TRAFFIC MODELS FOR WIRELESS COMMUNICATION NETWORKS

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Abstract

In this paper, we introduce a deterministic fluid model and two stochastic traffic models for wireless networks. The setting is a highway with multiple entrances and exits. Vehicles are classified as calling or non-calling, depending on whether they have calls in progress. The deterministic model ignores the behavior of individual vehicles and treats them as a continuous fluid, whereas the stochastic traffic models consider the random behavior of each vehicle. However, all three models use the same two coupled partial (or ordinary) differential equations to describe the system evolution. The call density and call handoff rate (or their expected values in the stochastic models) are readily computable by solving these equations. Numerical examples are presented to illustrate how the models can be used to investigate various aspects of time and space dynamics in wireless networks. These examples also show that the models can serve as useful tools for system engineering and planning.

1. Introduction

Unlike a fixed, terrestrial telephone network, a wireless network must support moving customers. Due to customer mobility, both the location and the length of a call in progress affect the network resources required to support the call. Customer mobility is presenting a major challenge to system designers of wireless networks [3,4]. Since wireless services are becoming more popular, there is an increasing need for mathematical models to help understand system dynamics and analyze the performance of wireless networks.

Motivated primarily by this need, a *Poisson-arrival-location model* (PALM) was introduced in [6], in which customers arrive according to a nonhomogeneous Poisson process and move independently through a general location state space according to a location stochastic process. The PALM is made tractable by assuming that different customers do not interact, although this behavior can be approximated indirectly. Similarly, the PALM can be used as an offered traffic model that serves to approximate important system capacity constraints (e.g., the number of available radio channels) indirectly. This is achieved by applying methods like the modified offered load technique in [2] to approximate blocking probability in these wireless networks. The PALM also provides a useful framework for representing both time-dependent behavior and customer mobility in wireless networks.

The general PALM in [6] is quite abstract. Further specification is needed to obtain practical models. Toward this end, a version of the PALM was constructed to study communicating mobiles on a highway in [7]. In this *highway PALM,* vehicles alternate between think and calling modes as they move along on a one-way, single-lane, semi-infinite highway according to a deterministic location function. More complicated highway networks are represented by superposing independent versions of these highway PALMs. The highway PALM enables us to characterize key quantities such as the call density, the handoff rate, the call-origination-rate density and the call-termination-rate density. In [7], it is shown that these quantities are related by two fundamental conservation equations, similar to relations in vehicular traffic theory [1]. These results thus bring together teletraffic theory and vehicular traffic theory. Other researchers have also observed the need for combining these two theories to study wireless networks [8-11].

While much more concrete than the general PALM in [6], the highway PALM in [7] is still quite abstract. Further specification is required to produce readily computable performance-related quantities. The purpose of this paper is to develop such a version of the highway PALM. We obtain such a more tractable highway PALM primarily by making additional Markov assumptions, in the spirit of Section 8 of [6]. Hence we call the main stochastic model introduced here the *Markovian highway PALM.*

In the Markovian highway PALM considered here, in addition to having arrivals occur according to a nonhomogeneous Poisson process, the state of each vehicle is assumed to evolve according to a nonstationary continuous-time Markov chain, while the vehicle moves deterministically along the highway. Each vehicle on the highway is classified as either a *calling* or *non-calling* vehicle, depending on whether it has a call in progress. Each calling (non-calling) vehicle becomes a non-calling (calling) vehicle randomly with specified deterministic intensity depending on time and space. Similarly, each calling (non-calling) vehicle leaves the highway with a deterministic intensity depending on time and space. Of course, our main interest is in the calling vehicles; they use the most resources in wireless networks. However, we also keep track of locations of non-calling vehicles because they are the source of future calling vehicles.

It turns out that the densities of the mean numbers of vehicles of each type are described by partial differential equations (PDE's), similar to the ones arising in the classic approach to modeling vehicular traffic [1]. With the Markovian assumption, the two PDE's are coupled due to the calling activities; i.e., a non-calling vehicle becomes a calling vehicle and vice versa, if it initiates (places and receives) or terminates a call. In this Markovian highway PALM, the PDE's relate the derivatives of the expected number of vehicles of each type, while the actual numbers of non-calling and calling vehicles in a given section of the highway have Poisson distributions, due to previous PALM results in [6, 7].

The PDE's can also be interpreted in another way. Instead of characterizing the *expected values* in a *stochastic model*, they can be regarded as characterizing the *actual values* in a *deterministic fluid model*. This deterministic fluid model neglects the behavior of individual vehicles (or customers), but is still capable of capturing the overall dynamics of the system. Calling and non-calling vehicles are treated as two types of continuous fluid. Such a model is appropriate and justifiable if the system has a large number of calling and non-calling vehicles, as discussed in Section 9 of [6]. Indeed, such deterministic differential equation models are common in vehicular traffic theory [1]. (Wright [11] also uses differential equations to capture the vehicle movement in a highway cellular system.)

In fact, three different models are considered here: the deterministic fluid model, the Markovian highway PALM, and a stochastic generalization of the Markovian highway PALM in which the arrival process need not be Poisson. As noted in Remark 2.3 of [6], the mean value formulas for a general PALM remain valid when the arrival process is *not* Poisson, if the arrival rate is still well defined and successive arrivals do not interact. We use the extra Poisson arrival assumption only to find the probability distributions of the quantities of interest; the mean values are determined from the PDE's.

