	1.4725	1.5132	1.5200	1.5232
$(1-\rho)E[W_y\|W_y>0]$	2.4915	2.7184	2.8364	2.9138	2.9444
$(1-\rho)SD[W_y\|W_y>0]$	1.5892	1.5830	1.5704	1.5442	1.5371

Table 52 Estimated mean $E[W_y]$ and standard deviation $SD(W_y)$ as a function of $1-\rho$ for five cases of the $(H_2)_t/M/1$ model at $y=0.0$ and $y=0.5$: $\mu=1, \bar{\lambda}=\rho$ and base parameter pair $(\beta,\gamma)=(1,2.5)$ using the scaling in (23) of the main paper.

$1-\rho$	0.16	0.08	0.04	0.02	0.01
$theta^*$	0.113	0.0548	0.0270	0.0134	0.00669
n_s	40,000	40,000	40,000	40,000	40,000
δ	0.001	0.002	0.004	0.008	0.016
b	41	86	173	345	691
$\mathbf{y=0}$					
$P(W_y>0)$	0.8617	0.9333	0.9668	0.9837	0.9918
s.e. of $P(W_y>0)$	6.16E-04	3.69E-04	2.39E-04	1.50E-04	1.05E-04
%95 CI of $P(W_y>0)$	[0.8605, 0.8629]	[0.9326, 0.9340]	[0.9663, 0.9673]	[0.9834, 0.9840]	[0.9916, 0.9920]
$E[W_y]$	6.636	14.715	30.874	63.199	127.735
std of $E[W_y]$	3.25E-03	6.41E-03	1.27E-02	2.53E-02	5.05E-02
%95 CI of $E[W_y]$	[6.629, 6.642]	[14.703, 14.728]	[30.849, 30.899]	[63.149, 63.248]	[127.636, 127.834]
$E[W_y\|W_y>0]$	7.701	15.767	31.934	64.246	128.786
%95 CI of $E[W_y\|W_y>0]$	[7.683, 7.719]	[15.742, 15.793]	[31.893, 31.976]	[64.176, 64.315]	[128.659, 128.912]
$E[W_y^2]$	110.805	504.944	2148.048	8845.680	35881.950
std of $E[W_y^2]$	5.24E-02	2.14E-01	8.74E-01	3.506	14.028
%95 CI of $E[W_y^2]$	[110.702, 110.908]	[504.524, 505.365]	[2146.336, 2149.760]	[8838.808, 8852.552]	[35854.456, 35909.445]
$SD[W_y]$	8.171	16.983	34.566	69.654	139.878
$P(W_y>0)/\rho$	1.0258	1.0144	1.0071	1.0038	1.0019
$(1-\rho)E[W_y]$	1.0617	1.1772	1.2350	1.2640	1.2773
$(1-\rho)SD[W_y]$	1.3074	1.3586	1.3827	1.3931	1.3988
$(1-\rho)E[W_y]/\rho$	1.2640	1.2796	1.2864	1.2898	1.2903
$(1-\rho)SD[W_y]/\rho$	1.0982	1.2499	1.3274	1.3652	1.3848
$(1-\rho)E[W_y\|W_y>0]$	1.2322	1.2614	1.2774	1.2849	1.2879
$(1-\rho)SD[W_y\|W_y>0]$	1.3318	1.3681	1.3868	1.3950	1.3997
$\mathbf{y=0.5}$					
$P(W_y>0)$	0.9123	0.9576	0.9802	0.9897	0.9950
s.e. of $P(W_y>0)$	6.97E-04	4.26E-04	2.89E-04	1.75E-04	1.31E-04
%95 CI of $P(W_y>0)$	[0.9109, 0.9136]	[0.9568, 0.9584]	[0.9796, 0.9807]	[0.9894, 0.9901]	[0.9948, 0.9953]
$E[W_y]$	9.615	20.988	43.720	89.079	180.034
std of $E[W_y]$	5.76E-03	1.07E-02	2.07E-02	4.07E-02	8.15E-02
%95 CI of $E[W_y]$	[9.604, 9.626]	[20.967, 21.009]	[43.679, 43.760]	[88.999, 89.159]	[179.874, 180.194]
$E[W_y\|W_y>0]$	10.540	21.917	44.603	90.005	180.934
%95 CI of $E[W_y\|W_y>0]$	[10.512, 10.568]	[21.876, 21.958]	[44.536, 44.671]	[89.893, 90.117]	[180.726, 181.141]
$E[W_y^2]$	185.574	836.287	3534.258	14511.739	58834.208
std of $E[W_y^2]$	9.24E-02	0.362	1.441	5.761	23.019
%95 CI of $E[W_y^2]$	[185.392, 185.755]	[835.578, 836.997]	[3531.433, 3537.082]	[14500.447, 14523.030]	[58789.091, 58879.324]
$SD[W_y]$	9.650	19.895	40.285	81.097	162.548
$P(W_y>0)/\rho$	1.0860	1.0409	1.0210	1.0099	1.0051
$(1-\rho)E[W_y]$	1.5384	1.6790	1.7488	1.7816	1.8003
$(1-\rho)SD[W_y]$	1.5440	1.5916	1.6114	1.6219	1.6255
$(1-\rho)E[W_y]/\rho$	1.8314	1.8250	1.8216	1.8179	1.8185
$(1-\rho)SD[W_y]/\rho$	1.2970	1.4643	1.5469	1.5895	1.6092
$(1-\rho)E[W_y\|W_y>0]$	1.6864	1.7533	1.7841	1.8001	1.8093
$(1-\rho)SD[W_y\|W_y>0]$	1.5375	1.5859	1.6081	1.6201	1.6245

