Selling Information

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Introduction

Research Question

- Mechanism design in selling information.
  - Buyers facing a general decision problem with unknown payoff relevant states.
  - Monopoly seller can design and commit to providing general Blackwell experiments to buyers.
  - Buyers have heterogeneous beliefs as private information.
  - What's the optimal menu for the seller?
- Motivations: industries selling information goods:
  - Research report, database access or articles are sold as bundled information. There will be many versions or add-ons that can be purchased at different prices.
  - Consulting service is sold as service good. Consumer can choose length and intensity of service to control quality and price.

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Research Question

- A parallel question in classic screening problem: When will a rich menu be optimal?
  - In classic screening problem: concavity of surplus function.
  - In current problem: richness of choice set.

Introduction

Summary of Results

- We assume a continuum of alternatives in underlying decision problem. Utility and distribution of beliefs are well behaved.
  - Optimal mechanism includes experiments with up to two signals and up to one partially revealing signal.
  - For decision makers with intermediate beliefs, fully revealing experiment will be sold at highest price.
  - For decision makers with more extreme beliefs, experiments with decreasing informativeness will be sold at decreasing price.
- When distribution of beliefs become more dispersed:
  - A wider interval of buyers will be sold the fully revealing experiment. A wider interval of buyers will be included in the market.
  - For any buyer type (prior belief), she is sold a Blackwell more informative experiment and has higher surplus.
  - The highest price decreases.

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Review of Literature

- Monopoly price discrimination:
    - [Mussa and Rosen, 1978] solved linear problem and [Maskin and Riley, 1984] solved non-linear problem.
    - We have a much richer mechanism space than one dimensional quality space.
    - Experiments also interact with the general decision problem.
- Selling information:
    - [Horner and Skrzypacz, 2011] studied optimal persuasion schedule with private type on provider side.
    - [Bergemann and Bonatti, 2013] has one section on optimal mechanism of selling consumer level matching value data.
    - In [Eső and Szentes, 2007], information seller can contract on buyer's actions.
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  - Similar to our setup except for our generality in decision problem.
  - With binary actions in decision problem, value of information is linear in “quality”. Because everyone is interested in an identical choice problem. $\Rightarrow$ Flat price of fully revealing experiment.
  - Convexity is a general property in decision problem which predicts a rich menu of experiments.
- Value of information:
  - [Blackwell et al., 1951]: No general answer.
  - [Cabrales et al., 2010]: related to entropy for limited decision problem.
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Setup

- Payoff related state: $\Theta = \{l.h\}$.
  - Doesn't play a key role in our result. But higher dimensional problems can hardly be solved.
- Decision problem:
  - Actions: $A = \{a_i\}$.
  - Payoff: $u(a_i, \theta)$
  - Private information: belief $\mu = Pr(h) \in [0, 1]$.
- Monopoly Information provider:
  - Belief: $\mu \sim f(\mu)$.
  - Products: Blackwell experiments $(S_j, g(s_j|\theta))$ with price P_j.
  - Zero marginal cost of production.
  - No multiple purchasing.
    - Combining experiments keeps incentives.
  - No reselling.
    - Seller can avoid reselling of experimentation machine.
    - Cheap talk in reselling game. We focus on pooling equilibrium.

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Setup

Buyer's Problem

- Stage two:
  - With realization of signal inducing posterior belief $\hat{\mu}$, buyer choose $a(\hat{\mu}) = \arg\max_A \hat{\mu} u(a,h) + (1-\hat{\mu}) u(a,l)$.
  - We write $V(\hat{\mu}) = \hat{\mu} u(a(\hat{\mu}),h) + (1-\hat{\mu}) u(a(\hat{\mu}),l)$.
- Stage one:

$$\max_{j \in \mathbb{J}} \left\{ E_{s_j}\left[V\left(\hat{\mu}\left(\mu, s_j\right)\right)\right] - P_j, V(\mu) \right\} \tag{1}$$

