Price Competition with Rationally Inattentive Consumers

Weijie Zhong

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1 Introduction

The main purpose of this paper is to study markets in which consumers are rationally inattentive to price settings. A classic assumption on price competition is that consumers have full knowledge about the market. Rational consumers gather information about price, quality, location and all related features of a product freely and make an optimal decision. This results in a highly competitive market and finally drives firm to sell at their marginal cost. However prices in real markets behave much more complicated than prediction of this model. Economists doubt that consumer's limited knowledge about the market might be a necessary friction that generates strategic pricing behaviors.

We start by altering the full information assumption in this model: consumers form a prior belief about firm's price but do not observe the exact realization of each firm's price before visiting the firm. This is a common but controversial assumption in most models with inattentive consumers. It's true that prices are becoming more and more transparent if one item is sold on an online platform. However this assumption still applies to other industries like automobile, real estate or fashion that require consumers visiting a firm to get the finalized price. Even in online market, price might be hard to discover because of unobservable add-ons or other post-purchase services. For example learning price of an iMac is simple because apple releases limited number of configurations, while learning price of a server is extremely hard before finalizing the purchase because most manufacturers provides thousands of combinations of configurations combined with bundle offers that make the price of a single part mysterious. After introducing our model, we will find that this phenomenon, i.e. reduced price dispersion in PC industry, can be explained by reduced studying cost. While at the same time, professional computation market still requires high skill to learn.

Instead of assuming any specific form of inattention or irrationality like what existing literature does, we gives consumer full flexibility in what they can learn about the market. Consumer can conduct a general Blackwell experiment to figure out the realized prices as long as they pay corresponding information processing cost. Information processing cost is defined as proportional to the mutual information between the price and the experiment. This is widely used in rational inattention models introduced by [Sims, 1998] and [Sims, 2003] as a linear quadratic normal version. Mutual information can be understood as the number of binary questions related to the interested object one need to ask to get a certain level of information.

The main result of our paper is a characterization of equilibrium price strategies of firms in a market with rationally inattentive consumers. We found that firms will adopt pure strategy on some sunspot random signals. Deterministic equilibrium with firms charging marginal cost always exists, while equilibrium with independent price schedules never exists. We studied a simple example and find the firm are more likely to mis match their prices, i.e. offer sales in turn in equilibrium. We also did comparative statics on the example.

1.1 Review of Literatures

This paper borrowed techniques from the rational inattention models. The version [Sims, 1998] developed involves continuous choice but restricted payoff and distribution function. We borrowed a discrete choice model of rational inattention from [Matejka et al., 2015], which provides solution to a general discrete choice rational inattention problem as a general logit model. Literatures that embeds rational inattention to price competition are rare. [Matejka and McKay, 2012] studies symmetric equilibrium in a model where consumers are uninformed about their valuation and established link to logit model.

Our model has a similar research question as the search models, to model and explain price dispersion in a homogeneous market. [Varian, 1980] started the search literature with a passive search model in which consumers have heterogeneous information about the market. [Stahl, 1989] endogenized the heterogeneous information in a active search model in which consumers have heterogeneous search cost. [Burdett and Judd, 1983] studies non-sequential search and identical consumers will adopt different search rules. Our model differs as search is a specific form of general Blackwell experiment and our cost of experiment is endogenously decided by randomness in the market.

Our model is also related to irrational or inattentive consumer models. [Spiegler, 2006] assumes consumer only observes lower bound of a firms' price schedules. [Gabaix and Laibson, 2006] assumes naive consumer neglects product's add-on prices. [De Roos and Smirnov, 2015] assumes naive consumer failing to observe firms' prices other than a market leader's price. [De Roos and Smirnov, 2015] studies a dynamic model where consumers have only limited memory of past prices. Our model is a completion of inattentive consumer model by endogenizing inattention as a rational consumer's optimal choice.

Our paper is organized as following. We setup the model in section 2. Then solve the model and characterize the equilibrium in section 3. We discuss some limiting results in section 4. In section 5, we discuss in detail how our paper relates to search and consumer inattention literature. In section 6 we solve a simple example and perform some comparative statics. Finally we summarize in section 7. Detailed proofs are contained in the appendix.

