Greed, Leverage, and Potential Losses: A Prospect Theory Perspective*

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Abstract

This paper quantifies the notion of greed, and explores its connection with leverage and potential losses, in the context of a continuous-time behavioral portfolio choice model under (cumulative) prospect theory. We argue that the reference point can serve as the critical parameter in defining greed. An asymptotic analysis on optimal trading behaviors when the pricing kernel is lognormal and the S-shaped utility function is a two-piece CRRA shows that both the level of leverage and the magnitude of potential losses will grow unbounded if the greed grows uncontrolled. However, the probability of ending with gains does *not* diminish to zero even as the greed approaches infinity. This explains why a sufficiently greedy behavioral agent, despite the risk of catastrophic losses, is still willing to gamble on potential gains because they have a positive probability of occurrence whereas the corresponding rewards are huge. As a result an effective way to contain human greed, from a regulatory point of view, is to impose *a priori* bounds on leverage and/or potential losses.

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1 Introduction

"Greed" as a non-technical term is fairly subjective and vague[1]. In economics literature the notion of greed probably dates back to Adam Smith in 1776 (Smith 1909–14) – although he did not explicitly use the term – via his vision of "invisible hand". To mathematically analyze greed it is important to first make precise the notion "greed". In this paper, we quantify greed, and explore its connection with leverage and potential losses, in the context of a behavioral portfolio choice model under Kahneman and Tversky's cumulative prospect theory (CPT). As Hersh Shefrin notes, "the notion of greed is usually shorthand for a series of distinct psychological phenomena" (Shefrin and Zhou 2009). Greed is a psychological phenomenon; so it is only natural to conceptualize and investigate it in the framework of behavioral finance, in particular CPT, which posits that emotions and cognitive errors influence our decisions when faced with uncertainties, causing us to behave in incompetent and irrational ways.

We will build our theory of greed upon the recent results of Jin and Zhou (2008), where a continuous-time CPT portfolio selection model featuring general S-shaped utility functions[2] and probability distortion (weighting) functions is formulated and solved. The study on continuous-time CPT portfolio selection is quite lacking in the literature; to the authors' best knowledge there exist only two papers, Berkelaar, Kouwenberg and Post (2004) and Jin and Zhou (2008). In both papers[3] it is concluded that, with an exogenously fixed reference point, a CPT agent will take gambling strategies, betting on the "good states of the world" while accepting a loss on the bad, if the reference point is sufficiently high (due to excessive aspiration, unrealistic optimism, high expectation or over-confidence). Moreover, such strategies *must* involve substantial leverage.

The reference point in CPT holds the key in defining and analyzing greed, because a *higher* reference point is consistent with the common perception on greed as a very strong wish to get *more* of something. However, a mere strong desire to get more than one's fair share is not what greed is all about. Greed is always accompanied by aggressive *actions* so as to fulfil the desire. The significance of the reference point in CPT is that it divides between the gains and losses, and hence dictates whether an agent is risk-averse or risk-seeking. In other words, the higher the reference point the more likely the agent is to be a risk-taker, and hence the greedier she is. This suggests

[1] *Oxford English Dictionary* defines "greed" as "intense and selfish desire for food, wealth, or power".

[2] These are called *value functions* in the Kahneman–Tversky terminology (Kahneman and Tversky 1979, Tversky and Kahneman 1992). In this paper we still use the term *utility function* so as to distinguish it from the term "value function" commonly used in dynamic programming.

[3] We base the greed analysis of the present paper on the model and results of the latter, which is more general – in particular it includes probability distortions – than the former.

that greed can be represented by the level of reference point and, *consequently*, the corresponding risk-seeking behavior[4].

The leverage and potential losses inherent in an optimal CPT trading strategy have been endogenously derived in Jin and Zhou (2008) where the reference point is fixed, which enables us to study their asymptotic properties as the greed becomes infinitely strong. In this paper, we carry out an asymptotic analysis on the benchmark case when the pricing kernel is lognormal and the S-shaped utility is a two-piece CRRA. This case is sufficiently representative to support the generality of the results drawn. The results show that both the level of leverage and the magnitude of potential losses will grow uncontrolled as the greed becomes infinitely strong, as one would naturally expect.

An intriguing finding is, however, that the probability of ending with good (gain) states does *not* diminish to zero even as the greed approaches infinity. This result is quite counter-intuitive. The gain is defined with respect to the reference point; hence ending up with a gain state gets more difficult as the greed (and hence the reference point) soars. As a result, it would seem only reasonable that the probability of achieving gain states should decline as greed grows. A closer examination, however, reveals that the agent's trading strategy would become more aggressive with a stronger greed, which offsets the increased difficulty of reaching a gain state. Hence the riskier consequence of a greedier agent's trading behavior is reflected by the increased *magnitudes* of potential losses, not by the increased *odds* of having losses. On the other hand, this result does explain why a sufficiently greedy behavioral agent, despite the risk of catastrophic losses, is still willing to gamble on the gain states because they have a positive probability of occurrence whereas the corresponding rewards are huge[5].

An economic interpretation of these asymptotic results is that leverage and potential losses will be unbounded if greed is allowed to grow unbounded. Consequently, an effective way to contain human greed, from a regulatory point of view, is to impose *a priori* bounds on either leverage or potential losses or both in a financial investment decision model.

The novelties of this paper compared to Jin and Zhou (2008) are the following. Conceptually, this paper quantifies the term "greed", and establishes its connection to "leverage" and "potential losses". Technically, the paper presents an asymptotic analysis when $E[\rho B] \to \infty$, based on the results of Jin and Zhou (2008), through rather involved probabilistic and analytic arguments. As a by-product we will also derive some new results; e.g. we will solve the two-piece CRRA case with different powers. In summary, this paper is significantly different from Jin and Zhou (2008) in motivations, techniques and results.

[4]One might argue that greed could be also quantified and analyzed via a neoclassical portfolio selection model, such as the expected utility maximization, by introducing an additional aspiration constraint (e.g. a very high mean target or a guaranteed probability of achieving a high wealth level). Such a neoclassical treatment of greed, however, would have a critical drawback that it does not capture the psychological anomaly – the risk-taking behavior – inevitably associated with greed.

[5]Think about what many banks and insurers had done before the 2007-2010 financial crisis.

The rest of this paper is arranged as follows. In Section 2 we review the CPT portfolio choice model and its optimal terminal wealth profile derived in Jin and Zhou (2008), which sets the stage for the subsequent analysis on greed. Section 3 motivates and gives precise definitions of greed, leverage and potential losses. In Section 4 we perform an asymptotic analysis on greed for a model when the pricing kernel is lognormal and the S-shaped utility is a two-piece CRRA. Depending on whether the powers of the two pieces of the utility function are the same or not, the analysis are quite different. Yet, the results have essentially the same economic interpretation: as the agent's greed becomes infinitely strong, the limiting probability of having gains is constant and positive, while both the leverage and potential losses diverge to infinity. Section 5 proposes a modified CPT portfolio selection model where leverage and/or potential losses are *a priori* capped. The paper is finally concluded in Section 6.

2 A Behavioral Agent's Strategies

In this section we briefly review the optimal terminal wealth profiles of a CPT agent, derived in Jin and Zhou (2008), and then motivate the problem of the present paper.

Let T be a fixed terminal time and $(\Omega, \mathcal{F}, P, \{\mathcal{F}_t\}_{t\geq 0})$ a filtered complete probability space on which is defined a standard $\mathcal{F}_t$-adapted m-dimensional Brownian motion $W(t) \equiv (W^1(t), \cdots, W^m(t))^T$ with $W(0) = 0$. It is assumed that $\mathcal{F}_t = \sigma\{W(s) : 0 \leq s \leq t\}$, augmented by all the null sets. Here and throughout the paper A^T denotes the transpose of a matrix A.

There is a market where there are $m + 1$ assets being traded continuously. One of the assets is a bank account whose price process $S_0(t)$ is subject to the following equation:

$$dS_0(t) = r(t)S_0(t)dt, \quad t \in [0, T]; \quad S_0(0) = s_0 > 0, \tag{2.1}$$

where the interest rate $r(\cdot)$ is an $\mathcal{F}_t$-progressively measurable, scalar-valued stochastic process with $\int_0^T |r(s)|ds < +\infty$, a.s.. The other m assets are stocks whose price processes $S_i(t)$, $i = 1, \cdots, m$, satisfy the following stochastic differential equation (SDE):

$$dS_i(t) = S_i(t)\big[\mu_i(t)dt + \sum_{j=1}^{m} \sigma_{ij}(t)dW^j(t)\big], \quad t \in [0, T]; \quad S_i(0) = s_i > 0, \tag{2.2}$$

where $\mu_i(\cdot)$ and $\sigma_{ij}(\cdot)$, the appreciation and volatility rates, respectively, are scalar-valued, $\mathcal{F}_t$-progressively measurable stochastic processes with

$$\int_0^T [\sum_{i=1}^{m} |\mu_i(t)| + \sum_{i,j=1}^{m} |\sigma_{ij}(t)|^2]dt < +\infty, \text{ a.s..}$$

Set the excess rate of return vector process

$$e(t) := (\mu_1(t) - r(t), \cdots, \mu_m(t) - r(t))^T$$

and define the volatility matrix process $\sigma(t) := (\sigma_{ij}(t))_{m\times m}$. Basic assumptions imposed on the market parameters throughout this paper are summarized as follows:

Assumption 1.