Table 53 Estimated mean $E[W_y]$ and standard deviation $SD(W_y)$ as a function of $1-\rho$ for five cases of the $M_t/H_2/1$ model at $y=0.0$ and $y=0.5$: $\mu=1, \bar{\lambda}=\rho$ and base parameter pair $(\beta,\gamma)=(1,2.5)$ using the scaling in (23) of the main paper.

$1-\rho$	0.16	0.08	0.04	0.02	0.01
θ^*	0.101	0.0519	0.0263	0.0132	0.00664
n	40,000	40,000	40,000	40,000	40,000
δ	0.001	0.002	0.004	0.008	0.016
b	41	86	173	345	691
y = 0					
$P(W_y>0)$	0.8071	0.9028	0.9511	0.9762	0.9878
s.e. of $P(W_y>0)$	9.33E-04	5.64E-04	3.41E-04	2.03E-04	1.35E-04
%95 CI of $P(W_y>0)$	[0.8052, 0.8089]	[0.9017, 0.9039]	[0.9505, 0.9518]	[0.9758, 0.9766]	[0.9876, 0.9881]
$E[W_y]$	6.698	14.779	30.943	63.250	127.753
std of $E[W_y]$	4.38E-03	6.75E-03	1.27E-02	2.53E-02	5.05E-02
%95 CI of $E[W_y]$	[6.689, 6.707]	[14.766, 14.792]	[30.918, 30.968]	[63.201, 63.300]	[127.654, 127.852]
$E[W_y\|W_y>0]$	8.299	16.369	32.532	64.794	129.328
%95 CI of $E[W_y\|W_y>0]$	[8.270, 8.329]	[16.335, 16.404]	[32.483, 32.581]	[64.717, 64.871]	[129.193, 129.463]
$E[W_y^2]$	126.556	539.343	2217.805	8990.031	36149.733
std of $E[W_y^2]$	7.55E-02	2.36E-01	8.95E-01	3.548	14.131
%95 CI of $E[W_y^2]$	[126.408, 126.704]	[538.880, 539.806]	[2216.051, 2219.559]	[8983.078, 8996.985]	[36122.036, 36177.429]
$SD[W_y]$	9.038	17.914	35.502	70.636	140.815
$P(W_y>0)/\rho$	0.9608	0.9813	0.9908	0.9961	0.9978
$(1-\rho)E[W_y]$	1.0717	1.1823	1.2377	1.2650	1.2775
$(1-\rho)SD[W_y]$	1.4461	1.4332	1.4201	1.4127	1.4082
$(1-\rho)E[W_y]/\rho$	1.2758	1.2851	1.2893	1.2908	1.2904
$(1-\rho)SD[W_y]/\rho$	1.2148	1.3185	1.3633	1.3845	1.3941
$(1-\rho)E[W_y\|W_y>0]$	1.3279	1.3096	1.3013	1.2959	1.2933
$(1-\rho)SD[W_y\|W_y>0]$	1.5004	1.4520	1.4274	1.4158	1.4096
y = 0.5					
$P(W_y>0)$	0.8771	0.9399	0.9699	0.9847	0.9924
s.e. of $P(W_y>0)$	9.68E-04	5.87E-04	3.76E-04	2.34E-04	1.64E-04
%95 CI of $P(W_y>0)$	[0.8752, 0.8790]	[0.9387, 0.9410]	[0.9691, 0.9706]	[0.9842, 0.9851]	[0.9921, 0.9928]
$E[W_y]$	9.558	20.905	43.593	88.977	179.983
std of $E[W_y]$	7.53E-03	1.16E-02	2.11E-02	4.12E-02	8.15E-02
%95 CI of $E[W_y]$	[9.543, 9.573]	[20.882, 20.927]	[43.552, 43.635]	[88.896, 89.058]	[179.823, 180.142]
$E[W_y\|W_y>0]$	10.897	22.241	44.948	90.364	181.352
%95 CI of $E[W_y\|W_y>0]$	[10.857, 10.938]	[22.190, 22.293]	[44.871, 45.025]	[90.240, 90.488]	[181.133, 181.572]
$E[W_y^2]$	201.796	870.147	3603.439	14652.678	59167.620
std of $E[W_y^2]$	1.30E-01	0.397	1.478	5.833	23.190
%95 CI of $E[W_y^2]$	[201.540, 202.051]	[869.368, 870.926]	[3600.542, 3606.336]	[14641.246, 14664.110]	[59122.168, 59213.072]
$SD[W_y]$	10.509	20.812	41.268	82.072	163.627
$P(W_y>0)/\rho$	1.0442	1.0216	1.0103	1.0047	1.0025
$(1-\rho)E[W_y]$	1.5293	1.6724	1.7437	1.7795	1.7998
$(1-\rho)SD[W_y]$	1.6815	1.6650	1.6507	1.6414	1.6363
$(1-\rho)E[W_y]/\rho$	1.8206	1.8178	1.8164	1.8159	1.8180
$(1-\rho)SD[W_y]/\rho$	1.4124	1.5318	1.5847	1.6086	1.6199
$(1-\rho)E[W_y\|W_y>0]$	1.7435	1.7793	1.7979	1.8073	1.8135
$(1-\rho)SD[W_y\|W_y>0]$	1.6882	1.6611	1.6469	1.6390	1.6349