2 Setup of Model

We consider a model in which consumers can not observe firm's price realization and need to conduct experiment on price distribution. The design of experiment can be flexible but consumers are required to pay a cost proportional to the mutual information between the experiment and price. The details of the model are as following:

- **Firms:**

  There are N firms in the market, indexed by $i = 1, \dots, N$. Each firm will receive a private signal t_i from a signal space T, all signals t satisfies joint distribution $f(t) \in \Delta T^N$, which is common knowledge to all players. Firms set their price before the realization of signals, they can design and commit to a price schedule $g_i(p_i|t_i) \in \Delta\mathbb{R} \times T$ contingent on their signal realization. Thus the price distribution will be $\pi(p) = \int_{t \in T} \Pi_i g_i(p_i|t_i) f(t) dt$.

  If a price p_i is realized and firm i gets visited by consumer, then the firm offer p_i as a take or leave it offer. Thus the strategy of firms can be summarized as price distributions $g = (g_i)$.

- **Consumers:** There are homogeneous consumers of mass one. Or equivalently we can assume there is one unique consumer. Consumer has valuation U for one unit of the good provided by firms. Consumer can observe firms' price strategy g but doesn't know the realization of prices. Consumer can conduct an experiment taking the form $(S, h(s|p))$ where S is an arbitrary chosen signal space and h is the conditional distribution of signal on price. Let's abuse notation a little and let S and T represent also random variable which are realization of s and t. Then consumer need to pay a cost $cI(S, T)$ for experiment (S, h).

  After observing the realization of signal, consumer chooses an best firm to visit according to $P_i(s) \in \Delta N \times S$ and then see the realization of price. Then he chooses whether to accept that offer (when $U \geq p_i$) and leaves the market. Thus consumers choice of accepting or rejecting offer is passive and we can summarize consumer's strategy as $(S, h, (P_i))$.

- **Timing of Game:**

  1. Firms choose price strategy g.
  2. Private signals are realized and prices are realized according to g.
  3. Consumer conduct experiment (S, h) and see the realization of outcome s.
  4. Consumer choose optimal firm to visit according to (P_i) and see the price.
  5. Consumer decide whether to accept the offer.

Remark.

- We assume that consumers are totally homogeneous ex ante to distinguish the price dispersion created by consumer heterogeneity and by rational inattention. Here even consumers are identical ex ante, the randomness in the outcome of experiment might creates heterogeneity ex post.

- We assume that firms can commit to their price strategy after observing realization of signals. We will later also solve the case without commitment power and compare them.

Now let's write down formally the optimization problem faced by consumer and firms. This game is effectively a three-stage game. In stage one, firms decide their price strategy and price is realized. In stage two, consumer decides experiment design and out come is realized. In stage three, consumer decides which firm to visit and offer choice.

2.1 Consumer's Problem

In stage three, the behavior after observing the price will be passive, consumer accepts offer if $U \geq p_i$ and gets surplus$\max\{U - p_i, 0\}$. Conditional on the outcome of experiment, consumer forms some belief about the price distribution according to Bayes rule. Let's denote this posterior belief by $\mu \in \Delta\mathbb{R}^N$. Thus consumer's expected surplus from choosing a firm i will be $\int_p \max\{U - p_i, 0\}\, d\mu$. We can summarize consumer's behavior in stage three as:

$$V(\mu) = \max_i \left\{ \int_p (U - p_i) d\mu, 0 \right\} \tag{1}$$

In stage two, given any prior on price $\pi(p)$, consumers design experiment (S, h) which generates posterior beliefs $\mu(p|s) = \frac{h(s|p)\pi(p)}{\int_p h(s|p)\pi(p)d\mu}$. The payoff contingent on signal s will be $V(\mu(\cdot|s))$ with V as defined in stage three. Thus consumer's problem can be defined as following:

$$\begin{aligned} \max_{S, h(s|p) \in \Delta S \times \mathbb{R}} & E_s\left[V(\mu(\cdot|s))\right] - cI(S, P) \\ s.t.\ \mu(p|s) &= \frac{h(s|p)\pi(p)}{\int_p h(s|p)\pi(p)dp} \end{aligned} \tag{2}$$

Here consumer's problem is depend on the prior distribution of price π. Given the optimal signal structure (S^*, h^*) and optimal choice rule $P_i(s)$, we can define consumer's purchasing probability as $P_i(p, \pi) = \int_s P_i(s)h^*(s|p)ds$, which depends both on the price distribution and realization. Here we also abuse notation a little by redefining P_i.