We believe that an interesting feature of this paper is the identification of the three related models. The important point is that all three models lead to the same PDE's. For the deterministic fluid model, we interpret the solutions to the PDE's as the actual numbers (which need not be integers), whereas for the stochastic traffic model, the solutions represent the expected values. With the additional Poisson-arrival assumption in the Markovian highway PALM, we can obtain the full probability distributions. For large systems, the three models tend to be almost fully consistent, because in the stochastic model, the true distributions will typically cluster relatively tightly about their means, by virtue of the law of large numbers; see Section 9 of [6]. The mean value interpretation in the stochastic models tends to be more general, because it does not require large populations.

The rest of this paper is organized as follows. In Section 2, we develop the deterministic fluid model. The model has two versions: a time-nonhomogeneous model and a time-homogeneous model. The two PDE's become ordinary differential equations (ODE's) in the time-homogeneous model. In Section 3, we present some numerical examples to illustrate the time and space dynamics captured by the models. These numerical examples show that the quantities of interest can readily be computed and that the proposed models can serve as valuable tools for system engineering and planning. In Section 4, we introduce the Markovian traffic model (without Poisson arrivals) and discuss its connection with the deterministic fluid model. In Section 5, we discuss the additional distributional results that can be obtained when arrivals occur according to a nonhomogeneous Poisson process. Finally, we present our conclusions in Section 6. Due to space limitation, we omit all proofs here and they can be found in [5].

2. The Deterministic Fluid Model

Our basic setting is a one-way, single-lane, semi-infinite highway. (As in [7], independent versions of these highways can be superposed to make richer models.) Thus, we can regard the location space as the interval $[0,\infty)$. There are two types of vehicles, calling and non-calling. Each vehicle is assumed to make at most one call at a time and each call occupies one radio channel for the duration of the call. As in [6, 7], we do not impose any capacity constraints, i.e., we assume that there are an infinite number of channels available so that all calls are accepted without blocking. Thus our model can be regarded as a way to quantify the offered load. It is possible to add capacity constraints to the model in various ways, as we illustrate in Section 3.

Vehicles of both types at location x and time t move forward on the highway according to a deterministic velocity field $v(x,t)$. To ensure vehicle flow in a single direction, it is assumed that $v(x,t) \geq 0$ for all x and t with $x \geq 0$ and $-\infty < t < \infty$. Additionally, in order to make sure that vehicles flow at all, $v(x,t) > 0$ at least for some $t \geq t_0$ for all times t_0 at each location $x \geq 0$. For full generality, it is also assumed that both calling and non-calling vehicles can enter and leave the highway at any location. (Cases with finitely many fixed entrances and exits are considered at the end of Section 2.1.)

In Section 2.1, we first present the model, which captures both the time-dependent behavior (e.g., nonhomogeneous arrivals of vehicles) and vehicle movement on the highway. Then in Section 2.2, we simplify the time-nonhomogeneous fluid model into a time-homogeneous model to capture only the spatial dynamics. The PDE's then become ODE's.

2.1 The Time-Nonhomogeneous Deterministic Fluid Model

Let $N(x,t)$ and $Q(x,t)$ be the *number* of non-calling and calling vehicles in location $(0,x]$ at time t, respectively. The model treats vehicles as a continuous fluid; $N(x,t)$ and $Q(x,t)$ are not necessarily integers, but any non-negative real numbers. In addition, let $n(x,t)$ and $q(x,t)$ be the *non-calling* density and *calling* density at location x and time t, respectively. That is, $n(x,t) \equiv \partial N(x,t)/\partial x$ and $q(x,t) \equiv \partial Q(x,t)/\partial x$. In this paper, we assume that all derivatives are well defined.

Furthermore, let $\mathbf{C}_n^+(x,t)$ and $\mathbf{C}_n^-(x,t)$ be the number of non-calling vehicles *entering* or *leaving* in location $(0,x]$ during time interval $(-\infty,t]$, respectively. Similarly, we use $\mathbf{C}_q^+(x,t)$ and $\mathbf{C}_q^-(x,t)$ to denote the respective number of calling vehicles entering or leaving in location $(0,x]$ in time $(-\infty,t]$. A non-calling (calling) vehicle is considered to be entering the system, if either: a) it is an actual arrival of a non-calling (calling) vehicle to the highway, or b) it was a calling (non-calling) vehicle existing on the highway but with its call just terminated (started). Likewise, a non-calling (calling) vehicle leaves if it departs from the highway or becomes a calling (non-calling) vehicle by initiating (terminating) a call. Finally, let us define the rate densities as

$$c_n^+(x,t) \equiv \partial^2 \mathbf{C}_n^+(x,t)/\partial x \partial t, \quad c_n^-(x,t) \equiv \partial^2 \mathbf{C}_n^-(x,t)/\partial x \partial t,$$

$$c_q^+(x,t) \equiv \partial^2 \mathbf{C}_q^+(x,t)/\partial x \partial t \quad \text{and} \quad c_q^-(x,t) \equiv \partial^2 \mathbf{C}_q^-(x,t)/\partial x \partial t.$$

Lemma 2.1: *The evolution of non-calling and calling vehicles on the highway is governed by the PDE's:*

$$\frac{\partial n(x,t)}{\partial t} + \frac{\partial}{\partial x}[n(x,t)v(x,t)] = c_n^+(x,t) - c_n^-(x,t) \quad (2.1)$$

and $$\frac{\partial q(x,t)}{\partial t} + \frac{\partial}{\partial x}[q(x,t)v(x,t)] = c_q^+(x,t) - c_q^-(x,t) \quad (2.2)$$

for $x \geq 0$ *and* $-\infty < t < \infty$.