$$s.t.\ \hat{\mu}(\mu, s_j) = \frac{g(s_j|h)\mu}{g(s_j|h)\mu + g(s_j|l)(1-\mu)}$$

$$\pi(s_j) = g(s_j|h)\mu + g(s_j|l)(1-\mu)$$

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Setup

Seller's Problem

- Applying revelation principle, seller's strategy can be represented by direct mechanism $\mathcal{M} = ((S_\mu, g_\mu), P(\mu))$.
- Seller's optimization problem:

$$\max_{\mathcal{M}} \int_0^1 P(\mu) f(\mu) d\mu \tag{2}$$

$$s.t.\ E_{s_\mu}\left[V\left(\hat{\mu}(\mu, s_\mu)\right)\right] - P(\mu) \geq E_{s_{\mu'}}\left[V\left(\hat{\mu}(\mu, s_{\mu'})\right)\right] - P(\mu') \tag{IC}$$

$$E_{s_\mu}\left[V\left(\hat{\mu}(\mu, s_\mu)\right)\right] - P(\mu) \geq V(\mu) \tag{IR}$$

Optimal Mechanism

Assumptions

1. We assume the distribution of buyer's beliefs $f(\mu) \in \mathcal{L}[0,1]$ satisfies $\forall\lambda$:

$$\frac{f(\mu)(1-\mu)}{\lambda + F(\mu) - 1} \text{ decreasing with } \mu > F^{-1}(1-\lambda).$$

$$\frac{f(\mu)\mu}{\lambda + F(\mu) - 1} \text{decreasing with } \mu < F^{-1}(1-\lambda) \; .$$

   In a symmetric problem, this is weaker than MLRP.

2. We assume $V \in C^2(0,1)$, $\bar{\mu}$ and $\underline{\mu}$ are largest and smallest element in $(0,1)$ such that:

$$V(\mu) + V'(\mu)(1-\mu) + V''(\mu)\frac{\mu(1-\mu)^2}{1-\bar{\mu}} \text{ is strictly increasing}$$

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   Local concavity of value function is bounded, otherwise types can't be well ranked.

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Optimal Mechanism

Assumptions

1. We assume $\bar{\lambda} \geq \underline{\lambda}$ are largest and smallest element in $(0,1)$ such that:
$$\left(1 + \frac{f(\underline{\mu})\underline{\mu}}{\bar{\lambda} + F(\underline{\mu}) - 1}\right)(V(\mu) + V'(\mu)(1-\mu)) + V''(\mu)\frac{\mu(1-\mu)^2}{1-\underline{\mu}}$$
is strictly increasing
$$\left(1 - \frac{f(\bar{\mu})(1-\bar{\mu})}{\underline{\lambda} + F(\bar{\mu}) - 1}\right)(V(\mu) - V'(\mu)\mu) + V''(\mu)\frac{\mu^2(1-\mu)}{\bar{\mu}}$$
is strictly decreasing

2. We assume $\frac{f(\bar{\mu}^*)(1-\bar{\mu}^*)}{\underline{\lambda}+F(\bar{\mu}^*)-1} = 1$ and $\frac{f(\underline{\mu}^*)\underline{\mu}^*}{\bar{\lambda}+F(\underline{\mu}^*)-1} = 1$ and π^* is the profit earned by seller using mechanism proposed by us, then:
$$\max\left\{\bar{\mu}^* V(1) + (1-\bar{\mu}^*)V(0) - V(\bar{\mu}^*), \underline{\mu}^* V(1) + (1-\underline{\mu}^*)V(0) - V(\underline{\mu}^*)\right\} < \pi^*$$

3. $E_{s_{\tilde{\mu}}}\left[V(\hat{\mu}(\mu, s_{\tilde{\mu}}))\right] - p(\tilde{\mu})$ is smooth in $\tilde{\mu}$.