3.6. The Impact of the Adjustment for the First Random Variable in §5.3 and §5.4

Tables 7 and 11 of the main paper for the $(H_2)_t/M/1$ model would be different if we ignored the adjustment for the exceptional first interarrival time in the rare-event algorithm that were introduced in §5.3 and §5.4 there. We now show the corresponding tables without this refinement. Consistent with intuition and the fact that the two processes have identical steady-state limits, we see that the difference disappears as ρ increases. Nevertheless, the difference is noticeable in all cases.

First, Table 54 shows analog of the results in Table 7 of the main paper for the $(H_2)_t/M/1$ model.

Table 54 **Simulation estimates of $\hat{p} \equiv P(W_y > b) \equiv A_y e^{-\theta^* b}$ in the $(H_2)_t/M/1$ model without the factor $m_{X_1}(\theta^*)$ in (48) of the main paper for $y = 0.0$ and $y = 0.5$ as a function of $1-\rho$ with base parameter triple $(\beta, \gamma, b) = (1, 2.5, 4)$ in (14) based on $40,000$ replications.**

$1-\rho$	0.16	0.08	0.04	0.02	0.01
$\theta^*(\rho)$	0.113	0.0548	0.0270	0.0134	0.00669
$\hat{p}$ for $y = 0.0$	0.041099	0.047976	0.051467	0.053499	0.054240
$e^{-\theta^* b}$	0.0593	0.0645	0.0670	0.0682	0.0689
A_y	0.693	0.744	0.768	0.784	0.788
A_y^-	0.504	0.546	0.567	0.577	0.582
A_y^+	0.887	0.945	0.973	0.987	0.993
s.e.	4.62E-05	4.68E-05	4.82E-05	1.72E-04	4.96E-05
95% CI (lb)	0.04101	0.04788	0.05137	0.05316	0.05414
(ub)	0.04119	0.04807	0.05156	0.05384	0.05434
r.e.	0.001125	0.000975	0.000936	0.003208	0.000914
$P(W_y > b)/P(W > b)$	0.78064	0.78762	0.78945	0.79463	0.79294
diff	0.01230	0.00532	0.00349	-0.00169	0.00000
abs diff	0.01230	0.00532	0.00349	0.00169	0.00000
A_y/ρ	0.82476	0.80897	0.80027	0.79995	0.79559
diff	-0.02916	-0.01337	-0.00467	-0.00436	0.00000
abs diff	0.02916	0.01337	0.00467	0.00436	0.00000
$\hat{p}$ for $y = 0.5$	0.075260	0.086414	0.092196	0.095157	0.096491
$e^{-\theta^* b}$	0.0593	0.0645	0.0670	0.0682	0.0689
A_y	1.269	1.341	1.376	1.394	1.401
A_y^- LB	0.887	0.945	0.973	0.987	0.993
A_y^+ UB	1.561	1.635	1.671	1.688	1.696
s.e.	8.03E-05	7.92E-05	8.02E-05	1.83E-04	8.25E-05
95% CI (lb)	0.07510	0.08626	0.09204	0.09480	0.09633