We remark that (2.2) corresponds to the fundamental conservation equation in (2.7) of [7]. To show how (2.1) and (2.2) are coupled due to calling activity, let $\mathbf{E}_n^+(x,t)$ and $\mathbf{E}_n^-(x,t)$ be the number of non-calling vehicles entering and leaving the highway in location $(0,x]$ during time interval $(-\infty,t]$, respectively. We use $\mathbf{E}_q^+(x,t)$ and $\mathbf{E}_q^-(x,t)$ to denote the respective number of calling vehicles entering and leaving the highway in location $(0,x]$ in time $(-\infty,t]$. The associated rate densities are

$$e_n^+(x,t) \equiv \partial^2 \mathbf{E}_n^+(x,t)/\partial x \partial t, \quad e_n^-(x,t) \equiv \partial^2 \mathbf{E}_n^-(x,t)/\partial x \partial t,$$

$$e_q^+(x,t) \equiv \partial^2 \mathbf{E}_q^+(x,t)/\partial x \partial t \quad \text{and} \quad e_q^-(x,t) \equiv \partial^2 \mathbf{E}_q^-(x,t)/\partial x \partial t.$$

Further, let $\beta(x,t)n(x,t)$ and $\gamma(x,t)q(x,t)$ be the rates at which non-calling and calling vehicles actually depart from the highway at location x at time t. In addition, let $\lambda(x,t)n(x,t)$ be the call-origination rate of non-calling vehicles and $\mu(x,t)q(x,t)$ be the call-termination rate of calling vehicles at location x and time t. (In the stochastic model, these are stochastic intensities for individual vehicles; here these are actual deterministic flow rates.)

The rate densities, $c_n^+(x,t)$, $c_n^-(x,t)$, $c_q^+(x,t)$, and $c_q^+(x,t)$ can be expressed in terms of these parameters as follows.

Lemma 2.2: *The four rate densities can be expressed as:*

$$(a) \quad c_n^+(x,t) = e_n^+(x,t) + \mu(x,t)q(x,t) \quad (2.3)$$

$$(b) \quad c_n^-(x,t) = \beta(x,t)n(x,t) + \lambda(x,t)n(x,t) \quad (2.4)$$

$$(c) \quad c_q^+(x,t) = e_q^+(x,t) + \lambda(x,t)n(x,t) \quad (2.5)$$

$$(d) \quad c_q^-(x,t) = \gamma(x,t)q(x,t) + \mu(x,t)q(x,t). \quad (2.6)$$

We now combine Lemmas 2.1 and 2.2 to obtain the following coupled PDE's characterizing the densities $n(x,t)$ and $q(x,t)$ in our model. These PDE's can be regarded as the deterministic fluid model.

Theorem 2.1: *The densities of non-calling and calling vehicles,* $n(x,t)$ *and* $q(x,t)$*, satisfy the coupled PDE's:*

$$\frac{\partial n(x,t)}{\partial t} + \frac{\partial}{\partial x}[n(x,t)v(x,t)] = e_n^+(x,t) + \mu(x,t)q(x,t) - [\beta(x,t) + \lambda(x,t)]n(x,t) \quad (2.7)$$

and $$\frac{\partial q(x,t)}{\partial t} + \frac{\partial}{\partial x}[q(x,t)v(x,t)] = e_q^+(x,t) + \lambda(x,t)n(x,t) - [\gamma(x,t) + \mu(x,t)]q(x,t) \quad (2.8)$$

With an additional assumption, the PDE's in (2.7) and (2.8) can be converted into a set of three ordinary differential equations (ODE's), which are easier to solve in some cases. (This is the classical method of characteristics.) For this purpose, let the location x as a time function, $x(t)$, be given by

$$\frac{dx(t)}{dt} = v(x(t),t). \quad (2.9)$$

Equation (2.9) is one of the three ODE's.

Lemma 2.3: *Given (2.9), the PDE's (2.7) and (2.8) are equivalent to*

$$\frac{dn(x(t),t)}{dt} = e_n^+(x(t),t) + \mu(x(t),t)\,q(x(t),t) - \left[\frac{\partial v(x,t)}{\partial x} + \beta(x(t),t) + \lambda(x(t),t)\right] n(x(t),t) \quad (2.10)$$

and $$\frac{dq(x(t),t)}{dt} = e_q^+(x(t),t) + \lambda(x(t),t)\,n(x(t),t) - \left[\frac{\partial v(x,t)}{\partial x} + \gamma(x(t),t) + \mu(x(t),t)\right] q(x(t),t) \quad (2.11)$$

Due to the partial derivative of $v(x,t)$ w.r.t. x on the r.h.s. of (2.10) and (2.11), they are ODE's if and only if $v(x,t)$ is not a function of $q(x,t)$ and $n(x,t)$. It is also noteworthy that, by choosing some τ with $-\infty<\tau<\infty$ such that $x(\tau)=0$ as the initial condition for (2.9), $n(x(t),t)$ and $q(x(t),t)$ can be solved for all $\tau \le t < \infty$ from (2.9)-(2.11), e.g., by a Runge-Kutta method. Of course, the solution depends on the initial conditions $n(0,\tau)$ and $q(0,\tau)$. Thus, if τ is selected properly, one can obtain $n(x,t)$ and $q(x,t)$ for the location and the time interval of interest.