2.2 Firms' Problems

In stage one, given any signal distribution $f(t)$, each firm choose pricing strategy contingent on signal realization $g_i(p_i|t)$. Combining all firms' strategies, we get the price distribution $\pi(p) = \Pi g_i(p_i|t)f(t)$. Firm i's expected gain with realized price vector p will be $P_i(p, \pi) \times p_i$. Thus firm i's problem can be defined as following:

$$\begin{aligned} \max_{g_i(p_i|t) \in \Delta\mathbb{R} \times T} & E_t\left[E_{p|t}\left[P_i(p, \pi) \times p_i | t\right]\right] \\ s.t.\ \pi(p) &= \Pi g_i(p_i|t)f(t) \end{aligned} \tag{3}$$

Now assume we have already solved the problem in stage two and three, then we can plug the response of consumer $P_i(p, \pi)$ into firms' problems and the game reduces to a static Baysian game among the firms. Thus we can define the equilibrium by borrowing the notion of Bayesian Nash Equilibrium:

Definition 1.

1. *A Bayesian Nash Equilibrium of this game is defined as a set of price distributions* $\{g_i(p_i|s_i)\}$ *solving problem 3 for each firm* i.
2. *An interior Bayesian Nash Equilibrium of this game is a BNE in which* $E_p [P_i(p, \pi)] > 0$ *for all* i.
3. *A Pure Strategy Bayesian Nash Equilibrium of this game is a BNE in which* $Var(p_i|s_i) = 0$ *for all* i.

In Definition 1, we define three notions of equilibria. BNE is simply the general definition for BNE. By interior BNE we restrict to the case where all firms will be visited by consumer with positive probability. Otherwise the equilibrium is non-interesting for the firm that is never visited and we can just erase that firm. By PSBNE, we effectively require firm to choose price as a function of private signals.

3 Characterizing Equilibrium

The three optimization problems defined in last section are clean in expression. But solving them involves theoretical difficulties because we need to deal with optimization with respect to general function space. Before proceeding to solving the problem, let's first derive a few simplifications of the problem.

Lemma 2. *In any interior BNE,* $\forall i, t$, $g_i(p_i|t)$ *will have support contained in* $[0, U]$.

Lemma 2 is intuitively easy to understand. First charging any price negative will be non-profitable for firms. Second, since consumer can reject offer when offered price $p_i > U$, charging over U will be non-profitable for firms. Thus firms should price only in $[0, U]$. However Lemma 2 is stronger than this argument by claiming that charging outside of $[0, U]$ will be strictly non-profitable. Technical details involved in the proof will be available in the appendix.

3.1 Solving Consumer's Problem

We solve the consumer's problem explicitly by providing the following two statements:

Lemma 3. *Consumer's problem is equivalently:*

$$\max_{P_i(p)\in[0,U]^N} \sum_i P_i(p)(U - p_i) - cI(I, P)$$
$$s.t. \sum P_i(p) = 1 \ \forall p$$

where random variable I *is defined as the decision of consumer.*

Lemma 3 transforms consumer's problem into a problem that can be explicitly solved. And the solution to this problem is:

Proposition 1. *In an interior BNE, consumer's optimal decision rule is defined as the unique solution to:*

$$\begin{aligned} P_i(p) &= \frac{P_i^0}{\sum P_j^0 e^{(p_i - p_j)/c}} \\ \int_p \frac{1}{\sum P_j^0 e^{(p_i - p_j)/c}} \pi(p) dp &= 1 \end{aligned} \tag{4}$$

Since $P_i(p)$ is used effectively as firms' demand functions in firms' problems, Proposition 1 characterizes firms' demands. We see that consumer responds to price set by the firm in an elastic way. This is true because in our model, consumer doesn't observe the realization of prices. In stead, consumers conduct experiment and get informative signals on firms' prices. Changing the realized price in a certain state by a firm will have two effect. First since the consumer is not totally unaware of price, this will shift consumer's choice. Second, since the value of information changed, this will shift consumer's experiment design. Combining these two effects, when there is some randomness in the price in the market, firms may avoid Bertrand competition and earn a positive margin.