(i) There exists $c \in \mathbf{R}$ such that $\int_0^T r(s)ds \geq c$, a.s..

(ii) There exists a unique $\mathbf{R}^m$-valued, uniformly bounded, $\mathcal{F}_t$-progressively measurable process $\theta(\cdot)$ such that $\sigma(t)\theta(t) = e(t)$, a.e.$t \in [0,T]$, a.s..

It is well known that under these assumptions there exists a unique risk-neutral (martingale) probability measure Q defined by $\frac{dQ}{dP}\Big|_{\mathcal{F}_t} = e^{\int_0^t r(s)ds}\rho(t)$, where

$$\rho(t) := \exp\left\{-\int_0^t \left[r(s) + \frac{1}{2}|\theta(s)|^2\right] ds - \int_0^t \theta(s)^T dW(s)\right\} \tag{2.3}$$

is the pricing kernel or state density price. Denote $\rho := \rho(T)$. It is clear that $0 < \rho < +\infty$ a.s., and $0 < E\rho < +\infty$. Furthermore, the following assumption is in force throughout this paper.

Assumption 2. ρ admits no atom.

Consider an agent, with an initial endowment $x_0 \in \mathbf{R}$ (fixed throughout this paper), whose total wealth at time $t \geq 0$ is denoted by $x(t)$. Assuming that the trading of shares takes place continuously in a self-financing fashion, $x(\cdot)$ satisfies

$$dx(t) = [r(t)x(t) + e(t)^T\pi(t)]dt + \pi(t)^T\sigma(t)dW(t), \quad t \in [0,T]; \quad x(0) = x_0, \tag{2.4}$$

where $\pi(\cdot) \equiv (\pi_1(\cdot), \cdots, \pi_m(\cdot))^T$ is the portfolio of the agent with $\pi_i(t), \quad i = 1, 2\cdots, m$, denoting the total market value of the agent's wealth in the i-th asset at time t. A portfolio $\pi(\cdot)$ is said to be *admissible* if it is an $\mathbf{R}^m$-valued, $\mathcal{F}_t$-progressively measurable process with

$$\int_0^T |\sigma(t)^T\pi(t)|^2 dt < +\infty \ \text{ and } \ \int_0^T |e(t)^T\pi(t)|dt < +\infty, \ \text{ a.s..}$$

An admissible portfolio $\pi(\cdot)$ is said to be tame if the corresponding discounted wealth process, $S_0(t)^{-1}x(t)$, is almost surely bounded from below (the bound may depend on $\pi(\cdot)$).

The market in this paper is arbitrage-free and complete having a linear pricing rule with the pricing kernel ρ (i.e. a contingent claim X paid at T is priced as $E[\rho X]$ at $t = 0$). Note that in our model the agent is a "small investor"; so her CPT preference only affects her own utility function – and hence *her* portfolio choice – but not the overall market. In particular, it does not affect the pricing rule, $E[\rho X]$, of the market[6].

[6]Asset pricing under behavioral preferences in continuous time remains a significant open problem, which is certainly beyond the scope of this paper.

The agent's risk preference is dictated by CPT. Specifically, she has a reference point B at the terminal time T, which is a lower bounded, $\mathcal{F}_T$-measurable random variable[7]. The reference point B determines whether a given terminal wealth position is a gain (excess over B) or a loss (shortfall from B). It could be interpreted as a liability the agent has to fulfil (e.g. a house downpayment), or an aspiration she strives to achieve (e.g. a target profit aspired by, or imposed on, a fund manager). The agent's utility (value) function is S-shaped: $u(x) = u_+(x^+)\mathbf{1}_{x\geq 0}(x) - u_-(x^-)\mathbf{1}_{x<0}(x)$, where the superscripts $^\pm$ denote the positive and negative parts of a real number, u_+, u_- are *concave* functions on $\mathbf{R}^+$ with $u_\pm(0) = 0$, reflecting risk-aversion on gains and risk-seeking on losses. There are also probability distortions on both gains and losses, which are captured by two nonlinear functions w_+, w_- from $[0, 1]$ onto $[0, 1]$, with $w_\pm(0) = 0, w_\pm(1) = 1$ and $w_\pm(p) > p$ (respectively $w_\pm(p) < p$) when p is close to 0 (respectively 1).

The agent's preference on a terminal wealth X (which is an $\mathcal{F}_T$-random variable) is measured by the prospective functional

$$V(X - B) := V_+((X - B)^+) - V_-((X - B)^-),$$

where $V_+(Y) := \int_0^{+\infty} w_+(P(u_+(Y) \geq y))dy,\ V_-(Y) := \int_0^{+\infty} w_-(P(u_-(Y) \geq y))dy$. Thus, the CPT portfolio choice problem is to

$$\text{(2.5)} \qquad \begin{array}{ll} \text{Maximize} & V(x(T) - B) \\ \text{subject to} & (x(\cdot), \pi(\cdot)) \text{ satisfies (2.4), and } \pi(\cdot) \text{ is admissible and tame.} \end{array}$$

To solve (2.5) we need only to find the optimal terminal wealth by solving

$$\text{(2.6)} \qquad \begin{array}{ll} \text{Maximize} & V(X - B) \\ \text{subject to} & \begin{cases} E[\rho X] = x_0 \\ X \text{ is } \mathcal{F}_T - \text{measurable and lower bounded.} \end{cases} \end{array}$$

If X^* solves (2.6), then the optimal portfolio to (2.5) is obtained by replicating X^*. Note the lower boundedness constraint in (2.6) corresponds to the requirement that the admissible portfolios be tame.

We introduce some notation related to the pricing kernel ρ. Let $F(\cdot)$ be the cumulative distribution function (CDF) of ρ, and $\bar{\rho}$ and $\underline{\rho}$ be respectively the essential lower and upper bounds of ρ, namely,

[7]In Jin and Zhou (2008) it is assumed that $B = 0$ without loss of generality as the reference point therein is *fixed*. In the present paper, a critical issue we want to address is how the reference point would affect the agent behavior and hence her strategies; so we need to take B as an explicitly present exogenous variable.

$$\bar{\rho} \equiv \text{esssup}\ \rho := \sup\left\{a \in \mathbf{R} : P\{\rho > a\} > 0\right\},$$
(2.7)
$$\underline{\rho} \equiv \text{essinf}\ \rho := \inf\left\{a \in \mathbf{R} : P\{\rho < a\} > 0\right\}.$$

The following assumption, inherited from Jin and Zhou (2008), will be henceforth enforced.

Assumption 3. $u_+(\cdot)$ is strictly increasing, strictly concave and twice differentiable, with the Inada conditions $u'_+(0+) = +\infty$ and $u'_+(+\infty) = 0$, and $u_-(\cdot)$ is strictly increasing, and strictly concave at 0. Both $w_+(\cdot)$ and $w_-(\cdot)$ are non-decreasing and differentiable. Moreover, $F^{-1}(z)/w'_+(z)$ is non-decreasing in $z \in (0,1]$, $\liminf_{x\to+\infty}\left(\frac{-xu''_+(x)}{u'_+(x)}\right) > 0$, and $E\left[u_+\left((u'_+)^{-1}(\frac{\rho}{w'_+(F(\rho))})\right)w'_+(F(\rho))\right] < +\infty$.

By and large, the monotonicity of the function $F^{-1}(z)/w'_+(z)$ can be interpreted economically as a requirement that the probability distortion w_+ on gains should not be too large in relation to the market (or, loosely speaking, the agent should not be over-optimistic about huge gains); see Jin and Zhou (2008), Section 6.2, for a detailed discussion. Other conditions in Assumption 3 are mild and/or economically motivated.

We now summarize the main results of Jin and Zhou (2008) relevant to this paper[8], which are stated in terms of the following two-dimensional mathematical program with the decision variables (c, x_+):

$$\text{Maximize} \quad v(c,x_+) = E\left[u_+\left((u'_+)^{-1}\left(\frac{\lambda(c,x_+)\rho}{w'_+(F(\rho))}\right)\right)w'_+(F(\rho))\mathbf{1}_{\rho\le c}\right]$$
$$-u_-\left(\frac{x_+-(x_0-E[\rho B])}{E[\rho\mathbf{1}_{\rho>c}]}\right)w_-(1-F(c))$$
(2.8)
$$\text{subject to} \quad \begin{cases} \underline{\rho} \le c \le \bar{\rho}, \quad x_+ \ge (x_0 - E[\rho B])^+, \\ x_+ = 0 \text{ when } c = \underline{\rho}, \quad x_+ = x_0 - E[\rho B] \text{ when } c = \bar{\rho}, \end{cases}$$

where $\lambda(c,x_+)$ satisfies $E[(u'_+)^{-1}(\frac{\lambda(c,x_+)\rho}{w'_+(F(\rho))})\rho\mathbf{1}_{\rho\le c}] = x_+$, and we use the following convention:

$$\text{(2.9)} \qquad u_-\left(\frac{x_+-(x_0-E[\rho B])}{E[\rho\mathbf{1}_{\rho>c}]}\right)w_-(1-F(c)) := 0 \quad \text{when } c = \bar{\rho} \text{ and } x_+ = x_0 - E[\rho B].$$

Theorem 1. *Let (c^*, x_+^*) be optimal for Problem (2.8). We have the following conclusions:*

(i) *If X^* is optimal for Problem (2.6), then $\{X^* \ge B\}$ and $\{\rho \le c^*\}$ are identical up to a zero probability event.*

[8] Some of the results there will actually be enhanced (with proofs) in the present paper.