Now suppose that the highway is divided into cells, indexed by $i=1,2,3,\cdots$. For $i>1$, let the boundary between cell $i-1$ and cell i be located at x_{i-1} and $x_0 \equiv 0$. Further, let $Q_i(t)$ be the instantaneous offered load (i.e., the number of calls in progress) in cell i at time t. Let $h_i(t)$ denote the rate of calls handed off from cell $i-1$ to cell i at time t.

Theorem 2.2: *For cell $i \ge 1$, its instantaneous offered load and call handoff rate at time t are*

$$Q_i(t) = \int_{x_{i-1}}^{x_i} q(x,t)\,dx \quad (2.12)$$

and $$h_i(t) = q(x_{i-1},t)\,v(x_{i-1},t), \quad (2.13)$$

respectively.

We remark that (2.13) corresponds to the conservation equation (2.6) in [7]. It gives a flow rate at a point, which does not actually require that cells be defined; i.e., (2.13) is valid for arbitrary x as well as x_{i-1}.

We conclude this subsection by commenting on the rate densities of vehicle entering and leaving the highway for the case where vehicles can enter or leave only at entrances/exits at fixed locations, as in real highway systems. Suppose that $\{y_i : i=1,2,3,...\}$ is the location of the i^{th} entrance/exit on the highway. Let us use $\xi_n^i(t)$ and $\xi_q^i(t)$ to denote the external arrival rate of non-calling and calling vehicles at the i^{th} entrance at time t, respectively. Then, we have

$$e_n^+(x,t) = \sum_i \xi_n^i(t)\,\delta(x-y_i) \quad (2.14)$$

and $$e_q^+(x,t) = \sum_i \xi_q^i(t)\,\delta(x-y_i) \quad (2.15)$$

where $\lim_{\epsilon \to 0} \int_{x-\epsilon}^{x+\epsilon} \delta(y)\,dy = 1$ if $x=0$ and 0 otherwise.

As for vehicles leaving the highway, we use $p_n^i(t)$ and $p_q^i(t)$ to denote the fraction of non-calling and calling vehicles departing when they pass by the i^{th} entrance/exit at time t, respectively. If these departing vehicles leave at the same velocity as they move forward along the highway, then

$$\beta(x,t) = v(x,t) \sum_i p_n^i(t)\,\delta(x-y_i) \quad (2.16)$$

and $$\gamma(x,t) = v(x,t) \sum_i p_q^i(t)\,\delta(x-y_i). \quad (2.17)$$

2.2 The Time-Homogeneous Deterministic Fluid Model

We now cease to focus on time dynamics, and assume that the system has reached a steady state with respect to time. As a result, all system variables and parameters become independent of time. For this reason, we simply drop the variable t from our previously defined notation, and use primes to denote derivatives w.r.t. x.

Theorem 2.3: *At temporal equilibrium, the densities of non-calling and calling vehicles, $n(x)$ and $q(x)$, at any location $x \ge 0$ satisfy the following ODE's:*

$$v(x)\,n'(x) = e_n^+(x) + \mu(x)\,q(x) - [\beta(x) + \lambda(x) + v'(x)]\,n(x) \quad (2.18)$$

and $$v(x)\,q'(x) = e_q^+(x) + \lambda(x)\,n(x) - [\gamma(x) + \mu(x) + v'(x)]\,q(x). \quad (2.19)$$

In general, $n(x)$ and $q(x)$ can be solved from the coupled ODE's in (2.18) and (2.19) plus initial conditions. However, under reasonable assumptions, the two ODE's can be combined into one, and $n(x)$ and $q(x)$ can be obtained explicitly. To prove this, we present the following proportionality result.

Lemma 2.4: *For some $x_0 \ge 0$, if*

(a) $\lambda(x)=\lambda,\ \mu(x)=\mu,\ \beta(x)=\gamma(x)$ *and* $e_q^+(x)/e_n^+(x)=\lambda/\mu$ *for all* $x \ge x_0$, *and* (2.20)

(b) $q(x_0)$ *is finite and* $q(x_0)/n(x_0)=\lambda/\mu$, (2.21)

then $q(x)/n(x)=\lambda/\mu$ *for all* $x \ge x_0$. (2.22)

We actually obtain a stronger proportionality result for a time-dependent setting in [5]. Lemma 2.4 can be viewed as a consequence of the time-dependent result since all quantities here are independent of time. We choose to present Lemma 2.4 here because, as explained below, its conditions have a clear physical meaning and are natural for the time-homogeneous model. When $\lambda(x)=\lambda$ and $\mu(x)=\mu$, vehicles initiate and terminate calls at rates independent of their locations. The condition $\beta(x)=\gamma(x)$ indicates that a vehicle departs from the highway at the same rate, regardless of whether it is a calling or non-calling vehicle. The ratio $e_q^+(x)/e_n^+(x)=q(x_0)/n(x_0)=\lambda/\mu$ means that the proportion of vehicles arriving to the highway at location x which are calling vehicles is identical to that of existing vehicles at location x_0, which in turn is equal to the ratio λ/μ.