3.2 Solving Firms' Problems

Let's first establish a non-trivial lemma which significantly simplifies the strategy space of firms.

Lemma 4. *All interior BNE are PSBNE.*

The proof of Lemma 4 involves solving the problem explicitly. However we can gain some vague intuition why firms are not willing to add extra randomness into their price. In our model, the total surplus among firm and consumers are constant as U. Thus a firm does not want consumer to waste effort in searching for its price. Instead, the firm can transfer the saved surplus by charging a higher fixed price given any signal. Thus the final optimization problem we need to solve is as following:

$$\begin{aligned} \max_{p_i(t_i)} & \int P_i(p(t)) p_i(t_i) f(t) dt \\ s.t.\ P_i(p) &= \frac{P_i^0}{\sum_j P_j^0 e^{(p_i - p_j)/c}} \\ \int & \frac{1}{\sum_j P_j^0 e^{(p_i(t_i) - p_j(t_j))/c}} f(t) dt = 1 \end{aligned} \tag{5}$$

Solving the problem 5 gives us the main theorem characterizing equilibrium pricing strategy.

Proposition 2. *Interior BNE of the game satisfies the following properties:*

1. *There always exists an unique deterministic equilibrium in which $p_i = 0\ \forall i$, that is, firms compete like in Bertrand.*

2. *In any equilibrium with price dispersion, price strategy $p(t)$ is the fixed point of the following equation:*

$$p_i(s_i) = c\frac{E_{t|t_i}\left[P_i(p)|t_i\right]}{E_{t|t_i}\left[P_i(p) - P_i(p)^2|t_i\right]} - \frac{E_t\left[p_i(s_i)\left(P_i(p) - P_i(p)^2\right)\right]}{E_s\left[P_i(p)^2\right]} \tag{6}$$

   where $P_i(p)$ defined as the solution to consumer's problem.

With the characterization derived in proposition 2, we can have a closer study in firms' pricing behaviors. First, deterministic equilibrium with each firm offering a constant price over states is qualitatively different from equilibrium with price dispersion. This is because when all firms are charging a deterministic price, the probability purchasing from a particular firm i will be discontinuous in price p_i. Thus the first order condition approach relying on smoothness is no longer valid. Specifically, while decreasing p_i to lower than other firms' prices, the denominator of the first term on RHS of (2) changes from negative infinity to zero. Thus we are effectively in a Bertrand case. This result is intuitive because when firms are charging deterministic price, consumer will have full knowledge of the market at zero cost.

Second, the two terms on RHS of (2) characterizes exactly the two effect of changing price in one state described after Proposition 1. The first term is a common term in monopolistic pricing. The nominator is demand and the denominator is negative first derivative of demand. Thus the whole term is demand times marginal change of price caused by changing demand by one unit. This term is gain because of consumer's experiment design gives them elastic demand on realized prices. This is exactly the first effect of changing price on consumer's decision rule.

The second term comes from the second effect of changing realized price. When a firm increase realized price in one state, this will cause an infinitely small negative effect on consumer's prior belief of which firm to visit. However, by doing this the gain from increased price is also infinitely small from an ex ante view. By comparing these two small effect, we find that they actually are of the same scale and will lead to underpricing comparing to case without this effect. To illustrate this point, let's focus on a slightly different model where firms don't have commitment power on their price. Thus firm are effectively optimizing on realization of each signal and will neglect the effect on whole price distribution:

Proposition 3. *In a game without commitment on price strategy, in any interior BNE with price dispersion, price strategy $p(t)$ is the fixed point of the following equation:*

$$p_i(s_i) = c\frac{E_{t|t_i}\left[P_i(p)|t_i\right]}{E_{t|t_i}\left[P_i(p) - P_i(p)^2|t_i\right]} \tag{7}$$

where $P_i(p)$ defined as the solution to consumer's problem.

4 Reducing to Bertrand

In this section, we are going discuss several limiting cases where the equilibrium of our model reduces to Bertrand equilibrium. These limiting cases are interesting because some of them are theoretically desirable. For example we want the model to reduce to Bertrand if information processing cost goes to zero. Some cases are useful for us to clarify the source of price randomness in a homogeneous market. For example the case with independent pricing.