(ii) *The following solution*

$$X^* = \left[(u'_+)^{-1}\left(\frac{\lambda\rho}{w'_+(F(\rho))}\right) + B\right]\mathbf{1}_{\rho\le c^*} - \left[\frac{x_+^* - (x_0 - E[\rho B])}{E[\rho\mathbf{1}_{\rho>c^*}]} - B\right]\mathbf{1}_{\rho>c^*} \tag{2.10}$$

is optimal for Problem (2.6).

This result is a part of Theorem 4.1, along with (4.6), in Jin and Zhou (2008). The explicit form of the optimal terminal wealth profile, X^*, is sufficiently informative to reveal the key qualitative and quantitative features of the corresponding optimal portfolio[9].

The following summarize the economical interpretations and implications[10] of Theorem 1, including those of c^* and x_+^*:

- The future world at $t = T$ is divided by two classes of states: "good" ones (having gains) or "bad" ones (having losses). Whether the agent ends up with a good state is *completely* determined by $\rho \le c^*$, which in statistical terms is a simple hypothesis test involving a constant c^*, *à la* Neyman–Pearson's lemma (see, e.g., Lehmann 1986).

- The optimal strategy is a *gambling* policy, betting on the good states while accepting a loss on the bad. Specifically, at $t = 0$ the agent needs to sell the "loss" lottery, $\left[\frac{x_+^*-(x_0-E[\rho B])}{E[\rho\mathbf{1}_{\rho>c^*}]} - B\right]\mathbf{1}_{\rho>c^*}$, in order to raise fund to purchase the "gain" lottery, $\left[(u'_+)^{-1}\left(\frac{\lambda\rho}{w'_+(F(\rho))}\right) + B\right]\mathbf{1}_{\rho\le c^*}$.

- The probability of finally reaching a good state is $P(\rho \le c^*) \equiv F(c^*)$, which in general depends on the reference point B, since c^* depends on B via (2.8). Equivalently, c^* is the quantile of the pricing kernel evaluated at the probability of good states.

- The magnitude of potential losses in the case of a bad state is a *constant* $\frac{x_+^*-(x_0-E[\rho B])}{E[\rho\mathbf{1}_{\rho>c^*}]} \ge 0$, which is endogenously *dependent* on B.

- $x_+^* + E[\rho B\mathbf{1}_{\rho\le c^*}]$ is the $t = 0$ price of the gain lottery. Hence, if B is set too high such that $x_0 < x_+^* + E[\rho B\mathbf{1}_{\rho\le c^*}]$, i.e., the initial wealth is not sufficient to purchase the gain lottery[11], then the optimal strategy *must* involve a leverage.

- If $x_0 < E[\rho B]$, then the optimal $c^* < \bar\rho$ (otherwise by the constraints of (2.8) it must hold that $x_+^* = x_0 - E[\rho B] < 0$ contradicting the non-negativeness of x_+^*); hence $P(\rho > c^*) > 0$. This shows that if the reference point is set too high compared with the initial endowment, then the odds are not zero that the agent ends up with a bad state.

[9]The optimal strategy is the one that replicates X^* in a Black–Schole way. However, we do not actually need the form of the optimal strategy in our study below.

[10]These have not been adequately elaborated in Jin and Zhou (2008).

[11]Later we will show that $P(\rho \le c^*)$ converges to a constant when B goes to infinity. So $x_+^* + E[\rho B\mathbf{1}_{\rho\le c^*}]$ will be sufficiently large when B is sufficiently large.

3 Defining Greed, Leverage and Potential Losses

Next we are to give precise definitions of greed, leverage, and potential losses in the setting of the CPT portfolio choice model formulated in Section 2.

Greed as a common term holds two defining features: 1) a high desire for wealth, and 2) the subsequent aggressive action to fulfil the desire. The reference point in CPT, therefore, provides a key in defining and analyzing greed, in that it divides between the gains and losses, and hence dictates whether an agent is risk-averse or risk-seeking. In other words, the higher the reference point the more likely the agent is to be a risk-taker. This suggests that greed can be captured by the level of reference point and the corresponding risk-seeking behavior.

Notice that if $x_0 \geq E[\rho B]$, i.e., the agent's aspiration is so moderate that she starts in the gain territory, then she is risk-averse as stipulated by CPT[12]. Thus the agent's greed becomes relevant and significant in portfolio choice only when $0 < x_0 < E[\rho B]$.

The preceding discussions suggest that the greed G ought to be quantified in such a way that it is applicable only when $x_0 < E[\rho B]$, and that it is a monotonically increasing function of the reference point B. There could be several ways of achieving these, but a natural and simple definition of greed is the ratio between what the agent is desperate to achieve – (the $t = 0$ value of) the reference point – and what she has to start with, i.e., x_0, when $x_0 < E[\rho B]$.

Our definition of greed per se depends on the market via ρ. One may argue that a notion of greed should depend on the investor's preferences alone *without* involving the financial market. While it is a valid point, we think that greed indeed interacts with investment opportunities, and the level of greed is *relative* to the overall market. An aspiration of 5% annual return is quite moderate in a bull market, but can be considered to be greedy in a bear one. A greedy person typically becomes greedier in a bull market (which seems to be one of the reasons behind bubbles and crashes). On the other hand, in this paper we are concerned with the asymptotic trading behaviors when G goes to infinity. So we are ultimately interested in the situation when B becomes sufficiently large, irrespective of the market.

Leverage, on the other hand, can be loosely defined (although there are several definitions) as the ratio between the borrowing amount and the equity in a venture. To motivate our definition below, we take for illustration one of the most commonly used financial devices – a mortgage – where leverage is inherent. Suppose one buys a house of \$500K, putting 10% downpayment and borrowing \$450K from a lender. Then

$$50K(\text{home buyer's equity}) = 500K(\text{total value}) - 450K(\text{borrowing amount}). \tag{3.1}$$

So leverage$= \dfrac{\text{borrowing amount}}{\text{home buyer's equity}} = \frac{450}{50} = 9$. In the context of behavioral portfolio choice when $x_0 < E[\rho B]$, since the initial endowment is not adequate to cover what is implied by the reference

[12]Jin and Zhou (2008), Theorem 9.1, shows that in the case of a two-piece CRRA utility the optimal strategy is reminiscent of that of a classical utility maximizing agent (albeit with a "distorted" asset allocation due to the probability distortions) if $x_0 \geq E[\rho B]$, where there is no gambling and leverage involved.

point, the agent needs to borrow money to fund her portfolios. Hence we can define the leverage of any given portfolio as the ratio between the $t = 0$ value of the borrowing amount and the initial endowment x_0. To do this we could examine the cash flow at the terminal time T and then discount it to $t = 0$. Specifically, let X be the terminal wealth of a given portfolio starting from x_0. Then we have the following *unique* decomposition based on gains and losses

$$X \equiv \big((X-B)^+ + B)\big)\,\mathbf{1}_{X\geq B} - \big((X-B)^- - B\big)\,\mathbf{1}_{X<B} := X_g - X_l. \tag{3.2}$$

Here, X_g is the payoff in a gain state while X_l is that in a loss one (see (2.10) for an example of such a decomposition). Hence (3.2) can be regarded as the agent shorting the amount X_l in order to fund the long position X_g. Therefore, the leverage is defined to be the ratio between the $t = 0$ value of X_l and x_0.

Finally, the potential loss (rate) can be defined simply as the expected ratio between the $t = 0$ value of the loss and x_0, *given* that a loss has occurred. Note that the potential loss is fundamentally different from the *expected* loss, since the former concerns the magnitude of the loss once a loss *does occur* while the latter simply averages out everything. So the potential loss could be disastrously large even though the expected loss is small or moderate. Indeed, Samuelson (1979) criticized the expected log utility model for its ignorance of the potential losses.

Motivated by the above discussions, we have the following definitions.

Definition 1. Given an agent with an initial endowment x_0, an investment horizon $[0, T]$, and a reference point B at T, her *greed* is defined as $G := \frac{E(\rho B)}{x_0}$. For any trading strategy leading to a terminal wealth position X that decomposes as in (3.2), its leverage is defined as $L := \frac{E(\rho X_l)}{x_0}$. Moreover the potential loss rate of the portfolio is defined as $l := E\left(\frac{\rho X_l}{x_0}\middle| X < B\right)$.

4 Asymptotic Analyses on Greed

This section explores how the leverage level, the probability of having losses, and the magnitude of potential losses change when greed monotonically expands to infinity in the setting of the CPT model formulated in Section 2. In particular we study the benchmark case where ρ is lognormal, i.e., $\log\rho \sim N(\mu, \sigma^2)$ with $\sigma > 0$, and the utility function is two-piece CRRA, i.e.,

$$u_+(x) = x^\alpha,\ \ u_-(x) = kx^\beta,\ x \geq 0$$

where $k > 0$ (the loss aversion coefficient) and $0 < \alpha, \beta < 1$ are constants. In this case $\bar{\rho} = +\infty$ and $\underline{\rho} = 0$. This setting is general enough to cover, for example, a market with a deterministic investment opportunity set and Kahneman–Tversky's utility functions (Tversky and Kahneman 1992).