If the conditions for the proportionality result are satisfied, one ODE is sufficient for describing the movement of non-calling and calling vehicles. For this purpose, let $L(x)$ be the total number of vehicles in location $(0,x]$ at steady state and $l(x)\equiv dL(x)/dx$. By definition, $L(x)=N(x)+Q(x)$ and $l(x)=n(x)+q(x)$.

Theorem 2.4: *If the conditions in Lemma 2.4 with $x_0=0$ are satisfied, then the vehicular density is given by the following ODE:*

$$\frac{dl(x)}{dx}=\frac{1}{v(x)}\left[-\frac{dv(x)}{dx}-\beta(x)\right]l(x)+e_q^+(x)\left[1+\frac{\mu}{\lambda}\right] \qquad (2.23)$$

whose solution is

$$l(x)=\frac{e^{-I(x)}}{v(x)}\left[1+\frac{\mu}{\lambda}\right]\int_0^x e_q^+(u)\,e^{I(u)}\,du \;+\; l(0)\,v(0) \qquad (2.24)$$

where $I(x)=\int_0^x \frac{\beta(u)}{v(u)}\,du$. Furthermore,

$$n(x) \;=\; \mu l(x)/(\lambda+\mu) \qquad (2.25)$$

and

$$q(x) \;=\; \lambda l(x)/(\lambda+\mu). \qquad (2.26)$$

Once $n(x)$ and $q(x)$ are computed from (2.18) and (2.19) or (2.25) and (2.26), the offered load and call handoff rate for cell i can be obtained from (2.12) and (2.13), with the variable t omitted.

3. Numerical Examples

In this section, we present numerical examples to illustrate the time and space dynamics captured by the fluid model in Section 2. As indicated in Section 1, these examples also apply to the stochastic models to be introduced later, but with a different interpretation.

The examples first assume no constraint on the number of available channels. Furthermore, the average think time (time before initiating a call) and call-holding time are 10 and 2 minutes, respectively. That is, $\lambda(x,t)=0.1$ and $\mu(x,t)=0.5$ for all $x\geq 0$ and $-\infty<t<\infty$. The highway has a single entrance at location 0 at which only non-calling vehicle arrive at a constant rate (denoted by α) of 30 cars/minute, and vehicles are assumed not to depart from the highway.

We consider a time-dependent case where the velocity field $v(x,t)=1$ km/min. for all $x\geq 0$ when $t\leq 35$ or $t>55$ min. However, for $35<t\leq 55$ min., the velocity field is

$$v(x,t) \;=\; \begin{cases} 1 & \text{if } x\leq 3 \\ 1 \;-\; 0.7(x-3) & \text{if } 3<x\leq 4 \\ 0.3 & \text{if } 4<x\leq 6 \\ 0.3 \;+\; 0.7(x-6) & \text{if } 6<x\leq 7 \\ 1 & \text{if } x>7. \end{cases} \qquad (3.1)$$

This velocity field is U-shaped as a function of location on the highway for $35<t\leq 55$, so that it can be used to simulate the slowing down of traffic in the time interval due to an accident. For this example, Lemma 2.3 can be applied to convert the PDE's (2.7) and (2.8) into the ODE's (2.9)-(2.11). We numerically solve the ODE's with the initial condition of $n(0,t)=\alpha/v(0,t)$ for all $t>0$ in order to obtain $n(x,t)$ and $q(x,t)$.

Figures 1-4 show the total vehicular density, the densities of non-calling and calling vehicles, and the call handoff rate as a function of location at time prior (Fig. 1), during (Fig. 2-3), and after (Fig. 4) the traffic accident. For $t=30$ in Figure 1, $n(x,t)$ and $q(x,t)$ reach their "equilibrium" solution for sufficiently large x. The reason for this is explained as follows. Recall that all vehicles arriving at $x=0$ are non-calling vehicles. They start to make calls as they move forward on the highway. As a result, the density of non-calling and calling vehicles decreases and increases, respectively, as x increases. Since vehicles initiate and terminate calls independently at constant rates of $\lambda(x,t)=0.1$ and $\mu(x,t)=0.5$, respectively, such decrease and increase of vehicular densities approach an equilibrium at locations farther down the highway. In fact, the ratio $n(x)/q(x)$ in this case tends to equal to $\mu(x,t)/\lambda(x,t)$ for sufficiently large x. Since $v(x,t)=1$ at $t=30$, according to (2.14), the density of calling vehicles equals handoff rate, so that their curves shown in the figure coincide.

At $t=40$ in Figure 2, vehicles start to build up sharply at location $(3,7]$ where velocity is relatively low. Accordingly, as indicated in the figure, the call density in this region is also higher than elsewhere because of the velocity reduction began at $t=35$. Note that the vehicular density in location $(6,12]$ is lower than that beyond location $x\geq 12$. This is because the vehicles that would have been at this location if there were no reduction in velocity starting at $t=35$ have been trapped in location $(3,7]$ due to the low velocity. At $t=50$ in Figure 3, the vehicular traffic and call density continue to build up in location $(3,7]$. In addition, the "dip" of vehicular density has shifted to the right from the position shown in Figure

2, as vehicles continue to move forward on the highway.