Corollary 4. *The only interior BNE of the game is Bertrand, that is, $p_i = 0$ $\forall i$ if any one the following is true:*

1. *Private signals are independent to each other: $f(t) = \Pi f_i(t_i)$.*
2. *Information processing cost is zero.*

The second condition of Corollary 4 is easiest to see. Applying $c = 0$ yields no solution with price dispersion in (6). The intuition behind it is also straight forward, when consumer has zero information processing cost, they can observe price realizations perfectly. As a result firms are effectively competing as Bertrand in every realized state. Thus in every possible state only equilibrium will be $p = 0$. The first condition is also easy to understand. It's a corollary to Lemma 4. If all private signals are independent to each other, then signals actually doesn't change a single firm's belief about other firms pricing strategy. By the logic of Lemma 4, a firm doesn't have incentive to add extra randomness, thus he will charge a fixed price independent to his own signal.

We also have the following conjectures as stronger version of Corollary 4:

Conjecture 5. *Considering a series of games indexed by k, The resulting price distribution $\pi_k(p)$ of the interior BNE of the game converges to degenerate distribution on $p = 0$ by (Probability, Distribution) if any one of these is true:*

1. *Holding $f(t)$ fixed, $c_k \to 0$*
2. *Holding c fixed, $f_k(t) \to f^I(t)$ by (Probability, Distribution) where $f^I(t)$ has independent signals*

The discussion in this section gives us some insight on the source of randomness of prices. Randomness in price is actually like a self sustaining sunspot in a homogeneous market. Given our assumption on rational inattentiveness of consumers, randomness is a product of randomness itself. If a firm has unified posterior belief about other firms strategy, then a firm will not offer random prices. And in a market where randomness in prices is low, it costs little for consumer to figure out the price, thus competition is highly strengthened and the market end up reaches Bertrand equilibrium.

This provides foundation for observed properties of sales in real life which are hard to sustain in other models. For example, sales are almost never independent. What's always happening is that firms tie their sale to some sun spot like events and holidays that are unrelated to the fundamental of the market. Take another example, when firms start price war, they lose unpredictability in their prices, thus consumers can easily discover prices and firms end up playing Bertrand. However, on the other hand, it's hard to find markets where firms coordinate on full extraction of surplus, which is a key prediction of search models.

5 Relation to Literatures

In this section, we compare our model with existing models concerning competitive market with non fully informed consumers. The literature mainly falls into two categories, one is search models which assume consumer pays search cast to survey prices in a market, the other is consumer inattention models which impose specific forms of inattentions. Our model of course differs from these in assumptions. We allow full flexibility in what consumer can learn about the market. The search model actually allows a special form of learning, which reveals the price perfectly for a single form in one search. And in the inattention models, the form of inattention, that is to say what a consumer can learn, is assumed by model. The most important difference is that our model creates randomness in price through a different mechanism and thus creates different but desirable implications

[Varian, 1980] is the first paper utilizing search friction to explain price dispersion. The model is simplest among search models but all following models adopt the same mechanism. Consumers are ex ante heterogeneous with fully informed shoppers and fully uninformed naive consumers. Thus firms have two offsetting incentives, if they charge high price, they can extract high surplus from naive consumers, if they charge low price, they can attract shoppers. Combining the two incentives make the firms indifferent in choosing price. There only exists mixed strategy equilibrium and [Varian, 1980] focused on symmetric independent equilibrium. Their prediction is high probability of monopolistic pricing. [Stahl, 1989] endogenized the information different consumer holds by assuming consumers with different search cost doing sequential search. Their prediction is very similar to [Varian, 1980]'s. The key mechanism creating price dispersion is heterogeneity in consumers, and elasticity created by search behavior. However, heterogeneity in consumer allows independent pricing, which is not possible in our model. Also the existence of effectively naive consumer leads to high probability of monopolistic pricing.