In this case, the crucial mathematical program (2.8) has the following more specific form (see Jin and Zhou 2008, eq. (9.3)):

$$
\text{(4.1)} \qquad
\begin{array}{ll}
\text{Maximize} & v(c,x_+) = \varphi(c)^{1-\alpha}x_+^{\alpha} - \frac{kw_-(1-F(c))}{(E[\rho\mathbf{1}_{\rho>c}])^{\beta}}(x_+ - \tilde{x}_0)^{\beta}, \\
\text{subject to} & \begin{cases} 0 \le c \le +\infty, \quad x_+ \ge \tilde{x}_0^+, \\ x_+ = 0 \text{ when } c = 0, \quad x_+ = \tilde{x}_0 \text{ when } c = +\infty, \end{cases}
\end{array}
$$

where $\tilde{x}_0 := x_0 - E[\rho B]$ and

$$
\varphi(c) := E\left[\left(\frac{w_+'(F(\rho))}{\rho}\right)^{1/(1-\alpha)}\rho\mathbf{1}_{\rho\le c}\right]\mathbf{1}_{c>0}, \quad 0 \le c \le +\infty.
$$

Note that Assumption 3 implies that $\varphi(+\infty) < +\infty$. Moreover, it follows from the dominated convergence theorem that $\lim_{c\downarrow 0}\varphi(c) = \varphi(0) = 0$. So φ is continuous on $[0, +\infty]$.

First of all, we note that if $\alpha > \beta$, then the objective function of (4.1) is unbounded, since it converges to infinity as x_+ goes to infinity. According to Jin and Zhou (2008), Proposition 5.1, our original CPT model (2.6) is *ill-posed* in this case, i.e., the prospective value is unbounded from above. In general a *maximization* problem is ill-posed if its objective function is unbounded from above (and hence the supremum value is $+\infty$). Economically, $\alpha > \beta$ implies that the joys associated with *large* gains (measured by $u_+(x) = x^{\alpha}$ for large x) far outweigh the pains of losses of the same magnitude ($u_-(x) = kx^{\beta}$) in the sense that $\lim_{x\to+\infty}\frac{u_+(x)}{u_-(x)} = +\infty$; hence the agent will take an infinite level of leverage leading to an infinitely high optimal prospective value[13]. Such a model sets wrong trade-offs among choices, and the agent is led by her criterion to undertake the most risky investment.

In view of this discussion, in what follows we consider only the case when $\alpha \le \beta$. The following function will be useful in our subsequent analysis:

$$
k(c) := \frac{kw_-(1-F(c))}{\varphi(c)^{1-\alpha}(E[\rho\mathbf{1}_{\rho>c}])^{\beta}} > 0, \quad c > 0.
$$

4.1 The case when $\alpha = \beta$

We first consider the case when $\alpha = \beta$ (this is the case proposed by Tversky and Kahneman 1992 with $\alpha = \beta = 0.88$). In this case both the mathematical program (4.1) and the corresponding CPT portfolio selection model have been solved explicitly by Jin and Zhou (2008). Here we reproduce the results for reader's convenience:

Theorem 2. *(Jin and Zhou 2008, Theorem 9.2) Assume that $\alpha = \beta$ and $x_0 < E[\rho B]$.*

[13]This statement is true so long as $\alpha > \beta$ (even if only slightly), no matter how large k may be. Only when $\alpha = \beta$ does the value of k become significant in the model well-posedness. See a discussion in Section 4.1 for details.

(i) *If* $\inf_{c>0} k(c) > 1$, *then the CPT portfolio selection model (2.6) is well-posed. Moreover, (2.6) admits an optimal solution if and only if the following optimization problem attains an optimal solution*

$$\text{(4.2)} \qquad \mathrm{Min}_{0\le c<+\infty} \quad \left[\left(\frac{kw_-(1-F(c))}{(E[\rho\mathbf{1}_{\rho>c}])^\alpha}\right)^{1/(1-\alpha)} - \varphi(c)\right].$$

Furthermore, if an optimal solution c^* *of (4.2) satisfies* $c^* > 0$, *then the optimal terminal wealth is*

$$\text{(4.3)} \qquad X^* = \frac{x_+^*}{\varphi(c^*)}\left(\frac{w_+'(F(\rho))}{\rho}\right)^{1/(1-\alpha)} \mathbf{1}_{\rho\le c^*} - \frac{x_+^* - (x_0 - E[\rho B])}{E[\rho\mathbf{1}_{\rho>c^*}]}\mathbf{1}_{\rho>c^*} + B,$$

where $x_+^* := \frac{-(x_0-E[\rho B])}{k(c^*)^{1/(1-\alpha)}-1}$.

(ii) *If* $\inf_{c>0} k(c) = 1$, *then the supremum value of (2.6) is* 0, *which is however not achievable.*

(iii) *If* $\inf_{c>0} k(c) < 1$, *then (2.6) is ill-posed.*

As seen from the preceding theorem the characterizing condition for well-posedness is $\inf_{c>0} k(c) \ge 1$, which is equivalent to

$$k \ge \left(\inf_{c>0} \frac{w_-(1-F(c))}{\varphi(c)^{1-\alpha}(E[\rho\mathbf{1}_{\rho>c}])^\alpha}\right)^{-1} := k_0.$$

Recall that k is the loss aversion level of the agent ($k = 2.25$ in Tversky and Kahneman 1992). Thus the agent must be *sufficiently* loss averse in order to have a well-posed portfolio choice model; otherwise the agent would simply take the maximum possible risky exposure even with a fixed, finite strength of greed.

As described by Theorem 2-(i), the solution of (2.6) relies on some attainability condition of a minimization problem (4.2), which is rather technical without clear economical interpretation. The following (new) Theorem 3, however, gives a sufficient condition in terms of the probability distortion on *losses*.

Theorem 3. *Assume that* $\alpha = \beta$, $x_0 < E[\rho B]$, *and* $\inf_{c>0} k(c) > 1$. *If there exists* $\gamma < \alpha$ *such that* $\liminf_{p\downarrow 0} \frac{w_-(p)}{p^\gamma} > 0$, *or equivalently (by l'Hôpital's rule),* $\liminf_{p\downarrow 0} \frac{w_-'(p)}{p^{\gamma-1}} > 0$, *then (4.2) must admit an optimal solution* $c^* > 0$ *and hence (4.3) solves (2.6).*

To prove this theorem we need a lemma. Denote $g(c) := \frac{w_-(1-F(c))}{(E[\rho\mathbf{1}_{\rho>c}])^\alpha}$, which is a continuous function in $c \in [0, +\infty)$.

Lemma 1. (i) *If* $w_-(1-F(c_0)) \le 1 - F(c_0)$ *for some* $c_0 \in (0, +\infty)$, *then* $g(0) > g(c_0)$.

(ii) *If there exists* $\gamma < \alpha$ *such that* $\liminf_{p\downarrow 0} \frac{w_-(p)}{p^\gamma} > 0$, *then* $\liminf_{c\to+\infty} g(c) = +\infty$.

(iii) *If* $\limsup_{p\downarrow 0} \frac{w_-(p)}{p^\alpha} < +\infty$, *then* $\limsup_{c\to+\infty} g(c) = 0$.

Proof:

(i) Noting $E[\rho \mathbf{1}_{\rho>c_0}] = E[\rho|\rho > c_0]P(\rho > c_0)$, we have

$$\begin{aligned} g(c_0) &\leq \frac{1-F(c_0)}{(E[\rho\mathbf{1}_{\rho>c_0}])^\alpha} = \frac{(E[\rho\mathbf{1}_{\rho>c_0}])^{1-\alpha}}{E[\rho|\rho>c_0]} \\ &< \frac{(E\rho)^{1-\alpha}}{E\rho} = g(0). \end{aligned}$$

(ii) Denote $b := \liminf_{c\to+\infty} \frac{w_-(1-F(c))}{(1-F(c))^\gamma} > 0$, and fix $n > 1$ such that $\gamma < \alpha/n$. By virtue of the Cauchy–Schwartz inequality there exists $m > 1$ such that $E[\rho\mathbf{1}_{\rho>c}] \leq (E\rho^m)^{1/m}(1 - F(c))^{1/n}$. Hence

$$\begin{aligned} \liminf_{c\to+\infty} g(c) = \liminf_{c\to+\infty} \frac{w_-(1-F(c))}{(E[\rho\mathbf{1}_{\rho>c}])^\alpha} &\geq \liminf_{c\to+\infty} \frac{w_-(1-F(c))}{(1-F(c))^\gamma} \liminf_{c\to+\infty} \frac{(1-F(c))^\gamma}{(E[\rho\mathbf{1}_{\rho>c}])^\alpha} \\ &\geq b \liminf_{c\to+\infty} \frac{(1-F(c))^\gamma}{(E\rho^m)^{\alpha/m}(1-F(c))^{\alpha/n}} \\ &= \frac{b}{(E\rho^m)^{\alpha/m}} \lim_{c\to+\infty} (1-F(c))^{\gamma-\alpha/n} = +\infty. \end{aligned}$$

(iii) Denote $b' := \limsup_{c\to+\infty} \frac{w_-(1-F(c))}{(1-F(c))^\alpha} < +\infty$. Then

$$\begin{aligned} \limsup_{c\to+\infty} g(c) = \limsup_{c\to+\infty} \frac{w_-(1-F(c))}{(E[\rho\mathbf{1}_{\rho>c}])^\alpha} &\leq \limsup_{c\to+\infty} \frac{w_-(1-F(c))}{(1-F(c))^\alpha} \limsup_{c\to+\infty} \frac{(1-F(c))^\alpha}{(E[\rho\mathbf{1}_{\rho>c})^\alpha} \\ &= b' \limsup_{c\to+\infty} \left(\frac{1-F(c))}{E[\rho\mathbf{1}_{\rho>c)}]}\right)^\alpha \\ &= b' \limsup_{c\to+\infty} \left(\frac{1}{E[\rho|\rho>c]}\right)^\alpha \\ &\leq b' \limsup_{c\to+\infty} c^{-\alpha} = 0. \end{aligned}$$

□

Proof of Theorem 3: Write the objective function in (4.2) as

$$\bar{g}(c) := \left(\frac{kw_-(1-F(c))}{(E[\rho\mathbf{1}_{\rho>c}])^\alpha}\right)^{1/(1-\alpha)} - \varphi(c), \quad 0 \leq c < +\infty.$$

This function is continuous on $[0, +\infty)$. So to prove that $\bar{g}$ admits a minimum point $c^* > 0$, it suffices to show that $\bar{g}$ is *coercive* (i.e., $\lim_{c\to+\infty} \bar{g}(c) = +\infty$), and that $\bar{g}(0) > \bar{g}(c)$ for some $c > 0$.

