Response to Comment on “Thou Shalt Buy and Hold”*

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This Version: November 21, 2008

In this note we point out that the problem studied in the comment written by Majumdar and Bouchard (2008) on our paper “Thou Shalt Buy and Hold”, Shiryaev *et al.* (2008), is *fundamentally* different (and technically much easier) than the one in our original paper, although the difference between the two problems may appear deceivingly little to non-specialists in optimal stopping. That said, we acknowledge that generally speaking the path integral methods being promoted in the comment could indeed be a useful tool in treating some problems in quantitative finance.

The problem (4), considered in the comment by Majumdar and Bouchard (2008), is a one-dimensional *deterministic optimisation problem* where the optimal time τ^* to be determined is known to be *deterministic* a priori. In contrast, the problem studied in the original paper Shiryaev *et al.* (2008) is an *optimal stopping problem* where the decision variable τ is a *random* time. The scale of the difference and difficulty of the latter problem compared with the former is, shall we say, enormous. A deterministic optimisation problem (sometimes called a mathematical programme especially if there are various constraints involved) can be solved by simple calculus, whereas optimal stopping remains an area where what we know is far less than what we do not; see Shiryaev (1978) and Peskir and Shiryaev (2006) for an account of what we know. One should note that determining a random time as in optimal stopping is a necessity in many decision-making problems facing uncertainties, because the optimal timing may need to respond to the

*We thank the Editors in Chief for inviting us to respond to the comment of S.N. Majumdar and J.-P. Bouchard on our paper “Thou Shalt Buy and Hold”. Shiryaev acknowledges financial support from Grants RFBI 08-01-00740 and 08-01-91205-YaF, while Zhou acknowledges financial support from a start-up fund of the University of Oxford.

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random enviroment and hence cannot be assumed upfront to be a deterministic quantity. In the context of selling a stock, what triggers one to sell could be all kinds of random events; so the optimal selling time is random in general . What is intriguing (and probably deceiving at the same time) with our original paper Shiryaev *et al.* (2008) is that with the *specific* formulation of a stock-selling problem it just so *happens* that the optimal times are either 0 or T (after long, tedious mathematical derivations); and hence deterministic. As a consequence, the solutions to the problem in Majumdar and Bouchard (2008) and the one in Shiryaev *et al.* (2008) *turn out* to be identical; but this is exactly the main contribution of Shiryaev *et al.* (2008), as it reveals the degeneracy of the optimal stopping problem under consideration, which is in general not true at all.

In conclusion, the claim in the abstract of Majumdar and Bouchard (2008) that it "extends" the results of Shiryaev *et al.* (2008) to the "entire parameter region" is not valid.

We have also the following minor comments:

- The distribution of t_m studied in section 5, Majumdar and Bouchard (2008), is well known in literature; see, e.g., Borodin and Salminen (2002), p. 266, 1.12.4.
- The optimality of $\tau^* = 0$ and $\tau^* = T$ is obtained by an inspection from Figure 4 in section 4.1, Majumdar and Bouchard (2008). We believe that it could be derived *analytically* by some (not necessarily involved) calculus.
- We do not agree with the discussion in the third paragraph of Majumdar and Bouchard (2008) on the case of $\alpha < 0.5$. Yes the value of annual volatility exceeding 40% is not uncommon (especially in some volatile period such as now), but the buy-and-hold policy is indeed for *long term* investors whereas volatility is smoothed out by time. Moreover, we also agree that the volatility of S&P500 is smaller due to diversification, but bear in mind the return rate gets smaller too for the same reason!

Having said all these, we acknowledge that the path integral methods mostly used by physicists are nice in deriving the joint probability (9) in Majumdar and Bouchard (2008).[1] In particular, the symmetry in eqaution (23) is an interesting observation. We do appreciate that methods employed in other fields may be surprisingly powerful in dealing with problems that cannot be otherwise solved by more familiar, "home" approaches – think of heat equations in solving option pricing problems. For this we are grateful to the two authors for their effort.

[1] Although we suspect, for lack of an exact reference, that an expression for (9) is available in literature. See Borodin and Salminen (2002), p. 251, 1.1.8, for at least the case when $\tau = T$.

As a final note, the results in Shiryaev *et al.* (2008) have been presented in various seminars and conferences around the world since October 2007, and we are very pleased (and almost flattered) by the warm (and, at times, overwhelming) responses to and interests in them. However, it is our hope that the reader do not view the paper as a "mathematical paper". There have been already many (indeed *too* many) papers around dealing with various variants of the problem of stopping a Brownian motion, and there is no point of adding yet another one to the high pile without fully understanding the underlying financial motivation, interpretation and significance. The main contributions of Shiryaev *et al.* (2008), to reiterate, are the formulation of the model and the discovery that the model has a degenerate solution (and hence consistent with the conventional wisdom of buy-and-hold), much more than the mathematical technique it develops.

References

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