Optimal Mechanism I

Simplification

- We study a relaxed problem ignoring global IC and IR.

$$\max_{\mathcal{M}} \int_0^1 p(\mu) f(\mu) d\mu \tag{3}$$

$$s.t. \ \frac{\partial}{\partial s_\mu} E_{s_\mu}\left[V(\hat{\mu}(\mu, s_\mu))\right] \frac{ds_\mu}{d\mu} = P'(\mu) \tag{IC1}$$

$$P(0) = P(1) = 0 \tag{IR1}$$

Lemma

Any solution of problem (3) satisfying IC and IR will be a solution of problem (2).

Optimal Mechanism II

Simplification

- Then we can integrate P' to eliminate price in optimization problem:

$$\max_{\mathcal{M}} \int_0^1 E_{s_\mu}\left[V(\hat{\mu}, s_\mu)\right] f(\mu) d\mu - \int_0^1 E_{s_\mu}\left[\frac{\partial}{\partial \mu}(V(\hat{\mu}, s_\mu) - V(\mu))\right](1 - F(\mu)) d\mu \quad (4)$$

$$s.t. \int_0^1 E_{s_\mu}\left[\frac{\partial}{\partial \mu}(V(\hat{\mu}, s_\mu) - V(\mu))\right] d\mu = 0$$

Optimal Mechanism III

Simplification

- Assume all experiments share same signal set $S = \{s_i\}$, each signal $g(s_i|h) = p_i, g(s_i|l) = q_i$. By Bayes rule: $\hat{\mu}(s_i, \mu) = \frac{p_i\mu}{p_i\mu + q_i(1-\mu)}$ and $\pi(s_i) = p_i\mu + q_i(1-\mu)$. We can write the parametrized problem:

$$\max_{\{p_i,q_i\}} \int_0^1 \left(\sum_i (p_i\mu + q_i(1-\mu))\, V(\frac{p_i\mu}{p_i\mu + q_i(1-\mu)}) - V(\mu) \right) f(\mu) d\mu \quad (5)$$

$$- \int_0^1 \frac{\partial}{\partial\mu} \left(\sum_i (p_i\mu + q_i(1-\mu))\, V(\frac{p_i\mu}{p_i\mu + q_i(1-\mu)}) - V(\mu) \right) (1 - F(\mu)) d\mu$$

$$s.t. \int_0^1 \frac{\partial}{\partial\mu} \left(\sum_i (p_i\mu + q_i(1-\mu))\, V(\frac{p_i\mu}{p_i\mu + q_i(1-\mu)}) - V(\mu) \right) d\mu = 0$$

$$\sum_i p_i = \sum_i q_i = 1$$

Optimal Mechanism IV

Simplification

- Assign multiplier λ to integral constraint, eliminate second constraint by replacing. We take first order condition w.r.t p_i and q_i:

$$
\begin{aligned}
\text{FOC for } p\text{: } & \left(f(\mu)\mu + (\lambda + F(\mu) - 1)\right)\left(V(\mu_i) + V'(\mu_i)(1-\mu_i)\right) \\
& + (\lambda + F(\mu) - 1)V''(\mu_i)\frac{\mu_i(1-\mu_i)^2}{1-\mu} \\
=& \left(f(\mu)\mu + (\lambda + F(\mu) - 1)\right)\left(V(\mu_j) + V'(\mu_j)(1-\mu_j)\right) \\
& + (\lambda + F(\mu) - 1)V''(\mu_j)\frac{\mu_j(1-\mu_j)^2}{1-\mu} + \gamma_p^+ - \gamma_p^- \\
\text{FOC for } q\text{: } & \left(f(\mu)(1-\mu) - (\lambda + F(\mu) - 1)\right)\left(V(\mu_i) - V'(\mu_i)\mu_i\right) \\
& - (\lambda + F(\mu) - 1)V''(\mu_i)\frac{\mu_i^2(1-\mu_i)}{\mu} \\
=& \left(f(\mu)(1-\mu) - (\lambda + F(\mu) - 1)\right)\left(V(\mu_j) + V'(\mu_j)\mu_j\right) \\
& - (\lambda + F(\mu) - 1)V''(\mu_j)\frac{\mu_j^2(1-\mu_j)}{\mu} + \gamma_q^+ - \gamma_q^-
\end{aligned}
$$