Indeed, it follows from Lemma 1-(ii) that

$$\begin{aligned}\lim_{c\to+\infty}\bar{g}(c) &\geq k^{1/(1-\alpha)}(\lim_{c\to+\infty}g(c))^{1/(1-\alpha)}-\varphi(+\infty)\\ &= +\infty.\end{aligned}$$

On the other hand, recall that $w_-(p) < p$ when p is close to 1. Fix such a $p_0 \in (0,1)$ and take $c_0 := F^{-1}(1-p_0) > 0$. Then $w_-(1-F(c_0)) \leq 1-F(c_0)$. According to Lemma 1-(i), $\frac{w_-(1-F(c_0))}{(E[\rho\mathbf{1}_{\rho>c_0}])^\alpha} < \frac{w_-(1-F(0))}{(E[\rho\mathbf{1}_{\rho>0}])^\alpha}$. So

$$\begin{aligned}\bar{g}(0) &= \left(\frac{kw_-(1-F(0))}{(E[\rho\mathbf{1}_{\rho>0}])^\alpha}\right)^{1/(1-\alpha)}-\varphi(0)\\ &> \left(\frac{kw_-(1-F(c_0))}{(E[\rho\mathbf{1}_{\rho>c_0}])^\alpha}\right)^{1/(1-\alpha)}-\varphi(c_0)\\ &= \bar{g}(c_0).\end{aligned}$$

The proof is complete. □

The conditions of Theorem 3 stipulate that the curvature of the probability distortion on losses around 0 must be sufficiently significant in relation to her risk-seeking level (characterized by α). In other words, the agent must have a strong fear on the event of huge losses, in the sense that she exaggerates its (usually) small probability, to the extent that it overrides her risk-seeking behavior in the loss domain.

If, on the other hand, the agent is not sufficiently fearful of big losses, then the risk-seeking part dominates and the problem is ill-posed, as stipulated in the following result.

Proposition 1. *Assume that* $\alpha=\beta$ *and* $x_0 < E[\rho B]$. *If there exists* $\gamma \geq \alpha$ *such that* $\limsup_{p\downarrow 0}\frac{w_-(p)}{p^\gamma} < +\infty$*, then* $\inf_{c\geq 0}k(c)=0<1$*, and hence Problem (2.6) is ill-posed.*

Proof: By Lemma 1-(iii), we have

$$\limsup_{c\to+\infty}k(c)=k\varphi(+\infty)^{\alpha-1}\limsup_{c\to+\infty}g(c)=0.$$

This implies that $\inf_{c\geq 0}k(c)=0<1$, and hence it follows from Theorem 2-(iii) that (2.6) is ill-posed. □

We highlight another very interesting feature of these results. In the current setting the threshold c^*, which determines the probability of ending up with a good state (as well as that of a bad one), turns out (as seen from (4.2)) to be *independent* of the reference point B or the greed G. Moreover, under the conditions of Theorem 3, $c^*>0$ exists and we have $P(X^*\geq B)=P(\rho\leq c^*)>0$. In other words, no matter how strong the agent's greed is, the good states of the world have a *fixed, positive* probability of occurrence. This makes perfect sense, of course, since otherwise the agent would not gamble on something whose chance of occurrence diminishes to zero.

However, both the leverage level and the *magnitude* of the potential losses do indeed increase to infinity if the greed goes to infinity, as shown in the following theorem.

Theorem 4. *Under the assumptions of Theorem 2-(i) or Theorem 3, we have the following conclusions:*

(i) *The leverage $L \to +\infty$ as the greed $G \to +\infty$.*

(ii) *The probability of ending with gains is $P(X^* < B) \equiv P(\rho > c^*)$, which is independent of the greed G and is strictly positive.*

(iii) *The potential loss rate $l \to +\infty$ as the greed $G \to +\infty$.*

Proof: First of all, the optimal solution is given in (4.3) by Theorem 2 or Theorem 3. Fitting (4.3) into the general decomposition (3.2) we have

$$X_l^* = \left(\frac{x_+^* - (x_0 - E[\rho B])}{E[\rho \mathbf{1}_{\rho > c^*}]} - B \right) \mathbf{1}_{\rho > c^*}.$$

Substituting $x_+^* := \frac{-(x_0 - E[\rho B])}{k(c^*)^{1/(1-\alpha)} - 1}$ into the above and noting that $k(c^*) \geq \inf_{c>0} k(c) > 1$ under the assumption, we have

$$\begin{aligned} \frac{x_+^* - (x_0 - E[\rho B])}{E[\rho \mathbf{1}_{\rho > c^*}]} - B &= \frac{-(x_0 - E[\rho B])}{E[\rho \mathbf{1}_{\rho > c^*}]} \frac{k(c^*)^{1/(1-\alpha)}}{k(c^*)^{1/(1-\alpha)} - 1} - B \\ &= \left(\frac{aE[\rho B]}{E[\rho \mathbf{1}_{\rho > c^*}]} - B \right) - \frac{a x_0}{E[\rho \mathbf{1}_{\rho > c^*}]}, \end{aligned}$$

where $a := \frac{k(c^*)^{1/(1-\alpha)}}{k(c^*)^{1/(1-\alpha)} - 1} > 1$. Therefore the leverage L as a function of the greed G is

$$\begin{aligned} L &= \frac{E(\rho X_l^*)}{x_0} = \frac{1}{x_0} E\left[\rho \left(\frac{x_+^* - (x_0 - E[\rho B])}{E[\rho \mathbf{1}_{\rho > c^*}]} - B \right) \mathbf{1}_{\rho > c^*} \right] \\ &= \frac{1}{x_0} E\left(aE[\rho B] - E[\rho B \mathbf{1}_{\rho > c^*}] \right) - a \\ &\geq (a-1) \frac{E(\rho B)}{x_0} - a \\ &= (a-1)G - a \to +\infty \text{ as } G \to +\infty. \end{aligned}$$

This proves (i). Next, the conclusion (ii) is evident.

Finally, the potential loss l is

$$\begin{aligned} l &= E\left(\frac{\rho X_l^*}{x_0} \Big| X^* < B \right) = E\left(\frac{\rho X_l^*}{x_0} \Big| \rho > c^* \right) \\ &= \frac{E(\frac{\rho X_l^*}{x_0})}{P(\rho > c^*)} \\ &\geq \frac{(a-1)G - a}{P(\rho > c^*)} \to +\infty \text{ as } G \to +\infty, \end{aligned}$$

where we have utilized the fact that $P(\rho > c^*)$ is independent of G. This proves (iii). $\square$

4.2 The case when $\alpha < \beta$

Next let us consider the case where $\alpha < \beta$, which implies that the pain associated with a *substantial* loss is *much* larger than the happiness with a gain of the same magnitude. So the agent is loss averse in a larger scale than the case when $\alpha = \beta$ and $k > 1$. Note that $\alpha < \beta$ is supported by some empirical evidences. For instance, Abdellaoui (2000) estimates the median of α and β to be 0.89 and 0.92 respectively.

No optimal solution, as explicit as that with the case $\alpha = \beta$, of the CPT model (2.6) has been obtained in Jin and Zhou (2008) or in any other literature for the case $\alpha < \beta$. Hence we have to first solve (2.6) before carrying out an asymptotic analysis on greed.

As discussed earlier we are interested only in the case when the agent is sufficiently greedy, namely, when $\tilde{x}_0 \equiv x_0 - E[\rho B] < 0$.

Define a function $h(c) = \frac{kw_-(1-F(c))}{(E[\rho \mathbf{1}_{\rho > c}])^\beta}$, $c > 0$. The point

$$c_1 := \sup\{c' \in [0, +\infty) : h(c') = \inf_{c \in [0,+\infty)} h(c)\}, \tag{4.4}$$

where we convent $\sup \emptyset := -\infty$, will be crucial in solving Problem (4.1) or (2.6). Notice also that c_1 depends only on the market (i.e., the pricing kernel ρ) and the agent behavioral parameters on *losses* (i.e. $w_-(\cdot)$, k and β), and is independent of the reference point or the level of greed G.