(ub)	0.07542	0.08657	0.09235	0.09552	0.09665
r.e.	0.001067	0.000916	0.000870	0.001921	0.000855
$P(W_y > b)/P(W > b)$	1.42950	1.41863	1.41419	1.41339	1.41060
diff	-0.01891	-0.00803	-0.00360	-0.00279	0.00000
abs diff	0.01891	0.00803	0.00360	0.00279	0.00000
A_y/ρ	1.51029	1.45708	1.43357	1.42285	1.41532
diff	-0.09497	-0.04176	-0.01825	-0.00753	0.00000
abs diff	0.09497	0.04176	0.01825	0.00753	0.00000

Second, Table 55 shows results related to Table 11 of the main paper.

Table 55 Estimated mean $E[W_y]$ and standard deviation $SD(W_y)$ as a function of $1-\rho$ for five cases of the $(H_2)_t/M/1$ queue without the factor $m_{X_1}(\theta^*)$ in (48) of the main paper at $y=0.5$: $\mu=1, \bar{\lambda}=\rho$ and base parameter pair $(\beta,\gamma)=(1,2.5)$.

$1-\rho$	0.16	0.08	0.04	0.02	0.01
$\theta^*(\rho)$	0.113	0.0548	0.0270	0.0134	0.00669
n_s	40,000	40,000	40,000	40,000	40,000
δ	0.001	0.002	0.004	0.008	0.016
largest b	41	86	173	345	691
$P(W_y>0)$	0.9123	0.9576	0.9802	0.9897	0.9950
s.e. of $P(W_y>0)$	6.97E-04	4.26E-04	2.89E-04	1.75E-04	1.31E-04
%95 CI of $P(W_y>0)$	[0.9109, 0.9136]	[0.9568, 0.9584]	[0.9796, 0.9807]	[0.9894, 0.9901]	[0.9948, 0.9953]
$E[W_y]$	9.615	20.988	43.720	89.079	180.034
std of $E[W_y]$	5.76E-03	1.07E-02	2.07E-02	4.07E-02	8.15E-02
%95 CI of $E[W_y]$	[9.604, 9.626]	[20.97, 21.01]	[43.68, 43.76]	[80.00, 89.16]	[179.87, 180.19]
$E[W_y\|W_y>0]$	10.540	21.917	44.603	90.005	180.934
%95 CI of $E[W_y\|W_y>0]$	[10.512, 10.568]	[21.876, 21.958]	[44.54, 44.67]	[89.89, 90.12]	[180.73, 181.14]
$E[W_y^2]$	185.574	836.287	3534.26	14,511.7	58,834.2
std of $E[W_y^2]$	9.24E-02	0.362	1.441	5.761	23.019
%95 CI of $E[W_y^2]$	[185.39, 185.76]	[835.58, 837.00]	[3531.4, 3537.1]	[14,500, 14,523]	[58,789, 58,879]
$SD[W_y]$	9.650	19.90	40.29	81.10	162.55
$P(W_y>0)/\rho$	1.0860	1.0409	1.0210	1.0099	1.0051
$(1-\rho)E[W_y]$	1.5384	1.6790	1.7488	1.7816	1.8003
$(1-\rho)SD[W_y]$	1.5440	1.5916	1.6114	1.6219	1.6255
$(1-\rho)E[W_y]/\rho$	1.8314	1.8250	1.8216	1.8179	1.8185
$(1-\rho)SD[W_y]/\rho$	1.2970	1.4643	1.5469	1.5895	1.6092
$(1-\rho)E[W_y\|W_y>0]$	1.6864	1.7533	1.7841	1.8001	1.8093
$(1-\rho)SD[W_y\|W_y>0]$	1.5375	1.5859	1.6081	1.6201	1.6245

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