Finally, Figure 4 shows that the whole density curves continue to propagate to the right at $t = 60$, as vehicles resume their original velocity of $v(x,t) = 1$ at all locations after the accident has been cleared at $t = 55$. In particular, those vehicles that were located at location $(3,7]$ at $t = 55$ now have moved into location $(8,12]$ at $t = 60$ at a constant velocity of $v(x,t) = 1$ after $t > 55$. Note that these results may not closely reflect the vehicle movement following a traffic accident because the model allows vehicles to resume moving at a specific velocity regardless of the high vehicular density. In real situation, with the vehicles close to location 7 first resuming the normal velocity after the accident, the vehicles accumulated in location $(3,7]$ will slowly "diffuse" to the right. Such diffusion type of movement can be captured, if the model is augmented with an appropriate relationship between velocity and vehicular density, as discussed in Section 6 of [5].

As pointed out earlier, the call density in a region should be treated as its offered load. To illustrate how the offered load results can be used for system engineering and planning purposes, let the highway be served by non-overlapping cells of fixed size where each cell covers 2 km of the highway. By (2.12), the offered load at a given time is obtained for each cell. Given the number of channels available at a cell, the blocking probability can be approximated by applying the offered load to the Erlang-B formula. This approximation is naturally supported by the stochastic model, indeed the full highway PALM in Section 5; also see Sections 5 and 7 of [7] for further discussion.

Table 1 presents the blocking probabilities at $t = 30$ and 50, for which the call density has been depicted in Figures 1 and 3, respectively. As shown in the table, assuming that each cell has 20 channels, the blocking probabilities at $t = 30$ are a fraction of a percent, which are satisfactory. However, due to the traffic congestion caused by the accident, the blocking probability in the cell at location $(4,6]$ at $t = 50$ increases to 37.1%! In fact, it is found that for the surge of offered load, the cell has to be equipped with 45 channels to maintain the blocking probability satisfactorily low. Similarly, the cell at location $(6,8]$ also requires five additional channels to handle the offered load adequately. These results show that the proposed traffic models can serve as a valuable tool for system engineering and planning.

We have used the time-homogeneous fluid model to consider only the space dynamics. Our general observation is that the space dynamics and calling patterns have significant impacts on the traffic loads even for systems in temporal steady state; see [5] for details. We have also considered other examples for time-homogeneous cases where the highway has multiple entrances and exits. In these cases, the ODE's are solved for segments of the highway between two successive entrances/exits. Based on the vehicular densities at the end of one segment (i.e., just before an entrance/exit), the probability of a vehicle leaving from the exit, and the flow of vehicles entering from the entrance, we can obtain the initial conditions for the next segment. Then, solving the ODE's with these initial conditions yields the vehicular densities in the next segment of the highway.

4. The Stochastic Traffic Model

In contrast to the deterministic fluid model introduced above, the stochastic traffic model considers the random calling status of each individual vehicle as it moves along on the highway. However, it turns out that the PDE's and ODE's which govern the expected values are identical to those of the deterministic fluid model. Hence, the numerical examples that we have just considered apply equally well to the stochastic model. In Figures 1-4, we must simply replace the actual values on the y-axis by expected values.

The stochastic model has the same highway setting. Unless stated otherwise, the same notation is used as for the deterministic fluid model. Using the same definitions, $N(x,t)$, $Q(x,t)$, $\mathbf{C}_n^+(x,t)$, $\mathbf{C}_n^-(x,t)$, $\mathbf{C}_q^+(x,t)$, $\mathbf{C}_q^-(x,t)$, $\mathbf{E}_n^+(x,t)$, $\mathbf{E}_n^-(x,t)$, $\mathbf{E}_q^+(x,t)$ and $\mathbf{E}_q^-(x,t)$ become integer-valued random variables in the stochastic model. Now, the densities $n(x,t)$ and $q(x,t)$ are defined as the partial derivatives of expected values; i.e., $n(x,t) \equiv \partial E[N(x,t)]/\partial x$ and $q(x,t) \equiv \partial E[Q(x,t)]/\partial x$, respectively, where $E[Y]$ denotes the expected value of Y. Correspondingly, we let the rate densities be the second partial derivatives of expected values; i.e.,

$$c_n^+(x,t) \equiv \partial^2 E[\mathbf{C}_n^+(x,t)]/\partial x \partial t, \quad c_n^-(x,t) \equiv \partial^2 E[\mathbf{C}_n^-(x,t)]/\partial x \partial t,$$

$$c_q^+(x,t) \equiv \partial^2 E[\mathbf{C}_q^+(x,t)]/\partial x \partial t, \quad c_q^-(x,t) \equiv \partial^2 E[\mathbf{C}_q^-(x,t)]/\partial x \partial t,$$

$$e_n^+(x,t) \equiv \partial^2 E[\mathbf{E}_n^+(x,t)]/\partial x \partial t, \quad e_n^-(x,t) \equiv \partial^2 E[\mathbf{E}_n^-(x,t)]/\partial x \partial t,$$

$$e_q^+(x,t) \equiv \partial^2 E[\mathbf{E}_q^+(x,t)]/\partial x \partial t \text{ and } e_q^-(x,t) \equiv \partial^2 E[\mathbf{E}_q^-(x,t)]/\partial x \partial t.$$

For the stochastic model, let $\alpha(t)$ be the total arrival rate of vehicles arriving to the highway at time t. Thus,

$$\alpha(t) \equiv \frac{\partial}{\partial t}\{E[\mathbf{E}_n^+(\infty,t)] + E[\mathbf{E}_q^+(\infty,t)]\}. \quad (4.1)$$

We also make the following assumptions:

1. Vehicles arrive to the highway according to a pair of two-dimensional stochastic jump processes $\mathbf{E}_n^+(x,t)$ and $\mathbf{E}_q^+(x,t)$ with nondecreasing sample paths having only unit jumps and deterministic

intensity functions $e_n^+(x,t)$ and $e_q^+(x,t)$, where the total arrival rate $\alpha(t)$ in (4.1) is integrable over all $-\infty < t < \infty$.