The following result characterizes the well-posedness of the problem in terms of the function $h(c)$.

Proposition 2. *Problem (4.1), and therefore Problem (2.6), is well-posed if and only if* $\liminf_{c \to +\infty} h(c) > 0$.

Proof: First of all, by Jin and Zhou (2008), Proposition 5.1, Problem (2.6) is well-posed if and only if Problem (4.1) is well-posed. Now, assume that $\liminf_{c \to +\infty} h(c) = 0$. For any $M > 0$, fix $x_+ > \tilde{x}_0^+$ such that $\varphi(1)^{1-\alpha} x_+^\alpha > 2M$. On the other hand, there is $c > 1$ such that $h(c)(x_+ - \tilde{x}_0)^\beta < M$. Hence, $v(c, x_+) > \varphi(c)^{1-\alpha} x_+^\alpha - M \geq \varphi(1)^{1-\alpha} x_+^\alpha - M > M$. So problem (4.1) is ill-posed.

Conversely, if $\liminf_{c \to +\infty} h(c) > 0$, then there are $\epsilon > 0$ and $N > 0$ so that $h(c) > \epsilon \; \forall c > N$. However, $\underline{h}_N := \inf_{0 \leq c \leq N} h(c) > 0$. Hence $\underline{h} := \inf_{0 \leq c < +\infty} h(c) > 0$. Consequently, for any feasible (c, x_+),

$$\begin{aligned} v(c, x_+) &\leq \varphi(+\infty) x_+^\alpha - \underline{h}(x_+ - \tilde{x}_0)^\beta \\ &< \varphi(+\infty) x_+^\alpha - \underline{h} x_+^\beta \\ &\leq \sup_{x \geq 0} \{\varphi(+\infty) x^\alpha - \underline{h} x^\beta\} < +\infty. \end{aligned}$$

So (4.1) is well-posed. $\square$

The next result excludes $(c, x_+) = (0, 0)$ from being an optimal solution of (4.1).

Proposition 3. *If* $\liminf_{c \to +\infty} h(c) > 0$*, then* $(0, 0)$ *can not be optimal for Problem (4.1).*

Proof: It is easy to check that

$$\begin{aligned}\frac{1}{k}h'(c) &= \frac{-w'_-(1-F(c))F'(c)(E[\rho\mathbf{1}_{\rho>c}])^{\beta} - w_-(1-F(c))\beta(E[\rho\mathbf{1}_{\rho>c}])^{\beta-1}[-cF'(c)]}{(E[\rho\mathbf{1}_{\rho>c}])^{2\beta}} \\ &= -\frac{F'(c)}{(E[\rho\mathbf{1}_{\rho>c}])^{\beta+1}}\left(w'_-(1-F(c))E[\rho\mathbf{1}_{\rho>c}] - w_-(1-F(c))\beta c\right).\end{aligned}$$

Since $w'_-(1-F(c)) \geq 1$, $E[\rho\mathbf{1}_{\rho>c}] \to E\rho > 0$, and $w_-(1-F(c)) \leq 1$ as $c \downarrow 0$, we have $h'(c) < 0$ when c is sufficiently close to 0. So $h(c)$ is strictly decreasing in a neighborhood of 0. This means there exists a $c > 0$ such that $h(c) < h(0)$, hence $v(c, 0) = -h(c)(-\tilde{x}_0)^{\beta} > -h(0)(-\tilde{x}_0)^{\beta} = v(0, 0)$. This shows $(0, 0)$ can not be optimal. $\square$

To find an optimal solution to (4.1), we first fix $c > 0$ and then find the optimal x_+ for $v(c, x_+) = \varphi(c)^{1-\alpha}\left(x_+^{\alpha} - k(c)(x_+ - \tilde{x}_0)^{\beta}\right)$.

Lemma 2. *For any* $c \in (0, +\infty)$*, we have* $\sup_{x\in[0,+\infty)}\left(x^{\alpha} - k(c)(x - \tilde{x}_0)^{\beta}\right) < +\infty$*, and there exists a unique maximizer*

$$x(c) = \text{argmax}_{x\in[0,+\infty)}\left(x^{\alpha} - k(c)(x - \tilde{x}_0)^{\beta}\right).$$

Moreover, we have the following relationship

$$k(c) = \frac{x(c)^{\alpha-1}\alpha}{(x(c) - \tilde{x}_0)^{\beta-1}\beta},$$

and hence $x(c)$ *is continuous in* c*.*

Proof: Denote $f(x) = x^{\alpha} - k(c)(x - \tilde{x}_0)^{\beta}$, $x \geq 0$. Since $\alpha < \beta$ and $k(c) > 0$, we have $\lim_{x\to+\infty} f(x) = -\infty$; and hence $\sup_{x\in[0,+\infty)} f(x) < +\infty$.

Now, $f'(x) = \alpha x^{\alpha-1} - \beta k(c)(x - \tilde{x}_0)^{\beta-1}$. Denoting $\tilde{k} := \beta k(c)/\alpha > 0$, we have

$$\begin{aligned} f'(x) = 0 \;&\Leftrightarrow\; x^{\alpha-1} = \tilde{k}(x - \tilde{x}_0)^{\beta-1} \\ &\Leftrightarrow\; (\alpha - 1)\ln x = \ln\tilde{k} + (\beta - 1)\ln(x - \tilde{x}_0) \\ &\Leftrightarrow\; (1 - \alpha)\ln x - (1 - \beta)\ln(x - \tilde{x}_0) = -\ln\tilde{k}.\end{aligned}$$

Set $g(x) = (1-\alpha)\ln x - (1-\beta)\ln(x - \tilde{x}_0)$, $x > 0$. Then $g'(x) = \frac{1-\alpha}{x} - \frac{1-\beta}{x-\tilde{x}_0} > \frac{\beta-\alpha}{x} > 0\; \forall x > 0$. Together with the facts that $g(0) = -\infty$, $g(+\infty) = +\infty$, we conclude that $f'(x) = 0$ admits a unique solution $x = x(c) > 0$ which is the unique maximizer of $f(x)$ over $x \geq 0$. Moreover, the expression of $k(c)$ is derived from $g'(x(c)) = 0$, and the continuity of $x(c)$ is seen from the standard implicit function theorem. $\square$

Recall the number c_1 defined in (4.4). In the proof of Proposition 3 we have established that $h(c)$ strictly decreases in a neighborhood of 0; hence $c_1 > 0$ if $c_1 \neq -\infty$. Meanwhile, the following result identifies $c_1 = \pm\infty$, or equivalently, $\liminf_{c\to+\infty} h(c) = \inf_{c\geq 0} h(c)$, as a pathological case.

Proposition 4. *If* $\liminf_{c\to+\infty} h(c) = \inf_{c\geq 0} h(c)$*, then Problem (4.1) admits no optimal solution for any* $\tilde{x}_0 < 0$.

Proof: If $\liminf_{c\to+\infty} h(c) = \inf_{c\geq 0} h(c)$, then for any $c_0 \in (0, +\infty)$, we can find $c > c_0$ such that $h(c) \leq h(c_0)$. Hence for any $x \geq 0$,

$$\begin{array}{rcl} v(c_0, x) & \leq & v(c_0, x(c_0)) \\ & = & \varphi(c_0)^{1-\alpha} x(c_0)^\alpha - h(c_0)(x(c_0) - \tilde{x}_0)^\beta \\ & < & \varphi(c)^{1-\alpha} x(c_0)^\alpha - h(c)(x(c_0) - \tilde{x}_0)^\beta \\ & \leq & \varphi(c)^{1-\alpha} x(c)^\alpha - h(c)(x(c) - \tilde{x}_0)^\beta \\ & = & v(c, x(c)), \end{array}$$

where $x(\cdot)$ is the maximizer as specified in Lemma 2 and the last inequality is due to the definition of $x(c)$. So if there exists any optimal solution pair (c^*, x_+^*), then $c^* = +\infty$. The constraints in Problem (4.1) dictate that $x_+^* = \tilde{x}_0 < 0$, which contracts the requirement that $x_+^* \geq 0$. $\square$

Proposition 5. *If* $\liminf_{c\to+\infty} h(c) > 0$ *and* $\liminf_{c\to+\infty} h(c) > \inf_{c\geq 0} h(c)$*, then Problem (4.1) admits optimal solutions when the agent is sufficiently greedy. Moreover, any optimal solution* $(c^*, x(c^*))$ *of (4.1) must satisfy* $c^* \in [c_1, +\infty)$.

Proof: First note that the agent being sufficiently greedy is equivalent to $-\tilde{x}_0 > 0$ being sufficiently large. In this case, $(c, x_+) = (+\infty, \tilde{x}_0)$ is not feasible. On the other hand, $(c, x_+) = (0, 0)$ is not optimal either according to Proposition 3. So we only need to consider $c \in (0, +\infty)$.

Given $\liminf_{c\to+\infty} h(c) > \inf_{c\geq 0} h(c)$, we have $c_1 \in [0, +\infty)$. For any $c < c_1$, we see that $\varphi(c) < \varphi(c_1), h(c) \geq h(c_1)$. The same analysis as in the proof of Proposition 4 yields $v(c, x(c)) < v(c_1, x(c_1))$; hence the optimal c must be in $[c_1, +\infty)$ if it exists.