Optimal Mechanism V

Simplification

- Suppose existence of 2 interior signals with $p, q \in (0,1)$, then multipliers γ are 0. Applying assumption 1, we can discuss μ in four regions to conclude that the two interior signals have to be the same.

$$
\begin{aligned}
H_1(\mu_i) &= \left(1 + \frac{f(\mu)\mu}{\lambda + F(\mu) - 1}\right)\left(V(\mu_i) + V'(\mu_i)(1-\mu_i)\right) \\
&\quad + V''(\mu_i)\frac{\mu_i(1-\mu_i)^2}{1-\mu} \\
H_2(\mu_i) &= \left(1 - \frac{f(\mu)(1-\mu)}{\lambda + F(\mu) - 1}\right)\left(V(\mu_i) - V'(\mu_i)\mu_i\right) \\
&\quad + V''(\mu_i)\frac{\mu_i^2(1-\mu_i)}{\mu}
\end{aligned}
$$

Optimal Mechanism VI

Simplification

Lemma

The optimal mechanism which solves problem (5) includes experiments with up to three signals, and up to one of them is partially informative.

- Now suppose existence of one interior signal. Then for another signal, only multiplier γ^- will be triggered. We can apply similar analysis to prove that the two fully revealing signals can not both exist in one experiment.

Lemma

The optimal mechanism which solves problem (5) includes experiments with up to two signals.

Optimal Mechanism VII

Simplification

- Thus we almost pin down the form of optimal mechanism. Applying monotonicity of the mechanism, we can finally reduce the problem into a one-dimensional problem:

Lemma

Let $\mu^0 = F^{-1}(1 - \lambda)$,

- *For* $\mu \in [0, \mu^0]$, *experiments revealing h are sold.*
- *For* $\mu \in [\mu^0, 1]$, *experiments revealing l are sold.*

Optimal Mechanism I

Solving Optimal Mechanism

- We parametrize the mechanism sending signals $\{H, L\}$:
  - For $\mu \in [\mu^0, 1]$, when $\theta = l$, $p(L) = q, p(H) = 1 - q$, when $\theta = h$, $p(L) = 0, p(H) = 1$.
  - For $\mu \in [0, \mu^0]$, when $\theta = h$, $p(L) = 1 - p, p(H) = p$, when $\theta = l$, $p(L) = 1, p(H) = 0$.
- Solving FOCs ($\mu_1 \in [\mu, 1]$,$\mu_2 \in [0, \mu]$):

$$\left(1 - \frac{(1-\mu)f(\mu)}{\lambda + F(\mu) - 1}\right)\Big(V(\mu_1) - V(0) - \mu_1 V'(\mu_1)\Big) + V''(\mu_1)\frac{\mu_1^2(1-\mu_1)}{\mu} = 0 \tag{6}$$

$$\left(1 + \frac{\mu f(\mu)}{\lambda + F(\mu) - 1}\right)(V(\mu_2) - V(1) + (1-\mu_2)V'(\mu_2)) + V''(\mu_2)\frac{\mu_2(1-\mu_2)^2}{1-\mu} = 0 \tag{7}$$

Optimal Mechanism II

Solving Optimal Mechanism

- We discuss solution to the FOCs in four regions:

$$\begin{cases} f(\mu^-)\mu^- + (\lambda + F(\mu^-) - 1) = 0 \\ f(\mu^+)(1-\mu^+) - (\lambda + F(\mu^+) - 1) = 0 \end{cases}$$

1. $\mu \in (0, \mu^-)$: $1 + \frac{\mu f(\mu)}{\lambda + F(\mu) - 1} > 0$. Equation (7) might have solution. For μ close enough to μ^-, there exists positive mass of μ at which equation (7) has solution.
2. $\mu \in [\mu^-, \mu^0]$: $1 + \frac{\mu f(\mu)}{\lambda + F(\mu) - 1} \leq 0$, fully revealing experiment is sold.
3. $\mu \in [\mu^0, \mu^+]$: $1 - \frac{(1-\mu) f(\mu)}{\lambda + F(\mu) - 1} \leq 0$, fully revealing experiment is sold.
4. $\mu \in (\mu^+, 1)$: $1 - \frac{(1-\mu) f(\mu)}{\lambda + F(\mu) - 1} > 0$. Equation (6) might have solution. For μ close enough to μ^+, there exists positive mass of μ at which equation (6) has solution.