[Burdett and Judd, 1983] dropped ex ante heterogeneity in consumers by assuming identical consumers doing non-sequential search. There might exist equilibrium in which consumers are indifferent ex ante in the number of firms they sample. Thus, ex ante homogeneous consumers will adopt heterogeneous experiments and this creates price dispersion. There model is close to our one in predicting diminishing of search when price becomes deterministic. However, they also have a sharply different implication of existence of monopolistic pricing equilibrium. This is because the cost of search is exogenous while in our model, the cost of learning is endogenous decided by the complexity of price schedules. Since study of deterministic price is no less costly, monopoly is sustained in the search model. In our model, studying deterministic price is costless, Bertrand is sustained. Also, although consumers are ex ante identical, they will have heterogeneous learning behavior, which allows existence of independent pricing. In our model, both consumers and their experiment designs are identical, heterogeneity is only ex post. Thus we predicts non-existence of both fully correlated equilibrium and independent equilibrium.

In the literature of consumer inattention, specific forms of inattention or irrationality are assumed. In [Spiegler, 2006], consumers are assumed to choose firm to visit according to lower bound of firm's pricing schedule but they actually have to pay the realized price. They characterized the resulting equilibrium in which price randomness increases with competition. In [Gabaix and Laibson, 2006], part of consumers are assumed to be naive about firm's add on price and each firm can choose whether to reveal the add on price. Their model also relies on heterogeneity in consumers and they found existence of shrouding equilibrium when ratio of naive consumers is high. [De Roos and Smirnov, 2015] embeds consumer inattention to multi markets. They assumed that naive consumer only observes a market leader's price setting and sophisticated consumer can investigate other firms' prices in a market. [Varian, 1980] studies an dynamic competition model where consumers have limited memory on history prices. A common feature of these models is that inattention is exogenous. Since inattention itself is inelastic, most models still need to relies on heterogeneous consumers to generate more interesting price strategies. Some models like [Spiegler, 2006] doesn't require heterogeneity but the assumption on irrationality is quite hand-waving.

Our model contributes to this branch by establishing a framework with rationally inattentive consumers who can perform a general study to complete his knowledge about the market. This assumption is not only more general but also models one dimension of market manipulation that hasn't been studied. We doesn't only allow firm to utilize consumer's inattention but also allows firm to manipulate consumer's inattention.

6 Examples

In this section, we study a simple example with two firms and binary signals that can be solved analytically. Thus we can perform some comparative statics on this example. We assume $N = 2$, $T = \{L, H\}$ are distributed symmetrically among firms. We assume U sufficiently large to avoid discussing the corner case. The probability matrix between signals is assumed as:

	L	H
L	a	b
H	b	a

Each element represents unconditional probability of a signal corresponding signal pair happens. We focus on symmetric equilibrium in which firms set p_L when $t_i = L$ and p_H when $t_i = H$. Thus we can assume $a \in [0, 0.5]$ to incorporate both positively correlated pricing $a > 0.25$ and negatively correlated pricing $a < 0.25$.

By applying equation 6, we can solve for equilibrium price pairs:

$$\Delta p = c\frac{4b(e^{2\Delta p/c}-1)}{a(1+e^{\Delta p/c})^2+4be^{\Delta p/c}}$$
$$\Sigma p = c\frac{2a(1+e^{\Delta p/c})^2+4b(1+e^{2\Delta p/c})}{a(1+e^{\Delta p/c})^2+4be^{\Delta p/c}}$$

where $\Sigma p = p_L + p_H$ and $\Delta p = p_H - p_L > 0$.

With these two expression, we can directly see the limiting behavior of price when $c \to 0$ will be both price doing to zero. When taking the distribution of signals to fully correlated distribution, we can see that it's hard for positively correlated pricing equilibrium (with $a \to 0.5$) to exist but negatively correlated equilibrium (with $a \to 0$) can still exist. This is because positively correlated equilibrium leads to more competition and finally firms fall into Bertrand. But for negatively correlated equilibrium, firm gain surplus by alternatively charging high prices.

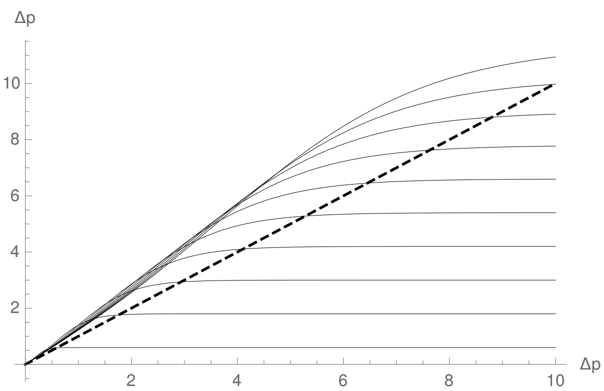

Figure 1: Δp with different c
The dashed line is 45-degree line, the crossing points with dashed line and solid lines are resulting Δp under different c. From up to down, c decreases from 2 to 0.2.