Denote $h_1 = \liminf_{c\to+\infty} h(c) > h(c_1)$. Then

$$\limsup_{c\to+\infty} v(c, x(c)) \leq \max_{x\in[0,+\infty)} \left[\varphi(+\infty)^{1-\alpha} x^\alpha - h_1(x - \tilde{x}_0)^\beta\right].$$

By Lemma 2, we can find $x_* = \operatorname{argmax}_{x\in[0,+\infty)} \left[\varphi(+\infty)^{1-\alpha} x^\alpha - h_1(x - \tilde{x}_0)^\beta\right]$. Notice that x_* depends on $\tilde{x}_0$. Setting $\tilde{k} := \frac{h_1}{\varphi(+\infty)^{1-\alpha}}$, then Lemma 2 gives

$$\tilde{k} = \frac{\alpha}{\beta} \frac{x_*^{\alpha-1}}{(x_* - \tilde{x}_0)^{\beta-1}} = \frac{\alpha}{\beta} \left(\frac{x_*}{x_* - \tilde{x}_0}\right)^{\alpha-1} (x_* - \tilde{x}_0)^{\alpha-\beta}.$$

Since $\tilde{k}$ is independent of $\tilde{x}_0$ and $0 < \alpha < \beta < 1$, we conclude that $\frac{x_*}{x_* - \tilde{x}_0} \to 0$, or equivalently $\frac{x_*}{-\tilde{x}_0} \to 0$ as $-\tilde{x}_0 \to +\infty$.

Denote $m = (\varphi(+\infty)/\varphi(c_1))^{(1-\alpha)/\alpha} > 1$ and $n = (h_1/h(c_1))^{1/\beta} > 1$. Then

$$\begin{aligned}
\limsup_{c\to+\infty} v(c, x(c)) &\leq \varphi(+\infty)^{1-\alpha} x_*^{\alpha} - h_1(x_* - \tilde{x}_0)^{\beta} \\
&= \varphi(c_1)^{1-\alpha}(mx_*)^{\alpha} - h(c_1)(nx_* - n\tilde{x}_0)^{\beta} \\
&= \varphi(c_1)^{1-\alpha}(mx_*)^{\alpha} - h(c_1)(mx_* - \tilde{x}_0)^{\beta} \\
&\quad + h(c_1)\left[(mx_* - \tilde{x}_0)^{\beta} - (nx_* - n\tilde{x}_0)^{\beta}\right] \\
&\leq v(c_1, x(c_1)) + h(c_1)\left[(mx_* - \tilde{x}_0)^{\beta} - (nx_* - n\tilde{x}_0)^{\beta}\right].
\end{aligned}$$

We have proved that $\lim_{-\tilde{x}_0\to+\infty} \frac{x^*}{-x_0} = 0$; so when $-\tilde{x}_0$ is large enough, $h(c_1)[(mx_* - \tilde{x}_0)^{\beta} - (nx_* - n\tilde{x}_0)^{\beta}] < 0$. In other words, $v(c, x(c))$ never achieves its infimum when c is sufficiently large. On the other hand, we have shown that any $c < c_1$ is not a maximizer of $v(c, x(c))$ either. Since $v(c, x(c))$ is continuous of c, it must attain its minimum at some $c^* \in [c_1, +\infty)$ for any fixed, sufficiently large $-\tilde{x}_0$ or sufficiently large greed G. □

Notice that Problem (4.1) may have multiple optimal solutions. It is sometimes convenient to consider the "maximal solution" of (4.1), denoted by (c^*, x_+^*), which is one of the optimal solutions satisfying

$$c^* = \sup\{c \in [0, +\infty) : (c, x_+) \text{ solves Problem (4.1)}\}.$$

The following result gives a complete solution to Problem (2.6) for the case when $\alpha < \beta$ and the reference point B (or equivalently the greed G) is sufficiently large.

Theorem 5. *Assume that $\alpha < \beta$ and $x_0 < E[\rho B]$.*

(i) *If $\liminf_{c\to+\infty} h(c) > 0$ and $\liminf_{c\to+\infty} h(c) > \inf_{c\geq 0} h(c)$, then the CPT portfolio selection model (2.6) admits an optimal solution if the agent's greed G is sufficiently large. Moreover, if $(c^*(G), x_+^*(G))$ is any maximal solution of Problem (4.1), then the optimal terminal wealth is*

$$X^*(G) = \frac{x_+^*(G)}{\varphi(c^*(G))}\left(\frac{w_+'(F(\rho))}{\rho}\right)^{1/(1-\alpha)} \mathbf{1}_{\rho\leq c^*(G)} - \frac{x_+^*(G) - (x_0 - E[\rho B])}{E[\rho \mathbf{1}_{\rho > c^*(G)}]} \mathbf{1}_{\rho > c^*(G)} + B.$$

(ii) *If $\liminf_{c\to+\infty} h(c) > 0$ and $\liminf_{c\to+\infty} h(c) = \inf_{c\geq 0} h(c)$, then (2.6) is well-posed, but it does not admit any optimal solution.*

(iii) *If $\liminf_{c\to+\infty} h(c) = 0$, then (2.6) is ill-posed.*

Proof: It follows from Propositions 2, 4, and 5. □

The following result is a counterpart of Theorem 3, which presents an easy-to-check sufficient condition for the assumption in Theorem 5-(i).

Theorem 6. *Assume that $\alpha < \beta$ and $x_0 < E[\rho B]$. If there exists $\gamma < \beta$ such that $\liminf_{p \downarrow 0} \frac{w'_-(p)}{p^{\gamma-1}} > 0$, then (2.6) admits an optimal solution for any greed G, which is expressed explicitly in Theorem 5-(i) via a maximal solution $(c^*(G), x_+^*(G))$ of (4.1).*

Proof: Under the assumptions of Theorem 6, a proof similar to that of Lemma 1-(ii) shows that $\liminf_{c\to+\infty} h(c) = +\infty$. Hence, trivially, $\liminf_{c\to+\infty} h(c) > 0$ and $\liminf_{c\to+\infty} h(c) > \inf_{c\geq 0} h(c)$. So Problem (2.6) is well-posed.

Next we show that there exists an optimal portfolio for *any* level of greed. To this end, we have for any fixed $\tilde{x}_0 < 0$:

$$\begin{aligned}
\limsup_{c\to+\infty} v(c, x(c)) &\leq \limsup_{c\to+\infty} \left\{\varphi(+\infty)^{1-\alpha} x(c)^\alpha - h(c)(x(c) - \tilde{x}_0)^\beta\right\} \\
&\leq \limsup_{c\to+\infty} \left\{\varphi(+\infty)^{1-\alpha}[(x(c) - \tilde{x}_0)^\beta + 1] - h(c)(x(c) - \tilde{x}_0)^\beta\right\} \\
&\leq \varphi(+\infty)^{1-\alpha} - \liminf_{c\to+\infty}\{(h(c) - \varphi(+\infty)^{1-\alpha})[(x(c) - \tilde{x}_0)^\beta\} \\
&\leq \varphi(+\infty)^{1-\alpha} - \liminf_{c\to+\infty}\{(h(c) - \varphi(+\infty)^{1-\alpha})(-\tilde{x}_0)^\beta\} \\
&= -\infty,
\end{aligned}$$

yielding that $v(c, x(c))$ is a coercive function in c. Thus it must attain a minimum at some $c^* \in [c_1, +\infty)$, proving the desired result. □

Now we set out to derive the asymptotic properties of the optimal solutions for Problem (4.1) when $G \to +\infty$. For a fixed $\tilde{x}_0$, define

$$c^*(\tilde{x}_0) = \sup\{c \in [c_1, +\infty) : (c, x(c)) \text{ solves Problem (4.1)}\};$$

namely $(c^*(\tilde{x}_0), x(c^*(\tilde{x}_0)))$ is a maximal solution of Problem (4.1).

Proposition 6. *Under the conditions of Theorem 5-(i) or Theorem 6, we have $\lim_{-\tilde{x}_0\to+\infty} c^*(\tilde{x}_0) = c_1$, $\lim_{-\tilde{x}_0\to+\infty} x(c^*(\tilde{x}_0)) = +\infty$, and $\lim_{-\tilde{x}_0\to+\infty} \frac{x(c^*(\tilde{x}_0))}{-\tilde{x}_0} = 0$.*

Proof: Recall that $c^*(\tilde{x}_0)$, when it exists, must be greater or equal to c_1. Hence to prove the first limit, it suffices to show that for any $\delta > 0$, $\sup_{c\in[c_1+\delta,+\infty)} v(c, x(c)) < v(c_1, x(c_1))$ when $-\tilde{x}_0$ is large enough.

Define $h_2 = \inf_{c\in[c_1+\delta,+\infty)} h(c)$. By the assumption that $\liminf_{c\to+\infty} h(c) > \inf_{c\geq 0} h(c)$, we know there exists $c_M > c_1 + \delta$ such that $h(c) > h(c_1 + \delta) + 1$ $\forall c \geq c_M$. Hence $h_2 = \inf_{c\in[c_1+\delta,c_M]} h(c) > h(c_1)$. Then

$$\sup_{c\in[c_1+\delta,+\infty)} v(c, x(c)) \leq \sup_{x\in[0,+\infty)} \left[\varphi(+\infty)^{1-\alpha} x^\alpha - h_2(x - \tilde{x}_0)^\beta\right].$$

An argument completely parallel to that in proving Proposition 5 reveals that

$$\sup_{c\in[c_1+\delta,+\infty)} v(c, x(c)) < v(c_1, x(c_1))$$

when $-\tilde{x}_0$ is sufficiently large.