Optimal Mechanism III

Solving Optimal Mechanism

- Verify single crossing difference condition:

$$\frac{\partial^2}{\partial\mu\partial p}\Delta V = -V(\mu_2) + V(1) - V'(\mu_2)(1-\mu_2) - V''(\mu_2)\frac{\mu_2(1-\mu_2)^2}{1-\mu} \quad (8)$$

$$\frac{\partial^2}{\partial\mu\partial q}\Delta V = V(\mu_1) - V(0) - V'(\mu_1)\mu_1 + V''(\mu_1)\frac{\mu_1^2(1-\mu_1)}{\mu} \quad (9)$$

- Monotonicity condition implies when $\mu < \mu^0$, $p(\mu)$ increasing, when $\mu > \mu^0$, $p(\mu)$ decreasing. Thus if solved mechanism satisfies this monotonicity (potentially after ironing), our mechanism satisfies global IC and IR.

Optimal Mechanism

Main Theorem

The optimal mechanism which solves problem (9) includes experiments with up to two signals defined as following: There exists an $\lambda \in (\underline{\lambda}, \bar{\lambda})$ such that:

$$\begin{cases} f(\mu^-)\mu^- + (\lambda + F(\mu^-) - 1) = 0 \\ f(\mu^+)(1 - \mu^+) - (\lambda + F(\mu^+) - 1) = 0 \end{cases}$$

- For $\mu \in [\mu^-, \mu^+]$, an experiment fully revealing both states is sold at flat price.
- For $\mu \in [0, \mu^-]$, experiments fully revealing state h will be sold. The other signal induces belief $\min\{\mu_1, \mu\}$ to buyer with belief μ_1 defined by:

$$\left(1 + \frac{f(\mu)\mu}{\lambda + F(\mu) - 1}\right)(V(\mu_1) - V(1) + V'(\mu_1)(1 - \mu_1)) + V''(\mu_1)\frac{\mu_1(1 - \mu_1)^2}{1 - \mu} =$$

- For $\mu \in [\mu^+, 1]$, experiments fully revealing state l will be sold. The other signal induces belief $\max\{\mu_1, \mu\}$ to buyer with belief μ_1 defined by:

$$\left(1 - \frac{f(\mu)(1 - \mu)}{\lambda + F(\mu) - 1}\right)(V(\mu_1) - V(0) - V'(\mu_1)\mu_1) + V''(\mu_1)\frac{\mu_1^2(1 - \mu_1)}{\mu} = 0$$

Optimal Mechanism

Example

- Quadratic utility, uniform distribution.
- This problem can be solved analytically:

$$1 + 2\mu^2 - 3.5\mu + (2\mu - 1)\mu_1 = 0$$

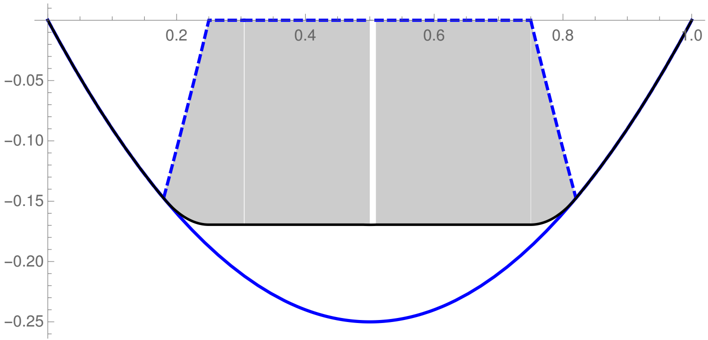

Figure: Optimal mechanism with quadratic utility and uniform distribution

Comparative Statics

Assumptions

- The intuition is not limited to symmetric environment but the analysis can be largely simplified. We assume $V(\mu) = V(1-\mu), f(\mu) = f(1-\mu)$.
- *Dispersive order:*
  $\mathcal{F} = \{f | f \in \Delta[0,1], f(x) = f(1-x)$, satisfying assumption 1$\}$, $\forall f, g \in \mathcal{F}$, F, G are corresponding CDF. We define f being more dispersed than g if for $\mu < \mu_g^-$:

$$\frac{f(\mu)}{0.5 - F(\mu)} \geq \frac{g(\mu)}{0.5 - G(\mu)}$$

  μ_g^- defined by solution of $g(\mu) + G(\mu) - 0.5 = 0$.

Comparative Statics

- The interval to which fully revealing experiment is sold has end point $g(\mu) + G(\mu) - 0.5 = 0$. By definition of dispersive order, μ^- moves monotonically with distribution becoming more dispersed.
- The interval to which partially revealing experiments are sold has end point defined by solution to:

$$(1 - \frac{g(\mu)\mu}{0.5 - G(\mu)})(V(\mu) - V(1) + V'(\mu)(1-\mu)) + V''(\mu)\mu(1-\mu) = 0$$

- Be definition of dispersive order, the end point must move at same direction of μ^-. Otherwise a crossing point leads to contradiction.
- Now we know that at starting point and end point of the discrimination region, experiments sold under more dispersed distribution induces more extreme posterior beliefs. Again by the no crossing argument, experiments sold should be pointwise Blackwell more informative.

Comparative Statics

Propositions

Given symmetric underlying problem V, when distribution of buyer's beliefs becomes more dispersed,

1. The interval to which fully revealing experiment is sold is expanding
2. The interval to which at least partially revealing experiments are sold is expanding.
3. Any buyer type is sold a Blackwell weakly more informative experiment.
4. Any buyer type's surplus is weakly higher and highest price decreases.

Comparative Statics

Example

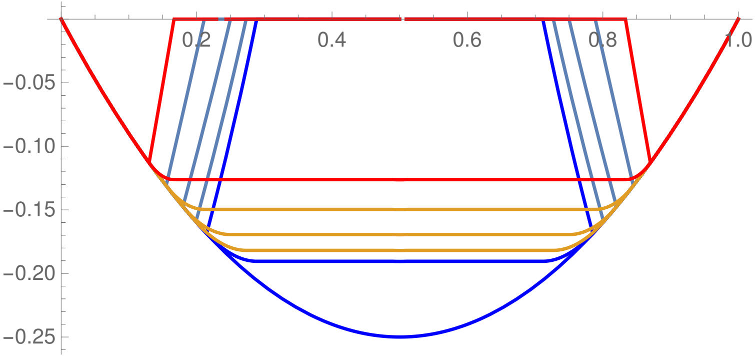

Figure: Optimal mechanism with quadratic utility and ordered distributions

Discussion

- Our paper and [Bergemann et al., 2014] covered two disjoint sets of the whole decision problem space: binary discrete choice and smooth continuum choice.
- What happens to decision problems in between?
- Existing methodologies don't apply. But the intuition that local concavity creates incentive for discrimination should apply

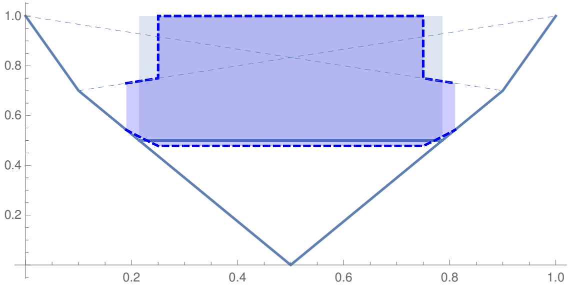

Figure: A mechanism dominating flat price

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