2. Vehicles move forward on the highway according to a deterministic velocity field $v(x,t)$.

3. The state of each vehicle after it arrives evolves as a nonstationary continuous-time Markov chain, while it moves deterministically down the highway. The Markov chains of different vehicles are conditionally stochastically independent given their arrival times. (The Markov chains are not unconditionally independent due to dependence induced through the arrival times, but once we condition upon the arrival times, there is no dependence left.) A calling vehicle becomes a non-calling vehicle and vice versa (due to call termination and initiation) randomly with intensity $\mu(x,t)$ and $\lambda(x,t)$, respectively. In addition, a calling (non-calling) vehicle leaves the highway randomly with intensity $\gamma(x,t)$ ($\beta(x,t)$).

4. Each cell has an infinite number of channels such that no call blocking occurs.

As in [6] and [7], we can construct $Q(x,t)$ by stochastic integration as follows. For $j \geq 1$, let $T_s^+(j)$ and $T_s^-(j)$ be the time when a vehicle arriving to the highway at time s initiates and terminates its j^{th} call, respectively. We have

$$s \leq T_s^+(1) \leq T_s^-(1) \leq T_s^+(2) \leq T_s^-(2) \leq \cdots$$

Then,
$$Q(x,t) = \int_{-\infty}^{t} 1_{\{L_s(t) \in (-\infty,x] \times \{1\}\}}\, dA(s), \tag{4.2}$$

where $A(t)$ counts the number of vehicles arriving to the highway up to time t, 1_B is an indicator function such that $1_B = 1$ if B is true and 0 otherwise, and the location process $L_s(t)$ specifies the position and calling status of the vehicle that arrived at time s. That is, $L_s(t) = (x,k)$ where x is the position on the highway and

$$k = \begin{cases} 1 & \text{if } t \in \bigcup_{j=1}^{\infty} [T_s^+(j),\, T_s^-(j)) \\ 0 & \text{otherwise.} \end{cases} \tag{4.3}$$

In this context, an analog of Lemma 2.1 holds, which is a natural extension of (2.7) of [7].

Lemma 4.1: *In the stochastic traffic model, the densities of non-calling and calling vehicles, $n(x,t)$ and $q(x,t)$, satisfy (2.1) and (2.2).*

Similarly, we have the following.

Lemma 4.2: *The results in Lemma 2.2, namely (2.3) to (2.6), hold for the stochastic traffic model.*

We can combine Lemmas 4.1 and 4.2 to obtain an analog of Theorem 2.1.

Theorem 4.1: *In the stochastic traffic model, the densities of non-calling and calling vehicles, $n(x,t)$ and $q(x,t)$, at any location $x \geq 0$ and time $-\infty < t < t$ satisfy (2.7) and (2.8).*

Since the PDE's for the stochastic model are identical to those for the deterministic fluid model, Lemma 2.3 remains applicable to convert them into ODE's.

Now suppose that the highway is divided into cells at some locations $\{x_0(\equiv 0), x_1, x_2, \cdots\}$, $Q_i(t)$ is the number of ongoing calls (i.e., offered load) in cell i at time t, and $H_i(t)$ is the number of calls handed off from cell $i-1$ to cell i before time t. The following is an analog of Theorem 2.2; it follows from Theorem 3.1 of [7].

Theorem 4.2: *In the stochastic traffic model,*
(a) For each cell $i \geq 1$ and at any given time $-\infty < t < \infty$, $Q_i(t)$ is a stochastic process with mean

$$E[Q_i(t)] = \int_{x_{i-1}}^{x_i} q(x,t)\, dx. \tag{4.4}$$

(b) For each cell $i \geq 1$, $H_i(t)$ is a stochastic process as a function of t with rate

$$h_i(t) \equiv dE[H_i(t)]/dt = q(x_{i-1},t)\, v(x_{i-1},t). \tag{4.5}$$

5. The Markovian Highway PALM

We obtain the full Markovian highway PALM simply by assuming, in addition to the assumptions of Section 4, that $\mathbf{E}_q^+(x,t)$ and $\mathbf{E}_n^+(x,t)$ are independent two-dimensional *Poisson* counting processes. Let $\mathbf{E}_q^+(B)$ and $\mathbf{E}_n^+(B)$ be random measures associated with the stochastic counting processes $\mathbf{E}_q^+(x,t)$ and $\mathbf{E}_n^+(x,t)$; i.e., $\mathbf{E}_{q/n}^+(B)$ counts the number of arrivals in the set B where B is a set of (x,t) pairs in $[0,\infty) \times \mathbf{R}$. The Poisson assumption means that the numbers of arrivals $\mathbf{E}_q^+(B_i)$ and $\mathbf{E}_n^+(B_i)$ of calling and non-calling vehicles in disjoints subsets $B_i, 1 \leq i \leq n$, of $[0,\infty) \times \mathbf{R}$ are mutually independent random variables with Poisson distributions determined by the deterministic intensity functions $e_q^+(x,t)$ and $e_n^+(x,t)$, respectively; e.g.,