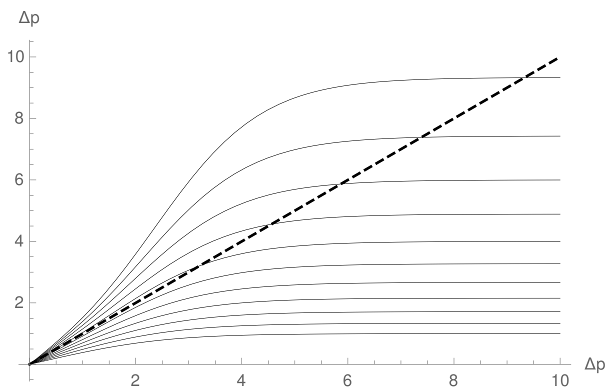

Figure 2: Δp with different a
The dashed line is 45-degree line, the crossing points with dashed line and solid lines are resulting Δp under different a. From up to down, a increases from 0.15 to 0.4.

We first numerically calculate the resulting price given different parameters. We choose $c = 1$ and $a = 0.2$ as a benchmark case. Figure 1 shows the comparative statics when changing information processing cost c. Decreasing information processing cost c decreases price dispersion. Figure 2 shows the comparative statics when changing correlation between signals. Decreasing correlation increases price dispersion. The intuition for the first result is that when information processing cost is high, consumer is less willing to conduct experiments even if the gain from identifying the state is high (Δp). When information processing cost decreases, under a previous equilibrium (Δp), consumer will conduct more informative experiment to pick the low price more often. The demand becomes more elastic, thus it's less profitable for firms to sustain large dispersion. For the second result, when a is closer to 0 or 0.5, competition is intensified because it's easier to predict the other firm's price when know one firm's price. Thus it's easier to sustain a larger negatively correlated price dispersion because firms effectively give up profit in one state to gain surplus in the other. It's harder to sustain a larger positively correlated price dispersion because firms will be competing more severely in each state.

Now we can do an analytical analysis on the effect of c and a on price. Assume $f_\Delta(\Delta p, a, c) = c\frac{(2-4a)(e^{2\Delta p/c}-1)}{a(1+e^{\Delta p/c})^2+(2-4a)e^{\Delta p/c}}$, $f_\Sigma(\Delta p, a, c) = c\frac{2a(1+e^{\Delta p/c})^2+(2-4a)(1+e^{2\Delta p/c})}{a(1+e^{\Delta p/c})^2+(2-4a)e^{\Delta p/c}}$, Thus Δp is the solution to $\Delta p = f_\Delta(\Delta P, a, c)$ and $\Sigma p = f_\Sigma(\Delta p, a, c)$. The following lemma gives a few properties on the two functions:

Lemma 5. *When $\Delta p > 0$, $\frac{\partial f_\Delta}{\partial \Delta p} > 0$, $\frac{\partial f_\Delta}{\partial a} < 0$, $\frac{\partial f_\Sigma}{\partial \Delta p} > 0$, $\frac{\partial f_\Sigma}{\partial a} < 0$.*

Proposition 6. *Given any interior BNE with price dispersion, when a decreases, both Δp and Σp strictly increase.*

The analysis with respect to c is more involved because even on figure 1 we can see that $f_\Delta(\Delta p, a, c)$ is not globally monotonic in c. Thus we will need equilibrium condition to analysis how the crossing point changes with c. We have the following conjecture:

Conjecture 7. *Given any interior BNE with price dispersion, when c increases, both Δp and Σp strictly increase.*

7 Conclusion

In this paper, we embedded rational inattentive consumer into price competition model. In Bayesian Nash Equilibrium, firm will play pure strategy contingent on a sunspot private signal. Deterministic equilibrium with firms charging marginal cost always exists and equilibrium with firms using independent price schedule never exists. With a simple example, we showed that firm are more likely to mismatch their prices when their signals are correlated. Increasing information processing cost or decreasing correlation both increases price dispersion.

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