Next, by Lemma 1, we have

$$k(c^*(\tilde{x}_0)) = \frac{\alpha}{\beta}\frac{x(c^*(\tilde{x}_0))^{\alpha-1}}{(x(c^*(\tilde{x}_0)) - \tilde{x}_0)^{\beta-1}} = \frac{\alpha}{\beta}\left(\frac{x(c^*(\tilde{x}_0))}{x(c^*(\tilde{x}_0)) - \tilde{x}_0}\right)^{\alpha-1}(x(c^*(\tilde{x}_0)) - \tilde{x}_0)^{\alpha-\beta}.$$

However, $\lim_{-\tilde{x}_0\to+\infty} k(c^*(\tilde{x}_0)) = k(c_1) > 0$; hence $k(c^*(\tilde{x}_0)) \in [k(c_1)/2, 2k(c_1)]$ when $-\tilde{x}_0$ is large enough. As a result, $\lim_{-\tilde{x}_0\to+\infty} \frac{x(c^*(\tilde{x}_0))}{x(c^*(\tilde{x}_0))-\tilde{x}_0} = 0$ or $\lim_{-\tilde{x}_0\to+\infty} \frac{x(c^*(\tilde{x}_0))}{-\tilde{x}_0} = 0$.

Finally, we can rewrite

$$k(c^*(\tilde{x}_0)) \quad = \quad \frac{\alpha}{\beta}\left(\frac{x(c^*(\tilde{x}_0))}{x(c^*(\tilde{x}_0)) - \tilde{x}_0}\right)^{\beta-1} x(c^*(\tilde{x}_0))^{\alpha-\beta}.$$

By the proved fact that $\lim_{-\tilde{x}_0\to+\infty} \frac{x(c^*(\tilde{x}_0))}{x(c^*(\tilde{x}_0))-\tilde{x}_0} = 0$ and that $\alpha < \beta$, we conclude that $\lim_{-\tilde{x}_0\to+\infty} x(c^*(\tilde{x}_0)) = +\infty$. □

Corollary 1. *We have*

$$\lim_{G\to+\infty} c^*(G) = c_1, \ \lim_{G\to+\infty} x_+^*(G) = +\infty, \ \lim_{G\to+\infty} \frac{x_+^*(G)}{G} = 0.$$

Proof: This is evident given that $-\tilde{x}_0 \to +\infty$ is equivalent to $G \to +\infty$. □

Theorem 7. *Under the assumptions of Theorem 5-(i) or Theorem 6, we have the following conclusions:*

(i) *The leverage $L \to +\infty$ as the greed $G \to +\infty$.*

(ii) *The asymptotic probability of ending with gains is $P(\rho < c_1) > 0$ as $G \to +\infty$.*

(iii) *The potential loss rate $l \to +\infty$ as the greed $G \to +\infty$.*

Proof: First of all, (ii) is evident as $\lim_{G\to+\infty} c^*(G) = c_1$.

Recall

$$X_l^*(G) = \left(\frac{x_+^*(G) - (x_0 - E[\rho B])}{E[\rho \mathbf{1}_{\rho>c^*(G)}]} - B\right)\mathbf{1}_{\rho>c^*(G)}.$$

Hence, the leverage L as a function of the greed G is

$$\begin{aligned} L(G) &= \frac{E(\rho X_l^*(G))}{x_0} \\ &= \frac{x_+^*(G) + E(\rho B)}{x_0} - \frac{1}{x_0}E[\rho B\mathbf{1}_{\rho>c^*(G)}] \\ &= \frac{x_+^*(G)}{x_0} + \frac{1}{x_0}E[\rho B\mathbf{1}_{\rho\le c^*(G)}] \\ &\to +\infty \text{ as } G \to +\infty. \end{aligned}$$

On the other hand, the potential loss l is

$$\begin{aligned} l(G) &= E\left(\frac{\rho X_l^*(G)}{x_0}\Big| X^*(G) < B\right) = E\left(\frac{\rho X_l^*(G)}{x_0}\Big| \rho > c^*(G)\right) \\ &= \frac{E(\frac{\rho X_l^*(G)}{x_0})}{P(\rho > c^*(G))} \\ &\to +\infty \text{ as } G \to +\infty. \end{aligned}$$

□

One of the most interesting implications of the preceding result is that, although for each fixed level of greed G, the probability of ending with good states does indeed depend on G (which is unlike the case when $\alpha = \beta$), the asymptotic probability when G gets infinitely large is fixed and strictly positive. Hence, as with the $\alpha = \beta$ case, the agent gambles on winning states with a positive probability of occurrence, even if she has an exceedingly strong greed. However, to do so she needs to take an incredibly high level of leverage and to risk catastrophic potential losses.

5 Models with Losses and/or Leverage Control

We have established that both leverage and potential losses will grow unbounded if human greed is allowed to grow unbounded. This suggests that, from either a loss-control viewpoint of an individual investor or from a regulatory perspective, one could contain the greed – if indirectly – by imposing *a priori* bounds on losses and/or on the level of leverage.

A CPT model with loss control can be formulated as follows:

$$(5.1) \qquad \begin{array}{ll} \text{Maximize} & V(X - B) \\ \text{subject to} & \begin{cases} E[\rho X] = x_0, \quad X \geq B - a \\ X \text{ is } \mathcal{F}_T - \text{measurable and lower bounded,} \end{cases} \end{array}$$

where a is a constant representing an exogenous cap on the losses allowed. This model is investigated in full in a companion paper Jin, Zhang and Zhou (2009). It is shown that the optimal wealth profile, in its greatest generality, depends on three – instead of two – classes of states of the world, with an intermediate class of states between the good and the bad. A moderate loss will be incurred in the intermediate states whereas the maximum allowable loss on the bad states. So the agent will still take leverage if her reference point is high, but she will be more cautious in doing so – by differentiating the loss states and controlling (indirectly) the leverage level.

Another possible model is to directly control the leverage instead of the loss, formulated as follows:

$$
(5.2) \qquad \begin{array}{ll} \text{Maximize} & V(X-B) \\ \text{subject to} & \begin{cases} E[\rho X] = x_0, \quad L \le b \\ X \text{ is } \mathcal{F}_T - \text{measurable and lower bounded,} \end{cases} \end{array}
$$

where L is the leverage and b is a pre-specified level. Notice that this model is not entirely the same as (5.1), because the correspondence between the loss and the leverage level depends on the specific form of a wealth profile. On the other hand, different agents may have different priorities in choosing their model specifications. For example, a regulator may be more concerned with the leverage level whereas an individual firm may stress on loss control. One could also impose explicit bounds on both the losses and the leverage.

One might argue that it would be simpler and more reasonable to introduce a bound directly on the level of greed (i.e. on the reference point according to our definition of greed), if the whole purpose is to contain the human greed within a reasonable range. The problem is that the reference point is an exogenous parameter which cannot be constrained in an optimization problem. More importantly, in reality an agent may not be aware of how high her reference point is – it is only *implied* by her risk attitude. Furthermore, a reference point does not stand still; it is a random variable depending on the states of the world. Indeed, it may be even dynamically changing (which is not modelled in this paper). Therefore, it does not seem to be sensible or feasible to directly pose an exogenous bound on reference point/greed.

6 Concluding Remarks

When one applies the neoclassical theory (e.g. utility maximization) to portfolio choice the results are to advise what people ought to do; namely they provide investment advices on the best investment strategies *assuming* that the investor is rational. In contrast, portfolio selection models based on behavioral theory (e.g. CPT) predict what people actually do; this is because irrationality is inherent in human behaviors. Therefore, the gambling behavior stipulated in an optimal strategy (see Theorems 1 and 5) tells a typical CPT agent's trading pattern, rather than an investment guide. In the same spirit, the main results of the paper, Theorems 4 and 7, describes what would happen should greed be allowed to expand infinitely. Neoclassical and behavioral theories hence fulfil separate but complementary needs in decision-making.

In this paper we have defined greed via the reference point under CPT. The underlying portfolio choice model is general enough to support the generality of the conclusions drawn. That said, the reference point (and CPT for that matter) is certainly by no means the only determinant of the notion of greed. As Shefrin (Shefrin and Zhou 2009) points out, the factors[14] contributing to greed

[14]See also Shefrin (2002, 2008) for detailed discussions on these factors in pieces.

include “excessive (or unrealistic) optimism; overconfidence in the sense of underestimating risk; high aspiration levels (high A in SP/A theory[15]) or high rho (the reference point) in prospect theory; strong hope/weak fear as emotions (as expressed in SP/A theory through the weighting function, which corresponds to significant curvature in the weighting functions for cumulative prospect theory)”. These factors (not necessarily all related to CPT) also warrant investigations in order to fully understand and deal with greed. In particular, the curvature of probability distortion (weighting) functions in CPT could be another dimension in analyzing greed, since greed is typically characterized by delusive and deceiving hope and fear, modelled through the exaggerations of small chances of huge gains/losses, namely the probability distortions. Some of the results in Section 4, albeit rather preliminary, shows the promise of this direction.

Having said all these, it is important to study how these factors contribute to the conceptualization and understanding of greed one at a time, and then in combination. It is our hope that a detailed analysis through reference point/CPT in this paper will motivate more quantitative behavioral research on human greed.

[15]Lopes (1987).

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