$$P(\mathbf{E}_n^+(B) = k) = [\gamma^+(B)^k\, e^{-\gamma^+(B)}]/k!, \tag{5.1}$$

where
$$\gamma^+(B) = \int\int_B e_n^+(u,v)\, du dv. \tag{5.2}$$

E.g., for $B = [0,x] \times (-\infty,t]$ and $\gamma^+(x,t) \equiv \gamma^+(B)$ for this B,

$$\gamma^+(x,t) = \int_0^x \int_{-\infty}^t e_n^+(u,v)\, du dv. \tag{5.3}$$

With this extra Poisson assumption, the PALM results in [6, 7] imply the following. (See Theorem 3.1 of [7].)

Theorem 5.1: *If, in addition to the assumptions in Section 4, $\mathbf{E}_n^+(x,t)$ and $\mathbf{E}_q^+(x,t)$ are independent two-*

dimensional Poisson counting processes, then the stochastic processes $\{Q(x,t) : x \geq 0\}$ *and* $\{H_i(t) : -\infty < t < \infty\}$ *are Poisson processes. Moreover,* $Q_i(t)$ *for* $i \geq 1$ *are mutually independent Poisson random variables with means in (4.4).*

If only the location dynamics at temporal steady state are of interest, the arrival process can be a time-homogeneous Poisson process. All results of the Markovian PALM presented above remain valid simply because a stationary Poisson process is a special case of time-homogeneous Poisson process.

6. Conclusions

We have presented a deterministic fluid model, a stochastic traffic model and a Markovian highway PALM for a wireless network along a highway. Vehicles can enter and leave the system at multiple entrances and exits, and they are classified as non-calling and calling vehicles, depending on whether they have calls in progress. All three models use the same two coupled PDE's or ODE's to describe the evolution of the system. The call density and call handoff rate are readily computable by solving the equations. Numerical examples were presented to illustrate the computability of our results and investigate various aspects of the time and space dynamics of wireless networks. The numerical results indicate that both the time-dependent behavior and the mobility of vehicles play important roles in determining the system performance. Furthermore, our numerical examples also show how the proposed models can be used to approximate blocking probabilities. Thus, the models are useful for planning and engineering wireless networks.

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REFERENCES

[1] R. Haberman, *Mathematical Models: Mechanical Vibrations, Population Dynamics, and Traffic Flow*, Prentice-Hall, New Jersey, 1977.

[2] D.L. Jagerman, "Nonstationary Blocking in Telephone Traffic," *Bell System Tech. J.* 54 (1975), 625-661.

[3] W.C.Y. Lee, *Mobile Cellular Telecommunications Systems,* McGraw-Hill, New York, 1989.

[4] W.C.Y. Lee, *Mobile communications Design Fundamentals,* Wiley, New York, 1993.

[5] K.K. Leung, W.A. Massey and W. Whitt, "Traffic Models for Wireless Communication Networks," to appear in *IEEE JSAC.*

[6] W.A. Massey and W. Whitt, "Networks of Infinite Server Queues with Nonstationary Poisson Input," *Queueing Systems*, 13 (1993), 183-250.

[7] W.A. Massey and W. Whitt, "A Stochastic Model to Capture Space and Time Dynamics in Wireless Communication Systems," submitted for publication.

[8] K.S. Meier-Hellstern, E. Alonso, D.R. O'Neil, "The Use of SS7 and GSM to Support High Density Personal Communication Systems," in *Third WINLAB Workshop on Third Generation Wireless Networks,* April 1992.

[9] G. Montenegro, M. Sengoku, Y. Yamaguchi, and T. Abe, "Time-Dependent Analysis of Mobile Communication Traffic in a Ring-Shaped Service Area with Nonuniform Vehicle Distribution," *IEEE Trans. Vehicular Tech.,* 41 (1992), 243-254.

[10] I. Seskar, S. Maric, J.M. Holtzman, and J. Wasserman, "Rate of Location Area Updates in Cellular Systems," in *Third WINLAB Workshop on Third Generation Wireless Networks,* April 1992.

[11] P.E. Wright, "A Vehicular-Traffic Based Model of Cellular Systems," Technical Memorandum, AT&T Bell Laboratories, Murray Hill, New Jersey, 1993.

Time	Cell Location	Offered Load (Erlang)	No. of Channels/Cell	Blocking Prob.
t=30	(2,4]	8.2455	20	0.00023
	(4,6]	9.4716	20	0.00107
	(6,8]	9.8408	20	0.00159
	(8,10]	9.9520	20	0.00178
t=50	(2,4]	11.2459	20	0.00565

	(4,6]	29.4977	20	0.37105
	(4,6]	29.4977	45	0.00179
	(6,8]	13.2327	20	0.02055
	(6,8]	13.2327	25	0.00127
	(8,10]	9.9980	20	0.00187

Table 1. Approximate Blocking Probabilities for the Fixed Cell Size.